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# ANALYSIS OF TRANSIENT ELECTROMAGNETIC SCATTERING WITH PLANE WAVE INCIDENCE USING MOD-FDM

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**Abstract**—Recently, a marching-on in degree finite difference method (MOD-FDM) was employed in the finite-difference time-domain (FDTD) formulation to obtain unconditionally stable transient responses. The objective of this work is to implement a plane wave excitation in the MOD-FDM formulation for scattering problems for an open region. This formulation has volume electric and magnetic current densities related to the incident field in Maxwell's equations explicitly. Numerical results computed by the proposed formulation are presented and compared with the solutions of the conventional FDTD method.

## 1. INTRODUCTION

The FDTD method has extensively been employed to analyze transient fields from conducting and dielectric objects using the time domain Maxwell's equations [1–8]. The MOD scheme with Laguerre basis functions was proposed to obtain an unconditionally stable solution in integral equations [9–14]. This methodology also has been successfully implemented for the FDTD method [15]. Based on this scheme, various works have been published [16–21].

To analyze a scattering problem in an open region, a plane wave is used as the source in general. In the traditional FDTD method, the plane wave incidence is included in several ways. For example, the incident electric field is added at each time step after computing the electric field by the Yee algorithm and employing the appropriate boundary conditions. Second, the spatial distribution of the electric and magnetic fields is given at the initial time step [22]. Lastly, a plane wave injector is employed [23]. But in the previous works for MOD-FDM [15–20], only the electric current density is considered as an electromagnetic source, so a plane wave cannot be considered in this way. Recently, a separate field formulation using the MOD scheme has been proposed to analyze a scattering problem [21], where the result of temporal testing total field/scattered field equation is modified by adding the temporal coefficient of the incident field.

In this paper, we present a compact MOD-FDM to obtain transient electromagnetic scattering responses using a plane wave injector method [22]. The formulation has the volume electric and magnetic current densities in Maxwell's equations in an explicit form. These current densities are expressed in term of the incident field. This paper is organized as follows. In the next section, for the sake of completeness, we present a MOD-FDM formulation with a plane wave injector for scattering problems. In Section 3, numerical results are presented to show the comparison between the presented and conventional FDTD methods. Finally, in Section 4, we present some conclusions drawn from this study.

## 2. FORMULATION

In the time domain, the general Maxwell's equations in a lossless media can be written as

$$\varepsilon(\boldsymbol{r})\frac{\partial \boldsymbol{E}(\boldsymbol{r},t)}{\partial t} = \nabla\times\boldsymbol{H}(\boldsymbol{r},t)-\boldsymbol{J}(\boldsymbol{r},t) \tag{1}$$

$$\mu(\boldsymbol{r})\frac{\partial \boldsymbol{H}(\boldsymbol{r},t)}{\partial t} = -\nabla\times\boldsymbol{E}(\boldsymbol{r},t)-\boldsymbol{M}(\boldsymbol{r},t) \tag{2}$$

where $\varepsilon$ is the permittivity and $\mu$ is the permeability. $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric field and magnetic field, and $\boldsymbol{J}$ and $\boldsymbol{M}$ are the volume electric and magnetic current densities, respectively. In the surface of the plane wave injector [23], the corresponding electric and magnetic surface current densities $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$ are expressed as

$$\boldsymbol{J}_s(\boldsymbol{r},t) = \boldsymbol{n}\times\boldsymbol{H}^{\mathrm{inc}}(\boldsymbol{r},t) \tag{3}$$

$$\boldsymbol{M}_s(\boldsymbol{r},t) = \boldsymbol{E}^{\mathrm{inc}}(\boldsymbol{r},t)\times\boldsymbol{n} \tag{4}$$

where $\boldsymbol{E}^{\mathrm{inc}}$ and $\boldsymbol{H}^{\mathrm{inc}}$ are the incident electric and magnetic fields, and $\boldsymbol{n}$ is the unit vector normal to this surface toward the propagating direction.

