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# AN ENHANCED FDTD MODEL FOR COMPLEX LUMPED CIRCUITS

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**Abstract**—One of most difficult challenges in simulating hybrid RF systems by finite-difference time-domain (FDTD) method is to construct the models for active and lumped elements. In this article, by combining the circuit equations with an integral transform, an enhanced FDTD method is proposed to model the hybrid lumped network which consists of nonlinear and high order linear elements with arbitrary connections. Based on this model, an active radiation system is analyzed. And it shows the application extension of the traditional FDTD method.

## 1. INTRODUCTION

Finite-difference time-domain (FDTD) method was firstly presented by Yee in 1966 and has been applied to analyze electromagnetic characteristics of various passive microwave and antenna structures [1]. For example, it was successfully used to analyze the guided wave problems [2–5], extract the propagation constant of uniform transmission line [6], study the radiation performance of microwave and millimeter wave antennas [7–11], simulate electromagnetic wave penetration into a complex structure as well as the scattering of the electric large object [12, 13].

Linear lumped circuits and active devices, such as integrated resistor, diode, FET and HEMT, play an important role in microwave integrated circuit (MIC) and microwave monolithic integrated circuit (MMIC). The FDTD method is also expected to be capable of analyzing these lumped and active devices. When the FDTD method is employed in this topic, one of the most difficult challenges is to construct the models for these circuit units. In order to simulate the linear lumped and active devices by the FDTD method, many efforts have been made. Literature [14] has proposed a FDTD method for modeling the linear lumped circuits. The linear lumped network is described with its impedance in Laplace domain, and then the impedance is transformed from Laplace domain into Z-domain by the bilinear transformation. Finally, the time domain terminal voltage of the linear lumped network required by Yee's cell can be obtained by using appropriate digital signal processing techniques. Subsequently, Wu et al. [15] have developed another FDTD model for incorporating a high order passive network into FDTD method. In order to obtain an explicit iteration format a complicated deductive procedure is used in this model. Shao et al. [16] have present an improved FDTD method based on matrix theory for linear lumped circuits. Whereas these methods can not address the active and nonlinear circuit issues. Based on the circuit equations, the authors in [17, 18] have reported a matrix method to model the complex lumped and active network. However, it can be seen that this matrix method is inapplicable to model the network with nonlinear and high order linear elements. In this paper, we present an enhanced FDTD method to overcome this challenge and a new matrix technique is built to solve the complex hybrid network.

## 2. THE BASIC THEORY OF THE ENHANCED FDTD MODEL

### 2.1. Matrix Technique for the Linear Lumped Network

The frequency dependent impedance of the linear lumped network is described in the integral form of the rational function as [15]

$$\frac{V_0(s)}{I_0(s)} = \frac{\sum_{r=0}^{R} a_r (1/s)^r}{\sum_{m=0}^{M} b_m (1/s)^m} \tag{1}$$

where $s = j\omega$, $V_0$ and $I_0$ are the total voltage and current across the linear lumped network, $R$ and $M$ are the highest order of the numerator and denominator of the rational function, respectively. The coefficients $a_r$ and $b_m$ are determined by the structure of the linear lumped network.

The formula (1) can be rewritten as

$$V_0(s) \sum_{m=0}^{M} b_m (1/s)^m = I_0(s) \sum_{r=0}^{R} a_r (1/s)^r \tag{2}$$

Utilizing the integral transform relationship between $s$-domain and time domain

$$F(s)/s \to \int_0^t f(\tau)d\tau \tag{3}$$

Eq. (2) can be transformed into the time domain integral form and discretized as follows

$$b_0 V_0^{n+1} + \sum_{m=1}^{M} b_m V_{0m}^{n+1} = a_0 I_0^{n+1/2} + \sum_{r=1}^{R} a_r I_{0r}^{n+1/2} \tag{4}$$

where $V_{01}^{n+1}, \ldots, V_{0M}^{n+1}$ and $I_{01}^{n+1/2}, \ldots, I_{0R}^{n+1/2}$ are given by

