NONLINEAR STABILITY ANALYSIS OF MICROWAVE OSCILLATORS USING THE PERIODIC AVERAGING METHOD

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Abstract—In this paper an approach for stability analysis of microwave oscillators is proposed. Using the perturbation theory and averaging method, a theorem which relates the oscillation stability to the stability of the periodic average of the circuit's Jacobian is mentioned. Using this theorem, a criterion for oscillation stability is devised. The proposed criterion is applied to the stability analysis of a negative resistance diode oscillators and a Colpitts oscillator. This method is readily applicable to microwave CAD routines.

1. INTRODUCTION

Oscillators are autonomous nonlinear circuits with different stability phases. Both stability and instability are present at different phases of oscillator operation. Instability of bias point, causes the oscillation start up while the stability of steady state oscillation is necessary for a physical and measurable oscillation. Oscillation stability is the sustainability of the oscillation amplitude and frequency in the presence of small perturbations [1, 2]. The instability of DC bias point can be studied by linearization of the circuit around its DC operating point. The stability of this linearized circuit can be studied using the stability criteria of linear circuits *e.g.*, Nyquist diagram. But for stability analysis of oscillation, the criteria of linear system stability can not be used [3]. For oscillation stability, it is ubiquitous to use the stability analysis method of nonlinear circuits [4].

Most oscillators are designed by means of linear concepts and their performances are satisfactory at least for basic application. But many

oscillatory behaviors have intrinsically nonlinear nature and they are becoming increasingly important in microwave applications *e.g.*, using the chaotic oscillators for secure spread spectrum communication and radar [5]. Also a fully nonlinear analysis can give a more accurate evaluation of the actual performance of the oscillator.

On the other hand, the microwave oscillators have some special problems which do not exist at lower frequency ones. For example at microwave and millimeter wave ranges, the device models change and some nonlinear dynamic elements like junction capacitances *e.g.*, C_{jc} and C_{je} in HBT transistors, could not be neglected. Nonetheless the harmonic balance "HB" method which is widely used for the analysis of nonlinear microwave circuits, has deficiency in stability analysis of its computed solutions. As shown in Fig. 1, in "HB", the circuit is divided into two distinct linear and nonlinear networks. The nonlinear network is analyzed in time domain while the linear network is analyzed in frequency domain. The Fourier transform and its inverse are used for transforming these two domains. Expanding the voltage at the ports of nonlinear networks in the Fourier series, the current components of the
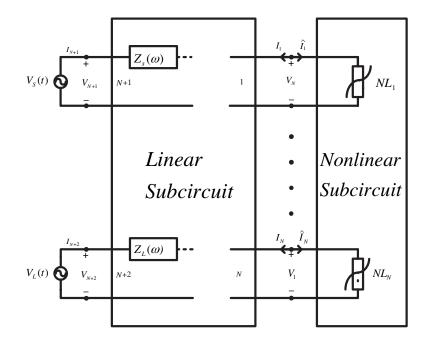

Figure 1. Separation of the linear and nonlinear networks in harmonic balance method.

linear network can be calculated using its admittance matrix. Using the ports voltage, the current waveforms and hence the components of the current of the nonlinear networks can be calculated. In fact, the main idea of harmonic balance approach is the KCL for the components of current carrying between the linear and nonlinear networks [6].

$$I + \hat{I} = 0 \tag{1}$$

where I and $\hat{I}$ are the current vectors of the linear and nonlinear networks. The current vector of the linear network is:

$$I = Y_S V_S + Y_{N \times N} V \tag{2}$$

where V_S is the voltage sources and Y_S and $Y_{N \times N}$ are the admittance matrixes which can be obtained using systematic approaches *e.g.* loop set matrix. The current vector of nonlinear networks at the common terminals is [6].

$$\hat{I} = I_G + j\Omega Q \tag{3}$$

where I_G is the vector of nonlinear current source and Q is the vector of the charge of nonlinear capacitors. The general "HB" equation can be expressed as (4).