For simplicity, we consider an one-dimensional problem with the field component $E_y$ and $H_z$ with a propagation along the $x$-direction. Then Maxwell's Equations (1) and (2) become

$$\varepsilon(x)\frac{\partial E_y(x,t)}{\partial t} = -\frac{\partial H_z(x,t)}{\partial x} - J_y(x,t) \tag{5}$$

$$\mu(x)\frac{\partial H_z(x,t)}{\partial t} = -\frac{\partial E_y(x,t)}{\partial x} - M_z(x,t). \tag{6}$$

To carry out the derivatives analytically, we expand all the temporal quantities in terms of the associate Laguerre polynomials given by

$$\phi_p(st) = e^{-st/2}L_p(st) \tag{7}$$

where $s$ is a time scale parameter which takes care of the units along the time axis [24], and $L_p$ is the Laguerre polynomial with degree $p$. This temporal basis functions are orthogonal as

$$\int_0^\infty \phi_p(st)\phi_q(st)d(st) = \delta_{pq} \tag{8}$$

where $\delta_{pq}$ is Kronecker delta with value 1 when $p = q$ and 0 otherwise. A continuous function $F(t)$ defined for any value of time $t \geq 0$ can be expanded by the Laguerre basis functions as

$$F(t) = \sum_{p=0}^{\infty} F_p\phi_p(st) \tag{9}$$

and the derivative of the basis functions can be written as [15]

$$\frac{d}{dt}F(t) = s\sum_{p=0}^{\infty}\left(\frac{1}{2}F_p + \sum_{m=0}^{p-1}F_m\right)\phi_p(st) \tag{10}$$

where $F_p$ is the coefficient which can be obtained from

$$F_p = \int_0^\infty F(t)\phi_p(st)d(st). \tag{11}$$

By expanding $E_y$ and $H_z$ with (9) and (10), and putting them in (5) we have

$$\varepsilon(x)s\sum_{p=0}^{\infty}\left(\frac{1}{2}E_y^p(x)+\sum_{m=0}^{p-1}E_y^m(x)\right)\phi_p(st)= -\frac{\partial}{\partial x}\sum_{p=0}^{\infty}H_z^p(x)\phi_p(st)-J_y(x,t) \tag{12}$$

where $E_y^p$ and $H_z^p$ are the coefficients of the Laguerre basis functions for $E_y$ and $H_z$, respectively.

To eliminate the variable $t$ and the infinite summation in (12), we test this equation in a Galerkin's methodology with $\phi_q(st)$. Due to the orthogonal property in (8) we have

$$\varepsilon(x)s\left(\frac{1}{2}E_y^q(x)+\sum_{m=0}^{q-1}E_y^m(x)\right)=-\frac{\partial H_z^q(x)}{\partial x}-J_y^q(x) \tag{13}$$

where

$$J_y^q(x)=\int_0^\infty J_y(x,t)\phi_q(st)d(st). \tag{14}$$

Similarly, from (6) we get

$$\mu(x)s\left(\frac{1}{2}H_z^q(x)+\sum_{m=0}^{q-1}H_z^m(x)\right)=-\frac{\partial E_y^q(x)}{\partial x}-M_z^q(x) \tag{15}$$

where

$$M_z^q(x)=\int_0^\infty M_y(x,t)\phi_q(st)d(st). \tag{16}$$

Using the finite difference in space to approximate the spatial derivatives is similar to the traditional FDTD method. By using spatial difference in (13) and (15), we have

$$E_y\left|_i^q\right.=\frac{2}{\varepsilon_i s\Delta x}\left(H_z\Big|_{i-1/2}^q-H_z\Big|_{i+1/2}^q\right)-2\sum_{m=0}^{q-1}E_y\left|_i^m\right.-\frac{2}{\varepsilon_i s}J_y\left|_i^q\right. \tag{17}$$

$$\begin{aligned}H_z\Big|_{i+1/2}^q &= \frac{2}{\mu_{i+1/2}s\Delta x}\left(E_y\left|_i^q\right.-E_y\Big|_{i+1}^q\right)-2\sum_{m=0}^{q-1}H_z\Big|_{i+1/2}^m\\ &\quad -\frac{2}{\mu_{i+1/2}s}M_z\Big|_{i+1/2}^q\end{aligned} \tag{18}$$

where $\Delta x$ is the cell size and $i$ is the grid number. By inserting (18) into (17), we have

$$\alpha_{i(i-1)}E_y\Big|_{i-1}^q+\alpha_{ii}E_y\left|_i^q\right.+\alpha_{i(i+1)}E_y\Big|_{i+1}^q=\beta_i^q \tag{19}$$

where

$$\alpha_{i(i-1)} = \frac{2}{\mu_{i-1/2}s\Delta x} \tag{20}$$

$$\alpha_{ii} = -\left(\frac{\varepsilon_i s\Delta x}{2} + \frac{2}{\mu_{i-1/2} s\Delta x} + \frac{2}{\mu_{i+1/2} s\Delta x}\right) \tag{21}$$

$$\alpha_{i(i+1)} = \frac{2}{\mu_{i+1/2} s\Delta x} \tag{22}$$

$$\begin{aligned}\beta_i^q = & \sum_{m=0}^{q-1}\left[\varepsilon_i s\Delta x E_y\big|_i^m + 2\left(H_z\Big|_{i-1/2}^m - H_z\Big|_{i+1/2}^m\right)\right] \\ & +\Delta x J_y\big|_i^q + \frac{2}{s}\left(\frac{M_z\Big|_{i-1/2}^q}{\mu_{i-1/2}} - \frac{M_z\Big|_{i+1/2}^q}{\mu_{i+1/2}}\right).\end{aligned} \tag{23}$$

We can get a matrix equation form from (19)–(23) with boundary conditions and solve this recursively in a MOD manner. We use the dispersion boundary conditions derived with Laguerre basis functions in [15].

