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USING PHOTON WAVE FUNCTION FOR THE TIME-DOMAIN ANALYSIS OF ELECTROMAGNETIC WAVE SCATTERING

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Abstract—In this paper, a generalized photon wave function (PWF) which is applicable to electromagnetic problems is introduced. The formulation treats the electromagnetics fields as quantum mechanical entities. The introduced PWF is especially useful for boundary-value problems. For instance,the reflection coefficient at a dielectric half space is calculated based on the concepts of PWF and quantum mechanics.

With the proposed method, inhomogeneous media, both isotropic and anisotropic, can also be analyzed. It is shown that by defining certain new variables, such as effective charges and effective currents, we will be able to describe the behavior of electromagnetic fields by the proposed photon wave function. At the end of this paper, a new FDTD method based on the notion of photon wave function is introduced and the resonance frequencies of a cubic cavity are obtained.

1. INTRODUCTION

Photons are mass-less, charge-less and spin-1 particles which are the carriers of electromagnetic force and obey the relativistic quantum field theory [1, 2]. On the other hand, in a semi-classical sense, the electromagnetic problems are fully described by Maxwell's equations. The relation between these two representations (quantum mechanical form and semi-classic one) is discussed in [3, 4] where it is stated that photons obey an equation similar to the Schrdinger and Dirac equations for electrons which is shown to be comparable to Maxwell's equations.

PWF in electromagnetics was introduced for the first time in [5], where Maxwell's equations are manipulated in order to find the stability condition for finite difference time domain (FDTD) method [5–11]. Also in [12], some other relations based on PWF are derived and discussed.

In [3, 4] the goal is to apply quantum mechanical formulation to PWF. In other words, by rearrangement of Maxwell's equations, one can achieve a formulation similar to quantum equations, thus, it will be possible to employ quantum mechanical formulations in electromagnetics. In [3, 4] the prediction of the behavior of the photon and the probability of its presence at a specific point in space is the main goal, but not the application of PWF to electromagnetics.

In [13], for solving Maxwell's equations by the Chebyshev method, a one-step FDTD algorithm are proposed. The idea is based on formulation of Maxwell's equations as a single-operator matrix equation. Due to symmetries in Maxwell's equations, they use:

$$\vec{X}(t) = \sqrt{\mu}\vec{H}(t) \qquad \vec{Y}(t) = \sqrt{\varepsilon}\vec{E}(t)$$
$$\frac{\partial}{\partial t}\begin{pmatrix} \vec{X}(t) \\ \vec{Y}(t) \end{pmatrix} = \Pi \begin{pmatrix} \vec{X}(t) \\ \vec{Y}(t) \end{pmatrix} - \frac{1}{\sqrt{\varepsilon}}\begin{pmatrix} 0 \\ \vec{J}(t) \end{pmatrix} \tag{1}$$

Where $\vec{J}(t)$ represents the source of the electric charge and Π denotes the operator:

$$\Pi = \begin{pmatrix} 0 & -\frac{1}{\sqrt{\mu}}\vec{\nabla} \times \frac{1}{\sqrt{\varepsilon}} \\ \frac{1}{\sqrt{\varepsilon}}\vec{\nabla} \times \frac{1}{\sqrt{\mu}} & 0 \end{pmatrix} \tag{2}$$

The variables ($\vec{E}(t)$ and $\vec{H}(t)$) are expressed as an infinite series. By imposing proper truncation, one can find the numerical solution of the problem.

The limitation of such method is the inaccuracy of Chebychev polynomial approximation in absorbing boundary conditions [13]. Also in this method, only lossless materials can be used.

Reference [14] also utilizes a comparable technique such that [13] is its special case. In [14] the initial value problem is solved by applying the Faber polynomial approximation. As we have mentioned above, we can abbreviate the electromagnetics equations as:

$$j\frac{\partial \Phi(t)}{\partial t} = \Pi\Phi(t) \tag{3}$$

with previously defined operator Π. If Φ_0 is the initial wave packet configuration, then $\Phi(t)$ can be found from:

$$\Phi(t) = \exp(-jt\Pi)\Phi_0 \tag{4}$$

The aforementioned equation provides a numerical solution of the initial-value problem. Time variant sources can't be directly computed by this method.

In [15] another use of operational form of Maxwells equations is introduced. In this paper, it is shown that by applying small and local perturbation theory we can treat electromagnetics as quantum mechanics and obtain wave propagation in eigen-frequencies of a system.

