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AMBIGUITY FUNCTIONS OF DIRECT CHAOTIC RADAR EMPLOYING MICROWAVE CHAOTIC COLPITTS OSCILLATOR

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Abstract—The ambiguity functions of a kind of direct chaotic radar system are investigated. In this radar system, a microwave chaotic Colpitts oscillator is employed to generate the source signal that is directly transmitted through a wideband antenna without modulation. The auto-ambiguity function of this radar system shows many sidelobes which makes the unambiguous detection difficult. It is because the spectrum of the chaotic signal is not very flat and smooth, with pulsation peaks in it. The cross-ambiguity functions of the direct radar system have also been investigated to evaluate the electronic counter countermeasure (ECCM) performance and the "multi-user" characteristic. It is shown that rather excellent ECCM capability can be achieved in this radar system with transmitting chaotic signals generated by circuits with same parameters but at different time or with slightly different circuit parameters. In addition, several possible methods to reshape the spectrum of the chaotic signal from microwave Colpitts oscillators to improve the unambiguous detection performance are suggested at the end of this paper.

1. INTRODUCTION

Radar has gone through a history for more than 100 years. In recent years, new techniques are continuously applied in various radar systems [1–5]. Among various kinds of radars, the random signal radar (RSR) refers to radars whose transmitted waveforms are random or random-like in contrast to conventional pulse, continuous wave (CW), frequency-modulation (FM) or FM/CW radars [6]. Because of the random transmitted signal, RSRs posses good ECCM capability, very low probability of intercept (LPI), and counter electronic support measure capability. In contrast, the ultra wide band (UWB) [7] radars have the merits of high range resolution, enhanced clutter-suppression capability, good penetration detection, anti-jamming and etc. [8]. Combining both the merits of RSR radar and UWB radar, an UWB RSR radar is of great importance to both military and civilian applications. However, in this type of radar system, how to generate the UWB random signal is regarded as a rather difficult problem.

Chaos is a kind of phenomena in deterministic nonlinear dynamical system, which is aperiodic and sensitively depends on its initial value. Traditionally, most of the applications for chaos study are restricted in communication fields, such as secure communications or broadband communications. In recent years, with their noise-like property in time domain and broadband characteristic in frequency domain, possibilities of exploiting chaotic signals in RSRs have been studied by some researchers. In year 1998, ambiguity functions of a chaotic phase modulated radar are investigated, which is probably the first work incorporating the concept of chaos into radar [9]. In year 2000, the authors in Ref. [10] raised the concept of "chaotic radar systems" for the first time. Performances of chaotic FM, amplitude-modulation (AM), phase-modulation (PM) and pulse modulation radars have been reported and these radar systems have been applied to vehicular collision-avoidance, through-the-wall imaging, homeland security and so on [9, 11–14]. In most of the above works, chaotic signals were generated by discrete map and employed into radar systems as baseband signal for modulation such as FM, PM, etc. As a result, the bandwidth of transmitting radar signals is limited to several MHz level, which can not meet the requirements of UWB RSR.

To utilize the unique merits of UWB RSR, one can employ chaotic signals generated in microwave band or laser band. Chaos can be generated in laser band rather easily, and the laser based chaotic radar system, along with its ambiguity functions, has been exploited [8, 15, 16]. Regarding the microwave chaotic signals, it is not until recent years that chaotic circuits with fundamental frequency more than 1 GHz were implemented [17–19], where the Colpitts oscillator is the major candidate due to its simple circuit structure. Details of design and implementation of microwave chaotic Colpitts oscillator and possibility of using the generated signal to design a direct chaotic radar have been discussed in [18, 20].

By using the microwave chaotic signals generated from Colpitts oscillator, besides the merits of UWB RSR, the direct chaotic radar has advantages over those with FM/AM/PM chaos modulation since the system structure can be simplified when no modulation/demodulation is required. Another possible advantage of the direct chaotic radar is, by varying the operating condition, the dynamics in a Colpitts oscillator can be easily switched among different states, and the corresponding diverse waveforms with different characteristics are highly desirable, since it provides the property of "multi-user", which is required when large number of radars with same scheme co-exist, in applications such as anti-collision vehicle-borne radars. When compared with laser-based chaotic radar, the direct chaotic radar employing Colpitts oscillator has the advantages of simple circuit structure and low fabrication cost.

