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MODEL THE ELECTROMAGNETIC SCATTERING FROM THREE-DIMENSIONAL PEC OBJECT BURIED UNDER ROUGH GROUND BY MOM AND MODIFIED PO HYBRID METHOD

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Abstract—In this paper, method of moment and modified physical optical hybrid method is used to analyze the scattering from 3-D PEC object buried under rough surface. The random rough ground surface is characterized with Gaussian statistics for surface height and for surface correlation function. The air-earth interface and the object are all replaced by the corresponding equivalent currents and the equivalent current on the ground surface is divided into two parts: the current caused by the incident wave which is named as incident current, and the current caused by buried object which is named as scattered current. The incident currents are obtained by PO approximation and the scattered currents are related to the current on the buried scatter by a modified PO method in this work. Only the current of scatter is considered as unknown and will be solved by MoM. After obtaining the current of scatter, the scattered current on the ground surface is calculated by the modified PO approximation. And the scatter field will be calculated by using the scattered current. In order to validate the hybrid method proposed in this paper, several numerical examples are given and compared with the results of MoM.

1. INTRODUCTION

Modeling the electromagnetic scattering from the object buried under ground has many important applications and has been of interested for a long time [1–4]. If the ground surface is flat, the Green's function can be formulated with its analytic form by Sommerfeld integral. Using the Green's function of stratified media to replace the free-space Green's function, we can solve the buried problem with the same technique used for free-space problem. Cui et al. combine the dyadic Green's function of stratified media and CG-FFT technique to analyze the scattering from object buried under planar multilayered ground [5, 6]. Basing on the Cui's work, Zhang et al. introduce the DCIM technique to accelerate the calculation of the Green's function of stratified media [7]. When the ground surface is arbitrary, for example, random rough surface, this problem becomes complicated. The transmission field in the bottom half-space is unknown and it's difficult to find the corresponding Green's function. In order to solve this problem, equivalent electric and magnetic currents should be used to replace the real ground surface. Obviously, it will lead to huge number of unknowns and it's difficult to solve this problem numerically with MoM because of the limitation of memory. In order to overcome this difficulty, some fast algorithms based on MoM have been used to analyze the buried problem of random rough ground. The sparse-matrix canonical-grid (SMCG) method was applied in [8] and the steepest descent fast multipole method (SDFMM) was adopted in [9, 10].

Different to the full wave numerical method, this paper will combine the MoM and modified PO method to model the EM scattering from buried object. As a powerful high-low frequency hybrid technique, MoM-PO hybrid method has been proposed and improved to solve many complex electromagnetic problems [11–18]. However, this hybrid technique is mostly used to deal with the scattering and radiation problem in free-space and few publications discuss the application of MoM-PO method for half-space problem. In our work, the MoM-PO method is modified to be suit for the buried problem, in which the buried object is considered as MoM region and ground surface is PO region.

Similar to [8–10], the air-earth interface and surface of buried object are all replaced by equivalent currents in this work. However, we only establish the electric field integral equation (EFIE) on the surface of the buried scatter. The equivalent currents on the ground surface are divided into two parts: the "incident currents" and the "scattered currents". The incident currents are caused by incident wave when the buried object is inexistence, while the scattered currents are caused by the buried object only. According to the equivalent theorem, the incident currents are equivalent to the transmission field from the top half-space to the bottom half-space and will be considered as the excited source in the integral equation. Rigour calculation of the incident currents and scattered currents will involve solving the PMCHW integral equation by MoM [9]. If the ground surface is not very rough, the Kirchhoff approximation (or named as PO approximation) can be used to simulate the scattering of the ground. In this paper, we use PO method to calculate the incident currents. The scattered current on the ground surface is what we are interested in, and will be used to calculate the scattered fields. In this paper, we use a modified PO approximation to calculate the scattered currents by the current of the buried object. Then the scattered currents will be related to the current of the buried object, and only the latter is unknown and will be solved by MoM in this work.

2. FORMULATION

As shown in Fig. 1, a PEC object is buried under rough ground surface, while the top half-space is the air and the bottom half-space is the earth. The complex permittivity and permeability of the top half-space and bottom half-space are $\varepsilon_{r1} = 1$, $\mu_{r1} = 1$ and $\varepsilon'_{r2} = \varepsilon'$, $\mu_{r2} = 1$ respectively. The ground surface is illuminated by a plane wave and the near scattered fields in the air region are what we care about in this paper.