In this paper, a simple form of PWF is suggested which is shown to be useful for treating certain boundary-value problems. A complex vector wave function for photon is defined and it is shown that electromagnetic formulation can be totally cast into this formulation. Besides, we can use the boundary conditions, such as PEC, PMC, or PML, for this formulation. Any kind of sources, either electric or magnetic currents or charges, can be simulated by this method.

In Section 2 of this paper, we derive the basic formulation of PWF for electromagnetics. In Section 3, we will review the Poynting theorem, duality, and equivalence principle in the light of the proposed PWF. Applying the PWF formulation to inhomogeneous media is demonstrated in Sections 4 and 5.

Finally, in Section 6, we will present a numerical technique similar to the finite difference time domain method. In this method, the two unknowns, i.e., electric and magnetic fields can be computed simultaneously at a reasonable computational cost.

2. DERIVING PHOTON WAVE FUNCTION IN HOMOGENEOUS MEDIA

In a homogeneous time-invariant medium with permittivity and permeability of $\varepsilon(r)$ and $\mu(r)$, respectively, Maxwell's equations are given by

$$\begin{cases} -\vec{\nabla} \times \vec{E}(\vec{r},t) = \mu(\vec{r})\dfrac{\partial}{\partial t}\vec{H}(\vec{r},t) + \vec{J}_m(\vec{r},t) \\ \vec{\nabla} \times \vec{H}(\vec{r},t) = \varepsilon(\vec{r})\dfrac{\partial}{\partial t}\vec{E}(\vec{r},t) + \vec{J}_e(\vec{r},t) \end{cases} \tag{5}$$

$$\begin{cases} \vec{\nabla} \cdot \left(\varepsilon(\vec{r})\vec{E}(\vec{r},t)\right) = \rho_e(\vec{r},t) \\ \vec{\nabla} \cdot \left(\mu(\vec{r})\vec{H}(\vec{r},t)\right) = \rho_m(\vec{r},t) \end{cases}$$

Defining photon wave function as

$$\vec{\psi} = \frac{\vec{E}}{\sqrt{\mu}} + j\frac{\vec{H}}{\sqrt{\varepsilon}}, \tag{6}$$

one can write the two curl equations in (1) in the following compact form:

$$\vec{\nabla} \times \vec{\psi}(\vec{r}, t) = j\frac{n}{c}\frac{\partial}{\partial t}\vec{\psi}(\vec{r}, t) + j\vec{G}(\vec{r}, t) \tag{7}$$

in which $c = 1/\sqrt{\mu_0\varepsilon_0}$ is the light velocity in free space and $n(\vec{r}) = \sqrt{\mu_r(\vec{r})\varepsilon_r(\vec{r})}$ denotes refraction index of the medium.

Similarly, the two divergence equations in (5) can be combined in a similar way to obtain

$$\vec{\nabla} \cdot \left(\frac{n}{c}\vec{\psi}\right) = g \tag{8}$$

In (7) and (8), combined current and charge sources are defined as:

$$\begin{aligned} \vec{G} &= \frac{\vec{J}_e}{\sqrt{\varepsilon}} + j\frac{\vec{J}_m}{\sqrt{\mu}} \\ g &= \frac{\rho_e}{\sqrt{\varepsilon}} + j\frac{\rho_m}{\sqrt{\mu}} \end{aligned} \tag{9}$$

The continuity equation obviously holds and can be derived either from (9) using the continuity equation for electric and magnetic sources or from substitution of (8) in divergence of (7). Thus the result is:

$$\vec{\nabla} \cdot \vec{G} + \frac{\partial}{\partial t} g = 0 \tag{10}$$

Equation (6) is the photon wave function we will use hereafter and equations (7) and (8) describe the dynamics of photon wave function. As shown in [3, 4] the source-free version of equation (7) can be obtained from relativistic quantum mechanics. Equation (7) describes evolution of photon wave function in time and space. It is similar to the Schrödinger equation in non-relativistic or the Dirac equation in relativistic quantum mechanics for electrons. In equations (9) and (10) we put current either for sources or for the currents resulted from conductivity (i.e., $\vec{G} = \sigma\vec{E}$).

Note that in this paper, we have assumed permittivity and permeability to be real numbers.

