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SURFACE MODES AT THE INTERFACES BETWEEN ISOTROPIC MEDIA AND UNIAXIAL PLASMA

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Abstract—A detailed study of surface TM modes at the interface between an isotropic medium and a uniaxial plasma is presented. Four cases for the isotropic medium, including normal, Left-handed, magnetic, and metallic media, are considered. The conditions for the existence of surface modes in each case are analyzed, showing that the existence is determined by the parameters of media, working frequency, and the direction of the principle axis. The Poynting vector along the propagating direction is also calculated. Depending on the media parameters and the frequency, the surface mode can have time-average Poynting vector in the opposite direction of the mode phase velocity.

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1. INTRODUCTION

Surface waves (SWs) have attracted the attention of many scientists since the work of Jonathan Zenneck in 1907 [1] and Lord Rayleigh on elastic solids [2]. They can exist under certain conditions on an entirely free surface bounded by air, or at the interface separating two semi-infinite half-spaces. They propagate along the interface and decay in the transverse direction exponentially while having the field maximum on the interface [3–8]. They are useful for studying of the physical properties of the surfaces [9, 10]. Thus the investigations of SWs are important from both a scientific point of view and a practical one.

The general case of surface waves at the interface between a uniaxial crystal and isotropic medium were analyzed in detail in previous publications [11–13]. They concluded that surface wave propagation is possible only for positive uniaxial materials and that too for a narrow angular range of propagation directions about the bisector of the angle between the optical axes. The case between a uniaxial crystal and a magnetic isotropic medium has also been studied [14]. Furthermore, Walker et al. analyzed surface wave propagation at the interface of an isotropic material and an arbitrarily oriented uniaxial or biaxial material [4]. They showed that if both optical axes of the biaxial material are tangential to the interface, surface wave can occur over a large range of propagation directions and will confine wave more tightly than biaxial materials with optical axes in other orientations or uniaxial materials. And more extended cases of the interface between isotropic media and biaxial by Alshits et al. [15] and between two uniaxial crystals by Darinskii and Furs et al. respectively [16, 17] were also studied. And quite recently, Sudarshan et al. investigated surface wave at the interface of identical biaxial crystals with a relative twist about the axis normal to the interface [18]. They showed that the selected type of surface wave is possible only for a restricted range of the twist angle, which depends on the ratio of the maximum and the minimum of the principal refractive indexes and the angle between the optical axes.

Characteristics of surface modes at the interface between an isotropic medium and an indefinite medium that has a dispersion relation of hyperbolic form are studied by Yan et al. [19]. They considered four cases for the isotropic medium, and gave the conditions for the existence of surface modes in each case, indicating that the existence of surface modes is determined by the nature of the indefinite medium as well as the orientation of the boundary surface of this anisotropic medium.

To our knowledge, however, no detailed analysis about surface modes at boundary of uniaxial medium, which has dispersion characteristics, has yet been given. In this paper, we consider the uniaxial medium as an anisotropic plasma externally applied with an infinitely strong DC magnetic field $\bar{B}_0$. We study four kinds of isotropic media: normal, Left-handed (with negative permeability and permittivity), magnetic (with negative permeability), and metallic medium (with negative permittivity), and obtain the existence conditions of surface modes. Moreover, to get deeper insight into the physics of such waves, we also calculate their Poynting vector and the energy flow along the propagating direction.

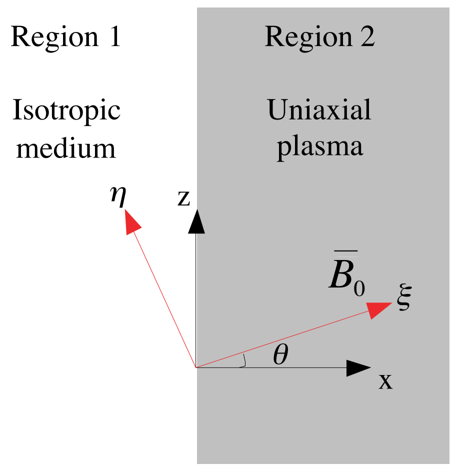


Figure 1. Diagram for an interface between a semi-infinite isotropic medium and a uniaxial plasma. The ξ axis indicates principle axis, also the direction of the external magnetic field which is infinitely strong in the uniaxial plasma.