$$F(V) = I_S + Y_{N \times N} V + I_G(V) + j\Omega Q(V) = 0 \tag{4}$$

Equation (4) represents an equation that should be solved to obtain the voltage V. There are various techniques to solve the harmonic-balance equation (4). Among them, the Newton technique is simpler, faster and has high gradient convergence [6]. "HB" method considers only the steady state solution and ignores its time evolution, hence the computed solution could be unstable [2]. The commercial microwave CADs have no tools for stability analysis of oscillation. These facts along with the modern expensive MMIC technology which requires "comprehensive simulation and ensuring the first pass design", clarify the importance and the need for the nonlinear stability analysis of the modern microwave oscillators. It should be also noted the differences between the stability of numerical method for analyzing the circuit and the stability of the circuit itself [7, 8].

There is a large amounts of studies in oscillation stability [2]. Using the characteristic matrix, Fluquet [9] derived a criterion for oscillation stability. This method is based on the characteristic matrix, but derivation of the characteristic matrix even for the simple circuits is cumbersome. Assuming the negative resistance model for the microwave oscillator and using the describing function method, Kurokawa derived a criterion for oscillator stability [10]. While

Kurokawa's criterion is widely used in texts, it has the inherent restriction of the describing function method *i.e.* it considers only the fundamental harmonic. Rizzoli and Lipparini proposed a general method for stability analysis of microwave circuits, but their method is very complex and difficult to implement [11–13]. Bifurcation is another tool for stability analysis of nonlinear circuits [14]. Based on the bifurcation theory, Suárez proposed a method for stability analysis. But this method is suitable only for the forced oscillators [15, 16]. Mons used the normalized determinant function for stability analysis of microwave circuits. He used auxiliary source in commercial microwave CAD to calculate it [17]. Mons method is suitable for forced circuits too.

In the following sections, the theoretical basis for an oscillator stability criterion has been reviewed and by stating some theorems, this criterion has been proposed. The main idea of this method is mapping the oscillator stability problem to the stability analysis of the equilibrium point of a linear time invariant system which is derived by averaging the Jacobian matrix of the oscillator state equations over one period of oscillation.

2. THEORETICAL BASIS

Periodic averaging is a suitable method for studying the stability of (5):

$$\dot{x} = \varepsilon f(t, x, \varepsilon) \qquad (5)$$

where $f([0, \infty) \times D \times [0, \varepsilon_0]) \to \Re^n$ is a vector of bounded functions with continuous first and second order derivatives, "D is the domain of x". f is also periodic in time, *i.e.*:

$$\forall n \in \mathbf{N} \ f(t, x, \varepsilon) = f(t + nT, x, \varepsilon) \qquad (6)$$

in which ε is a small positive parameter.

If $x_s(t, \varepsilon)$ and $x_p(t, \varepsilon)$ are the steady state and perturbed responses of (5), the dynamic system (5) is stable if and only if [18]:

$$\lim_{\substack{t \to \infty \\ \varepsilon \to 0}} \|x_s(t, \varepsilon) - x_p(t, \varepsilon)\| \to \quad 0 \qquad (7)$$

Averaging is a suitable method for verifying (7). The averaged system of (5) is:

$$\dot{\bar{x}} = \varepsilon f_{av}(\bar{x}) \qquad (8)$$

where

$$f_{av}(\bar{x}) = \frac{1}{T} \int_0^T f(t, x, \varepsilon = 0) dt \qquad (9)$$

By these preliminary definitions, it has been proved [19, 20] that:

Theorem 1: Suppose that $f(t, x, \varepsilon)$ is a function on $[0, \infty) \times D \times [0, \varepsilon_0]$ with continuous and bounded partial derivatives. Also $f(t, x, \varepsilon)$ is periodic in time with period of T and ε is a small positive number. If $x(t, \varepsilon)$ and $x_{av}(t, \varepsilon)$ be the solutions of (10) and (11) respectively

$$\dot{x} = \varepsilon f(t, x, \varepsilon) \tag{10}$$

and

$$\dot{\bar{x}} = \varepsilon f_{av}(x) \tag{11}$$

and if $p^* \in D$ be the asymptotic stable equilibrium point of the averaged system (11), then the positive constant ε^* exists so that if $0 < \varepsilon < \varepsilon^*$, then the equation $\dot{x} = \varepsilon f(x, t, \varepsilon)$ has a unique asymptotic stable periodic solution with period of T.