3. SOME BASIC THEOREMS

3.1. Poynting's Theorem

To study the above interpretations further, let us calculate the probability of detecting a photon at a position in space. Based on quantum mechanics rules, the probability of finding photon in a specified position can be obtained by:

$$P(\vec{r},t) = \vec{\psi} \cdot \vec{\psi}^* = \frac{|\vec{E}|^2}{\mu} + \frac{|\vec{H}|^2}{\varepsilon} = 2c^2 \left(\frac{1}{2}\varepsilon|\vec{E}|^2 + \frac{1}{2}\mu|\vec{H}|^2 \right) \tag{11}$$

Thus, we can deduce that:

$$P(\vec{r},t) = 2c^2 u(\vec{r},t) \tag{12}$$

where $u(\vec{r},t)$ is the time varying local density of electromagnetics energy. Consequently, $P(\vec{r},t)$ is proportional to the local density of electromagnetic energy. Accordingly, we see that a probability interpretation will be possible if we divide $\vec{\psi}$ by the normalization factor $(2c^2\hbar\omega)$. Corresponding to this probability density, a probability current density is defined in quantum mechanics [16] which satisfies continuity equation. If we take time derivative of spatial probability function, upon substituting the time derivation from (7) we have:

$$\frac{\partial}{\partial t}P(\vec{r},t) = \left(\frac{\partial}{\partial t}\vec{\psi}\right) \cdot \vec{\psi}^* + \vec{\psi} \cdot \left(\frac{\partial}{\partial t}\vec{\psi}^*\right) = -jc\vec{\nabla} \cdot \left(\vec{\psi} \times \vec{\psi}^*\right) \tag{13}$$

Since

$$\vec{\nabla} \cdot \left(\vec{\psi} \times \vec{\psi}^*\right) = -2jc\vec{\nabla} \cdot \vec{S} \tag{14}$$

where $\vec{S}$ stands for the Poynting vector, now the Poynting theorem will read:

$$\vec{\nabla} \cdot \vec{S} + \frac{\partial}{\partial t}u(\vec{r},t) = 0 \tag{15}$$

Again, if we divide $2c\vec{S}$ by normalization factor $2c^2\hbar\omega$, the probability current density is obtained and the last equation becomes the familiar probability continuity theorem in quantum mechanics.

3.2. Duality Transformation

Assume that pair belongs to:

$$(a,b) \in \left\{ (\vec{E}, \eta\vec{H}), (\eta\vec{D}, \vec{B}), (\eta\rho_e, \rho_m), (\eta J_e, J_m) \right\} \tag{16}$$

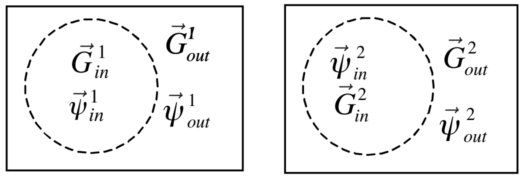

Figure 1. Two distinct inhomogeneous media.

($\eta = \sqrt{\mu/\varepsilon}$ is the medium characteristic impedance). There is a well known theorem in electrodynamics [17], known as duality theorem, which tells that Maxwell's equations are invariant if each of the above parameter pairs change under the following transformation:

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{17}$$

Duality theorem can be readily expressed in our formulation. Defining complex function x that belongs to the set:

$$x \in \left\{\vec{\psi}, g, \vec{G}\right\} \tag{18}$$

It can be easily shown that since (7) and (8) are linear, they are invariant under the transformation:

$$x' = xe^{-j\xi} \tag{19}$$

Hence, duality transformation is a trivial result of linearity of photon wave equations. Measurable parameters, such as acceleration, are independent of this transformation and are therefore in a quadratic form. For this reason, one nontrivial application of this theorem will be the prediction of magnetic charge in an electromagnetic field.

3.3. Equivalence Theorem

The equivalence theorem for time-harmonic field [18] can be similarly formulated. Given the two cases which are shown in Fig. 1.

A third one (depicted in Fig. 2) can be introduced from the cases shown in Fig. 1 and to support this solution, the surface current $\vec{K} = \hat{n} \times (\vec{\psi}^2_{out} - \vec{\psi}^1_{in})$ must be placed on the interface.