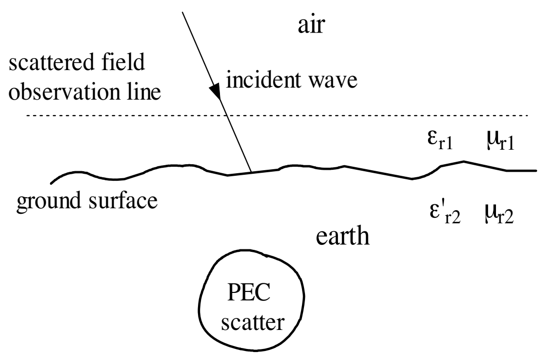

Figure 1. PEC object buried under ground surface.

Applying the equivalent theorem, we can use the equivalent currents to replace the real boundary of the ground surface and the object, as shown in Fig. 2. The electric current and magnetic current of the upper surface of the ground are denoted as $\mathbf{J}_e$, $\mathbf{J}_m$ respectively, while the electric current of the scatter is $\mathbf{J}_s$. The out normal vector of the ground surface and the scatter are $\mathbf{n}_1$, $\mathbf{n}_2$ respectively. By applying the boundary conditions at the surface of the buried scatter, we have the integral equations as follows:

$$\boldsymbol{n}_2 \times \left[\boldsymbol{L}^e_{E,2}\left(-\boldsymbol{J}_e\right) + \boldsymbol{L}^m_{E,2}\left(-\boldsymbol{J}_m\right) + \boldsymbol{L}^e_{E,2}\left(\boldsymbol{J}_s\right)\right] = 0 \tag{1}$$

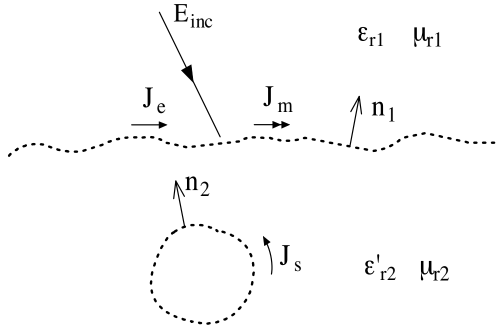

Figure 2. The corresponding equivalent problem of Fig. 1.

Four types vector operators are introduced here, that $\boldsymbol{L}_{E,i}^{e}$ means the electric field in region i caused by electric current; $\boldsymbol{L}_{E,i}^{m}$ means the electric field in region i caused by magnetic current; $\boldsymbol{L}_{H,i}^{e}$ means the magnetic field in region i caused by electric current and $\boldsymbol{L}_{H,i}^{m}$ means the magnetic field in region i caused by magnetic current, where $i$equals to 1 (in the air) or 2 (in the earth).

There are three unknown functions in the integral Equation (1): the electric current and magnetic current of ground surface and the electric current of the buried object. In order to solve this integral equation, we should eliminate the equivalent currents of the ground surface. Divide the currents of ground surface into two parts: incident currents $\boldsymbol{J}_{e0}$, $\boldsymbol{J}_{m0}$ caused by incident wave and scattered currents $\boldsymbol{J}_{e1}$, $\boldsymbol{J}_{m1}$ caused by the buried scatter. With PO approximation, the incident currents can be formulated as:

$$\boldsymbol{J}_{e0} = (\boldsymbol{n}_1 \times \boldsymbol{e}_\perp)\left(1+R_{TM}^1\right)(\boldsymbol{e}_\perp \cdot \boldsymbol{H}_{in}) + \boldsymbol{e}_\perp\left(-1+R_{TE}^1\right)\left(\boldsymbol{e}_{//} \cdot \boldsymbol{H}_{in}\right)\cos\theta_0 \quad (2)$$

$$\boldsymbol{J}_{m0} = \boldsymbol{e}_\perp\left(1-R_{TM}^1\right)\cos\theta_0\left(\boldsymbol{e}_{//} \cdot \boldsymbol{E}_{in}\right) - (\boldsymbol{n}_1 \times \boldsymbol{e}_\perp)\left(1+R_{TE}^1\right)(\boldsymbol{e}_\perp \cdot \boldsymbol{E}_{in}) \quad (3)$$