Generally, ambiguity function is used to describe the detection property of a radar system in both the range and range rate domains. The purpose of this paper is to study the ambiguity functions of the direct chaotic radar employing a microwave Colpitts oscillator to see the feasibility of such system. We borrowed the research method from [8] to study the auto-ambiguity function and cross-ambiguity function. The direct radar structure is described in Section 2, and ambiguity functions will be analyzed in Section 3. Section 4 presents discussions and conclusions.

2. DIRECT CHAOTIC RADAR

The direct chaotic radar employing a microwave chaotic oscillation source is shown in Fig. 1. Chaotic signal from the oscillation source is split into two channels, the signal and the reference. The signal waveform is isolated and driven by a buffer amplifier, and then directly radiated by a transmitter antenna after a power amplifier. If there is a target in air, the backscattered signal (echo) will be received by the receiver antenna, and after going through a low noise amplifier and an adaptive filter, the chaotic signal with noise and delay reaches the chaos synchronization circuit, as illustrated in [20], where a delayed copy of the source chaotic signal will be re-generated. Then the delayed version of chaotic signal and the reference waveform will be processed by a signal processing unit, where target detection and localization can be

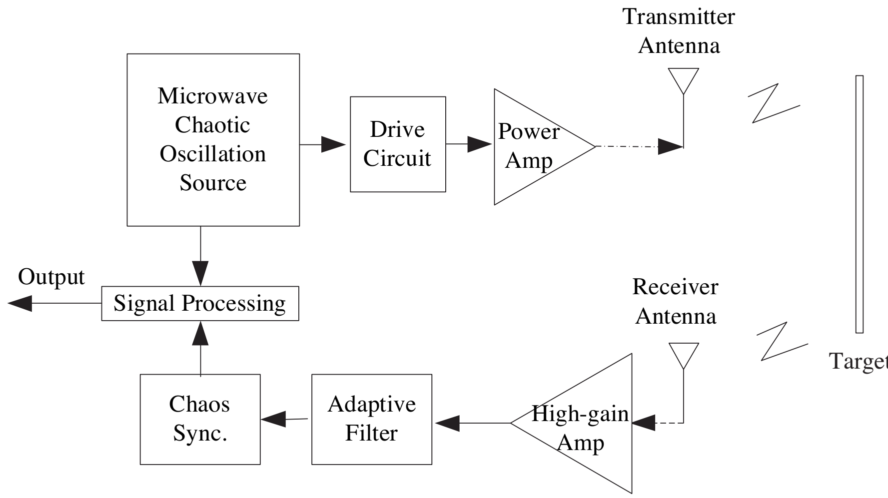

Figure 1. Architecture of the direct chaotic radar.

accomplished. The signal from the microwave chaotic oscillation source can be used as the transmitting signal without further modulation, and directly radiated to free space with wideband antenna, which has been proved feasible in [21].

In this paper, we study the ambiguity functions of the direct chaotic radar system shown in Fig. 1, where a chaotic Colpitts oscillator is used as the microwave chaotic oscillation source. The basic configuration of a Colpitts oscillator is shown in Fig. 2. It contains a bipolar junction transistor (BJT) as the gain element and a resonant network consisting of an inductor and a pair of capacitors [22]. The transistor is modelled with a voltage-controlled nonlinear resistor R_E and a linear current-controlled current source, neglecting the base current. The driving-point characteristics of the nonlinear resistor R_E can be expressed as:

$$I_E = f(V_{BE}) = I_s \left(\exp\left(\frac{V_{BE}}{V_T} - 1 \right) \right) \tag{1}$$

where I_s is the inverse saturation current and $V_T \simeq 26\,\mathrm{mV}$ at room temperature. The state equations for the Colpitts oscillator shown in Fig. 1 are:

$$\begin{aligned} C_1 \frac{dV_{C_1}}{dt} &= -f(-V_{C_2}) + I_L \\ C_2 \frac{dV_{C_2}}{dt} &= I_L - \frac{V_{C_1} + Vee}{Re} \end{aligned} \tag{2}$$