Corollary 1: If both (10) and (11), have equilibrium point at $x = 0$ and if it be the asymptotically stable equilibrium point of (11), then the origin is asymptotically stable equilibrium point of (10).

The theorem 1 and its corollary express a strong relation between the stability of the periodic response of a system with the stability of its averaged system over one period. Now we apply the above results to the special case (12):

$$\dot{x} = \varepsilon f(t, x, \varepsilon) = \varepsilon A(t)x \tag{12}$$

in this case f_{av} is the termwise average of $A(t)$ *i.e.*,

$$f_{av}(x) = \bar{A}x$$
$$\bar{A} = \frac{1}{T} \int_0^T A(t)dt \tag{13}$$

The averaged system is a time invariant system which is expressed by matrix $\bar{A}$ and according to the above theorem (12) is asymptotic stable if all the eigenvalues of $\bar{A}$ have negative real parts. Here another corollary has been stated and proved which is the base of the proceeding discussions.

Corollary 2 : The linear time varying system (14)

$$\dot{x} = A(t)x \tag{14}$$

where $A(t)$ is periodic with period of T, is stable if all the eigenvalues of $\bar{A}$, have negative real parts. The proof of this corrollary has been given in Appendix A.

3. FORMULATION OF OSCILLATOR STABILITY

The corollary 2 can be used for stability analysis of free running oscillators. Free running oscillator is an autonomous dynamic system and using the state variables formulation, it can be expressed as (15):

$$\dot{x} = f(x) \qquad (15)$$

Here it is assumed that (15) has an oscillatory solution. Even though finding the oscillatory response of the circuits, is a complex problem [21], but in this paper it is assumed that an oscillatory solution of (15) has been found by HB and solely the stability of this solution should be studied. For stability analysis, a small perturbation should be exerted to the steady state response of (15). Suppose that $x_s(t)$ is the steady state solution and $\xi(t)$ is a small perturbation, the perturbed circuit is (16):

$$\dot{x}_s + \dot{\xi} = f(x_s + \xi) \quad \Rightarrow \quad \dot{\xi} = \frac{\partial f}{\partial x}(x_s(t))\xi(t) \qquad (16)$$

where $\frac{\partial f}{\partial x}$ is the Jacobian matrix of (16) and henceforth we name it $Jacob_f(t)$. It is important to note that (16) is a linear time varying ordinary differential equation. Oscillation stability of (15) is equivalent to the stability of the equilibrium point "$\xi(t) = 0$" of (17):

$$\dot{\xi} = Jacob_f(t)\xi(t) \qquad (17)$$

As $x_s(t)$ is periodic with period of T, the Jacobian in 21 is periodic too *i.e.*:

$$Jacob_f(t) = Jacob_f(t + T) \qquad (18)$$

Using the previous discussions, the following method for stability analysis of microwave oscillators can be proposed.

 (i) Derive the state equations of the oscillator 15.
 (ii) Calculate the steady state response of (15) by the harmonic balance method.
(iii) Perturb the system and derive the perturbed circuit equations of (17).
(iv) Compute the averaged Jacobian matrix of oscillator over one period.
 (v) Check the average matrix $\overline{Jacob_f}$ for stability *e.g.*, test its eigenvalues or use the Nyquist diagram.
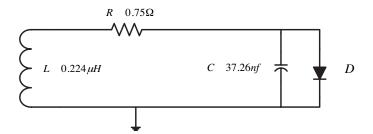
$$R \quad 0.75\Omega$$

$$L \quad 0.224\,\mu H \qquad C \quad 37.26nf \qquad D$$

Figure 2. Diode oscillator.

Briefly it can be said that in the proposed method, the oscillation stability can be deduced from the stability analysis of averaged Jacobian matrix which describes a linear time invariant system. In the next section the proposed method has been applied to a diode and a Colpitts oscillator.