2. SURFACE MODE DISPERSION LAWS

We consider a special case of uniaxial medium: an electron plasma applied with an external infinitely strong magnetic filed. In this case, $\omega_c \rightarrow \infty$ and the medium becomes a uniaxial plasma which has dispersion characteristics, and the permittivity tensor $\bar{\bar{\varepsilon}}_2$, takes the following form in the principal coordinate system: [20]

$$\bar{\bar{\varepsilon}}_{2\xi\zeta\eta} = \begin{bmatrix} \varepsilon(\omega) & 0 & 0 \\ 0 & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_0 \end{bmatrix}, \tag{1}$$

where $\varepsilon = \varepsilon_0(1 - \frac{\omega_p^2}{\omega^2})$ and ξ axis is in the same direction of the external magnetic filed $\bar{B}_0$. Furthermore, we consider electromagnetic waves propagating along the interface between an isotropic medium and the uniaxial plasma, as shown in Fig. 1. The $\xi\zeta\eta$ coordinate system is defined according to the principle axis of the uniaxial plasma, also the direction of the external magnetic field $\bar{B}_0$. Region 1 is the isotropic medium, with permittivity ε_1 and permeability μ_1; region 2 is the uniaxial plasma, with permeability μ_2 and permittivity tensor $\bar{\bar{\varepsilon}}_2$.

To get the solution of the fields in the two media, we establish a xyz coordinate system so that the interface is in the y-z plane and the x axis is perpendicular to it. For the xyz coordinate system, the permittivity tensor can be transformed to

$$\bar{\bar{\varepsilon}}_2 = \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_y & 0 \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{bmatrix}, \tag{2}$$

with

$$\varepsilon_{xx} = \varepsilon \cos^2\theta + \varepsilon_0 \sin^2\theta, \tag{3.1}$$
$$\varepsilon_{xz} = (\varepsilon - \varepsilon_0)\sin\theta\cos\theta, \tag{3.2}$$
$$\varepsilon_{xz} = \varepsilon_{zx}, \tag{3.3}$$
$$\varepsilon_{zz} = \varepsilon \sin^2\theta + \varepsilon_0 \cos^2\theta, \tag{3.4}$$
$$\varepsilon_y = \varepsilon_0, \tag{3.5}$$

where θ is the angle made by the magnetic field and the normal of the interface (see Fig. 1) and $-\pi/2 \leq \theta \leq \pi/2$.

The surface waves propagate along the z-axis with a wave number k_z, and the attenuation of the waves in the x-direction are defined by the quantities α_1, α_2. The wave vector and electrical field in the two regions can be written as

$$\left.\begin{aligned} \bar{k}_1 &= \hat{x}(-i\alpha_1) + \hat{z}k_z \\ \bar{E}_1 &= (\hat{x}E_{1x} + \hat{y}E_{1y} + \hat{z}E_{1z})e^{ik_z z + \alpha_1 x} \end{aligned}\right\} (x < 0) \tag{4}$$

$$\left.\begin{aligned} \bar{k}_2 &= \hat{x}(i\alpha_2) + \hat{z}k_z \\ \bar{E}_2 &= (\hat{x}E_{2x} + \hat{y}E_{2y} + \hat{z}E_{2z})e^{ik_z z - \alpha_2 x} \end{aligned}\right\} (x > 0) \tag{5}$$

