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HIGH FREQUENCY EXPRESSIONS FOR THE FIELD IN THE CAUSTIC REGION OF A CYLINDRICAL REFLECTOR PLACED IN CHIRAL MEDIUM

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Abstract—High frequency field expressions for a two dimensional reflector are derived. The reflector is placed in a homogenous and reciprocal chiral medium. Since geometrical optics fails at caustics so Maslovs method has been used to find the field expressions which are also valid around caustics. Examples of parabolic and circular reflectors have been considered.

1. INTRODUCTION

Asymptotic ray theory (ART) or the geometrical optics approximation is widely used to study various kinds of problems in the areas of electromagnetics, acoustic waves, seismic waves, etc. [1–3]. It is also well known that the geometrical optics fails in the vicinity of caustic. So, in order to study the field behavior near caustic, other approach is required. Maslov proposed a method to predict the field in the caustic region [4]. Maslov's method combines the simplicity of asymptotic ray theory and the generality of the Fourier transform method. This is achieved by representing the geometrical optics fields in terms of mixed coordinates consisting of space coordinates and wave vector coordinates. That is by representing the field in terms of six

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coordinates. It may be noted that information of ray trajectories is included in both space coordinates $\mathbf{R} = (x, y, z)$ and wave vector coordinates $\mathbf{P} = (p_x, p_y, p_z)$.

Solving the Hamiltonian equations under the prescribed initial conditions, one can construct the geometrical optics field in space R, which is valid except in the vicinity of caustic. Near the caustic, the expression for the geometrical optics field in spatial space is rewritten in mixed domain. The expression in mixed domain is related to the original domain R through the asymptotic Fourier transform.

Focusing systems have been analyzed by many authors [5–21]. A number of research problems have been analyzed using the Maslov's method [10–21]. In present discussion, our interest is to study GO fields associated with a cylindrical reflector placed in a lossless, homogeneous and reciprocal chiral medium. Number of caustics, locations of the caustics, and finite field around the caustics is of interest. In Section 2, we have discussed geometrical optics (GO) method and Maslov's method in ordinary medium. Extension of GO to reciprocal, homogenous and lossless medium is discussed in Section 3. In section 4 we have determined GO field around a cylindrical reflector placed in chiral medium, and have determined finite field around around caustics using Maslov's Method. In Sections 4.1 and 4.2 we have considered the parabolic and circular cylindrical reflectors as examples. Results and plots around caustics are discussed in Section 5. Concluding remarks are presented in Section 6.

2. GEOMETRICAL OPTICS AND MASLOV'S METHOD IN ORDINARY MEDIUM

Consider the scalar wave equation

$$(\nabla^2 + k^2 n^2) u(r) = 0 \tag{1}$$

where $r = (x, z)$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$, $k = \omega\sqrt{\mu\epsilon}$ is wavenumber, n is index of refraction of the medium, which in our problem is a constant. Medium is homogeneous and isotropic. Solution of (1) may be assumed in the form of asymptotic series in powers of $(jk)^{-1}$ as

$$u(r) = \sum_{m=0}^{\infty} \frac{A^m(r)}{(jk)^m} \exp(-jk\Phi) \tag{2}$$

Substituting (2) in (1) and equating coefficients of k^2 we get eikonal equation as [22]

$$(\nabla\Phi)^2 - n^2 = 0 \tag{3}$$

Equating co-efficient of k we get transport equation

$$2\nabla A.\nabla\Phi + A\nabla^2\Phi = 0 \tag{4}$$

where we have retained only A^0 and has denoted it with A. Since we have assumed k to be large so we have neglected higher order terms.

We define wave vector $\mathbf{p} = \nabla\Phi$ and Hamiltonian $H = (\mathbf{p}.\mathbf{p} - n^2)/2$, so the eikonal equation becomes $H = 0$. Equation (3) can be solved by method of characteristic, as follow

$$\frac{dx}{d\tau} = p_x \tag{5a}$$

$$\frac{dz}{d\tau} = p_z \tag{5b}$$

$$\frac{dp_x}{d\tau} = \frac{\partial H}{\partial x} = 0 \tag{5c}$$

$$\frac{dp_z}{d\tau} = \frac{\partial H}{\partial z} = 0 \tag{5d}$$

where τ is parameter along the ray. Solution of Hamilton's equations (5) is

$$x = \xi + p_x\tau \tag{6a}$$

$$z = \zeta + p_z\tau \tag{6b}$$

$$p_x = p_{x0} \tag{6c}$$

$$p_z = p_{z0} \tag{6d}$$

where (ξ, ζ) and (p_{x0}, p_{z0}) are initial values of (x, z) and (p_x, p_z) respectively. The phase function is given by

$$\Phi = \Phi_0(\xi) + \int_0^\tau n^2 d\tau = \Phi_0(\xi) + n^2\tau \tag{7}$$

where $\Phi_0(\xi)$ is initial phase at (ξ, ζ). The solution of transport equation is [22]

$$A(r) = A_0(\xi)J^{-1/2} \tag{8}$$

where $A_0(\xi)$ is initial field and $J = \frac{D(\tau)}{D(0)}$, where $D(\tau) = \partial(x, z)/\partial(\xi, \tau)$ is the Jacobian of transformation from ray co-ordinates (ξ, τ) to space co-ordinates (x, z). Now we can write approximate solution of (1), known as GO solution, as

$$u(r) = u_0(\xi)J^{-1/2}\exp(-jkn^2\tau) \tag{9}$$

where $u_0(\xi) = A_0(\xi)\exp\{-jk\Phi_0(\xi)\}$. Equation (9) predicts infinite amplitudes at points where $J = 0$ which is not realistic. This situation can be avoided and finite field around caustic may be found as follow using Maslov's Method [4, 11].

The solution may be assumed in the form of the Fourier transform and is given as

$$u(r) = \int_{-\infty}^{\infty} T(x, p_z)\exp[-jk\Psi(x, p_z) + p_z z]dp_z \tag{10}$$

Equation (10) can be solved using stationary phase method in the region much away from the caustic. The resulting solution is GO solution so it should agree with GO solution given by (9). Identifying the solution with (9), explicit expression for $\Psi(x, p_z)$ and $T(x, p_z)$ can be obtained. Although solution agrees at stationary points of p_z but we assume that they agree for all values of p_z. The field near caustics is given as

$$u(r) = \sqrt{\frac{k}{j2\pi}}\int_{-\infty}^{\infty} A_0(\xi)\left[J\frac{\partial p_z}{\partial z}\right]^{-1/2}\exp[-jk\{\Phi_0+n^2\tau-z_0p_z+zp_z\}]dp_z \tag{11}$$

where z_0 is the value of z at stationary point of p_z, given by (6a). The expression $J\frac{\partial p_z}{\partial z}$ can be calculated more simply as

$$J\frac{\partial p_z}{\partial z} = \frac{1}{D(0)}\frac{\partial(x, p_z)}{\partial(\xi, \tau)} \tag{12}$$

3. GEOMETRICAL OPTICS IN CHIRAL MEDIUM

Chiral medium supports left circularly polarized (LCP) and right circularly polarized (RCP) modes. There are more than one ways to define constitutive relations of chiral medium and we will use DBF constitutive relations as follow [23]

$$\mathbf{D} = \epsilon(\mathbf{E} + \beta\nabla\times\mathbf{E}) \tag{13a}$$
$$\mathbf{B} = \mu(\mathbf{H} + \beta\nabla\times\mathbf{H}) \tag{13b}$$

where ϵ, μ and β is permittivity, permeability and chirality constants of the medium. ϵ and μ has usual dimensions and β has the dimension of length. Using these constitutive relations, solution of Maxwell's equations results in coupled differential equations. If we use the following transformation [23], we can have uncoupled differential equations

$$\mathbf{E} = \mathbf{Q}_L - j\sqrt{\frac{\mu}{\epsilon}}\mathbf{Q}_R \tag{14a}$$

$$\mathbf{H} = \mathbf{Q}_R - j\sqrt{\frac{\epsilon}{\mu}}\mathbf{Q}_L \tag{14b}$$

where $\mathbf{Q}_L$ and $\mathbf{Q}_R$, represents LCP and RCP waves, respectively, and satisfy the following equations

$$\left(\nabla^2 + k^2 n_1^2\right)\mathbf{Q}_L = 0 \tag{15a}$$

$$\left(\nabla^2 + k^2 n_2^2\right)\mathbf{Q}_R = 0 \tag{15b}$$

where $n_1 = \frac{1}{1-k\beta}$ and $n_2 = \frac{1}{1+k\beta}$ are equivalent refractive indices of the medium seen by LCP and RCP waves respectively, and $k = \omega\sqrt{\mu\epsilon}$.