$$\begin{aligned} V_{01}^{n+1} &= \int_0^{(n+1)\Delta t} V_0(\tau)d\tau \\ &= \int_0^{n\Delta t} V_0(\tau)d\tau + \int_{n\Delta t}^{(n+1)\Delta t} V_0(\tau)d\tau \\ &= V_{01}^{n} + \Delta t V_0^{n+1} && \text{(5a)} \\ V_{02}^{n+1} &= V_{02}^{n} + \Delta t V_{01}^{n+1} && \text{(5b)} \\ &\cdots \\ V_{0M}^{n+1} &= V_{0M}^{n} + \Delta t V_{0(M-1)}^{n+1} && \text{(5c)} \end{aligned}$$

$$\begin{aligned} I_{01}^{n+1/2} &= \int_0^{(n+1/2)\Delta t} I_0(\tau)d\tau \\ &= I_{01}^{n-1/2} + \Delta t I_0^{n+1/2} && \text{(6a)} \\ I_{02}^{n+1/2} &= I_{02}^{n-1/2} + \Delta t I_{01}^{n+1/2} && \text{(6b)} \\ &\cdots \\ I_{0R}^{n+1/2} &= I_{0R}^{n-1/2} + \Delta t I_{0(R-1)}^{n+1/2} && \text{(6c)} \end{aligned}$$

where $\Delta t$ is the time step length. If the medium is linear, lossless and homogeneous, the electromagnetic fields and current density $\boldsymbol{J}$ can be coupled by the following Maxwell curl equation

$$\varepsilon \boldsymbol{\partial E}/\boldsymbol{\partial} t = \boldsymbol{\nabla} \times \boldsymbol{H} - \boldsymbol{J} \tag{7}$$

where $\varepsilon$ is the permittivity. We can assume that the lumped network occupies only one FDTD cell and it is along the $x$-direction. According to Eq. (7), the relationship between electromagnetic fields and current $I_0$ at the location of the lumped network can be written as

$$E_x^{n+1} = E_x^n + \Delta t/\varepsilon \left(\boldsymbol{\nabla} \times \boldsymbol{H}\right)_x^{n+1/2} - \Delta t/\varepsilon \cdot I_0^{n+1/2}/(\Delta y \Delta z) \tag{8}$$

The local electric field $E_x$ and the terminal voltage $V_0$ of the network satisfy

$$V_0^{n+1} = E_x^{n+1} \Delta x \tag{9}$$

where $\Delta x$, $\Delta y$, and $\Delta z$ are the spatial step lengths along the $x$-, $y$- and $z$-directions, respectively. In Eqs. (4)–(9), there are $(M + R + 3)$ unknowns $(E_x^{n+1}, V_0^{n+1}, V_{01}^{n+1}, \ldots, V_{0M}^{n+1}, I_0^{n+1/2}, I_{01}^{n+1/2}, \ldots, I_{0R}^{n+1/2})$ and $(M + R + 3)$ equations. The system of equations constructed by Eqs. (4)–(9) might be written as a matrix format

$$\boldsymbol{AB} = \boldsymbol{C}_0 \tag{10}$$

where, $\boldsymbol{A}$ is $(M + R + 3) \times (M + R + 3)$ coefficient matrix, $\boldsymbol{B}$ is the unknown column vector $(E_x^{n+1}, V_0^{n+1}, V_{01}^{n+1}, \ldots, V_{0M}^{n+1}, I_0^{n+1/2}, I_{01}^{n+1/2}, \ldots, I_{0R}^{n+1/2})^T$, and $\boldsymbol{C}_0$ is the known column vector, respectively. The matrices $\boldsymbol{A}$ and $\boldsymbol{C}_0$ can be written as

$$\boldsymbol{A} = \left[\begin{array}{ccccccc|ccccc}
1 & 0 & \cdots & & & & & \Delta t/\varepsilon/(\Delta y \Delta z) & 0 & & & \\
-\Delta x & 1 & 0 & \cdots & & & & & & & & \\
0 & -\Delta t & 1 & 0 & \cdots & & & 0 & & & & \\
0 & 0 & -\Delta t & 1 & 0 & \cdots & & 0 & \cdots & & & \\
 & & & & & & \cdots & & & & & \\
0 & 0 & & \cdots & & -\Delta t & 1 & 0 & & & & \\
\hline
0 & b_0 & b_1 & \cdots & & & b_M & -a_0 & -a_1 & \cdots & & -a_R \\
0 & 0 & & \cdots & & & 0 & -\Delta t & 1 & 0 & \cdots & 0 \\
0 & 0 & & \cdots & & & & 0 & -\Delta t & 1 & 0 & \cdots \\
 & & \cdots & & & & & & & & & \\
0 & & \cdots & & & & & & & 0 & -\Delta t & 1
\end{array}\right] \tag{11}$$

and

$$C_0 = \left[ E_x^n + \frac{\Delta t}{\varepsilon} (\boldsymbol{\nabla} \times \boldsymbol{H})_x^{n+1/2} \quad 0 \quad V_{01}^n \cdots \quad V_{0M}^n \quad 0 \quad I_{01}^{n-1/2} \cdots \quad I_{0R}^{n-1/2} \right]^T \tag{12}$$