For the uniaxial plasma, notice that the off-diagonal permittivity tensor, shown in Eq. (2), is symmetrical and there is only one element in y-direction. According to the Maxwell Equations, the electrical field components depend upon each other in the following way

$$\begin{pmatrix} -k_z^2 + \omega^2\mu_2\varepsilon_{xx} & i\alpha_2 k_z + \omega^2\mu_2\varepsilon_{xz} \\ i\alpha_2 k_z + \omega^2\mu_2\varepsilon_{zx} & \alpha_2^2 + \omega^2\mu_2\varepsilon_{zz} \end{pmatrix} \begin{pmatrix} E_{2x} \\ E_{2z} \end{pmatrix} = 0 \tag{6.1}$$

$$(\alpha_2^2 - k_z^2 + \omega^2\mu_2\varepsilon_y)E_{2y} = 0 \tag{6.2}$$

Equations (6.1) and (6.2) show that the TE mode (E_y, H_x, H_z) is not coupled to the TM mode (H_y, E_x, E_z) and it is the TM mode that is affected by the dispersion. That is why we focus on the TM wave in this paper.

Furthermore, by setting the determinant of the matrix in Eq. (6.1) equal to zero, we obtain the dispersion relation of the TM wave in the uniaxial plasma, which is

$$\omega^2 \mu_2 = \frac{(i\alpha_2 \cos\theta + k_z \sin\theta)^2}{\varepsilon_0} + \frac{(-i\alpha_2 \sin\theta + k_z \cos\theta)^2}{\varepsilon}. \tag{7}$$

And it is known that the dispersion relation of the isotropic medium is

$$k_z^2 - \alpha_1^2 = \omega^2 \mu_1 \varepsilon_1. \tag{8}$$

For TM wave, the magnetic field in the two regions can be written as

$$\bar{H}_1 = \hat{y} A e^{ik_z z + \alpha_1 x} \quad (x < 0), \tag{9}$$

$$\bar{H}_2 = \hat{y} A e^{ik_z z - \alpha_2 x} \quad (x > 0). \tag{10}$$

So, substituting Eqs. (9) and (10) into Maxwell Equations, the electrical fields can be obtained as

$$\bar{E}_1 = \frac{1}{\omega\varepsilon_1} (\hat{x} k_z + \hat{z} i\alpha_1) A e^{ik_z z + \alpha_1 x} \ (x < 0), \tag{11}$$

$$\bar{E}_2 = \frac{A}{\omega}\left(\hat{x}\frac{-\varepsilon_{zz}k_z - i\varepsilon_{xz}\alpha_2}{\varepsilon_{xz}^2 - \varepsilon_{xx}\varepsilon_{zz}} + \hat{z}\frac{\varepsilon_{xz}k_z + i\varepsilon_{xx}\alpha_2}{\varepsilon_{xz}^2 - \varepsilon_{xx}\varepsilon_{zz}}\right) e^{ik_z z - \alpha_2 x} \ (x > 0). \tag{12}$$

By matching the boundary conditions at interface of $x = 0$, i.e., $E_{1z} = E_{2z}$, we obtain the relation of propagation constant k_z and α_1, α_2, which is

$$\frac{i\alpha_1}{\varepsilon_1} = \frac{\varepsilon_{xz}k_z + i\varepsilon_{xx}\alpha_2}{\varepsilon_{xz}^2 - \varepsilon_{xx}\varepsilon_{zz}} = \frac{\varepsilon_{xz}k_z + i\varepsilon_{xx}\alpha_2}{-\varepsilon\varepsilon_0}. \tag{13}$$

Finally, solutions of equations (7), (8) and (13) are [19]

$$k_z^2 = \omega^2 \frac{\varepsilon_1(\varepsilon_1 \varepsilon_{xx} \mu_2 - \varepsilon\varepsilon_0 \mu_1)}{\varepsilon_1^2 - \varepsilon\varepsilon_0}, \tag{14}$$

$$\alpha_1 = \omega\,|\varepsilon_1| \sqrt{\frac{(\varepsilon_{xx}\mu_2 - \varepsilon_1 \mu_1)}{\varepsilon_1^2 - \varepsilon\varepsilon_0}}, \tag{15}$$

$$\alpha_2 = \alpha_{2r} + i\alpha_{2i} = \frac{-\alpha_1 \varepsilon\varepsilon_0}{\varepsilon_1 \varepsilon_{xx}} + i\frac{\varepsilon_{xz}k_z}{\varepsilon_{xx}}, \tag{16}$$

where both α_{2r}, α_{2i} are real.