$$L\frac{dI_L}{dt} = -V_{C_1} - V_{C_2} - I_L R + V_{CC}$$

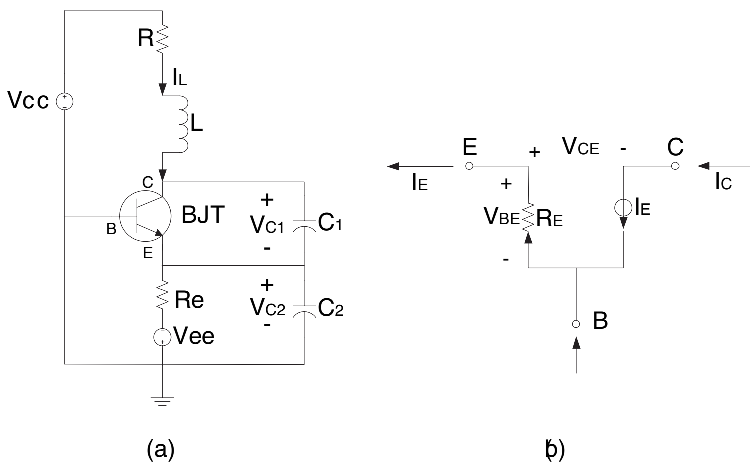

Figure 2. (a) Colpitts oscillator and (b) equivalent circuit model of the BJT in (a).

The modelling, nonlinear dynamics of the Chaotic Colpitts oscillator have been studied in detail [22], and the design and implementation of microwave chaotic oscillators have been discussed by several researchers [17–19, 23]. The randomness characteristic of chaotic signal from microwave Colpitts oscillator has also been investigated [24]. It has been verified that chaotic signal with fundamental frequency more than 1 GHz can be generated by fabricated circuit and the experimental results agree well with the circuit simulation in Advanced Design System (ADS) simulator. Since it is difficult to sample the chaotic signal with fundamental frequency more than 1 GHz, we use the simulated waveform from ADS to conduct the ambiguity function research of the direct chaotic radar system.

The BJT used in the Colpitts oscillator in Fig. 2 is Philips BFG425W, whose threshold frequency is 25 GHz. The parameters of circuit components, namely the resistance R, Re, the inductance L, the capacitance C_1, C_2, the voltage source Vcc and Vee, are critical because they determine whether the chaotic oscillation can be achieved and the fundamental frequency of the oscillation. These parameters can be selected under the guidance of Ref. [24], given the prescribed fundamental frequency. In our simulations, the circuit parameters are listed as follows: $Vcc = 10\,\text{V}$, $Vee = -10\,\text{V}$, $R = 25\,\text{Ohm}$, $Re = 1.6\,\text{kOhm}$, $L = 4.5\,\text{nH}$, $C_1 = 4\,\text{pF}$, $C_2 = 4\,\text{pF}$. Simulations of

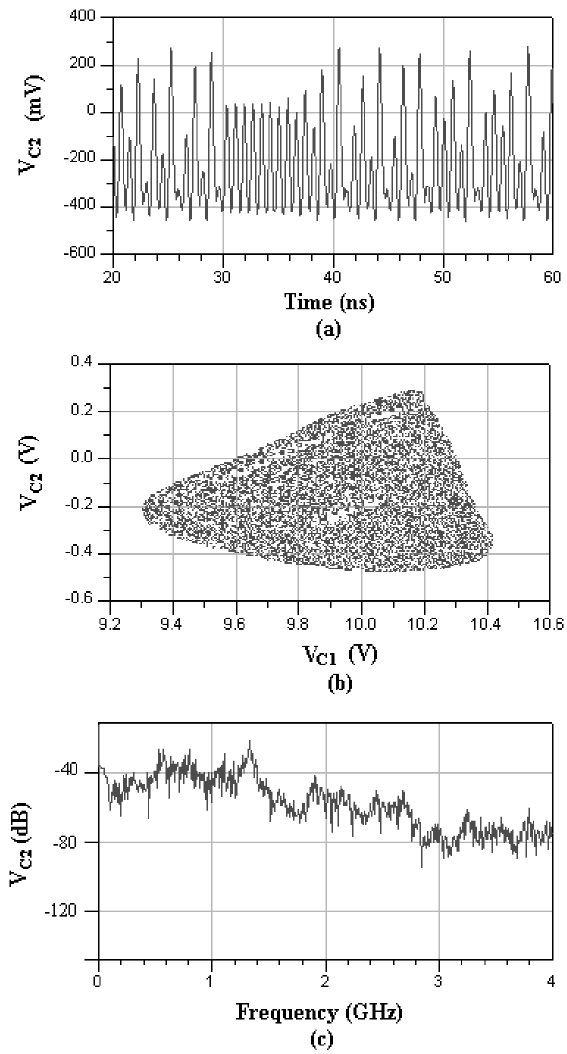

Figure 3. (a) Time domain waveform of V_{C_2}, (b) attractor in V_{C_2}-V_{C_1} plane and (c) spectrum of signal V_{C_2}.