It is clear that the local electric field on the FDTD cell occupied by the linear lumped network can be solved very easily by the new matrix technique. However, in order to obtain an explicit iteration format a complicated deductive procedure is used in Literature [15].

## 2.2. Matrix Technique for the Network with Nonlinear and High-order Linear Elements

The complex networks with linear and nonlinear elements have been extensively used in various microwave circuits and active antenna structures. For instance, diode and FET are frequently used in the microwave mixer and can be equivalent to a hybrid network with nonlinear sources and linear lumped elements. To model such network, an FDTD model has been presented in [17, 18]. However, if some one-order elements, i.e., $R, L$ or $C$, are replaced by complex high-order ones in the hybrid network, the method presented in [17, 18] is non-effective. In addition, the introduced matrix technique in Subsection 2.1 is only suitable for the linear lumped networks. Therefore, it is necessary to develop an effective method to model the arbitrary complex hybrid network. By using Eqs. (2)–(12) in conjunction with Kirchhoff's current law at circuit nodes and Kirchhoff's voltage law in branch loop circuits,

$$\sum I_{i_b}^{p} = 0 \tag{13a}$$

$$\sum V_{j_l}^{q} = 0 \tag{13b}$$

a new matrix technique can be built to solve this bothersome problem. In Eq. (13a), $p$ and $i_b$ denote the $p$-th node and the $i_b$-th branch current at the $p$-th node, respectively. In Eq. (13b), $q$ and $j_l$ denote the $q$-th loop circuit and the $j_l$-th element voltage in the $q$-th loop circuit, respectively. During the analysis, the high order lumped elements are regarded as a circuit subsystem and its impedance $Z_h$ can be expressed as

$$Z_h = \frac{\sum\limits_{x=0}^{X} f_x (1/s)^x}{\sum\limits_{y=0}^{Y} g_y (1/s)^y} \tag{14}$$

where $X$ and $Y$ are the highest order of the numerator and denominator of the rational function, respectively. Based on Eqs. (13)–(14), a system of nonlinear equations in $s$-domain can be established for complex hybrid network. The system of nonlinear equations needs to be transformed into time domain using Eq. (3). Then the transforms (5) and (6) are used to discretize the time domain integral expressions which exist in this time domain system of nonlinear equations. Finally, combining the system of the discretized equations with Eqs. (8)–(9), a new system of equations, like (10), is constructed. However, unlike the matrix equation of the linear lumped network, the new matrix equation is the nonlinear one and its coefficient matrix $\boldsymbol{A}$ depends on time. Its solution can be obtained by Newton-Raphson method. In the following section, we will show an illustrative example to demonstrate the applicability of the proposed approach.

## 3. THE APPLICATION OF THE ENHANCED FDTD MODEL

An example of active radiation system is analyzed using the proposed FDTD model. The active radiation system consists of two microstrip antenna elements. The two elements are connected by a coupling line. To constitute an active system, Gunn diodes $D_1$ and $D_2$ are integrated in two radiation element, respectively. The detailed circuit layout of this system is shown in Fig. 1. The microstrip substrate has a thickness of $h = 0.79$ mm and its relative permittivity is $\varepsilon_r = 2.33$. The geometry parameters are fixed as: $L_1 = 10.8$ mm, $L_2 = 0.24$ mm, $L_3 = 2.16$ mm, $L_4 = 0.48$ mm, $L_5 = 17.76$ mm, $W_1 = 8.37$ mm, $W_2 = 5.27$ mm, $W_3 = 3.72$ mm, $W_4 = 2.48$ mm and $W_5 = 5.58$ mm. This system has been presented in [19].