3. CONDITIONS FOR EXISTENCE OF TM SURFACE MODES

The necessary condition of surface mode existence is k_z^2, α_1 and α_{2r} must be positive. We will discuss the condition of surface TM mode existence for four cases of isotropic medium: (i) $\varepsilon_1 > 0$, $\mu_1 > 0$; (ii) $\varepsilon_1 < 0$, $\mu_1 < 0$; (iii) $\varepsilon_1 > 0$, $\mu_1 < 0$; (iv) $\varepsilon_1 < 0$, $\mu_1 > 0$, which correspond to normal, left-handed, magnetic, and metallic media respectively. Without losing any generality, we assume that the permeability of the uniaxial plasma is positive, i.e., $\mu_2 > 0$.

The conditions of surface TM mode existence for the four cases are shown in Table 1. And the critical angle θ_{c1} and θ_{c2} are defined as

$$\cos^2\theta_{c1} = \left(1 - \frac{\varepsilon_1\mu_1}{\varepsilon_0\mu_2}\right)\left(\frac{\omega^2}{\omega_p^2}\right) \tag{17}$$

$$\cos^2\theta_{c2} = \left[1 - \frac{\varepsilon_0\mu_1}{\varepsilon_1\mu_2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)\right]\left(\frac{\omega^2}{\omega_p^2}\right) \tag{18}$$

Table 1. Conditions for the existence of the surface TM mode.

Case	Parameter Condition		ω	θ
(i) $\varepsilon_1 > 0$, $\mu_1 > 0$	$\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1$		$\omega < \omega_p$	$\pi/2 > \lvert\theta\rvert > \theta_{c1}$
(ii) $\varepsilon_1 < 0$, $\mu_1 < 0$	$\lvert\varepsilon_1\rvert > \varepsilon_0, \varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1$		$\omega > \omega_p$	$\pi/2 > \lvert\theta\rvert > \theta_{c1}$
	$\lvert\varepsilon_1\rvert < \varepsilon_0$	$\varepsilon_1\mu_1/(\varepsilon_0\mu_2)<1$	$\omega\in(\omega_p, \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2-\varepsilon_1^2})$	$\pi/2 > \lvert\theta\rvert > \theta_{c1}$
		$\varepsilon_1\mu_1/(\varepsilon_0\mu_2) > 1/[1-(\omega_p^2/\omega^2)]$	$\omega > \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2-\varepsilon_1^2}$	$\lvert\theta\rvert < \theta_{c1}$
(iii) $\varepsilon_1 > 0$, $\mu_1 < 0$	$\varepsilon_0\mu_1/(\varepsilon_1\mu_2)>1/[1-(\omega_p^2/\omega^2)]$		$\omega < \omega_p$	$\pi/2 > \lvert\theta\rvert > \theta_{c2}$
(iv) $\varepsilon_1 < 0$, $\mu_1 > 0$	$\lvert\varepsilon_1\rvert > \varepsilon_0$		$\omega > \omega_p$	$\pi/2 > \lvert\theta\rvert > 0$
	$\lvert\varepsilon_1\rvert < \varepsilon_0$		$\omega\in(\omega_p, \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2-\varepsilon_1^2})$	$\pi/2 > \lvert\theta\rvert > 0$