4. INHOMOGENEOUS MEDIA

In this section we use the same form of Maxwell's equations as we did in Section 2, but $\varepsilon(\vec{r})$ and $\mu(\vec{r})$ are not constant functions of space. If we

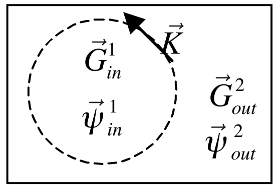

Figure 2. Mixing two distinct conditions by placing field-based surface currents on the interface.

again make use of (7) for photon wave function in the inhomogeneous case, we have:

$$\begin{aligned} -\vec{\nabla}\times\frac{\vec{E}}{\sqrt{\mu}} &= \sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\frac{\vec{H}}{\sqrt{\varepsilon}} - \vec{\nabla}\left(\frac{1}{\sqrt{\mu}}\right)\times\vec{E} + \frac{\vec{J}_m}{\sqrt{\mu}} \\ \vec{\nabla}\times\frac{\vec{H}}{\sqrt{\varepsilon}} &= \sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\frac{\vec{E}}{\sqrt{\mu}} + \vec{\nabla}\left(\frac{1}{\sqrt{\varepsilon}}\right)\times\vec{H} + \frac{\vec{J}_e}{\sqrt{\varepsilon}} \end{aligned} \tag{20}$$

After manipulation of the last two terms [Appendix A] and using the fact that for any variable a, we have: $\vec{\nabla}a^2/a^2 = 2\vec{\nabla}a/a$, we arrive at:

$$\vec{\nabla}\times\vec{\psi} = j\frac{n}{c}\frac{\partial}{\partial t}\vec{\psi} + j\vec{G} + \frac{1}{2}\left(\vec{\psi}\times\frac{\vec{\nabla}n}{n} + \vec{\psi}^{*}\times\frac{\vec{\nabla}\eta}{\eta}\right) \tag{21}$$

Likewise, the divergence equation can be shown to be

$$\vec{\nabla}\cdot\vec{\psi} = \frac{c}{n}g - \frac{1}{2}\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n} + \vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\right) \tag{22}$$

However, in this form there is no apparent connection between (21) and (22). The latter can be modified with a little algebra to yield:

$$\vec{\nabla}\cdot\left(\frac{n}{c}\vec{\psi}\right) = g + \frac{n}{2c}\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n} - \vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\right) \tag{23}$$

5. EFFECTIVE CHARGE AND CURRENT DENSITY

From comparing (21) with (7), it is evident that the last term can be regarded as an effective current density:

$$\vec{G}_{eff} = \frac{1}{2j}\left(\vec{\psi}\times\frac{\vec{\nabla}n}{n} + \vec{\psi}^{*}\times\frac{\vec{\nabla}\eta}{\eta}\right) \tag{24}$$

Similarly the last term in (22) can be modified to define the effective charge density as:

$$\vec{g}_{eff} = \frac{n}{2c}\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n} - \vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\right) \tag{25}$$

We also define a new parameter:

$$\vec{\Psi} = \frac{n}{c}\vec{\psi} = \frac{\vec{D}}{\sqrt{\varepsilon}} + j\frac{\vec{B}}{\sqrt{\mu}} \tag{26}$$

With these definitions, the set of Maxwell's equations in inhomogeneous media becomes:

$$\begin{aligned} \vec{\nabla}\times\vec{\psi} &= j\frac{\partial}{\partial t}\vec{\Psi} + j\left(\vec{G}+\vec{G}_{eff}\right) \\ \vec{\nabla}\cdot\vec{\Psi} &= g + g_{eff} \end{aligned} \tag{27}$$

Effective charge and current densities do not satisfy the continuity equation but it can readily be seen that total charge and total current densities satisfy it:

$$\vec{\nabla}\cdot\left(\vec{G}+\vec{G}_{eff}\right) + \frac{\partial}{\partial t}\left(g + g_{eff}\right) = 0 \tag{28}$$

To understand the meaning and origin of effective charge and current in (25) and (26) let us calculate them on an interface between two different materials. From (25) and (26) we see that there is a surface current and surface charge on the interface, given by:

$$\vec{K}_{eff} = -j\left(\vec{\psi}(\vec{r}\,')\frac{n_2-n_1}{n_2+n_1} + \vec{\psi}^{*}(\vec{r}\,')\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)\delta(\vec{r}-\vec{r}\,') \tag{29}$$

$$g_{eff} = \frac{n_1+n_2}{4c}\cdot\left(\hat{n}\cdot\vec{\psi}(\vec{r}\,')\frac{n_2-n_1}{n_2+n_1} + \hat{n}\cdot\vec{\psi}^{*}(\vec{r}\,')\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right) \tag{30}$$

The second terms in each of the above boundary sources are easy to be interpreted. Remembering that $\psi^{*}(x,-t)$ is equivalent to reflected wave from the interface, by considering its coefficient it is clear that the coefficient of the term $\psi^{*}(x,-t)$ is due to reflection of the incident wave from the boundary. The first term in both of (29) and (30) are, however, due to lensing (amplifying) effect of boundary transition on incoming wave from the other medium.