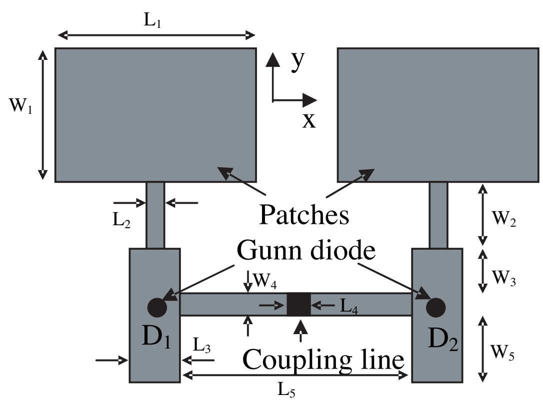


**Figure 1.** The layout of the active radiation system.

The equivalent circuit of Gunn diodes is shown in Fig. 2 and that of the quasi ideal-packaged Gunn diodes $D_1$ and $D_2$ is described by the part enclosed in dotted box. By using the new matrix technique, the quasi ideal-packaged Gunn diodes are simulated. In this equivalent circuit, $I_s$ is a nonlinear current source and its expression is

$$I_s = -G_1 V_c + G_3 V_c^3 \tag{15}$$

The circuit equations of the quasi ideal-packaged Gunn diode are:

$$I_2 - I_3 - I_s = 0 \tag{16a}$$
$$V_0 - V_c - I_2 R_0 = 0 \tag{16b}$$
$$C_0 V_c - \int I_3 dt = 0 \tag{16c}$$

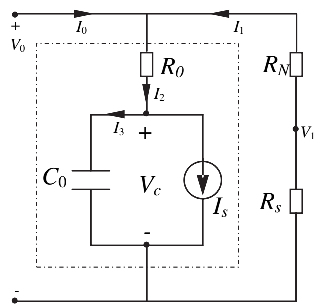


**Figure 2.** Equivalent circuit model of the Gunn diode.

where $I_2$ and $V_0$ are the total current and voltage across the quasi ideal-packaged diodes, respectively. Eqs. (16a) and (16b) can be discretized as

$$I_2^{n+1/2} - I_3^{n+1/2} - I_s^{n+1/2} = 0 \tag{17a}$$

$$\left(V_0^{n+1} - V_c^{n+1}\right)/2 - I_2^{n+1/2} R_0 + \left(V_0^n - V_c^n\right)/2 = 0 \tag{17b}$$

According to Eqs. (3)–(6), Eq. (16c) should be discretized as,

$$C_0 V_c^{n+1} - I_{31}^{n+1/2} = 0 \tag{18a}$$

$$I_{31}^{n+1/2} - \Delta t I_3^{n+1/2} - I_{31}^{n-1/2} = 0 \tag{18b}$$

Then, combining Eq. (15), Eqs. (17)–(18) with Eqs. (8)–(9), a discretized system of nonlinear equations can be constructed. Solving this system of equations by Newton-Raphson method in each time step, the required electric fields at the location of the diodes in FDTD cell can be obtained.

The circuit parameters in Eqs. (15), (17) and (18) are: $G_1 = 0.0252\,\Omega^{-1}$, $G_3 = 0.0265\,\Omega^{-1}\text{V}^{-1}$, $\text{R}_0 = 1.0\,\Omega$ and $C_0 = 0.2\,\text{pF}$. Here, we only consider the case that the coupling line between two elements is ideal metal pad. As an excitement, a small amount of numerical noise is introduced into the FDTD mesh. The simulated total voltage across the diode $D_1$ is shown in Fig. 3. The resonant frequency of 12.38 GHz is extracted by using the Fourier transformation and the resonant voltage amplitude of 1.125 V is observed directly from Fig. 3. The time developments of the voltage across $D_1$ and $D_2$ indicate that the two radiation elements operate with the out-of-phase mode. These results are the same as those shown in [19] and also indicate the validity of the enhanced FDTD method for complex hybrid circuits.