A. Case (i): $\varepsilon_1 > 0$, $\mu_1 > 0$ (normal medium)

First, we consider the case of $\varepsilon_1 > \varepsilon_0$. As $(\varepsilon_1^2 - \varepsilon\varepsilon_0) = [\varepsilon_1^2 - \varepsilon_0^2(1 - \omega_p^2/\omega^2)] > 0$, according to Eq. (14), the positiveness of k_z^2 requires $\varepsilon_{xx} > \varepsilon\varepsilon_0\mu_1/(\varepsilon_1\mu_2)$. According to Eqs. (15) and (16), the positiveness of α_1 and α_{2r} requires $\varepsilon_{xx} > \varepsilon_1\mu_1/\mu_2$, $\varepsilon_{xx}\varepsilon < 0$ respectively. Because ε is the function of the working frequency ω, the existence condition is based on the frequency. When $\omega > \omega_p$, then $\varepsilon > 0$, positiveness of α_{2r} requires $\varepsilon_{xx} < 0$, while positiveness of α_1 requires $\varepsilon_{xx} > \varepsilon_1\mu_1/\mu_2 > 0$, thus there is no surface wave under this condition, no matter what the angle θ is. When $\omega < \omega_p$, $\varepsilon < 0$, the solution of these inequalities is $\varepsilon_{xx} > \varepsilon_1\mu_1/\mu_2$. According to Eq. (3.1), the solution can be transformed as $(\varepsilon\cos^2\theta + \varepsilon_0\sin^2\theta) > \varepsilon_1\mu_1/\mu_2$, i.e., $(\varepsilon - \varepsilon_0)\cos^2\theta > (\varepsilon_1\mu_1/\mu_2 - \varepsilon_0)$. Hence, the condition for existence can be expressed as $\cos^2\theta < \left(1 - \frac{\varepsilon_1\mu_1}{\varepsilon_0\mu_2}\right)\left(\frac{\omega^2}{\omega_p^2}\right)$. Since $0 \le \cos^2\theta \le 1$, there is a necessary condition for the surface wave existence, which is $\left(1 - \frac{\varepsilon_1\mu_1}{\varepsilon_0\mu_2}\right)\left(\frac{\omega^2}{\omega_p^2}\right) > 0$, i.e., $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1$. It means that if $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) > 1$, there is no surface wave at all, no matter what the angle θ is.

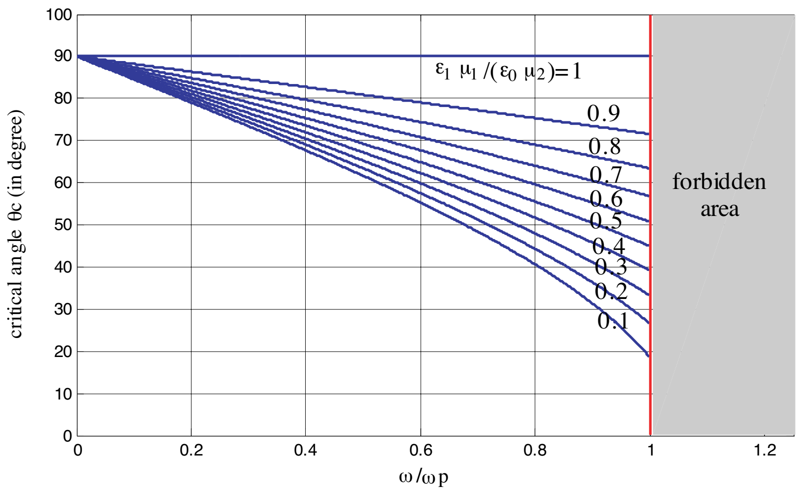
Second, we consider the other case, i.e., $0 < \varepsilon_1 < \varepsilon_0$. When $\omega < \omega_p$, then $\varepsilon < 0$ and $(\varepsilon_1^2 - \varepsilon\varepsilon_0) > 0$, the condition for existence is the same as $\omega < \omega_p$ in case of $\varepsilon_1 > \varepsilon_0$. When $\omega_p < \omega < \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2}$, we find $\varepsilon > 0$ and $(\varepsilon_1^2 - \varepsilon\varepsilon_0) > 0$, the same as $\omega > \omega_p$ in case of $\varepsilon_1 > \varepsilon_0$, so there is no surface wave either. When $\omega > \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2}$, we get $\varepsilon > 0$ and $(\varepsilon_1^2 - \varepsilon\varepsilon_0) < 0$. Based on the Eq. (14), (15) and (16), the condition is $\varepsilon_{xx} < 0$, i.e., $(\varepsilon\cos^2\theta + \varepsilon_0\sin^2\theta) < 0$, which can not be satisfied because $\varepsilon > 0$. So, there is no surface wave under this condition.