4. APPLICATION OF THE PROPOSED METHOD TO STABILITY ANALYSIS OF OSCILLATORS

As the first example, the proposed criterion is applied to stability analysis of a diode oscillator. Although its frequency is not in the microwave range, it is widely studied in literature [2, 10] and it is added here only for the comparison of results with the literature. A simple model of this oscillator has been shown in Fig. 2. The I-V characteristic of the diode is:

$$I_d = av_d + dv_d^3 \tag{19}$$

where v_d is the diode voltage, a is negative and equals to -0.2 and $d = 0.0375$. Using the capacitor voltage and inductance current as the state variables, the state equations are:

$$\begin{aligned} \frac{di_L}{dt} &= -\frac{R}{L}i_L + \frac{1}{L}v_C \\ \frac{dv_C}{dt} &= -\frac{1}{C}i_L - \frac{1}{C}(a.v_C + d.v_C^3) \end{aligned} \tag{20}$$

The Jacobian matrix of this system is:

$$J = \begin{pmatrix} -\dfrac{R}{L} & \dfrac{1}{L} \\ -\dfrac{1}{C} & -\dfrac{1}{C}(a + 3d.v_C^2) \end{pmatrix} \tag{21}$$

To study the stability of this oscillator, the average of (21) should be calculated over one period of its steady state response. The steady state response of this circuit can be calculated using the harmonic balance. If $v_d(t)$ and $i_d(t)$ be the diode voltage and current and $i(t)$ be the linear part's current, the Fourier series expansion of these quantities are:

$$
\begin{aligned}
v_d(t) &= \sum_{k=-N}^{k=N} V_k e^{-jk\omega} \\
i_d(t) &= \sum_{k=-N}^{k=N} ID_k e^{-jk\omega} \\
i(t) &= \sum_{k=-N}^{k=N} I_k e^{-jk\omega}
\end{aligned}
\tag{22}
$$

The admittance of the linear part is:

$$
Y_l(j\omega) = jC\omega + \frac{1}{R + jL\omega}
\tag{23}
$$

Using these expansions, the harmonic balance equations of this circuit is:

$$
\begin{pmatrix}
y_l(j0\omega) & 0 & \cdots & 0 \\
0 & y_l(j\omega) & \cdots & 0 \\
\cdot & \cdot & \cdot & \cdot \\
0 & \cdots & 0 & y_l(jN\omega)
\end{pmatrix}
\begin{pmatrix}
V_0 \\ V_1 \\ \vdots \\ V_N
\end{pmatrix}
+
\begin{pmatrix}
ID_0 \\ ID_1 \\ \vdots \\ ID_N
\end{pmatrix}
=
\begin{pmatrix}
0 \\ 0 \\ \vdots \\ 0
\end{pmatrix}
\tag{24}
$$

Using the oscillation frequency as the optimization parameter and by 5 harmonics and Newton-Raphson method, the steady state solution has been derived. This solution has been shown in Fig. 3. To check the oscillation stability, the proposed approach of the previous section has been used. The averaged Jacobian matrix over one period of steady state solution is:

$$
J_{av} = 10^7 \begin{pmatrix} -0.3348 & 0.4464 \\ -2.6817 & 0.1330 \end{pmatrix}
\tag{25}
$$

The eigenvalues of (25) are:

$$
\begin{aligned}
\lambda_1 &= 10^7 * (-0.1009 + i * 1.0689) \\
\lambda_2 &= 10^7 * (-0.1009 - i * 1.0689)
\end{aligned}
\tag{26}
$$

As the real parts of both eigenvalues are negative, the averaged system is asymptotically stable and hence the oscillator is stable. The stability of this oscillator has been shown in [2] using the time domain analysis.

In the next example, the stability of a Colpitts oscillator at 5.2 GHz has been studied. The instability of this circuit can cause chaotic behavior [22]. Using the Ebers Moll II model [24] for the HBT transistor, the oscillator and its equivalent circuit has been shown in Fig. 4 and Fig. 5.