circuit by using ADS are plotted in Fig. 3.

Fig. 3(a) shows the time-domain waveform of one of the voltage node V_{C_2}, which is a noise-like signal. Fig. 3(b) plots the projection of the attractor in the phase space into the V_{C_2}-V_{C_1} plane. This is a typical chaotic attractor of Colpitts oscillator. Fig. 3(c) shows the broadband continuous spectrum of the signal V_{C_2}.

3. AMBIGUITY FUNCTION ANALYSIS

In the direct chaotic radar system employing a microwave Colpitts oscillator, as shown in Fig. 2, the broadband microwave chaotic waveforms are transmitted and received as baseband signal. Without modulating the baseband signal with a carrier as in conventional radars, the Doppler effect will cause only the compression or stretch of the transmitted signals when the targets are moving. The delay rate for a point target is defined as $\alpha = 2v/(c - v) \simeq 2v/c$, where v is the constant velocity and c is the speed of light. As in [8], the ambiguity function is defined as:

$$\langle \chi(\tau, \alpha, T) \rangle = \int_{t}^{t+T} u_r(t) u_s((1 + \alpha)t - \tau) dt \tag{3}$$

where u_r and u_s are the reference and the signal waveforms, τ is the relative delay time between the reference and the signal waveforms, and T is the correlation interval.

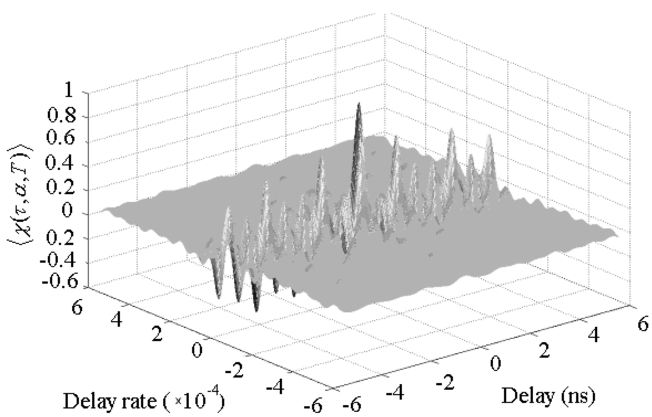

Figure 4. Auto-ambiguity function of the direct chaotic radar employing a microwave Colpitts oscillator.

Fig. 4 shows the auto-ambiguity function of the direct chaotic radar employing the chaotic Colpitts oscillator for waveform generation. The auto-ambiguity function is obtained by auto-correlating the chaotic waveforms shown in Fig. 3(a) with a correlation interval T of 10 us. As can be seen, the auto-ambiguity function shown in Fig. 4 has many sidelobes that make unambiguous detection difficult. Fig. 5 shows the contours of the auto-ambiguity function shown in Fig. 4. It can be seen that many islands are found along the zero delay rate axis in the contour of the ambiguity function.

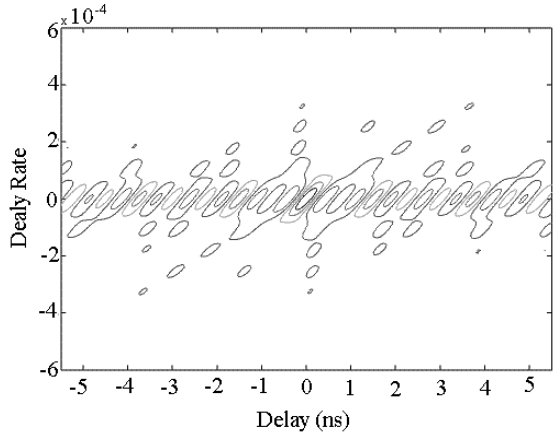

Figure 5. Contours of the auto-ambiguity function of the direct chaotic radar employing a microwave Colpitts oscillator.