Equation (15) shows that fields in chiral medium may be treated in a manner similar to ordinary medium if we use the transformation (14). So GO solution for chiral medium can be obtained in a manner similar to ordinary medium as is discussed in Section 2. Now we have two types of polarizations so, we have to solve for both waves independently. The total field will be the superposition of two contributions.

4. GEOMETRIC OPTICS FIELD OF A CYLINDRICAL REFLECTOR PLACED IN CHIRAL MEDIUM

Reflection of plane waves traveling in chiral medium has been considered by [24]. We recapitulate it here to introduce our notations and to present it in a form suitable for our present work. Consider reflection of RCP wave from perfect electric conducting (PEC) plane lying along xy-plane as shown in Figure 1. An RCP wave traveling with phase velocity $\frac{\omega}{kn_2}$ and amplitude unity, is incident on the plane making angle ψ with z-axis. Reflected wave is composed of two waves with opposite handedness. An LCP wave is reflected making angle

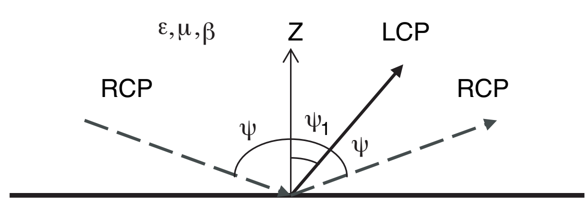

Figure 1. Reflection of RCP waves from PEC plane.

$\psi_1 = \sin^{-1}(\frac{n_2}{n_1}\sin\psi)$ and with amplitude of $\frac{2\cos\psi}{\cos\psi+\cos\psi_1}$. The phase velocity of LCP wave is $\frac{\omega}{kn_1}$. An RCP wave is reflected making angle ψ and amplitude $\frac{\cos\psi-\cos\psi_1}{\cos\psi+\cos\psi_1}$. If we take $\beta > 0$, then $n_1 > n_2$ and $\psi_1 < \psi$ i.e. LCP wave bends towards normal, this is because it is slower than RCP. If $\beta < 0$, the situation reverses, and in the rest of paper we will consider only $\beta > 0$. Two points are to be noted here, firstly, when $\beta = 0$, then $n_1 = n_2 = 1$ and $\psi_1 = \psi$, so RCP wave diminishes and LCP wave has amplitude unity. This is the case of reflection of RCP wave from PEC plane in simple isotropic medium. Secondly, greater the value of β, the greater is the difference between ψ and ψ_1.

Similarly when an LCP wave with unit amplitude is incident on PEC plane making angle ψ with z-axis, as shown in Figure 2, we get two reflected waves of opposite handedness. An RCP wave is reflected at angle $\psi_2 = \sin^{-1}(\frac{n_1}{n_2}\sin\psi)$ with amplitude $\frac{2\cos\psi}{\cos\psi+\cos\psi_2}$, and an LCP wave at angle ψ and amplitude $\frac{\cos\psi-\cos\psi_2}{\cos\psi+\cos\psi_2}$. Since $\psi_2 > \psi$, the RCP wave bends away from z-axis because it is faster than LCP. Again we can see that, firstly, for $\beta = 0$, we get only an RCP wave with amplitude unity and $\psi_2 = \psi$. Secondly as β increases, the difference between ψ_2 and ψ increases.

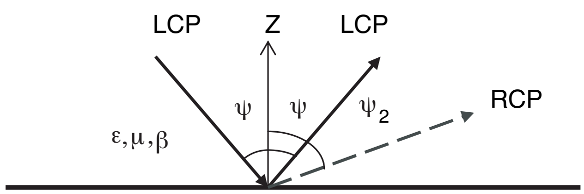

Figure 2. Reflection of LCP waves from PEC plane.

When both LCP and RCP hit PEC plane boundary there are four reflected waves. We will calculate the GO field for these four waves separately. To find the reflected wave vectors of these waves we will consider cylindrical boundary as a plane, locally. Henceforth, we will designate these waves as RR, RL, LL and LR. Quantities designated as RR and RL are RCP and LCP reflected wave components, respectively, when RCP is incident. Quantities designated as LL and LR are LCP and RCP reflected waves, respectively, when LCP is incident wave.

Consider a two dimensional conducting curved surface which is uniform in y-direction and is defined by $\zeta = g(\xi)$, as shown in Figure 3. The reflector is placed in homogenous and reciprocal chiral medium defined by constitutive relations (1). Let two plane waves are traveling

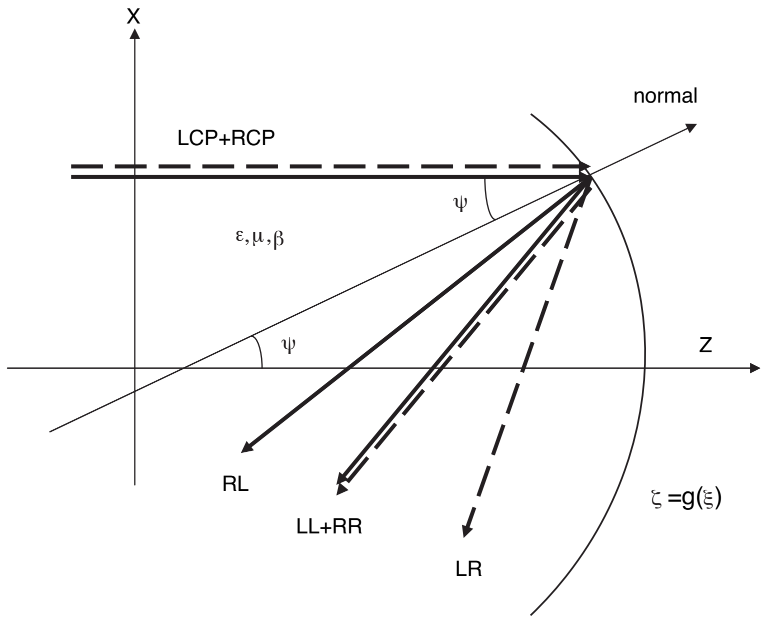

Figure 3. Cylindrical reflector in chiral media.