Although the circuit has 4 capacitors and 1 inductor, as they construct two capacitive loops, there are only 3 independent state equations. Using v_1, v_2 and i_L as the state variables, the state equations is (27):

$$
\begin{aligned}
L\frac{di_L}{dt} &= v_1 + v_2 + i_L \\
&\quad -C_1\frac{dv_1}{dt} + \left(C_2 + C_{be}(-v_2) - (v_2)C'_{be}(-v_2)\right)\frac{dv_2}{dt} \\
&= I_S\left(e^{-\frac{v_2}{V_T}} - 1\right) - \alpha_R I_S\left(e^{-\frac{v_1+v_2}{V_T} - \frac{v_2+V_{DD}}{R_E}} - 1\right) \\
&\quad \left(C_1 + C_{bc}(-v_1 - v_2) - (v_1 + v_2)C'_{bc}(-v_1 - v_2)\right)\frac{dv_1}{dt} \\
&\quad + \left(C_{bc}(-v_1 - v_2) - (v_1 + v_2)C'_{bc}(-v_1 - v_2)\right)\frac{dv_2}{dt} \\
&= -i_L - \frac{v_1+v_2}{R_L} - \alpha_F I_S\left(e^{-\frac{v_2}{V_T}} - 1\right) I_S\left(e^{-\frac{v_1+v_2}{V_T}} - 1\right)
\end{aligned}
\tag{27}
$$

To apply the proposed method for the stability analysis of this oscillator, the steady state response should be calculated using the "HB" method. The procedure for applying "HB" method to this circuit is shown in Fig. 6. The linear part is determined by the admittance matrix:

$$
Y_n = \begin{pmatrix}
jc_1\omega + \frac{1}{R_C} + \frac{1}{jL\omega} - & jc_1\omega & -\frac{1}{R_C} - \frac{1}{jL\omega} & 0 \\
-jc_1\omega & jc_1\omega + jc_2\omega + \frac{1}{R_E} & 0 & -\frac{1}{R_E} \\
-\frac{1}{R_C} - \frac{1}{jL\omega} & 0 & \frac{1}{R_E} + \frac{1}{jL\omega} & 0 \\
0 & -\frac{1}{R_E} & 0 & \frac{1}{R_E}
\end{pmatrix}
\tag{28}
$$

The nonlinear elements have similar configurations. They consist of a diode, a junction capacitor and a dependent current source. Then if the voltage at the nonlinear ports be v_1 and v_2, the current at each
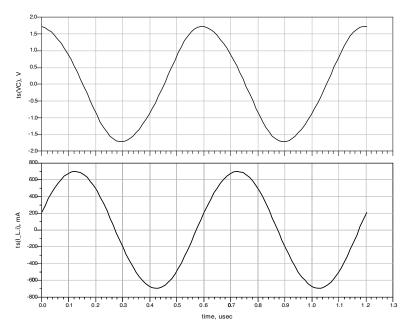
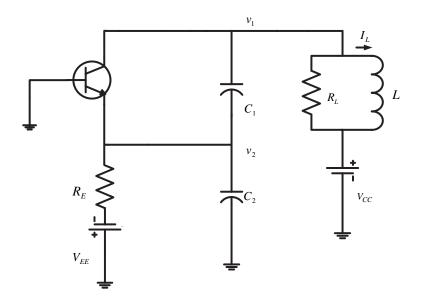**Figure 3.** Steady state response of diode oscillator.
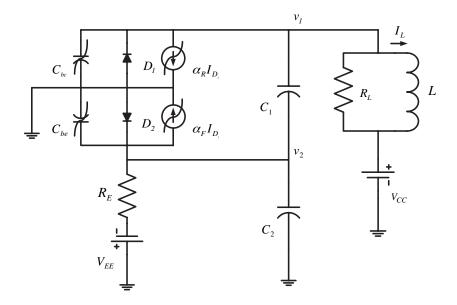

Figure 4. Colpitts oscillator.