6. A NEW FDTD ALGORITHM BASED ON THE NOTION OF PHOTON WAVE FUNCTION

Since its introduction, FDTD has been going through a number of improvements. Even in recent years, novel algorithms for optimization of FDTD are developed and applied to different structures [19]. Most of these methods are based on the conventional form of FDTD. In this paper, we introduce a new FDTD based on PWF. The main idea is to prepare a finite difference time domain algorithm, based on the basic formula for "Photon Wave Function."

$$\vec{\nabla} \times \vec{\psi} = j\frac{n}{c}\frac{\partial}{\partial t}\vec{\psi} + j\vec{G} \tag{31}$$

To achieve this aim, we separate the x, y, and z parts of the function $\vec{\psi}$ and make a central difference equation for each part in order to calculate $\vec{\psi}$ (and thus $\vec{E}$ and $\vec{H}$) in a specific time. By this idea, we will be able to calculate both electric and magnetic fields of each direction with one-dimensional equations, in each time slot, in contrast with the common FDTD method [5] that calculates the electric and magnetic fields in asynchronous time slots.

6.1. Boundary Condition Simulation

To simulate various boundary conditions is an important case in different FDTD methods [19]; here we will describe the way to impose boundary conditions in this method.

The first and simplest form is PEC (perfect electric conductivity). In this case, we should make tangential electric field and normal magnetic field zero in the neighbor of the boundary; as the numerical coding for the x-part of the wave function in all three aspects of a cube will be:

$$\begin{aligned}
\psi_x(1,:,:) &= real(\psi_x(1,:,:));\\
\psi_x(M,:,:) &= real(\psi_x(M,:,:));\\
\psi_x(:,1,:) &= i \times imag(\psi_x(:,1,:));\\
\psi_x(:,M,:) &= i \times imag(\psi_x(:,M,:));\\
\psi_x(:,:,1) &= i \times imag(\psi_x(:,:,1));\\
\psi_x(:,:,M) &= i \times imag(\psi_x(:,:,M));
\end{aligned}$$

Where M is the number of cells in each direction.

As we know, for PML (perfectly matched layer), in common notation we should separate electric and magnetic fields in two parts with anisotropic conductivity ($\sigma_i\,(i = x, y, z)$). Based on known PML formulation, we are familiar with 12 known formulas [5].

By regarding the PML equations of common FDTD, using the concept of PWF, and separating photon wave function as we do for electric and magnetic fields, and also by assuming that all conductivities are real numbers, can be abbreviated like:

$$\begin{aligned}
\frac{\partial}{\partial y}\left(\vec{\psi}_{zx}+\vec{\psi}_{zy}\right)-\eta c\sigma_y\vec{\psi}_{xy} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{xy}\right)+j\vec{G}_x \\
-\frac{\partial}{\partial z}\left(\vec{\psi}_{yx}+\vec{\psi}_{yz}\right)-\eta c\sigma_z\vec{\psi}_{xz} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{xz}\right)+j\vec{G}_x \\
\frac{\partial}{\partial z}\left(\vec{\psi}_{xz}+\vec{\psi}_{xy}\right)-\eta c\sigma_z\vec{\psi}_{yz} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{yz}\right)+j\vec{G}_y \\
-\frac{\partial}{\partial x}\left(\vec{\psi}_{zx}+\vec{\psi}_{zy}\right)-\eta c\sigma_x\vec{\psi}_{yx} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{yx}\right)+j\vec{G}_y \\
\frac{\partial}{\partial x}\left(\vec{\psi}_{yx}+\vec{\psi}_{yz}\right)-\eta c\sigma_x\vec{\psi}_{zx} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{zx}\right)+j\vec{G}_z \\
-\frac{\partial}{\partial y}\left(\vec{\psi}_{xz}+\vec{\psi}_{xy}\right)-\eta c\sigma_y\vec{\psi}_{zy} &= j\frac{c}{n}\frac{\partial}{\partial t}\left(\vec{\psi}_{zy}\right)+j\vec{G}_z
\end{aligned} \tag{32}$$

Thus, we have been able to reduce the number of needed equations, from 12 to 6, which will increase the efficiency of our numerical method compared to regular FDTD technique.