To sum up, the condition of surface TM wave existence for case (i) is found as

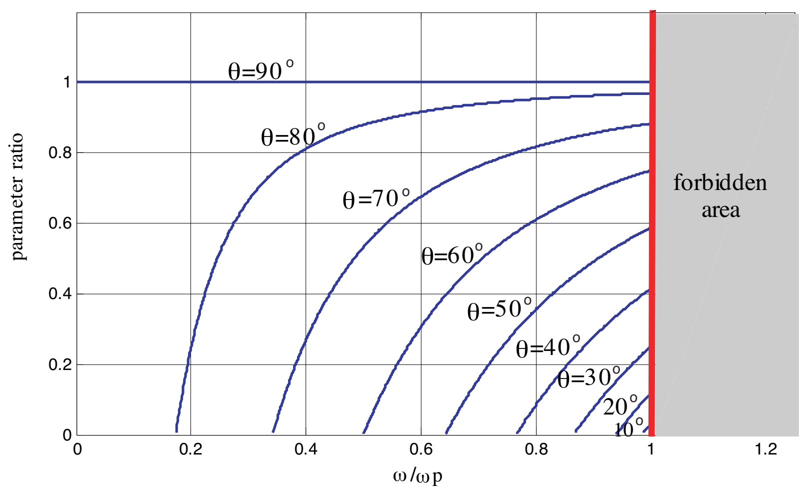
$$\cos^2\theta < \left(1 - \frac{\varepsilon_1\mu_1}{\varepsilon_0\mu_2}\right)\left(\frac{\omega^2}{\omega_p^2}\right), \tag{19}$$

with the necessary condition of $\omega < \omega_p$, $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1$.

Figure 2 is the plot of critical angle θ_c for the existence of the surface TM mode in case (i). Fig. 3 is the dispersion characteristics for the surface TM modes in case (i). In Fig. 2(a), we can see that the line $\theta_c = 45°$ has intersections with lines $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) = 0.1, 0.2, 0.3$ and 0.4. That means that when $\varepsilon_1\mu_1/(\varepsilon_0\mu_2)$ equals to those values, there must exist surface mode, and the frequency of the intersection in Fig. 2 is the cut-off frequency under such situation. Those can be verified clearly by Fig. 3(a).

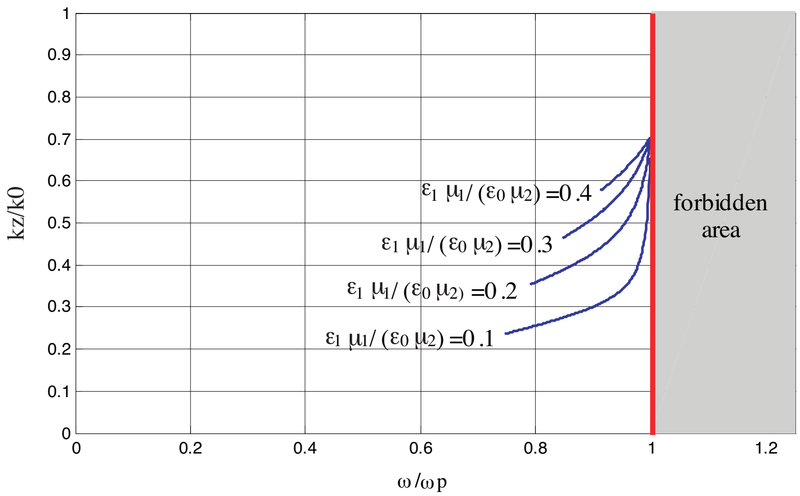


(a)