in chiral medium along z-axis of opposite handedness. Solutions of (15) for plane waves traveling along positive z-axis can be found as

$$\mathbf{Q}_L = (\mathbf{a}_x + j\mathbf{a}_y)\exp(-jkn_1 z)$$
$$\mathbf{Q}_R = (\mathbf{a}_x - j\mathbf{a}_y)\exp(-jkn_2 z)$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are unit vectors along x and y axis respectively. We suppress the polarization, and henceforth it will remain suppressed, and take the incident fields to be of unit amplitudes as follow

$$Q_L = \exp(-jkn_1 z) \tag{16a}$$
$$Q_R = \exp(-jkn_2 z) \tag{16b}$$

Consider the case of normal incidence such that these waves are incident at angle ψ with surface normal $\mathbf{a}_n$ of the surface of the reflector. The surface normal is given by

$$\mathbf{a}_n = \sin\psi\mathbf{a}_x + \cos\psi\mathbf{a}_z \tag{17}$$

where ψ is the angle made by the normal with z-axis, and is given by

$$\sin\psi = -\frac{g'(\xi)}{\sqrt{1 + g'^2(\xi)}} \tag{18a}$$

$$\cos\psi = \frac{1}{\sqrt{1+g'^2(\xi)}} \tag{18b}$$

Prime represents the derivative. The reflected wave vectors of LL, RR, RL and LR waves are given as

$$\mathbf{p}_{LL} = -n_1 \sin 2\psi \mathbf{a}_x - n_1 \cos 2\psi \mathbf{a}_z \tag{19a}$$
$$\mathbf{p}_{RR} = -n_2 \sin 2\psi \mathbf{a}_x - n_2 \cos 2\psi \mathbf{a}_z \tag{19b}$$
$$\mathbf{p}_{RL} = -n_1 \sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\mathbf{a}_x - n_1 \cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\mathbf{a}_z \tag{19c}$$
$$\mathbf{p}_{LR} = -n_2 \sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\mathbf{a}_x - n_2 \cos\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\mathbf{a}_z \tag{19d}$$

The initial fields and initial phases on the surface of the reflector for these rays are given as

$$A_{0LL} = \frac{\cos\psi - \cos\psi_2}{\cos\psi + \cos\psi_2} \tag{20a}$$
$$A_{0RR} = \frac{\cos\psi - \cos\psi_1}{\cos\psi + \cos\psi_1} \tag{20b}$$
$$A_{0RL} = \frac{2\cos\psi}{\cos\psi + \cos\psi_1} \tag{20c}$$
$$A_{0LR} = \frac{2\cos\psi}{\cos\psi + \cos\psi_2} \tag{20d}$$
$$\Phi_{0LL} = n_1\zeta \tag{20e}$$
$$\Phi_{0RR} = n_2\zeta \tag{20f}$$
$$\Phi_{0RL} = n_2\zeta \tag{20g}$$
$$\Phi_{0LR} = n_1\zeta \tag{20h}$$

The Jacobian of transformations for these rays are given as

$$J_{LL} = 1 - 2n_1\tau\frac{\partial\psi}{\partial\xi} \tag{21a}$$
$$J_{RR} = 1 - 2n_2\tau\frac{\partial\psi}{\partial\xi} \tag{21b}$$
$$J_{RL} = 1 - \frac{\partial\psi}{\partial\xi}\frac{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}{\sqrt{n_1^2 - n_2^2\sin^2\psi}}$$

$$\times \frac{n_1 \tau}{\cos\left\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\right\} + \tan\psi \sin\left\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\right\}} \tag{21c}$$

$$J_{LR} = 1 - \frac{\partial\psi}{\partial\xi} \frac{\sqrt{n_2^2 - n_1^2 \sin^2\psi} + n_1\cos\psi}{\sqrt{n_2^2 - n_1^2\sin^2\psi}} \times \frac{n_2 \tau}{\cos\left\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\right\} + \tan\psi \sin\left\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\right\}} \tag{21d}$$

and the GO field for each ray can now be written as

$$u_{LL}(r) = A_{0LL}(\xi)\exp\left\{-jk(\Phi_{0LL}(\xi) + n_1^2\tau)\right\} J_{LL}^{-1/2} \tag{22a}$$

$$u_{RR}(r) = A_{0RR}(\xi)\exp\left\{-jk(\Phi_{0RR}(\xi) + n_2^2\tau)\right\} J_{RR}^{-1/2} \tag{22b}$$

$$u_{RL}(r) = A_{0RL}(\xi)\exp\left\{-jk(\Phi_{0RL}(\xi) + n_1^2\tau)\right\} J_{RL}^{-1/2} \tag{22c}$$

$$u_{LR}(r) = A_{0LR}(\xi)\exp\left\{-jk(\Phi_{0LR}(\xi) + n_2^2\tau)\right\} J_{LR}^{-1/2} \tag{22d}$$

The equations of caustics where Jacobian becomes zero are given by

$$n_1\tau = \frac{1}{2}\frac{\partial\xi}{\partial\psi} \tag{23a}$$

$$n_2\tau = \frac{1}{2}\frac{\partial\xi}{\partial\psi} \tag{23b}$$

$$n_1\tau = \left\{\cos\left\{\psi + \sin^{-1}\left(\frac{n_2}{n_1}\sin\psi\right)\right\} + \tan\psi\sin\left\{\psi + \sin^{-1}\left(\frac{n_2}{n_1}\sin\psi\right)\right\}\right\} \times \frac{\sqrt{n_1^2 - n_2^2\sin^2\psi}}{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}\frac{\partial\xi}{\partial\psi} \tag{23c}$$

$$n_2\tau = \left\{\cos\left\{\psi + \sin^{-1}\left(\frac{n_1}{n_2}\sin\psi\right)\right\} + \tan\psi\sin\left\{\psi + \sin^{-1}\left(\frac{n_1}{n_2}\sin\psi\right)\right\}\right\} \times \frac{\sqrt{n_2^2 - n_1^2\sin^2\psi}}{\sqrt{n_2^2 - n_1^2\sin^2\psi} + n_1\cos\psi}\frac{\partial\xi}{\partial\psi} \tag{23d}$$

Since GO becomes infinite at caustics, so we find approximate field at caustics by Maslov's Method. To calculate field at caustic by (11) we need expression (12) which are found below

$$J_{LL}\frac{\partial p_{zLL}}{\partial z} = 2n_1\sin^2 2\psi\frac{\partial\psi}{\partial\xi} \tag{24a}$$

$$J_{RR}\frac{\partial p_{zRR}}{\partial z} = 2n_2\sin^2 2\psi\frac{\partial\psi}{\partial\xi} \tag{24b}$$

$$J_{RL}\frac{\partial p_{zRL}}{\partial z} = \frac{n_1\sin^2\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \times\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}{\sqrt{n_1^2-n_2^2\sin^2\psi}}\frac{\partial\psi}{\partial\xi} \tag{24c}$$

$$J_{LR}\frac{\partial p_{zLR}}{\partial z} = \frac{n_2\sin^2\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \times\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{\sqrt{n_2^2-n_1^2\sin^2\psi}}\frac{\partial\psi}{\partial\xi} \tag{24d}$$

and the finite fields around the caustics, using (11) and (24) are given by

$$u_{LL}(r) = \sqrt{\frac{k}{j2\pi}}\int_{-\infty}^{\infty}A_{0LL}(\xi)\left[J_{LL}\frac{\partial p_{zLL}}{\partial z}\right]^{-1/2} \times\exp\left[-jk\left\{\Phi_{0LL}+n_1^2\tau-z_{0LL}p_{zLL}+zp_{zLL}\right\}\right]dp_{zLL} \tag{25a}$$

$$u_{RR}(r) = \sqrt{\frac{k}{j2\pi}}\int_{-\infty}^{\infty}A_{0RR}(\xi)\left[J_{RR}\frac{\partial p_{zRR}}{\partial z}\right]^{-1/2} \times\exp\left[-jk\left\{\Phi_{0RR}+n_2^2\tau-z_{0RR}p_{zRR}+zp_{zRR}\right\}\right]dp_{zRR} \tag{25b}$$

$$u_{RL}(r) = \sqrt{\frac{k}{j2\pi}}\int_{-\infty}^{\infty}A_{0RL}(\xi)\left[J_{RL}\frac{\partial p_{zRL}}{\partial z}\right]^{-1/2} \times\exp\left[-jk\left\{\Phi_{0RL}+n_1^2\tau-z_{0RL}p_{zRL}+zp_{zRL}\right\}\right]dp_{zRL} \tag{25c}$$

$$u_{LR}(r) = \sqrt{\frac{k}{j2\pi}}\int_{-\infty}^{\infty}A_{0LR}(\xi)\left[J_{LR}\frac{\partial p_{zLR}}{\partial z}\right]^{-1/2} \times\exp\left[-jk\left\{\Phi_{0LR}+n_2^2\tau-z_{0LR}p_{zLR}+zp_{zLR}\right\}\right]dp_{zLR} \tag{25d}$$