where θ_0 is the incident angle. $\boldsymbol{e}_\perp$ is the unit vector which is vertical to the incident plane and $\boldsymbol{e}_{//}$ is the unit vector which is parallel to the incident plane. R_{TM}^1 and R_{TE}^1 are the geometry-optical (GO) reflection coefficients of the TM and TE waves at the air-earth interface when the incident wave propagate from the air to the earth. The reflection coefficients can be written as

$$R_{TM}^1 = \frac{\cos\theta_0 - \sqrt{\varepsilon_r' - \sin^2\theta_0}}{\cos\theta_0 + \sqrt{\varepsilon_r' - \sin^2\theta_0}} \quad (4)$$

$$R_{TE}^1 = \frac{\varepsilon_r \cos\theta_0 - \sqrt{\varepsilon_r' - \sin^2\theta_0}}{\varepsilon_r \cos\theta_0 + \sqrt{\varepsilon_r' - \sin^2\theta_0}} \tag{5}$$

In order to solve the integral Equation (1) with MoM method, the infinity ground surface is approximated by a $L \times L$ open surface. The buried scatter has a projective region on the ground surface, and we will name this region as projective region in this paper. Obviously, the truncation size should be selected large than the projective region. On the other hand, because of the loss of the earth, the transmission fields illuminating on the buried scatter mostly depend on the incident currents which are near to the projective region. And the scattered currents will vanish fast when the observation point is far away from this region. So the effect of the truncation of the ground surface to the scattered field can be neglected.

We mesh the ground surface and the object with triangle patch and expand all the currents with RWG basis function.

$$\boldsymbol{J}_{e0} = \sum_{n=1}^{N_{ground}} J_{e0,n}\boldsymbol{f}_n \quad \boldsymbol{J}_{m0} = \sum_{n=1}^{N_{ground}} J_{m0,n}\boldsymbol{f}_n \tag{6}$$

$$\boldsymbol{J}_{e1} = \sum_{n=1}^{N_{ground}} J_{e1,n}\boldsymbol{f}_n \quad \boldsymbol{J}_{m1} = \sum_{n=1}^{N_{ground}} J_{m1,n}\boldsymbol{f}_n \tag{7}$$

$$\boldsymbol{J}_s = \sum_{n=1}^{N_{object}} J_{s,n}\boldsymbol{f}_n \tag{8}$$

where $\boldsymbol{f}_n$ is RWG basis function [19]. $J_{e0,n}$, $J_{m0,n}$, $J_{e1,n}$, $J_{m1,n}$, $J_{s,n}$ are the expanded coefficients of corresponding currents. Applying the point-match method proposed in [11], the expanded coefficients of the incident currents can be obtained by the incident field at center point of the public edge of the ground.

The scattered currents of the ground can be considered as the sum of contribution from every element at the buried object. To the mth public edge on ground surface, if the scattered wave from every element of the buried object is approximated by plane wave, the scattered currents at center point of this edge can be written as

$$\boldsymbol{J}_{e1}^m = \sum_{n=1}^{N_{object}} J_{s,n}\boldsymbol{J}_{e1}^{mn} \tag{9}$$

$$\boldsymbol{J}_{m1}^m = \sum_{n=1}^{N_{object}} J_{s,n}\boldsymbol{J}_{m1}^{mn} \tag{10}$$

where $\boldsymbol{J}_{e1}^{mn}$ and $\boldsymbol{J}_{m1}^{mn}$ are the scattered currents at the mth public edge of ground surface caused by the nth basis function of the object.

$$\begin{aligned}\boldsymbol{J}_{e1}^{mn} &= (-\boldsymbol{n}_1 \times \boldsymbol{e}_\perp)\left(1+R_{TM}^2\right)(\boldsymbol{e}_\perp \cdot \boldsymbol{H}_{sn}) \\ &\quad +\boldsymbol{e}_\perp\left(-1+R_{TE}^2\right)\left(\boldsymbol{e}_{//} \cdot \boldsymbol{H}_{sn}\right)\cos\theta_0' \qquad (11)\\ \boldsymbol{J}_{m1}^{mn} &= \boldsymbol{e}_\perp\left(1-R_{TM}^2\right)\cos\theta_0'\left(\boldsymbol{e}_{//} \cdot \boldsymbol{E}_{sn}\right) \\ &\quad +(\boldsymbol{n}_1 \times \boldsymbol{e}_\perp)\left(1+R_{TE}^2\right)(\boldsymbol{e}_\perp \cdot \boldsymbol{E}_{sn}) \qquad (12)\end{aligned}$$