The information in Fig. 4 and Fig. 5 is somewhat contrary to one's intuition that the direct chaotic radar should has an ideal thumbtack-like ambiguity function. The reason for this can be interpreted as follows. Theoretically, ideal thumbtack-like auto-ambiguity function can only be obtained if the transmitted signal is strictly random, characterized by a very smooth and flat spectrum with broad bandwidth. From the spectrum of chaotic signal of the Colpitts oscillator shown in Fig. 3(b), it can been seen that the spectrum is not very flat and smooth, on the contrary, it is spiky with a major pulsation frequency at around 1.3 GHz. The 1.3 GHz is generally referred as the fundamental frequency f_0 of the chaotic Colpitts oscillator. If we define the quasi-period of the oscillator as τ_0, the rough relationship between the quasi-period and the fundamental frequency is:

$$\tau_0 = 1/f_0 \tag{4}$$

For $f_0 = 1.3\,\text{GHz}$, it is easily obtained that $\tau_0 = 0.77\,\text{ns}$. From the time-domain chaotic signal shown in Fig. 3(a), similarity between signals with a time distance of τ_0 can be easily found. Here the similarity means that both the signals have positive or negative values when the DC components are removed. When the time distance is about $2\tau_0$, $3\tau_0$, ..., $n\tau_0$, similarity also exists but will decay with the increment of time distance. It is exactly this similarity that results in the sidelobes along the zero delay rate axis in the ambiguity function shown in Fig. 4.

To our knowledge, for most kinds of analog chaotic circuits, there will be pulsation peaks in the continuous but not very smooth and flat spectrum, and this may cause the difficulty of unambiguity detection when using them as the transmitted signal in direct chaotic radar systems. To improve the performance of the auto-ambiguity function in such systems, the spectrum of output chaotic signals should be optimized.

The above analysis can be confirmed by the research in [8], where two kinds of chaotic optical signals generated by OEF and OI schemes were used in the CRADAR system and the ambiguity functions were studied. The chaotic waveform generated by the OI scheme has a rather flat and smooth broadband spectrum and better ambiguity function plot while the chaotic waveform from the OEF scheme has a spiky spectrum and thus it has many sideslobes in the auto-ambiguity function. In addition, in [9, 12], since the chaotic signals generated by digital chaotic circuit have rather smooth and flat spectrum, the corresponding ambiguity functions can have ideal thumbtack-like shapes.

Note that the island in Fig. 5 is skewed along the same direction. The skewness in the direct chaotic radar system is caused by the coupling between the delay time τ and the delay rate α as shown in Equation (3). This is different from the ridges seen in linear FM and stepped frequency radar systems which are due to the coupling between the Doppler frequency shift and the modulation frequency generated. The skewness characteristic results in that the radar has a slightly better unambiguity detection performance in the second and fourth quadrants of the delay-delay rate plane than in the first and the third quadrants.

In modern radar systems, besides considering the unambiguous detection performance, another very important performance parameter is the ECCM capability. Specifically, when anti-collision vehicle-borne radars is considered, it has to be co-existent with a number of radars with the same schemes, or it should be with the property of "multi-user", similar to a large-volume wireless communication system, such as a cellular system. Chaotic signals are noise-like, however, they are not real white noise and not necessarily uncorrelated with each other for a given time interval. Actually, they strongly depend on their complexity and predictability, which are mainly determined by the largest positive Lyapunov exponent.

To study the ECCM capability and the "multi-user" property of the direct chaotic radar employing microwave chaotic Colpitts oscillator, Figs. 6(a) and (b) plot the cross-ambiguity functions of chaotic waveforms with same and slightly different circuit parameters. Fig. 6(a) is obtained by cross-correlating the chaotic signal shown in

