Progress In Electromagnetics Research, PIER 76, 467–475, 2007

ELECTROMAGNETIC WAVE PROPAGATION THROUGH A SLAB WAVEGUIDE OF UNIAXIALLY ANISOTROPIC DISPERSIVE METAMATERIAL

S.-H. Liu, C.-H. Liang, W. Ding, L. Chen, and W.-T. Pan

National Key Laboratory of Antennas and Microwave Technology
Xidian University
Xi'an 710071, China

Abstract—The characteristics of the guided electromagnetic wave propagation through a slab waveguide of uniaxially anisotropic dispersive metamaterial are investigated. Taking the cold plasma media model with $\omega_{mpz} < \omega_{mp\perp} < \omega_{ep\perp}$ as an example, the mode classification established in terms of the operating angular frequency ω_0 of the slab waveguide. The results indicate that the mode properties are closely dependent on the frequency. When $\omega_{mpz}^2 < \omega_0^2 < \omega_{mp\perp}^2$ there are infinite guided modes. It is also found that when $\omega_{em}^2 < \omega_0^2 < \omega_{mpz}^2$, there may be multiple solutions of the propagating mode with imaginary transverse wave number in a slab waveguide with thickness less than a certain value.

1. INTRODUCTION

Materials with negative effective permittivity (ε) and permeability (μ) in a certain band of frequency, referred to as metamaterial, double negative material, left-handed material (LHM), and so on, have received much attention [1–7]. The first double negative medium was proposed when periodic arrays of split ring resonators (SRRs) and wire strips were combined and the phenomenon of the negative refraction was experimentally verified [4]. One of the most important applications of such a material is to make a perfect lens [5]. The amplification of the evanescent waves to achieve sub-wavelength imaging is related to the guided modes in the slab waveguide. Thereafter, the propagating modes with both real and imaginary transverse wave numbers in the isotropic LHM slab have been investigated extensively [8–14].

However, the LHM that has been made are actually anisotropic, and it may be difficult to prepare an isotropic LHM. It has been shown theoretically that under some conditions the amplitude of the evanescent wave would be amplified exponentially when it is transmitted through a slab of uniaxially anisotropic metamaterial [15]. Thus, the analysis of guided modes in an anisotropic slab waveguide that exhibits the negative refractive index is also of great interest and importance. Recently, Lindell and Ilvonen studied the guided waves on a slab of uniaxial backward-wave medium [16]. Cheng and Cui discussed the guidance conditions for both bulk and surface modes in different waveguide structures of biaxially anisotropic metamaterial under the excitation of a line source [17, 18]. Nevertheless, such an anisotropic LHM must be dispersive [1]. The effect of dispersion on mode properties is complicated and has useful applications in several respects.

In this paper, we discuss the guided modes in a slab waveguide of uniaxially anisotropic dispersive metamaterial. We show that the mode properties are closely dependent on the operating frequency ω_0 of the slab waveguide. Furthermore, the mode cutoffs are analyzed under the assumption that the ratio of slab thickness d to free-space wavelength λ_0, defined as the normalized frequency in [19], is only determined by the slab thickness d.

2. DERIVATION OF THE GUIDANCE CONDITIONS

Consider a slab waveguide with thickness $d = d_2 - d_1$ in free space as shown in Fig. 1.

Region 1 is filled with uniaxially anisotropic dispersive media with

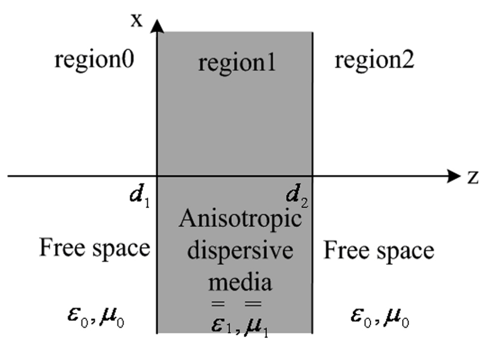

Figure 1. Geometry of an anisotropic dispersive metamaterial slab with thickness $d = d_2 - d_1$.