To extend the application of the enhanced FDTD model, another parasitic effect on each Gunn diode is introduced. The additional

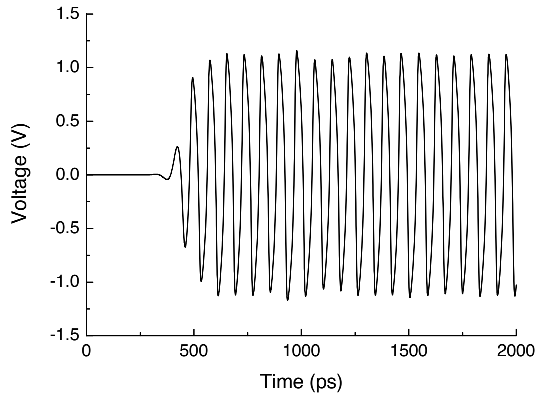


**Figure 3.** The time development of the total voltage across the diode $D_1$ when the quasi ideal-packaged diode model is used.

parasitic effect is equivalent to a shunt circuit, as shown in Fig. 2. This parasitism can be caused by a non-ideal packaging. In this shunt circuit, we assume $R_s = 60\,\Omega$, $R_N$ is a high order linear network and is expressed as,

$$R_N = \frac{A_0 + A_2 \times \left(\frac{1}{j\omega}\right)^2}{B_1 \frac{1}{j\omega}} \tag{19}$$

where $\omega$ is the angular frequency, $A_0 = 0.2 \times 10^{-9}$, $A_2 = 3.8 \times 10^{11}$ and $B_1 = 1$, respectively. It is very difficult to model these hybrid equivalent networks by the traditional FDTD method. However, it is straightforward to model this device using the enhanced FDTD technique. Besides Eqs. (8)–(9), (15) and (17)–(18), some additional circuit equations are needed to simulate this complicated network, i.e.,

$$V_1 - V_0 - R_N I_1 = 0 \tag{20a}$$
$$I_0 + I_1 - I_2 = 0 \tag{20b}$$
$$V_1 + I_1 R_s = 0 \tag{20c}$$

Eqs. (20b) and (20c) may be discretized like Eqs. (16a)–(16b). According to Eqs. (3)–(6), Eq. (20a) should be discretized by the following equations,

$$\left(B_1 V_{11}^{n+1} - B_1 V_{01}^{n+1}\right)/2 - A_0 I_1^{n+1/2} - A_2 I_{12}^{n+1/2} + (B_1 V_{11}^{n} - B_1 V_{01}^{n})/2 = 0 \tag{21a}$$

$$V_{11}^{n+1} - \Delta t V_1^{n+1} - V_{11}^n = 0 \tag{21b}$$
$$V_{01}^{n+1} - \Delta t V_0^{n+1} - V_{01}^n = 0 \tag{21c}$$
$$I_{11}^{n+1/2} - \Delta t I_1^{n+1/2} - I_{11}^{n-1/2} = 0 \tag{21d}$$
$$I_{12}^{n+1/2} - \Delta t I_2^{n+1/2} - I_{12}^{n-1/2} = 0 \tag{21e}$$

The established new system of equations can be solved by Newton-Raphson method. Fig. 4 shows the simulated total voltage across the Gunn diode $D_1$. Owing to the effect of the additional parasitic shunt circuit, the resonant voltage amplitude changes from 1.125 V to 0.66 V, and the resonant frequency has also shifted from 12.38 GHz to 13.26 GHz. The resonant voltage amplitude and the resonant frequency of $D_2$ are the same as those of $D_1$. Two radiation elements operate with the out-of-phase mode. This example indicates that the application of the traditional FDTD method is extended by this enhanced FDTD method.

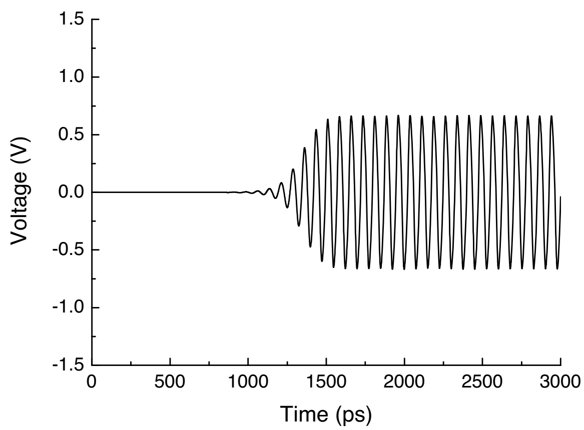


**Figure 4.** The time development of the total voltage across the diode $D_1$ when another parasitic shunt circuit is added.

## 4. CONCLUSION

Based on the circuit equations and the integral transform, an enhanced FDTD technique has been proposed in this article. This proposed method uses a matrix equation to model the complex hybrid network with nonlinear and high order linear elements with arbitrary connections. A typical numerical example has shown the validity and applicability of the proposed method.

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