4.1. Parabolic Cylindrical Reflector

Consider the parabolic reflector defined by $\zeta = f - \xi^2/4f$ where f is focal length of parabola. Using (18)

$$\cos\psi = \frac{2f}{\sqrt{\xi^2+4f^2}} \tag{26a}$$

$$\sin\psi = \frac{\xi}{\sqrt{\xi^2+4f^2}} \tag{26b}$$

$$\frac{\partial\psi}{\partial\xi} = \frac{\cos^2\psi}{2f} \tag{26c}$$

Substituting (26b) in (21) we get Jacobians as follow

$$J_{LL} = 1 - n_1\tau\frac{\cos^2\psi}{f} \tag{27a}$$

$$J_{RR} = 1 - n_2\tau\frac{\cos^2\psi}{f} \tag{27b}$$

$$J_{RL} = 1 - \frac{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}{2f\sqrt{n_1^2 - n_2^2\sin^2\psi}} \times \frac{n_1\tau\cos^2\psi}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \tag{27c}$$

$$J_{LR} = 1 - \frac{\sqrt{n_2^2 - n_1^2\sin^2\psi} + n_1\cos\psi}{2f\sqrt{n_2^2 - n_1^2\sin^2\psi}} \times \frac{n_2\tau\cos^2\psi}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \tag{27d}$$

and the GO field is found using (20) and (27) in (22) as

$$u_{LL}(r) = \left[\frac{\cos\psi - \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\psi + \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]\exp(-jkn_1\zeta) \times \left[1 - n_1\tau\frac{\cos^2\psi}{f}\right]^{-1/2}\exp(-jkn_1^2\tau) \tag{28a}$$

$$u_{RR}(r) = \left[\frac{\cos\psi - \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\psi + \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right]\exp(-jkn_2\zeta)$$

$$\times\left[1-n_2\tau\frac{\cos^2\psi}{f}\right]^{-1/2}\exp(-jkn_2^2\tau) \tag{28b}$$

$$u_{RL}(r)=\left[\frac{2\cos\psi}{\cos\psi+\cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right]\exp(-jkn_2\zeta)$$
$$\times[1-\frac{n_1\tau\cos^2\psi}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}$$
$$\times\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}{2f\sqrt{n_1^2-n_2^2\sin^2\psi}}]^{-1/2}\exp(-jkn_1^2\tau) \tag{28c}$$

$$u_{LR}(r)=\left[\frac{2\cos\psi}{\cos\psi+\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]\exp(-jkn_1\zeta)$$
$$\times[1-\frac{n_2\tau\cos^2\psi}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}$$
$$\times\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{2f\sqrt{n_2^2-n_1^2\sin^2\psi}}]^{-1/2}\exp(-jkn_2^2\tau) \tag{28d}$$

equations of caustics are written using (23) and (26b) as

$$n_1\tau = \frac{f}{\cos^2\psi} \tag{29a}$$
$$n_2\tau = \frac{f}{\cos^2\psi} \tag{29b}$$
$$n_1\tau = \frac{2f}{\cos^2\psi}\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}}{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}$$
$$\times\left\{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\right\} \tag{29c}$$
$$n_2\tau = \frac{2f}{\cos^2\psi}\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}}{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}$$
$$\times\left\{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\right\} \tag{29d}$$

and (24) becomes

$$J_{LL}\frac{\partial p_{zLL}}{\partial z}=n_1\sin^2 2\psi\frac{\cos^2\psi}{f} \tag{30a}$$

$$J_{RR}\frac{\partial p_{zRR}}{\partial z}=n_2\sin^2 2\psi\frac{\cos^2\psi}{f} \tag{30b}$$

$$J_{RL}\frac{\partial p_{zRL}}{\partial z}=\frac{n_1\cos^2\psi\sin^2\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \times\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}{2f\sqrt{n_1^2-n_2^2\sin^2\psi}} \tag{30c}$$

$$J_{LR}\frac{\partial p_{zLR}}{\partial z}=\frac{n_2\cos^2\psi\sin^2\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \times\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{2f\sqrt{n_2^2-n_1^2\sin^2\psi}} \tag{30d}$$

The finite fields around caustics are found for each ray as follow.

Using (26a)

$$\xi=2f\tan\psi,\qquad \zeta=f\frac{\cos 2\psi}{\cos^2\psi} \tag{31}$$

For LL ray,

$$\tau=\frac{x-\xi}{p_{xLL}}=\frac{-x+2f\tan\psi}{n_1\sin 2\psi} \tag{32a}$$

$$p_{zLL}=-n_1\cos 2\psi \tag{32b}$$

$$dp_{zLL}=2n_1\sin 2\psi d\psi \tag{32c}$$

putting (20), (30a) and (32) in (25a), we get finite field around caustic for LL ray as

$$u_{LL}(r)=\sqrt{\frac{2kfn_1}{j\pi}}\int_{-\infty}^{\infty}\left[\frac{\cos\psi-\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\psi+\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]\sec\psi \times\exp\left[-jkn_1\left\{2f-x\sin 2\psi-z\cos 2\psi\right\}\right]d\psi \tag{33}$$

For RR ray

$$\tau=\frac{x-\xi}{p_{xRR}}=\frac{-x+2f\tan\psi}{n_2\sin 2\psi} \tag{34a}$$

$$p_{zRR}=-n_2\cos 2\psi \tag{34b}$$

$$dp_{zRR}=2n_2\sin 2\psi d\psi \tag{34c}$$

Using (20), (30b) and (34) in (25b) we get

$$u_{RR}(r) = \sqrt{\frac{2kfn_2}{j\pi}} \int_{-\infty}^{\infty} \left[\frac{\cos\psi - \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\psi + \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right] \sec\psi \times \exp\left[-jkn_2\left\{2f - x\sin 2\psi - z\cos 2\psi\right\}\right] d\psi \tag{35}$$

For RL ray

$$p_{zRL} = -n_1\cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\} \tag{36a}$$

$$dp_{zRL} = n_1 \frac{\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi\}}{\sqrt{n_1^2 - n_2^2\sin^2\psi}} d\psi \tag{36b}$$

$$\tau = \frac{x-\xi}{p_{xRL}} = \frac{-x + 2f\tan\psi}{n_1\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \tag{36c}$$

Using (20), (30c) and (36) in (25c) we get

$$u_{RL}(r) = \sqrt{\frac{kfn_1}{j\pi}} \int_{-\infty}^{\infty} \left[\frac{2}{\cos\psi + \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right] \times \left[\cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\} + \tan\psi\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\right]^{1/2} \times \left[\frac{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}{\sqrt{n_1^2 - n_2^2\sin^2\psi}}\right]^{1/2} \exp[-jkn_1\{\Psi_{RL} + zp_{zRL}\}]d\psi \tag{37}$$

where

$$\Psi_{RL} + zp_{zRL} = f\frac{n_2\cos 2\psi}{n_1\cos^2\psi} - (x - 2f\tan\psi)\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\} - \left\{z - f\frac{\cos 2\psi}{\cos^2\psi}\right\}\cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}$$