the permittivity and permeability tensors as follows:

$$\overline{\overline{\varepsilon}}_1 = \begin{bmatrix} \varepsilon_\perp(\omega) & & \\ & \varepsilon_\perp(\omega) & \\ & & \varepsilon_z(\omega) \end{bmatrix}, \quad \overline{\overline{\mu}}_1 = \begin{bmatrix} \mu_\perp(\omega) & & \\ & \mu_\perp(\omega) & \\ & & \mu_z(\omega) \end{bmatrix} \tag{1}$$

Besides, part or all of the components in the $\overline{\overline{\varepsilon}}_1$ and $\overline{\overline{\mu}}_1$ tensors are negative. Under the time-harmonic excitation with $e^{-j\omega t}$, for guided transverse electric (TE) waves, the electric field vectors in the three regions can be written as

$$\vec{E}_0 = \hat{y} E_0^- e^{\alpha_{0z} z} e^{jk_x x} \tag{2}$$

$$\vec{E}_1 = \hat{y}\left(E_1^+ e^{jk_{1z} z} + E_1^- e^{-jk_{1z} z}\right) e^{jk_x x} \tag{3}$$

$$\vec{E}_2 = \hat{y} E_2^+ e^{-\alpha_{0z} z} e^{jk_x x} \tag{4}$$

The magnetic field vectors in source-free regions are governed by Faraday's law, which gives

$$\vec{H}_0 = \frac{1}{j\omega\mu_0}\left(\hat{z} j k_x - \hat{x}\alpha_{0z}\right) E_0^- e^{\alpha_{0z} z} e^{jk_x x} \tag{5}$$

$$\begin{aligned} \vec{H}_1 = & \frac{1}{\omega}\left(\hat{z}\frac{k_x}{\mu_z} - \hat{x}\frac{k_{1z}}{\mu_\perp}\right) E_1^+ e^{jk_{1z} z} e^{jk_x x} \\ & + \frac{1}{\omega}\left(\hat{z}\frac{k_x}{\mu_z} + \hat{x}\frac{k_{1z}}{\mu_\perp}\right) E_1^- e^{-jk_{1z} z} e^{jk_x x} \end{aligned} \tag{6}$$

$$\vec{H}_2 = \frac{1}{j\omega\mu_0}\left(\hat{z} j k_x + \hat{x}\alpha_{0z}\right) E_2^+ e^{-\alpha_{0z} z} e^{jk_x x} \tag{7}$$

where k_x is the wave vector component in the $\hat{x}$ direction, k_{1z} is the transverse wave number in region 1, which can either be real or imaginary. In order to restrict the waves to propagate within the slab, α_{0z} should be positive real. The dispersion relations in regions 0 and 1 are separately

$$(k_x d)^2 - (\alpha_{0z} d)^2 = (k_0 d)^2 \tag{8}$$

$$(k_x d)^2 + \frac{\mu_z}{\mu_\perp}(k_{1z} d)^2 = \frac{\varepsilon_\perp \mu_z}{\varepsilon_0 \mu_0}(k_0 d)^2 \tag{9}$$

where k_0 is the free-space wave number. Eliminating $(k_x d)^2$ from the Eqs. (8) and (9), we obtain

$$(\alpha_{0z} d)^2 + \frac{\mu_z}{\mu_\perp}(k_{1z} d)^2 = \left(\frac{\varepsilon_\perp \mu_z}{\varepsilon_0 \mu_0} - 1\right)(k_0 d)^2 \tag{10}$$

It is seen that the curve shape that Eq. (10) represents is related to both the sign and the value of the tensor components, which are dependent on the operating angular frequency ω_0 of the anisotropic dispersive metamaterial. Plus, provided that d/λ_0 remains constant, Eq. (10) is independent on the frequency [20].

Matching the boundary conditions at $z = d_1$ and $z = d_2$, we obtain the guidance conditions for even and odd bulk modes with real transverse wave number k_{1z} as follows:

$$\alpha_{0z}d = \frac{\mu_0}{\mu_\perp}(k_{1z}d)\tan\left(\frac{k_{1z}d}{2}\right) \tag{11}$$

$$\alpha_{0z}d = -\frac{\mu_0}{\mu_\perp}(k_{1z}d)\cot\left(\frac{k_{1z}d}{2}\right) \tag{12}$$