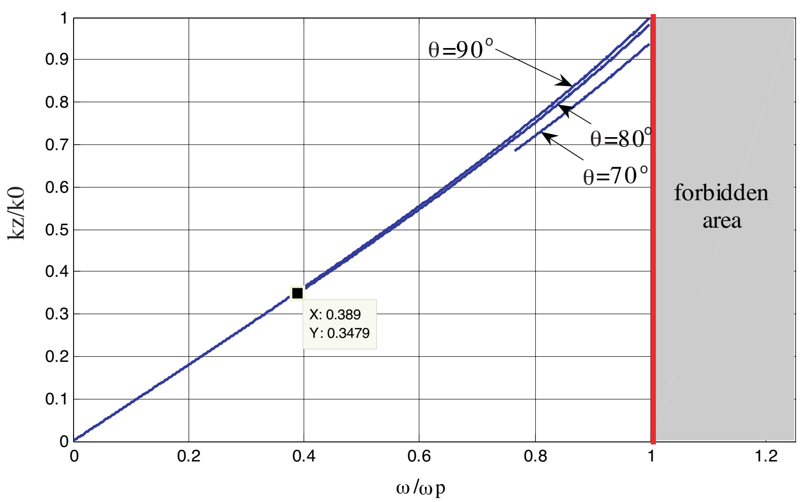


(b)

Figure 2. Contour plot of critical angle θ_c for the existence of the surface TM mode in case (i). The surface mode can exist when $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1$ and $\pi/2 > |\theta| > \theta_c$ for (a) different parameter ratio $\varepsilon_1\mu_1/(\varepsilon_0\mu_2)$; (b) different external magnetic field direction (θ is the angle between the magnetic field and the normal of the interface). Shaded area in each figure indicates the frequency domain where the surface waves are forbidden.



(a)



(b)

Figure 3. Dispersion characteristics for the surface TM modes in case (i): (a) when the direction of the external magnetic field is fixed ($\theta = 45°$); (b) when the parameter ratio is fixed ($\varepsilon_1\mu_1/(\varepsilon_0\mu_2) = 0.8$). Each case has a cut-off frequency except $\theta = 90°$. Shaded area in each figure indicates the frequency domain where the surface waves are forbidden.

Similarly, in Fig. 2(b), the line $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) = 0.8$ has intersections with curves $\theta = 80°$, $\theta = 70°$. According to the existence conditions, i.e., $\theta > \theta_c$, we can see that when $\varepsilon_1\mu_1/(\varepsilon_0\mu_2) = 0.8$, if $\theta = 70°$ or $80°$, there must exist SW with the cut-off frequency equals to that of the intersection. When $\theta = 90°$, there is SW without cut-off frequency. Also, those can be verified by Fig. 3(b).

B. Case (ii): $\varepsilon_1 < 0$, $\mu_1 < 0$ (left-handed medium)

Taking the procedure similar to that in case (i), we can find that the condition for the existence of surface wave, shown in Table 1. The condition depends on the comparison of absolute value of ε_1 with ε_0. For different situation, there is different frequency domain and angle range needed.

C. Case (iii): $\varepsilon_1 > 0$, $\mu_1 < 0$ (magnetic medium)

When the region 1 is a magnetic medium, (see Table 1) the frequency must be lower than ω_p, and there is also further restraint to the medium parameter and angle, similar to case (i), except the expression of critical angle and medium parameter condition.

D. Case (iv): $\varepsilon_1 < 0$, $\mu_1 > 0$ (metallic medium)

In this case, the existence of the surface wave is independent of the direction of the magnetic field, i.e., there is no restraint to the angle θ, shown in Table 1. That means that once the parameter and frequency conditions are satisfied, there will be a surface mode, no matter what the direction of the principle axis of the uniaxial plasma is.

4. POYNTING VECTOR AND ENERGY

In order to get deeper insight into the physics of wave process, we proceed with calculation of energy flow along the surface. Without losing any generality, the propagating direction of the wave is assumed to be z-direction, i.e., $k_z > 0$ in the following.