For LR ray

$$p_{zLR} = -n_2\cos\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\} \tag{38a}$$

$$dp_{zLR} = n_2 \frac{\sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\{\sqrt{n_2^2 - n_1^2\sin^2\psi} + n_1\cos\psi\}}{\sqrt{n_2^2 - n_1^2\sin^2\psi}} d\psi$$

(38b)

$$\tau = \frac{x-\xi}{p_{xLR}} = \frac{-x+2f\tan\psi}{n_2\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \tag{38c}$$

using (20), (30d) and (38) in (25d) we get

$$\begin{aligned} u_{LR}(r) \;=\; & \sqrt{\frac{kfn_2}{j\pi}}\int_{-\infty}^{\infty}\left[\frac{2}{\cos\psi+\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right] \\ & \times\left[\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\right]^{1/2} \\ & \times\left[\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{\sqrt{n_2^2-n_1^2\sin^2\psi}}\right]^{1/2}\exp[-jkn_2\{\Psi_{LR}+zp_{zLR}\}]d\psi \end{aligned} \tag{39}$$

where

$$\begin{aligned} \Psi_{LR}+zp_{zLR} \;=\; & f\frac{n_1\cos 2\psi}{n_2\cos^2\psi}-(x-2f\tan\psi)\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\} \\ & -\left\{z-f\frac{\cos 2\psi}{\cos^2\psi}\right\}\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\} \end{aligned}$$

4.2. Circular Cylindrical Reflector

As an other example consider circular cylindrical reflector defined by

$$\zeta = \sqrt{a^2-\xi^2}$$

where a is the radius of cylinder. Using (18)

$$\cos\psi = \frac{\zeta}{a} \tag{40a}$$

$$\sin\psi = \frac{\xi}{a} \tag{40b}$$

$$\frac{\partial\psi}{\partial\xi} = \frac{1}{\zeta} \tag{40c}$$

Substituting (40b) in (21) we get Jacobians as follow

$$J_{LL} = 1-2\frac{n_1\tau}{a\cos\psi} \tag{41a}$$

$$J_{RR} = 1 - 2\frac{n_2\tau}{a\cos\psi} \tag{41b}$$

$$J_{RL} = 1 - \frac{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}{a\cos\psi\sqrt{n_1^2 - n_2^2\sin^2\psi}} \times \frac{n_1\tau}{\cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\} + \tan\psi\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \tag{41c}$$

$$J_{LR} = 1 - \frac{\sqrt{n_2^2 - n_1^2\sin^2\psi} + n_1\cos\psi}{a\cos\psi\sqrt{n_2^2 - n_1^2\sin^2\psi}} \times \frac{n_2\tau}{\cos\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\} + \tan\psi\sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \tag{41d}$$

GO field using (41) and (20) in (22) is given as

$$u_{LL}(r) = \left[\frac{\cos\psi - \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\psi + \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]\exp(-jkn_1\zeta) \times \left[1 - 2\frac{n_1\tau}{a\cos\psi}\right]^{-1/2}\exp(-jkn_1^2\tau) \tag{42a}$$

$$u_{RR}(r) = \left[\frac{\cos\psi - \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\psi + \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right]\exp(-jkn_2\zeta) \times \left[1 - 2\frac{n_2\tau}{a\cos\psi}\right]^{-1/2}\exp(-jkn_2^2\tau) \tag{42b}$$

$$u_{RL}(r) = \left[\frac{2\cos\psi}{\cos\psi + \cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right]\exp(-jkn_2\zeta) \times [1 - \frac{n_1\tau}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \times \frac{\sqrt{n_1^2 - n_2^2\sin^2\psi} + n_2\cos\psi}{a\cos\psi\sqrt{n_1^2 - n_2^2\sin^2\psi}}]^{-1/2}\exp(-jkn_1^2\tau) \tag{42c}$$

$$u_{LR}(r) = \left[\frac{2\cos\psi}{\cos\psi + \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]\exp(-jkn_1\zeta) \times [1 - \frac{n_2\tau}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}$$

$$\times\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{a\cos\psi\sqrt{n_2^2-n_1^2\sin^2\psi}}]^{-1/2}\exp(-jkn_2^2\tau) \qquad (42d)$$

equations of caustics are found using (40b) in (23) as

$$n_1\tau = \frac{1}{2}a\cos\psi \qquad (43a)$$

$$n_2\tau = \frac{1}{2}a\cos\psi \qquad (43b)$$

$$n_1\tau = \left\{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\right\} \times\frac{a\cos\psi\sqrt{n_1^2-n_2^2\sin^2\psi}}{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi} \qquad (43c)$$

$$n_2\tau = \left\{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\right\} \times\frac{a\cos\psi\sqrt{n_2^2-n_1^2\sin^2\psi}}{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi} \qquad (43d)$$

and using (40b) in (24) we have

$$J_{LL}\frac{\partial p_{zLL}}{\partial z} = 2\frac{n_1\sin^2 2\psi}{a\cos\psi} \qquad (44a)$$

$$J_{RR}\frac{\partial p_{zRR}}{\partial z} = 2\frac{n_2\sin^2 2\psi}{a\cos\psi} \qquad (44b)$$

$$J_{RL}\frac{\partial p_{zRL}}{\partial z} = \frac{n_1\sin^2\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \times\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}{a\cos\psi\sqrt{n_1^2-n_2^2\sin^2\psi}} \qquad (44c)$$

$$J_{LR}\frac{\partial p_{zLR}}{\partial z} = \frac{n_2\sin^2\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \times\frac{\sqrt{n_2^2-n_1^2\sin^2\psi}+n_1\cos\psi}{a\cos\psi\sqrt{n_2^2-n_1^2\sin^2\psi}} \qquad (44d)$$

Finite field around caustics are found as follow.

From (40a)

$$\xi = a\sin\psi, \qquad \zeta = a\cos\psi \tag{45}$$

For LL ray,

$$\tau = \frac{x-\xi}{p_{xLL}} = \frac{-x+a\sin\psi}{n_1\sin 2\psi} \tag{46}$$

using (20), (32b), (44a) and (46) in (25a) we get

$$\begin{aligned} u_{LL}(r) \;=\; & \sqrt{\frac{kan_1}{j\pi}}\int_{-\infty}^{\infty}\left[\frac{\cos\psi-\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}{\cos\psi+\cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right][\cos\psi]^{1/2} \\ & \times\exp[-jkn_1\{2a\cos\psi - x\sin 2\psi - z\cos 2\psi\}]d\psi \end{aligned} \tag{47}$$

For RR ray

$$\tau = \frac{x-\xi}{p_{xRR}} = \frac{-x+a\sin\psi}{n_2\sin 2\psi} \tag{48}$$

using (20), (34b),(44a) and (48) in (25b)we get

$$\begin{aligned} u_{RR}(r) \;=\; & \sqrt{\frac{kan_2}{j\pi}}\int_{-\infty}^{\infty}\left[\frac{\cos\psi-\cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}{\cos\psi+\cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right][\cos\psi]^{1/2} \\ & \times\exp[-jkn_2\{2a\cos\psi - x\sin 2\psi - z\cos 2\psi\}]d\psi \end{aligned} \tag{49}$$

For RL ray

$$\tau = \frac{x-\xi}{p_{xRL}} = \frac{-x+a\sin\psi}{n_1\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}} \tag{50}$$

using (20), (36b), (44b) and (50) in (25c) we get

$$\begin{aligned} u_{RL}(r) = & \sqrt{\frac{kan_1}{j2\pi}}\int_{-\infty}^{\infty}\left[\frac{2\cos^{\frac{3}{2}}\psi}{\cos\psi+\cos\{\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}}\right] \\ & \times\left[\cos\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}+\tan\psi\sin\{\psi+\sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}\right]^{1/2} \\ & \times\left[\frac{\sqrt{n_1^2-n_2^2\sin^2\psi}+n_2\cos\psi}{\sqrt{n_1^2-n_2^2\sin^2\psi}}\right]^{1/2}\exp\left[-jkn_1\left\{\Psi_{RL}+zp_{zRL}\right\}\right]d\psi \end{aligned} \tag{51}$$

where

$$\Psi_{RL} + zp_{zRL} = \frac{n_2}{n_1} a\cos\psi - (x - a\sin\psi)\sin\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}$$
$$- \{z - a\cos\psi\}\}\cos\{\psi + \sin^{-1}(\frac{n_2}{n_1}\sin\psi)\}$$