When a plane wave with $y$-polarization is incident to $x$-direction, using (3) and (4) the current densities in (14) and (16) are given as

$$J_y(x,t) = -\frac{H^{\text{inc}}(x,t)}{\Delta x} = -\frac{E^{\text{inc}}(x,t)}{\eta\Delta x} \tag{24}$$

$$M_z(x,t) = -\frac{E^{\text{inc}}(x,t)}{\Delta x} \tag{25}$$

where $\eta$ is the wave impedance of free space. Here we note that (23) has the magnetic current density as well as electric current density as the excitation source in solving the matrix equation, and (18) also includes the magnetic current density related to the incident plane wave at the computation of the magnetic field coefficients.

## 3. NUMERICAL EXAMPLES

The geometry to be analyzed here is a one-dimensional dielectric slab with 9 cm thick, which was considered in [22]. This slab has relative permittivity 4. The problem space consists of 512 cells with $\Delta x = 1.5$ mm and 250–309 cells for the slab. The incident field in (24) and (25) used in this work is the Gaussian pulse defined as [25]

$$E^{\text{inc}}(x,t) = E_0\frac{4}{T\sqrt{\pi}}\exp\left[\frac{4}{T}(ct - ct_0 - x + x_s)^2\right] \tag{26}$$

where $T$ is the pulse width, $c$ is the velocity of propagation in free space, $t_0$ is a time delay which represents the time at which the pulse peak at the origin, and $x_s$ is the source position. In the computation of the conventional FDTD method, we set the time step size $\Delta t = \Delta x/2c$. Here, the pulse width is $T/c = 400\,\mathrm{ps}$, and $x_s/\Delta x = 50$. In the numerical computation, we choose the number of Laguerre basis functions as 500 and the time scale parameter is $s = 1.1 \times 10^{10}$.

As the first example, Fig. 1 shows the electric field along $x$-position for an incident Gaussian pulse at 800 and 1000 time steps of the FDTD computation. We set $E_0 = T\sqrt{\pi}/4$. Fig. 2 shows the transient electric field at $x = 30\,\mathrm{cm}$ and $x = 42\,\mathrm{cm}$. The agreement between the

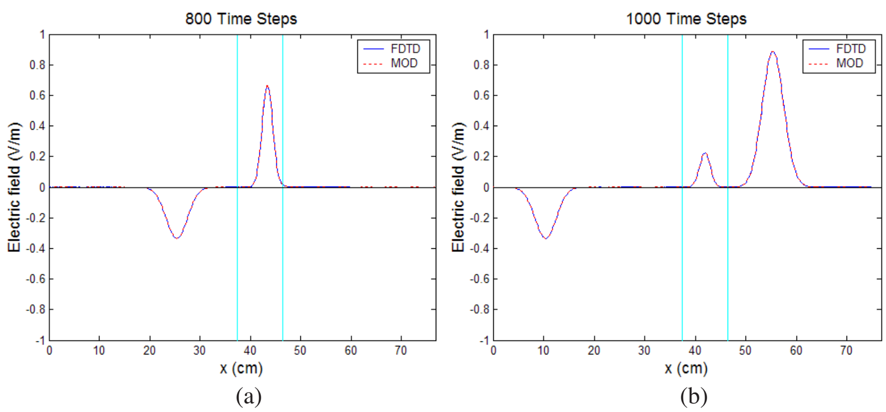


**Figure 1.** Electric field along $x$-position with the incidence of Gaussian pulse plane wave. (a) 800 time steps, (b) 1000 time steps.