Figure 5. Equivalent model of Colpitts oscillator.

port can be expressed as:

$$i_1 = I_s\left(e^{\frac{v_1}{V_T}} - 1\right) + \frac{d}{dt}\left(\frac{C_{0jc}v_1}{\sqrt{1 - \frac{v_1}{\varphi_0}}}\right) + \alpha_R I_s\left(e^{\frac{v_2}{V_T}} - 1\right)$$

$$i_2 = I_s\left(e^{\frac{v_2}{V_T}} - 1\right) + \frac{d}{dt}\left(\frac{C_{0je}v_2}{\sqrt{1 - \frac{v_2}{\varphi_0}}}\right) + \alpha_F I_s\left(e^{\frac{v_1}{V_T}} - 1\right) \qquad (29)$$

Using the above descriptions of the nonlinear ports, the harmonic balance equations have been derived and the steady state responses of v_1, v_2 and i_L are shown in Fig. 7. Analytic derivation of the Jacobian matrix of (27) is cumbersome and it has been calculated numerically. The eigenvalues of the averaged Jacobian matrix are:

$$
\begin{aligned}
\lambda_1 &= -10^7 * (1.67) \\
\lambda_2 &= -10^7 * (0.5298) \qquad (30)\\
\lambda_3 &= -10^7 * (0.05351)
\end{aligned}
$$

As all the eigenvalues have negative real parts, the averaged system is stable. Therefore using the proposed criterion, the oscillation is stable.
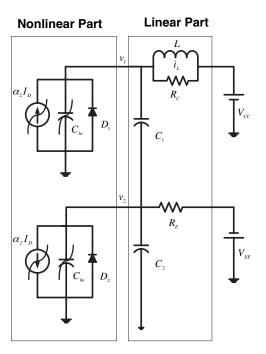
Figure 6. Separation of linear and nonlinear part for harmonic balance analysis.
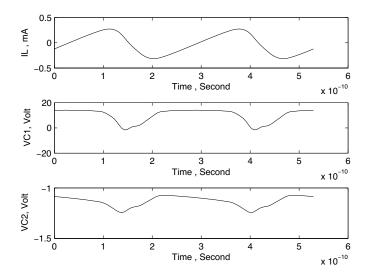

Figure 7. Steady state response of Colpitz oscillator.

The oscillation stability of this oscillator has been verified using the time domain analysis [2].

5. CONCLUSION

In this paper a method for stability analysis of microwave oscillator has been proposed and its theoretical basis has been discussed. It is based on the perturbation theory and averaging method. By deriving the perturbed system which is a linear time varying with periodic behavior and checking the stability of its average, the oscillation stability can be verified. The feasibility of the proposed method has been shown by analyzing the stability of a diode and a Colpitts oscillators. Because of its theoretical basis and its simplicity, it is promising for attaining a general yet simple stability analysis of microwave oscillator.

ACKNOWLEDGMENT

APPENDIX A. PROOF OF COROLLARY 2

Proof: It has been shown [24] that every stable system has a Lyapunov function like $V(x)$ where:

$$
\begin{aligned}
V(x) &> 0 \qquad \text{and} \\
\dot{V}(x) &< 0
\end{aligned}
\tag{A1}
$$

Expanding the time derivative of Lyapunov function $V(x)$:

$$
\dot{V}(x) < 0 \Rightarrow \dot{x}\nabla V < 0 \Rightarrow
\tag{A2}
$$

where ∇ is the gradient of V. Now consider the system $\dot{x} = \varepsilon A(t)x$ where ε is a small positive number and use the same Lyapunov function as used in (A1). To avoid confusing we call it $V_1(x)$ which is obviously positive definite and

$$
\begin{aligned}
\dot{V}_1(x) &< 0 \Rightarrow \dot{x}\nabla V_1 < 0 \Rightarrow \\
\varepsilon A(t)x\nabla V_1 &< 0 \qquad \text{where} \qquad \varepsilon > 0
\end{aligned}
\tag{A3}
$$

As ε is a positive number, it follows from (A2) that $\dot{V}_1(t) < 0$. This means that the system is stable in the sense of Lyapunov. The above corollary states that to show the stability the equilibrium point of a linear time varying system (14) with periodic $A(t)$ it suffices to show the stability of its averaged system on its period.

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