For LR ray

$$\tau = \frac{x-\xi}{p_{xLR}} = \frac{-x + a\sin\psi}{n_2\sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}} \tag{52}$$

using (20), (38b), (44c) and (52) in (25d) we get

$$u_{LR}(r) = \sqrt{\frac{kan_2}{j2\pi}} \int_{-\infty}^{\infty} \left[\frac{2\cos^{\frac{3}{2}}\psi}{\cos\psi + \cos\{\sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}}\right]$$
$$\times \left[\cos\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\} + \tan\psi\sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}\right]^{1/2}$$
$$\times \left[\frac{\sqrt{n_2^2 - n_1^2\sin^2\psi} + n_1\cos\psi}{\sqrt{n_2^2 - n_1^2\sin^2\psi}}\right]^{1/2} \exp[-jkn_2\{\Psi_{LR} + zp_{zLR}\}]d\psi \tag{53}$$

where

$$\Psi_{LR} + zp_{zLR} = \frac{n_1}{n_2} a\cos\psi - (x - a\sin\psi)\sin\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}$$
$$- \{z - a\cos\psi\}\}\cos\{\psi + \sin^{-1}(\frac{n_1}{n_2}\sin\psi)\}$$

5. RESULTS AND DISCUSSION

Contour plots of (33), (35), (37), (39), (47), (49), (51) and (53) are given in Figure 4 to Figure 11, respectively. Each figure contains four plots, that is, for $k\beta$ =0, 0.01, 0.05 and 0.1. We have taken $kf = 100$ for parabolic reflector and $ka = 100$ for circular reflector. Integration has been performed from $\psi = -\pi/4$ to $\psi = \pi/4$. In these plots horizontal axis is kz and vertical axis is kx. Variation of magnitude of the fields is shown versus kz in Figure 12 to Figure 19.

From equation (6)

$$n_1\tau = n_2\tau = \sqrt{(x-\xi)^2 + (z-\zeta)^2} \tag{54}$$

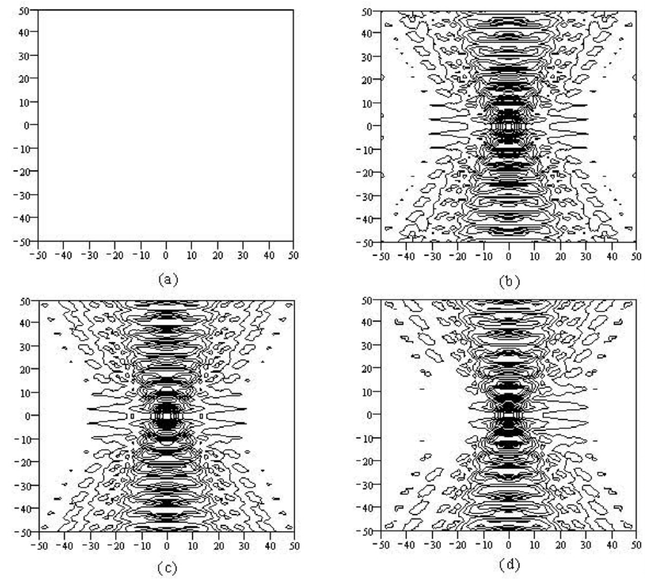

Figure 4. Equi-amplitude plots of $|u_{LL}|$ of parabolic cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

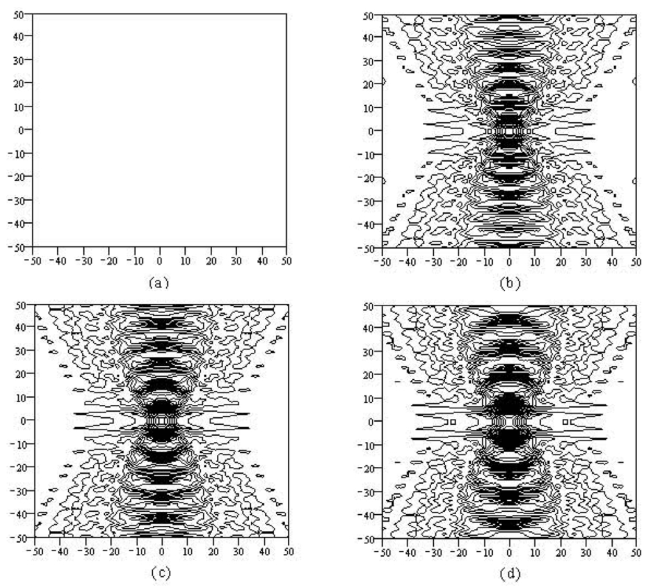

Figure 5. Equi-amplitude plots of $|u_{RR}|$ of parabolic cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

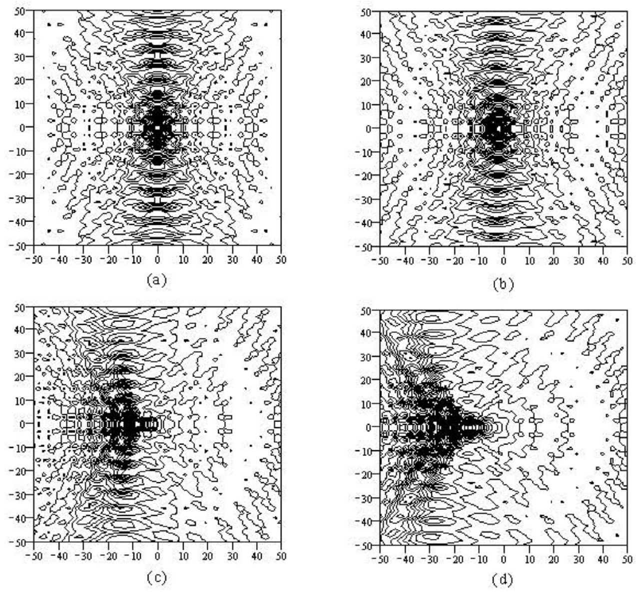

Figure 6. Equi-amplitude plots of $|u_{RL}|$ of parabolic cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

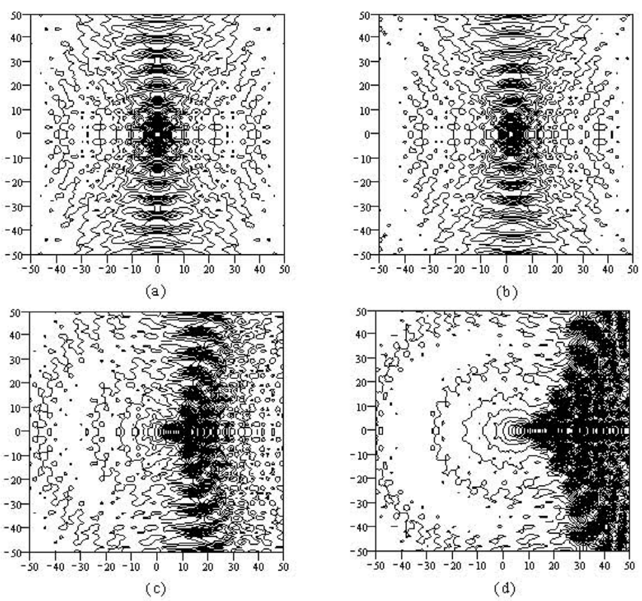

Figure 7. Equi-amplitude plots of $|u_{LR}|$ of parabolic cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