where θ_0' is the incident angle from the nth element of buried object to the mth public edge of ground surface. R_{TM}^2 and R_{TE}^2 are the reflection coefficients at the observation point when the plane wave is from the earth to the air. $\boldsymbol{E}_{sn}$ and $\boldsymbol{H}_{sn}$ are the scattered fields caused by the nth basis function buried object.

It is known that when a spherical wave source is buried in loss media, the PO method is not a good approximation for the field on the air-earth interface. It is because that the lateral wave [20] will become to the dominant part. In our work, it is found that if the quasi-static reflection coefficients are used to replace the GO reflection coefficients, the results of PO can be improved and accordant to the result of MoM when the observation position is near to the projective region. So the reflection coefficients in formulation (6–7) will be chosen as follows.

$$R_{TM}^2 = \frac{1-\sqrt{\varepsilon_r'}}{1+\sqrt{\varepsilon_r'}} \qquad (13)$$

$$R_{TE}^2 = \frac{\sqrt{\varepsilon_r'}-1}{\sqrt{\varepsilon_r'}+1} \qquad (14)$$

Obviously, the formulations (13–14) are the reflection coefficients when the incident angle is equal to 0. Using the similar process of MoM-PO method proposed in [11], we can obtain the matrix equation of the current on object surface as follow

$$\left(\bar{\bar{Z}}_{oo} + \bar{\bar{Z}}_{og}^{ee}\bar{\bar{R}}_{go}^{ee} + \bar{\bar{Z}}_{og}^{me}\bar{\bar{R}}_{go}^{me}\right)\bar{J}_s = \bar{\bar{Z}}_{og}^{ee}\bar{J}_{e0} + \bar{\bar{Z}}_{og}^{me}\bar{J}_{m0} \qquad (15)$$

where $\bar{\bar{Z}}_{oo}$ is the self-impedance matrix of the object, $\bar{\bar{Z}}_{og}^{ee}$ is the mutual-impedance matrix between electric current of object and electric current of ground and $\bar{\bar{Z}}_{og}^{ee}$ is the mutual-impedance matrix between electric current of object and magnetic current of ground. $\bar{\bar{R}}_{go}^{ee}$ is the matrix that relate the electric current of ground to the electric current of the object and $\bar{\bar{R}}_{go}^{me}$ is the matrix that relate the magnetic current of ground to the electric current of the object. The elements of related matrixes $\bar{\bar{R}}_{go}^{ee}$ and $\bar{\bar{R}}_{go}^{me}$ can be calculated with similar technique proposed in [11].

3. NUMERICAL RESULTS

In order to evaluate the efficiency and accuracy of the hybrid method proposed in this paper, several examples are discussed in this section and the results of MoM and hybrid method are compared. The MoM results are based on the PMCHW integral equation. In these examples, the relative permittivity of the earth is $\varepsilon' = 4-0.5j$. Truncation size of the ground surface L is chosen to be $4\lambda_0$. The random rough ground surface is created by Monte Carlo method, in which the root mean square height is σ and correlation length is l_c.

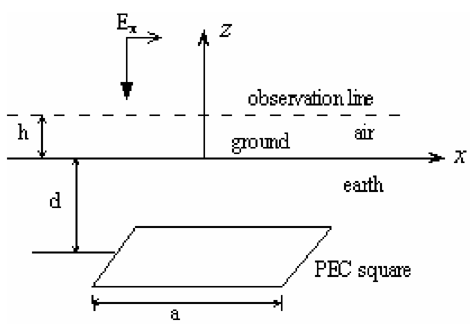

Figure 3. PEC square buried under flat plane.