6.2. Numerical Example

In order to verify the numerical results of this method, we decided to calculate the resonance frequencies of a cubic cavity. To attain this goal, the electric and magnetic fields of a cavity by the mentioned method were calculated. Afterwards, FFT transform of the results were computed.

Here, we explain an example which is simulated with MATLAB software. Assume a cavity with PEC (perfect electric condition) as its boundary condition. We set imaginary part of $\vec{\psi}$ normal to the direction of cavity faces zero to make normal magnetic field zero on PEC sides.

For space discretion, we have assumed that $\vec{\psi}$ is a 3×3, with each dimension contains desired cells (for this problem, we have assumed the number of discretion in each dimension is 120). Thus mathematical operations, such as the main equation (7), or imposing boundary conditions are just simple matrix operations.

We have applied the below specifications for our numerical program:

1) $dt = 0.9\times\left(1/\sqrt{((1/dx^2)+(1/dy^2)+(1/dz^2))}/c\right)$, which dx and dy and dz are unit cell lengths (90% of Taflove margin)

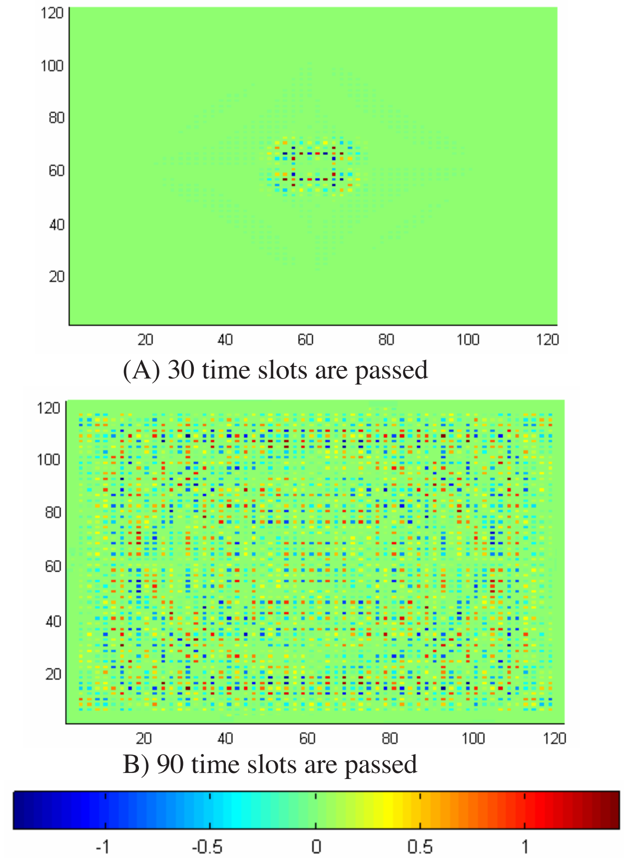

Figure 3. Demonstrates the propagation of Ex-wave in x-y plane stimulated by a current which is parallel to z-axis in the middle space slot of the cavity.

2) The conductivity of inside medium = 0.0001 (a small attenuation for stabilizing the stability of the system)
3) The number of iterations: 8000
4) Each side of the civility is $120 \times dl$, where dl is the length of the cubic cell in each dimension. ($dl = dx$ or $dl = dy$ or $dl = dz$).

6.2.1. The Amount of Energy vs. Time (an Indication of Stability)

One indication of stability is the amount of energy in the system vs. time. The energy in each time slot was computed. Figure 4 shows

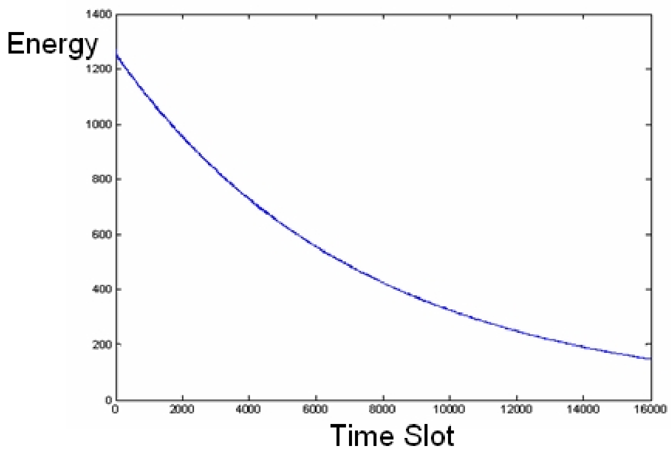

Figure 4. Stability indication, energy decreases versus time.