If the transverse wave number inside a slab of anisotropic media is imaginary ($k_{1z} = j\alpha_{1z}$, α_{1z} is positive real), the guidance conditions for even and odd surface modes are separately

$$\alpha_{0z}d = -\frac{\mu_0}{\mu_\perp}(\alpha_{1z}d)\tanh\left(\frac{\alpha_{1z}d}{2}\right) \tag{13}$$

$$\alpha_{0z}d = -\frac{\mu_0}{\mu_\perp}(\alpha_{1z}d)\coth\left(\frac{\alpha_{1z}d}{2}\right) \tag{14}$$

The relation between $\alpha_{0z}d$ and $\alpha_{1z}d$ is given by

$$(\alpha_{0z}d)^2 - \frac{\mu_z}{\mu_\perp}(\alpha_{1z}d)^2 = \left(\frac{\varepsilon_\perp\mu_z}{\varepsilon_0\mu_0} - 1\right)(k_0d)^2 \tag{15}$$

3. RESULTS AND DISCUSSION

A graphical approach is useful in determining the even mode solutions for guided TE waves, whose properties are only related to μ_z, $\mu_\perp$ and $\varepsilon_\perp$. For the convenience, we consider the cold plasma media model and the losses are assumed negligible. The three component parameters in Eq. (1) have the following form [5, 21, 22]:

$$\varepsilon_\perp(\omega) = \varepsilon_0\left(1 - \omega_{ep\perp}^2/\omega^2\right) \tag{16a}$$

$$\mu_\perp(\omega) = \mu_0\left(1 - \omega_{mp\perp}^2/\omega^2\right) \tag{16b}$$

$$\mu_z(\omega) = \mu_0\left(1 - \omega_{mpz}^2/\omega^2\right) \tag{16c}$$

Here, ω_{ep} and ω_{mp} are electric and magnetic plasma frequencies. Since the metamaterial is uniaxially anisotropic, we have $\omega_{mp\perp} \neq \omega_{mpz}$.

Without loss of generality, let the parameters satisfy the following relation:

$$\omega_{mpz} < \omega_{mp\perp} < \omega_{ep\perp} \tag{17}$$

In this paper, we choose $\omega_{ep\perp}/2\pi = 2\sqrt{3}\,\text{GHz}$, $\omega_{mp\perp}/2\pi = \sqrt{6}\,\text{GHz}$, $\omega_{mpz}/2\pi = 2\,\text{GHz}$ and meanwhile define $t_1 = \varepsilon_\perp\mu_z/\varepsilon_0\mu_0 - 1$, $t_2 = \mu_z/\mu_\perp$ for simplicity in later analysis. Different combinations of their signs will affect the mode properties greatly. In unixially anisotropic dispersive media, the signs are determined by the operating angular frequency of the slab waveguide. In terms of the ranges of ω_0, they are classified into the following three cases. The odd mode properties can be obtained similarly and will not be analyzed in this section.

Case I: $\omega_{mpz}^2 < \omega_0^2 < \omega_{mp\perp}^2$. In such a case, $t_1 < 0$, $t_2 < 0$ can be readily obtained. The curves that Eq. (10) stands for are hyperbola lines with the real axis x. The values of $\alpha_{0z}d$ and $k_{1z}d$ given by the intersections of the hyperbola lines with intercept $(k_0 d)\sqrt{t_1/t_2}$ and tanlike lines derived from Eq. (11) are the solutions of guided modes, which represent the possible propagating TE even modes, as shown in Fig. 2(a). Here, we assume that the intercepts are only determined by the slab thickness d, since the anisotropic metamaterial is dispersive. The corresponding $k_x d$ can be determined in turn from either Eq. (8) or Eq. (9). The calculated values of propagation constant k_x normalized by k_0 are seen in Fig. 2(b) as a function of the normalized frequency d/λ_0 where λ_0 is a free-space wavelength. The lowest even mode is the TE_2 mode. At any fixed normalized frequency point d/λ_0 there are