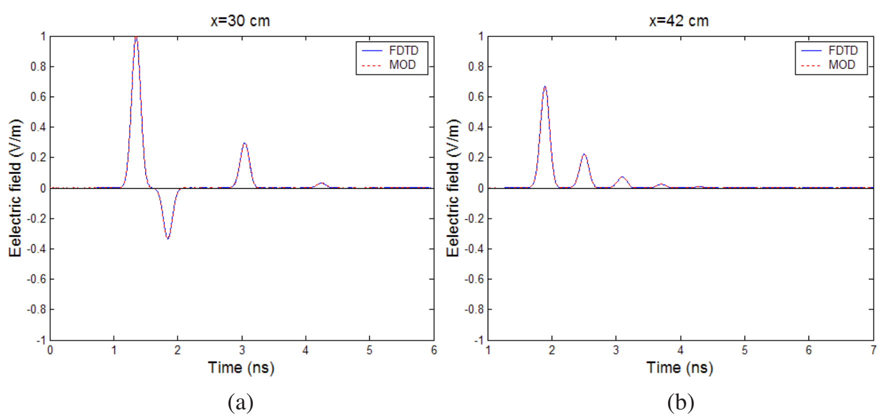


**Figure 2.** Transient electric field versus time with the incidence of Gaussian pulse plane wave. (a) $x = 30\,\mathrm{cm}$, (b) $x = 42\,\mathrm{cm}$.

conventional FDTD method and the proposed method is very good. As the next example, Fig. 3 shows the electric field along $x$-position at 800 and 1000 time steps when the derivative of Gaussian pulse with $E_0 = T^2\sqrt{\pi}/(32c)$ is incident to the slab. Fig. 4 shows the transient electric field at $x = 30\,\text{cm}$ and $x = 42\,\text{cm}$ for an incidence of derivative form of Gaussian pulse. The two solutions agree well as is evident from the figures.

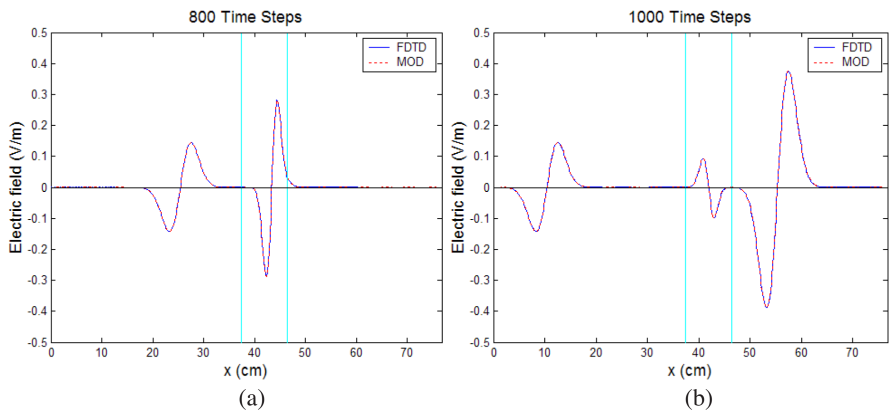


**Figure 3.** Electric field along $x$-position with the incidence of derivative Gaussian pulse plane wave. (a) 800 time steps, (b) 1000 time steps.

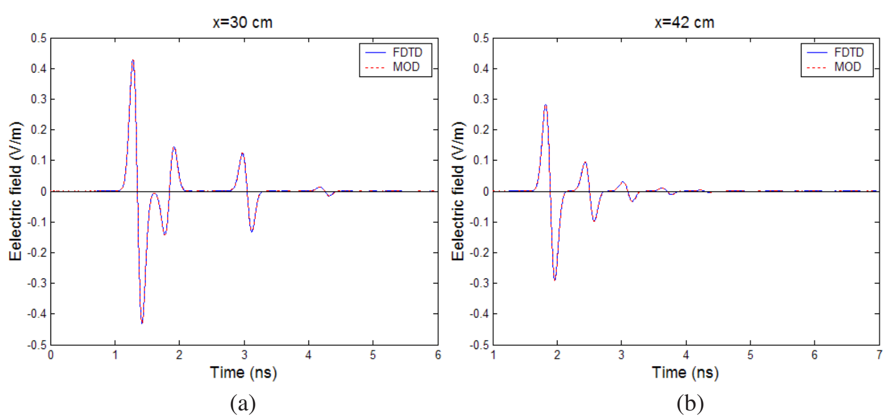


**Figure 4.** Transient electric field versus time with the incidence of derivative Gaussian pulse plane wave. (a) $x = 30\,\text{cm}$, (b) $x = 42\,\text{cm}$.

## 4. CONCLUSION

The conclusion of this work is that the plane wave injector scheme is well implemented with MOD-FDM. This formulation has volume electric and magnetic current densities explicitly. With the plane wave injector in the Maxwell's equations the plane wave incidence can be treated simply. The agreement between the solutions obtained using the proposed method and the traditional FDTD solution is excellent at both the spatial and the time axis.

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