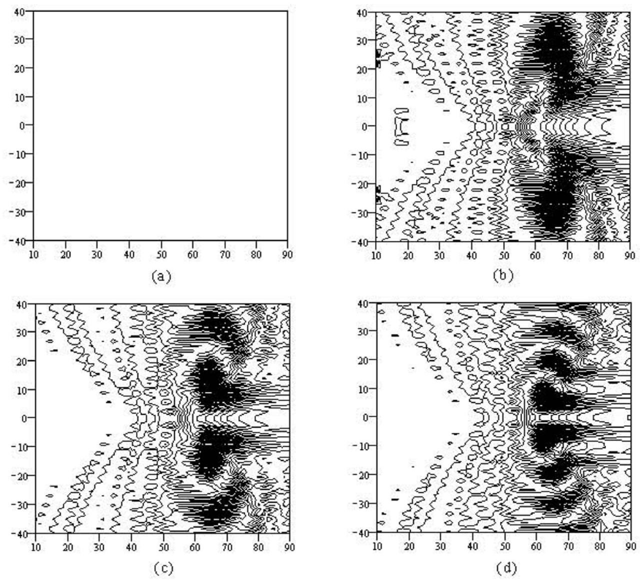

Figure 8. Equi-amplitude plots of $|u_{LL}|$ of circular cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

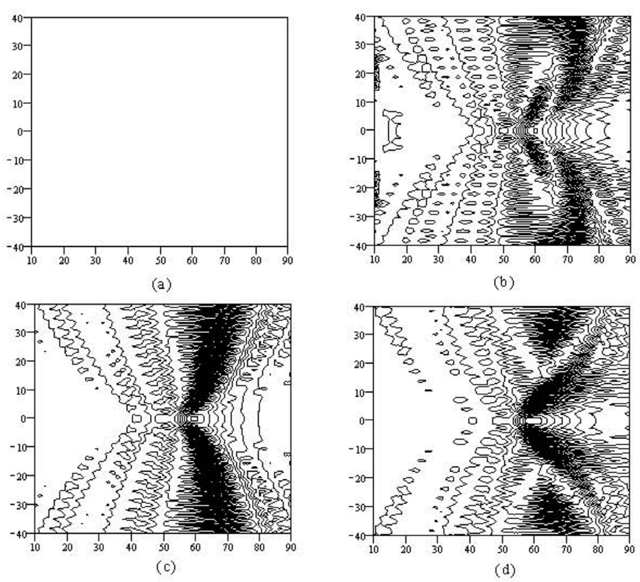

Figure 9. Equi-amplitude plots of $|u_{RR}|$ of circular cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

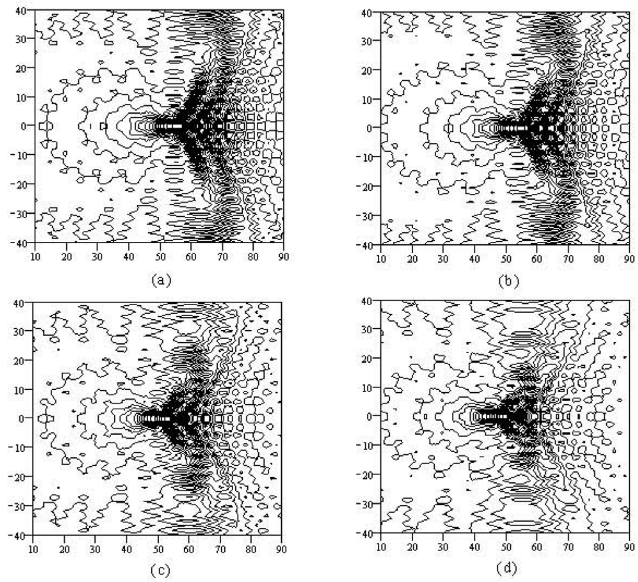

Figure 10. Equi-amplitude plots of $|u_{RL}|$ of circular cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

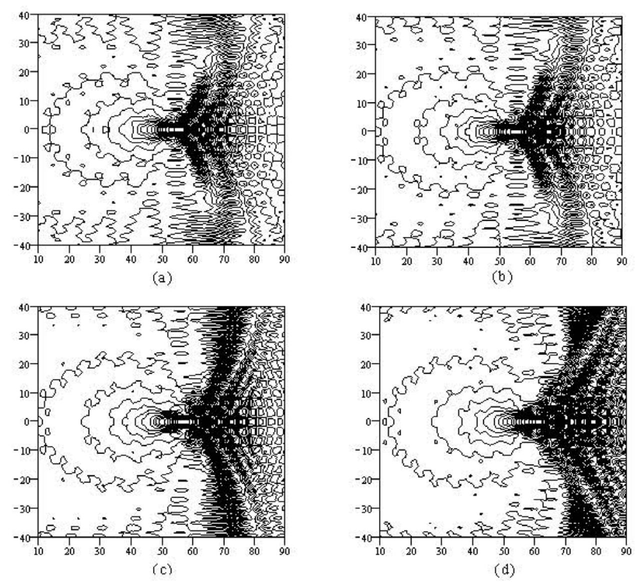

Figure 11. Equi-amplitude plots of $|u_{LR}|$ of circular cylindrical reflector with $kf = 100$ for (a) $k\beta = 0$, (b) $k\beta = 0.01$, (c) $k\beta = 0.05$ and (d) $k\beta = 0.1$.

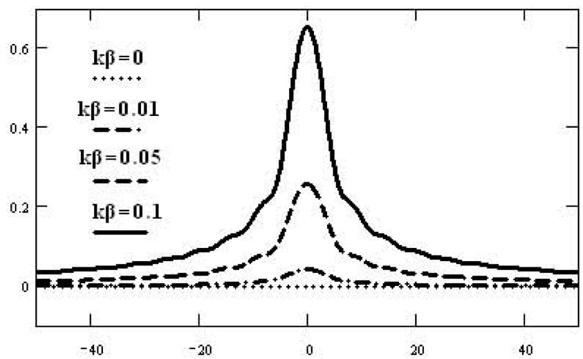

Figure 12. Plots of $|u_{LL}|$ of parabolic cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

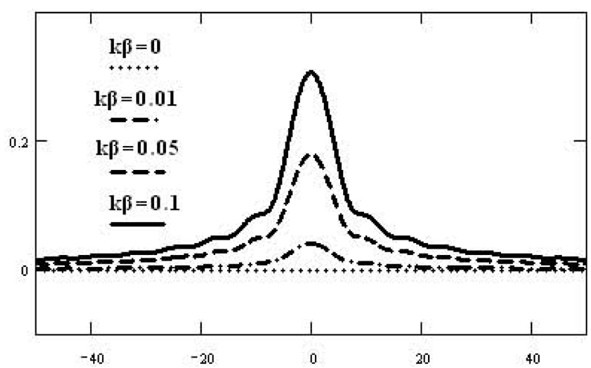

Figure 13. Plots of $|u_{RR}|$ of parabolic cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

so equations of caustics for u_{LL} and u_{RR} of parabolic cylindrical reflector, (29a) and (29b), are same and is given below using (19a) and (19b) in (6a) and (6b).