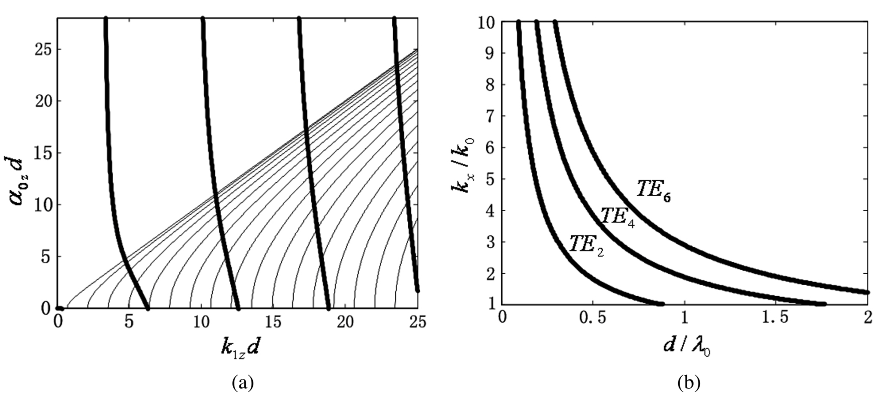

Figure 2. Even bulk modes determined by Eqs. (10) and (11) when $\omega_0/2\pi = \sqrt{5}\,\text{GHz}$, (a) Curves of $\alpha_{0z}d$ versus $k_{1z}d$, (b) Curves of k_x/k_0 versus d/λ_0.

infinite values of k_x. That is, we will have infinite propagation modes in the slab waveguide, which is obviously different from the case that only finite modes can propagate simultaneously in the conventional slab waveguide. Besides, high-normalized frequency cutoffs exist for all the even modes. For imaginary transverse wave numbers, there are no values of $\alpha_{0z}d$ and $\alpha_{1z}d$ satisfying Eq. (15). The curves describing the dispersion relation do not exist. Thus, the propagating mode with the imaginary transverse wave number does not exist.

Case II: $\omega_0^2 < \omega_{em}^2$, where $\omega_{em}^2 = \omega_{mpz}^2 \cdot [\omega_{ep\perp}^2/(\omega_{ep\perp}^2 + \omega_{mpz}^2)]$. In this case, $t_1 > 0$, $t_2 > 0$. Eq. (10) becomes ellipse lines. The curves of $\alpha_{0z}d$ versus $k_{1z}d$ given are shown in Fig. 3(a). All the even modes exhibit a similar behavior of low-normalized frequency cutoffs. The graphs of k_x/k_0 as a function of d/λ_0 can further be obtained, as shown in Fig. 3(b). From the dispersion relation described by Eq. (9) we know that the normalized propagation constant k_x/k_0 approaches $\sqrt{\varepsilon_\perp\mu_z/\varepsilon_0\mu_0}$ as the normalized frequency d/λ_0increases. For imaginary transverse wave numbers, substituting Eq. (13) in Eq. (15) and defining $F = t_1 \cdot (k_0 d)^2$ we find

$$F = \left[(\mu_0^2/\mu_\perp\mu_z)\tanh^2(\alpha_{1z}d/2) - 1\right] \cdot t_2 \cdot (\alpha_{1z}d)^2 \tag{18}$$

The sign of F is determined only by t_1. This equation describes a family of curves for different values of $\mu_0^2/\mu_\perp\mu_z$, which means ω_0^2 varies from 0 to ω_{em}^2. The F versus $\alpha_{1z}d$ diagram is plotted in Fig. 4(a). It's seen that in the region where $F > 0$, which corresponds to the anisotropic slab with the component parameters satisfying $t_1 > 0$,

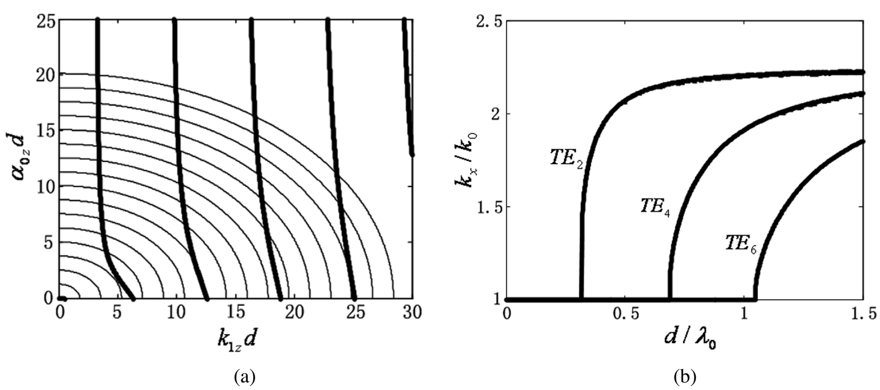

Figure 3. Even bulk modes determined by Eqs. (10) and (11) when $\omega_0/2\pi = \sqrt{2}$ GHz, (a) Curves of $\alpha_{0z}d$ versus $k_{1z}d$, (b) Curves of k_x/k_0 versus d/λ_0.