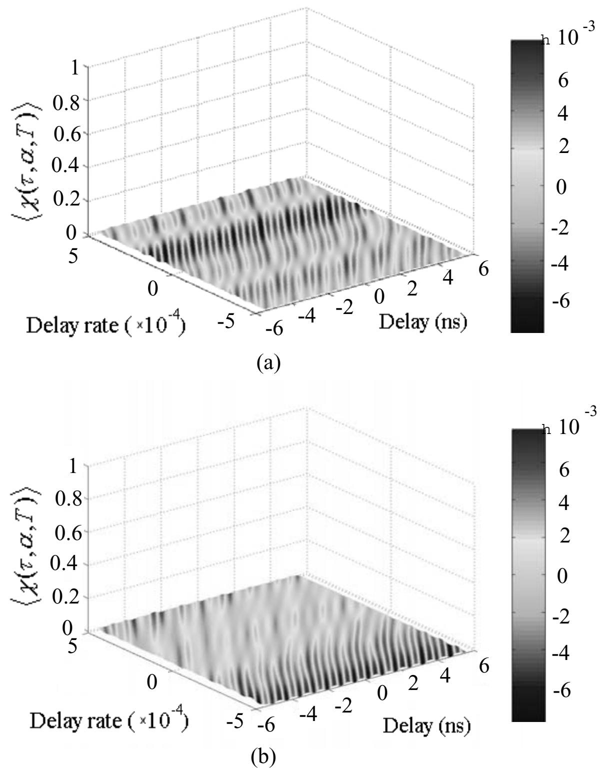

Figure 6. (a) Cross-ambiguity function with same circuit parameters and (b) Cross-ambiguity function with slightly different circuit parameters.

Fig. 3(a) with the same circuit parameters but at a different time with a correlation interval of $T = 10\,\text{us}$. Fig. 6(b) is obtained using the same method but from two Colpitts oscillators with slightly different parameters. One circuit has the same parameters as listed above, but the other changes the value of R ($R = 27\,\text{Ohm}$). The above simulations were conducted to understand the situations that one such direct chaotic radar is interfered with another similar scheme radar system.

From Fig. 6, it can be seen that only very small ripples appear on the surfaces and there are no discernible peaks. The rather excellent cross-ambiguity functions indicate that the chaotic signals generated by circuits with same parameters but at large different time or with slightly different circuit parameters has non-repetitive random features. Since they are not repetitive, they barely correlated with each other. Thus, rather excellent ECCM capability can be achieved in the direct chaotic radar systems. This is similar to the case in laser-based chaotic radar in [8].

4. DISCUSSIONS AND CONCLUSIONS

In this paper, the ambiguity functions of a kind of direct chaotic radar system, which incorporates a microwave chaotic Colpitts oscillator, have been studied. The time-domain, frequency domain chaotic signal and chaotic attractor of the Colpitts oscillator are presented for illustration. The auto-ambiguity function of direct chaotic radar system using chaotic signal from the Colpitts oscillator shows many sidelobes which makes the unambiguous detection difficult. It is because that the spectrum of the chaotic signals is not very flat and smooth, with pulsation peaks in it. From the time-domain view, the chaotic signals with time distance of τ_0, $2\tau_0$, ..., $n\tau_0$ (n is small) have similarities which result in the sidelobes. To improve the unambiguous detection performance, the spectrum of chaotic signal from the microwave Colpitts oscillator has to be optimized, that is, the randomness characteristics of the microwave chaotic signal still need improvement.

The cross-ambiguity functions of the direct radar system have also been investigated to evaluate the ECCM performance and the "multi-user" characteristic when the radar system is used as anti-collision vehicle-borne radar. Rather excellent ECCM capability can be achieved in the direct chaotic radar system with transmitting chaotic signals generated by circuits with same parameters but at different time or with slightly different circuit parameters.

To improve the unambiguous detection of the direct chaotic radar system, the following possible methods are suggested to optimize the spectrum of chaotic signals generated by the microwave Colpitts oscillator. The most direct and intuitive way is to adjust the circuit parameters of the microwave Colpitts oscillator. In the different parts of the bifurcation diagram of Colpitts oscillator, the dynamic characteristics are different, especially when they belong to different types of chaos region, namely the Feigenbaum chaos or the Shil'nikov chaos region [22]. Finding operating condition with more abound dynamic characteristics can help improve the signal spectrum.

The second method is to use analog filter to reshape the spectrum and make it more flat and smooth. The feasibility of this method has been suggested and confirmed by [25]. The third way is to introduce one or more passive devices into suitable positions in the Colpitts oscillator to improve the nonlinear dynamics, as suggested in [26].

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