$$x = z = 0 \tag{55}$$

This is the same equation as given in [13] for the case of normal incidence when parabolic reflector is placed in ordinary medium. This is also true for circular reflector as is evident from (43a) and (43b) and equation of caustic is

$$x = \frac{a}{4}(3\sin\psi - \sin 3\psi) \tag{56a}$$

$$x = \frac{a}{4}(3\cos\psi - \cos 3\psi) \tag{56b}$$

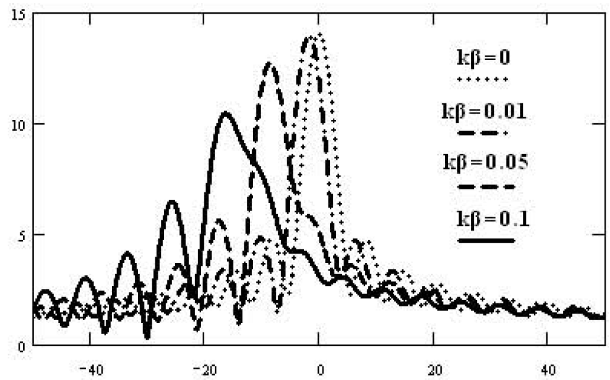

Figure 14. Plots of $|u_{RL}|$ of parabolic cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

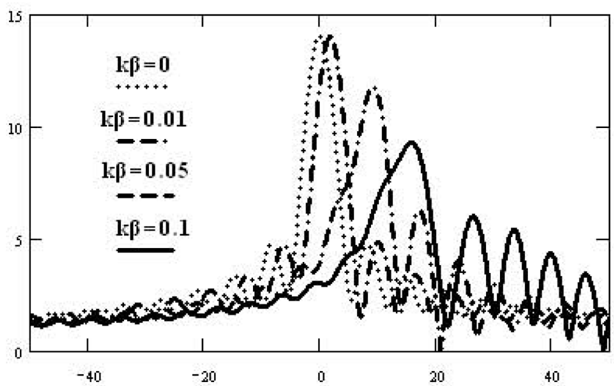

Figure 15. Plots of $|u_{LR}|$ of parabolic cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

Again these are the same equations as given in [13] for the case of normal incidence when circular reflector is placed in ordinary medium. So caustics for LL and RR rays coincide for all values of $k\beta$. This behavior is depicted in Figures 4, 5, 12 and 13 for parabolic cylindrical reflector and in Figures 8, 9, 16 and 17 for circular reflector. For $k\beta = 0$, $n_1 = n_2 = 1$ and

$$u_{LL} = u_{RR} = 0 \tag{57}$$

for both circular and cylindrical reflectors. As value of $k\beta$ increases, magnitude of the field around caustic increases.

Equations of caustics of RL and LR rays are given by (29c) and (29d) for parabolic and by (43c) and (43d) for circular reflector,

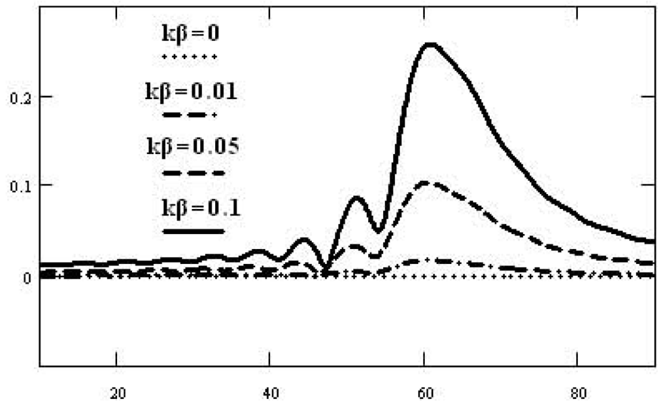

Figure 16. Plots of $|u_{LL}|$ of circular cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

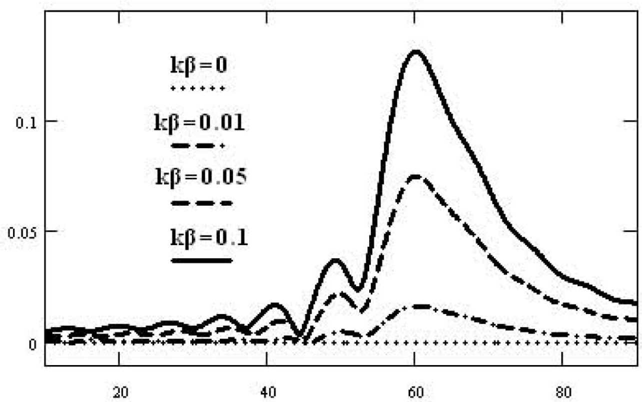

Figure 17. Plots of $|u_{RR}|$ of circular cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

respectively. From Figures 6, 7, 10, 11, 14, 15, 18 and 19, it can be seen that as $k\beta$ increases, caustic of RL ray moves to left and caustic of LR ray moves to right. The greater is $k\beta$, the larger is the gap between focal region of RL and LR ray. The behavior of the field around caustic is shown in the figures for different values of $k\beta$. When $k\beta = 0$

$$u_{RL} = u_{LR} = \sqrt{\frac{2kf}{j\pi}} \int_{-\infty}^{\infty} \sec\psi \exp\{-jk(2f - x\sin 2\psi - z\cos 2\psi)\} d\psi \tag{58}$$

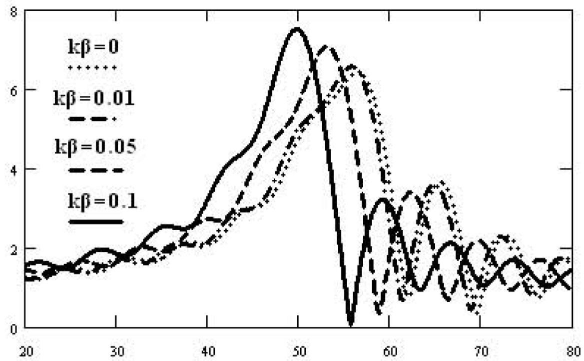

Figure 18. Plots of $|u_{RL}|$ of circular cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

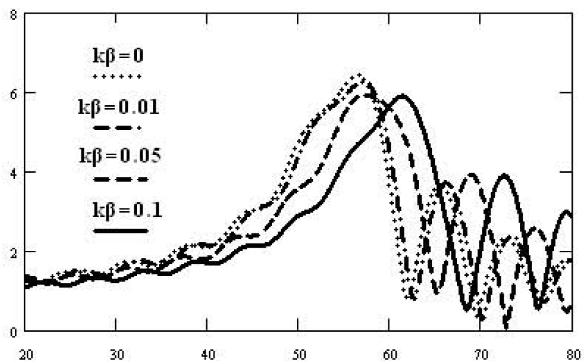

Figure 19. Plots of $|u_{LR}|$ of circular cylindrical reflector at $x = 0$ for $k\beta = 0, 0.01, 0.05, 0.1$.

for parabolic and

$$u_{RL} = u_{LR} = \sqrt{\frac{ka}{j\pi}} \int_{-\infty}^{\infty} \sqrt{\cos\psi}\, \exp\{-jk(2a\cos\psi - x\sin 2\psi - z\cos 2\psi)\} d\psi \quad (59)$$

for circular reflector. The equations of caustics reduce to (55) and (56) for parabolic and circular reflector respectively. Which is the case of an ordinary medium. The field expression (58) and (59) are in agreement with [13] for normal incidence.

6. CONCLUSIONS

It is found that excitation of a cylindrical reflector, placed in reciprocal and homogenous chiral medium, by plane wave may yield four focal points. Two of them are located at the same location as if the reflector is placed in ordinary medium. Other two focal points are on the opposite sides of caustic located at ordinary medium location. If LCP waves is moving slower than RCP (for $\beta > 0$) then it is focused near to reflector and RCP wave is focused away from the reflector. The situation is reversed for $\beta < 0$. It is also noted that if medium becomes achiral $\beta = 0$, field of LL and RR becomes zero and that of RL and LR reduces to the case of ordinary medium. As the chirality parameter increases, the gap among the caustics increases.

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