the values of $\alpha_{1z}d$ characterize the propagating mode with imaginary transverse wave number. The mode properties are related to the values of the component parameters. Thus for $\omega_{z\perp}^2 < \omega_0^2 < \omega_{em}^2$, where $\omega_{z\perp}^2 = \omega_{mpz}^2 \cdot [\omega_{mp\perp}^2/(\omega_{mp\perp}^2 + \omega_{mpz}^2)]$, we have $\mu_0^2/\mu_\perp\mu_z > 1$. The propagating mode with the imaginary transverse wave number exists and has no cutoff. For $\omega_0^2 \leq \omega_{z\perp}^2$, we have $\mu_0^2/\mu_\perp\mu_z \leq 1$. The guided mode does not exist in this case.

Case III: $\omega_{em}^2 < \omega_0^2 < \omega_{mpz}^2$ or $\omega_0^2 > \omega_{mp\perp}^2$. In such a case, $t_1 < 0$, $t_2 > 0$. The values of $\alpha_{0z}d$and $k_{1z}d$satisfying Eq. (10) do not exist. That is, the guided modes do not exist. For imaginary transverse wave numbers, when $\omega_{em}^2 < \omega_0^2 < \omega_{mpz}^2$, $\mu_0^2/\mu_\perp\mu_z > 1$ and $\mu_\perp < 0$are obtained. Fig. 4(b) shows that in the region where $F < 0$, there are multiple values of $\alpha_{1z}d$. The propagating mode has the high-normalized frequency cutoff. On the other hand, when $\omega_0^2 > \omega_{mp\perp}^2$, we have $\mu_\perp > 0$. The solutions of the guided modes obtained are not coincident with the practical physical performance, so they should be excluded.

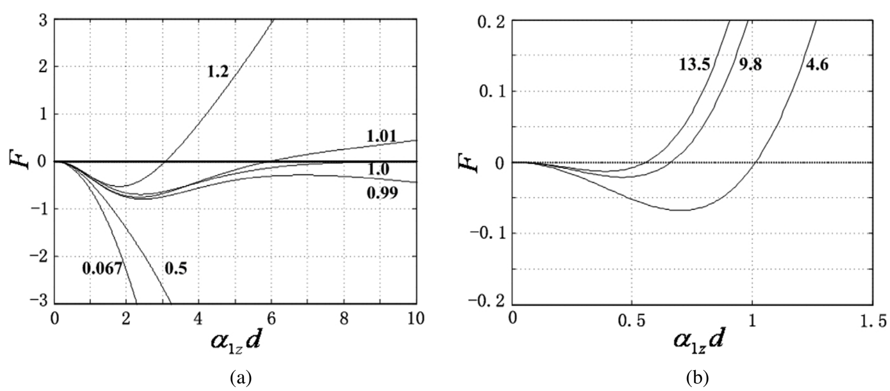

Figure 4. Even surface modes for different values of $\mu_0^2/\mu_\perp\mu_z$ determined by Eq. (18) in terms of F versus $\alpha_{1z}d$, (a) $\omega_0/2\pi < \sqrt{3}\,\text{GHz}$, (b) $\sqrt{3}\,\text{GHz} < \omega_0/2\pi < 2\,\text{GHz}$.

4. CONCLUSION

In conclusion, we have discussed the effect of the operating angular frequency ω_0 and the slab thickness d on the mode properties in detail. When the assumption of the relation in cold plasma media model $\omega_{mpz} < \omega_{mp\perp} < \omega_{ep\perp}$ changes, the mode properties can also be obtained by using the analysis method in the present study. The investigation will be helpful for the choice of the operating frequency and the thickness of the metamaterial slab waveguide in practical application.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China under contract No. 60601028.

REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics USPEKHI*, Vol. 10, No. 4, 509–514, 1968.
2. Ding, W., L. Chen, and C. H. Liang, "Characteristics of electromagnetic wave propagation in biaxially anisotropic left-handed materials," *Progress In Electromagnetics Research*, PIER 70, 37–52, 2007.
3. Chen, L., W. Ding, X. J. Dang, and C. H. Liang, "Counter-propagating energy-flows in nonlinear left-handed metamaterials," *Progress In Electromagnetics Research*, PIER 70, 257–267, 2007.
4. Shelby, R. A., D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, 77–79, April 2001.
5. Pendry, J. B., "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, Oct. 2000.
6. Xu, W., L. W. Li, H. Y. Yao, and T. S. Yeo, "Left-handed material effects on waves modes and resonant frequencies: filled waveguide structures and substrate-loaded patch antennas," *J. of Electromagn. Waves and Appl.*, Vol. 19, 2033–2047, 2005.
7. Grzegorczyk, T. M. and J. A. Kong, "Review of left-handed metamaterials: evolution from theoretical and numerical studies to potential applications," *J. of Electromagn. Waves and Appl.*, Vol. 20, No. 14, 2053–2064, 2006.
8. Ruppin, R., "Surface polaritons of a left-handed medium," *Phys. Lett. A*, Vol. 277, No. 1, 61–64, 2000.
9. Michael, W. F., P. J. Bevelacqua, and J. B. Schneider, "Effects of surface waves on the behavior of perfect lenses," *Phys. Rev. B*, Vol. 66, No. 3, 0351131, July 2002.

10. Darmanyan, S. A., M. Neviere, and A. A. Zakhidov, "Surface modes at the interface of conventional and left-handed media," *Optics Communications*, Vol. 225, No. 4, 233–240, Oct. 2003.
11. Cory, H. and A. Barger, "Surface-wave propagation along a metamaterial slab," *Microwave and Optical Technology Letters*, Vol. 38, No. 5, 392–395, Sep. 2003.
12. Baccarelli, P., P. Burghignoli, F. Frezza, A. Galli, P. Lampariello, G. Lovat, and S. Paulotto, "Fundamental modal properties of surface waves on metamaterial grounded slabs," *IEEE Trans. Microwave Theory Tech.*, Vol. 53, No. 4, 1431–1442, April 2005.
13. Wu, B.-I., T. M. Grzegorczyk, Y. Zhang, and J. A. Kong, "Guided modes with imaginary transverse wave number in a slab waveguide with negative permittivity and permeability," *Journal of Applied Physics*, Vol. 93, No. 11, 9386–9388, June 2003.
14. Lu, J., B.-I. Wu, and J. A. Kong, "guided modes with a linearly varying transverse field inside a left-handed dielectric slab," *J. of Electromagn. Waves and Appl.*, Vol. 20, No. 5, 689–697, 2006.
15. Hu, L. and S. T. Chui, "Characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed materials," *Phys. Rev. B*, Vol. 66, 085108, 2002.
16. Lindell, I. V. and S. Ilvonen, "Waves in a slab of uniaxial BW medium," *J. Electromagn. Waves Appl.*, Vol. 16, No. 3, 303–318, 2002.
17. Cheng, Q. and T. J. Cui, "Guided modes in a planar anisotropic biaxial slab with partially negative permittivity and permeability," *Applied Physics Letters*, Vol. 87, No. 17, 174102, Oct. 2005.
18. Cheng, Q. and T. J. Cui, "Infinite guided modes in a planar waveguide with a biaxially anisotropic metamaterial," *J. Opt. Soc. Am. A*, Vol. 23, No. 8, 1989–1993, Aug. 2006.
19. Satomura, Y., M. Matsuhara, and N. Kumagai, "Analysis of electromagnetic-wave modes in anisotropic slab waveguide," *IEEE Trans. Microwave Theory Tech.*, Vol. 22, No. 2, 86–92, Feb. 1974.
20. Collin, R. E., *Field Theory of Guided Waves*, McGraw-Hill, New York, 1960.
21. Ziolkowski, R. W., "Superluminal transmission of information through an electromagnetic metamaterial," *Phys. Rev. E*, Vol. 63, No. 4, 046604, 2001.
22. Cui, T. J. and J. A. Kong, "Time-domain electromagnetic energy in a frequency-dispersive left-handed medium," *Phys. Rev. B*, Vol. 70, 205106, 2004.