Example 1

First, we consider a $0.3\lambda_0 \times 0.3\lambda_0$ PEC plate buried under a flat ground as shown in Fig. 3, in which $d = 1\lambda_0$. The ground is illuminated by an x-polarization plane wave and the incident angle $\theta_i = 0$. The height of observation line is $h = 0.1\lambda_0$ and along the x-direction. The calculated results of x-direction scattered electric field are shown in Fig. 4. The computation efficiency of MoM and the hybrid method are compared and listed in Table 1.

It can be seen from Fig. 4 that the results of hybrid method are in good agreement with MoM when the observation position is near to the projective region. In application of buried object exploration technique, the near scattered fields in this region are of interested.

Table 1. Comparison of the computation efficiency between MoM and hybrid method of example 1.

Method	Unknown	Computational time (s)	Memory (Mb)
MoM	5400	1366	350
Hybrid method	96	53	10

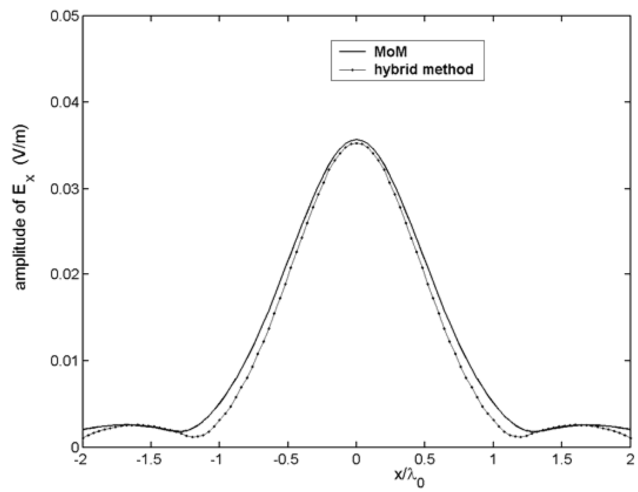

Figure 4. The scattered field from PEC plate buried under flat ground illuminated by x-polarization plane wave.

Example 2

Then the ground surface is changed to be random rough. The observation height is $h = 0.3\lambda_0$ relative to the mean plane of rough ground. Fig. 5 and Fig. 6 show the single realization results of scattered fields. In Fig. 5, the root mean square height $\sigma = 0.2\lambda_0$ and the correlation length $l_c = 0.25$, while $\sigma = 0.2$ and $l_c = 0.5$ in Fig. 6. It can be seen that the results of hybrid method are more accordant to that of MoM when the correlation length is increased. It is because that the PO approximation is more accurate for the rough surface with large correlation length.

Example 3

Next, a PEC sphere buried under rough surface shown in Fig. 7 is discussed. The radius of the sphere is $0.3\lambda_0$ and the buried depth d between the mean plane of ground and sphere center is $1\lambda_0$. The ground is illuminated by an x-polarization plane wave and the incident

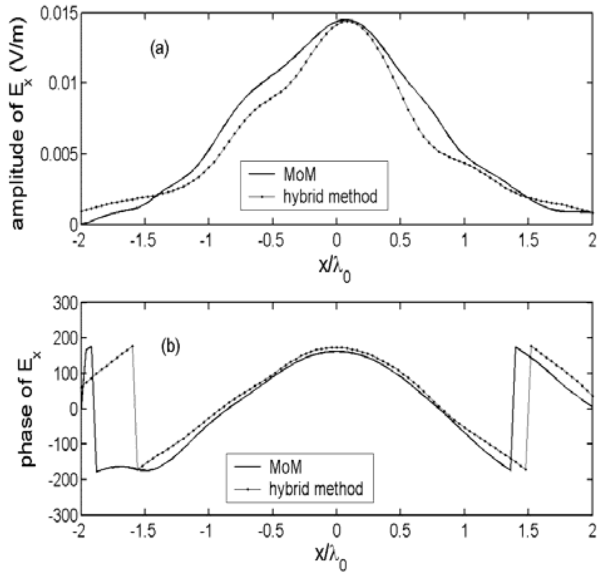

Figure 5. The scattered fields from PEC plate buried under rough surface (single realization results). The root mean square height $\sigma = 0.2\lambda_0$ and the correlation length $l_c = 0.25$.