the result of computation of energy for the mentioned problem. Since we have used a small dissipation (sigma $= 0.0001\,\Omega^{-1}\mathrm{m}^{-1}$) to reduce numerical noise of the system, the decreasing the energy by passing time is seen. If we had instability in each part of $\vec{\psi}$, the energy would be infinity. As a consequence, the decaying energy certifies the stability of the proposed numerical technique, using applying Taflove stability condition [5]. Especially the positive conductivity acts as a negative feedback in the main formulation (7), in each step of the iteration and thus confirms the stability.

6.2.2. FFT Calculation of the Numerical Result

It is known [20] that the resonance frequencies can be calculated by the formula:

$$fc = \frac{c}{2} \times \sqrt{\left(\frac{m}{l_x}\right)^2 + \left(\frac{n}{l_y}\right)^2 + \left(\frac{p}{l_z}\right)^2} \tag{33}$$

Where l_x, l_y, and l_z are the dimensions and m, n, and p are the mode indices and c denotes the velocity of light in space. For our example, the first five frequencies are given in the Table 1.

As it seems from the table above, there are rather large simulation errors in this problem. The main reason of this error is regarding only 120 cells for each direction of the cavity; besides that, the loss of the medium affected the FFT results, by reshaping the picks of

Table 1. Resonance frequencies calculated by formulas and the difference percentage with numerical method.

fc (GHz) (obtained by numerical Method)	1.58	2.15	2.91	3.48
fc (GHz) (obtained by exact formula)	1.68	2.37	2.90	3.36
Error Percentage	6%	9%	3%	4%

the diagram of the FFT calculation; it should be mentioned that for numerical calculation, we have considered the worst case of the pick points.

7. CONCLUSION

In this paper, development of "Photon Wave Function" was introduced, for both homogeneous and inhomogeneous problems. This method is efficient especially in analyzing wave propagation and calculation of resonance frequencies in many-sided structures. By utilizing quantum mechanics formulation and rules for the introduced photon wave function, we are able to analyze electromagnetic problems more efficiently.

A numerical method similar to the FDTD algorithm was introduced based on the PWF by defining a complex variable, which consists of both electric and magnetic fields as its real and imaginary part, respectively; thus we can obtain both mentioned fields in each unit cell that means fewer unit cells for the same precision as we obtain in the conventional FDTD method. Another advantage of this numerical method is that a general boundary condition can be imposed on both fields. These features result in a more efficient numerical method.

The method and formulation in this paper are not only useful for numerical application, but also for demonstration of many similarities between quantum mechanics and electromagnetics and thus the same physical concept behind both realms.

APPENDIX A.

By combining the below equation:

$$-\vec{\nabla} \times \frac{\vec{E}}{\sqrt{\mu}} = \sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\frac{\vec{H}}{\sqrt{\varepsilon}} - \vec{\nabla}\left(\frac{1}{\sqrt{\mu}}\right) \times \vec{E} + \frac{\vec{J}_m}{\sqrt{\mu}} \tag{A1}$$

With

$$\vec{\nabla}\times\frac{\vec{H}}{\sqrt{\varepsilon}}=\sqrt{\mu\varepsilon}\frac{\partial}{\partial t}\frac{\vec{E}}{\sqrt{\mu}}+\vec{\nabla}\left(\frac{1}{\sqrt{\varepsilon}}\right)\times\vec{H}+\frac{\vec{J}_e}{\sqrt{\varepsilon}} \tag{A2}$$

We can produce:

$$\begin{aligned}\vec{\nabla}\times\left(\frac{\vec{E}}{\sqrt{\mu}}+j\frac{\vec{H}}{\sqrt{\varepsilon}}\right)&=j\frac{n}{c}\frac{\partial}{\partial t}\left(\frac{\vec{E}}{\sqrt{\mu}}+j\frac{\vec{H}}{\sqrt{\varepsilon}}\right)\\&\quad+j\left(\frac{\vec{J}_e}{\sqrt{\varepsilon}}+j\frac{\vec{J}_m}{\sqrt{\mu}}\right)+j\vec{\nabla}\left(\frac{1}{\sqrt{\mu}}\right)\times\vec{E}+j\vec{\nabla}\left(\frac{1}{\sqrt{\varepsilon}}\right)\times\vec{H}\end{aligned} \tag{A3}$$