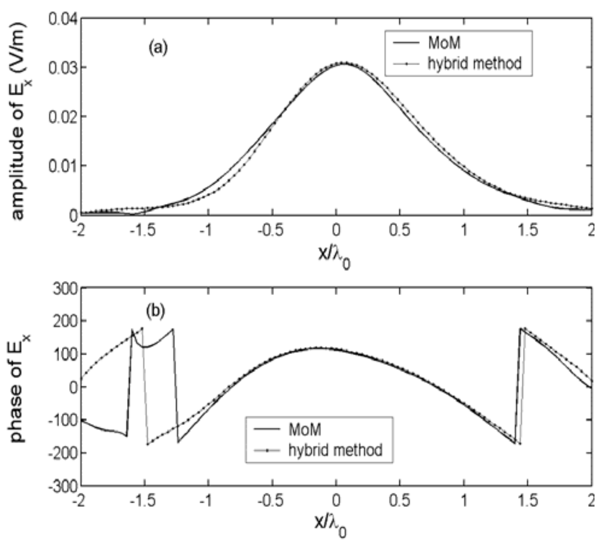

Figure 6. The scattered fields from PEC plate buried under rough surface (single realization results). The root mean square height $\sigma = 0.2\lambda_0$ and the correlation length $l_c = 0.5$.

angle $\theta_i = 0$. The height of observation line is $h = 0.3\lambda_0$. The root mean square height $\sigma = 0.2\lambda_0$ and the correlation length $l_c = 0.5$. The results of scattered fields obtained by hybrid method are shown in Fig. 8 and are in good agreement with the MoM.

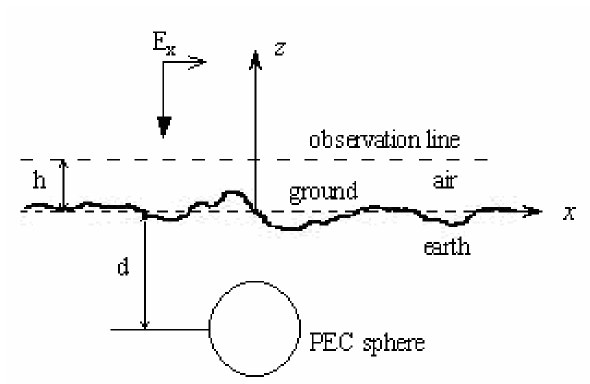

Figure 7. PEC sphere buried under random rough surface.

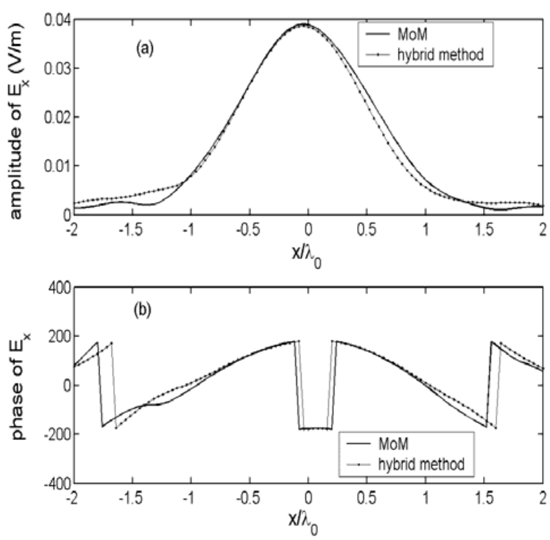

Figure 8. The scattered fields from PEC sphere buried under rough surface (single realization results). The root mean square height $\sigma = 0.2\lambda_0$ and the correlation length $l_c = 0.5$.

4. CONCLUSION

In this paper, MoM and PO hybrid method is proposed to simulate the EM scattering from 3-D PEC object buried under random rough ground. With this method, equivalent currents of the ground are divided into incident currents caused by incident wave and scattered currents caused by buried object. PO approximation is applied to calculate the transmission field from the air to the earth. The relation between currents of object and scattered currents on ground surface are simplified by a modified PO method, in which the quasi-static reflection coefficients are used to replace the GO reflection coefficients. Several examples are discussed in this paper to evaluate the computation accuracy and efficiency of the proposed hybrid method. The comparison between proposed method and MoM shows that the accuracy of the hybrid method is acceptable for near observation region while the computation efficiency is much high than the MoM. With this hybrid method, the EM scattering problems of 3-D buried object can be solved more effectively, which is helpful to understand the effect of the rough ground surface in buried object exploration.

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