With a little manipulation of the last two terms we have

$$\begin{aligned}\vec{\nabla}\left(\frac{1}{\sqrt{\mu}}\right)\times\vec{E}+j\vec{\nabla}\left(\frac{1}{\sqrt{\varepsilon}}\right)\times\vec{H}&=\frac{-1}{2}\left[\frac{\vec{\nabla}\mu}{\mu}\times\frac{\vec{E}}{\sqrt{\mu}}+\frac{\vec{\nabla}\varepsilon}{\varepsilon}\times j\frac{\vec{H}}{\sqrt{\varepsilon}}\right]\\&=\frac{1}{4}\left[\vec{\psi}\times\left(\frac{\vec{\nabla}\mu}{\mu}+\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)+\vec{\psi}^*\times\left(\frac{\vec{\nabla}\mu}{\mu}-\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)\right]\\&=\frac{1}{2}\left[\vec{\psi}\times\left(\frac{\vec{\nabla}n}{n}\right)+\vec{\psi}^*\times\left(\frac{\vec{\nabla}\eta}{\eta}\right)\right]\end{aligned} \tag{A4}$$

This completes the proof for equation (21).

Also, if we compute the divergence of both sides in equation (7), we will obtain:

$$\frac{\partial}{\partial t}\left[\frac{n}{c}\vec{\nabla}\cdot\vec{\psi}+\vec{\nabla}\left(\frac{n}{c}\right)\cdot\vec{\psi}-g\right]=0 \tag{A5}$$

By assuming the time variant system, or regarding the bias charge as zero (as a result of what is mentioned in duality transformation),

$$\frac{n}{c}\vec{\nabla}\cdot\vec{\psi}+\vec{\nabla}\left(\frac{n}{c}\right)\cdot\vec{\psi}-g=0 \tag{A6}$$

Thus,

$$\begin{aligned}\vec{\nabla}\cdot\vec{\psi}&=\frac{c}{n}g-\frac{c}{n}\vec{\nabla}\left(\frac{n}{c}\right)\cdot\vec{\psi}=\frac{c}{n}g-\frac{c}{2n}\left(\sqrt{\varepsilon}\frac{\vec{\nabla}\mu}{\sqrt{\mu}}+\sqrt{\mu}\frac{\vec{\nabla}\varepsilon}{\sqrt{\varepsilon}}\right)\cdot\vec{\psi}\\&=\frac{c}{n}g-\frac{1}{2}\left(\frac{\vec{\nabla}\mu}{\mu}+\frac{\vec{\nabla}\varepsilon}{\varepsilon}\right)\cdot\vec{\psi}\end{aligned} \tag{A7}$$

With manipulation, as (A4), we can obtain (equation (22)):

$$\vec{\nabla}\cdot\vec{\psi}=\frac{c}{n}g-\frac{1}{2}\left(\frac{\vec{\nabla}n}{n}\cdot\vec{\psi}+\frac{\vec{\nabla}\eta}{\eta}\cdot\vec{\psi}^*\right) \tag{A8}$$

APPENDIX B.

$$\begin{aligned}\vec{\nabla}\cdot\left(\frac{n}{c}\vec{\psi}\right) &= \vec{\nabla}\left(\frac{n}{c}\right)\cdot\vec{\psi}+\frac{n}{c}\vec{\nabla}\cdot\vec{\psi}\\ &= \vec{\nabla}\left(\frac{n}{c}\right)\cdot\vec{\psi}+g-\frac{n}{2c}\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n}+\vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\right)\\ &= g+\frac{n}{c}\frac{\vec{\nabla}n}{n}\cdot\vec{\psi}-\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n}\right)\frac{n}{2c}-\frac{n}{2c}\vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\\ &= g+\frac{n}{2c}\left(\vec{\psi}\cdot\frac{\vec{\nabla}n}{n}-\vec{\psi}^{*}\cdot\frac{\vec{\nabla}\eta}{\eta}\right)\end{aligned}\tag{B1}$$

This results in equation (23).

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