The time-averaged Poynting vectors of surface wave are

$$< \bar{S}_1 > = \hat{z} < \bar{S}_{1z} > = \hat{z}\frac{k_z\left|A\right|^2}{2\omega\varepsilon_1}e^{2\alpha_1 x} \quad (x < 0), \tag{20}$$

$$< \bar{S}_2 > = \hat{z} < \bar{S}_{2z} > = \hat{z}\frac{k_z\left|A\right|^2}{2\omega\varepsilon_{xx}}e^{-2\alpha_{2r} x} \ (x > 0). \tag{21}$$

Furthermore, the corresponding total energy flow associated with the whole mode is determined by integration over the x-coordinate

$$\begin{aligned} <\bar{S}> &= \int_{-\infty}^{0} <\bar{S}_1> dx + \int_{0}^{\infty} <\bar{S}_2> dx \\ &= \hat{z}\frac{k_z|A|^2}{4\omega}\left(\frac{1}{\varepsilon_1\alpha_1} + \frac{1}{\varepsilon_{xx}\alpha_{2r}}\right) \\ &= \hat{z}\frac{k_z|A|^2}{4\omega\alpha_1}\frac{\varepsilon\varepsilon_0 - \varepsilon_1^2}{\varepsilon_1\varepsilon\varepsilon_0} \end{aligned} \tag{22}$$

According to Eq. (16), if there exists surface wave, both α_1 and α_{2r} are positive, then $\varepsilon_1\varepsilon_{xx}\varepsilon < 0$.

Notice that in both case (i) and (iii), $\varepsilon_1 > 0$ and $\omega < \omega_p$, which causes $\varepsilon < 0$. So we can get $\varepsilon_{xx} > 0$. Hence, from Eqs. (20) and (21), we can see that the Poynting vectors in both media are in the same direction, i.e., the propagating direction. They are both forward waves, and the guided energy flows in both regions are always in the same direction, which can be seen obviously in Eq. (22).

For case (ii) and case (iv), $\varepsilon_1 < 0$ and $\omega > \omega_p$ (see Table 2), so $\varepsilon > 0$. Then we have $\varepsilon_{xx} > 0$. So $S_{1z} < 0$ corresponding to $\varepsilon_1 < 0$, and $S_{2z} > 0$ corresponding to $\varepsilon_{xx} > 0$. That means that in the isotropic medium of region 1, the Poynting vector is in the opposite direction of propagation, and the surface wave is a backward wave; while in the uniaxial plasma of region 2, the directions of Poynting

Table 2. Direction of the poynting vector and energy flow of the surface TM mode (The direction of the wave propagating is assumed to be z-direction.)

<table>
<tr><th>Case</th><th colspan="2">Parameter and Frequency Condition</th><th>$<\bar{S}_1>$</th><th>$<\bar{S}_2>$</th><th>$<\bar{S}>$</th></tr>
<tr><td>(i) $\varepsilon_1>0$, $\mu_1>0$</td><td colspan="2">$\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1,\ \omega < \omega_p$</td><td>$\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
<tr><td rowspan="3">(ii) $\varepsilon_1<0$, $\mu_1<0$</td><td colspan="2">$|\varepsilon_1| > \varepsilon_0,\ \varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1,\ \omega > \omega_p$</td><td>$-\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
<tr><td rowspan="2">$|\varepsilon_1|<\varepsilon_0$</td><td>$\varepsilon_1\mu_1/(\varepsilon_0\mu_2) < 1,\ \omega \in (\omega_p, \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2})$</td><td>$-\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
<tr><td>$\varepsilon_1\mu_1/(\varepsilon_0\mu_2)<1/[1-(\omega_p^2/\omega^2)], \omega > \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2-\varepsilon_1^2})$</td><td>$-\hat{z}$</td><td>$\hat{z}$</td><td>$-\hat{z}$</td></tr>
<tr><td>(iii) $\varepsilon_1>0$, $\mu_1<0$</td><td colspan="2">$\varepsilon_0\mu_1/(\varepsilon_1\mu_2) > 1/[1 - (\omega_p^2/\omega^2)],\ \omega < \omega_p$</td><td>$\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
<tr><td rowspan="2">(iv) $\varepsilon_1<0$, $\mu_1>0$</td><td colspan="2">$|\varepsilon_1| > \varepsilon_0,\ \omega > \omega_p$</td><td>$-\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
<tr><td colspan="2">$|\varepsilon_1| < \varepsilon_0,\ \omega \in (\omega_p, \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2})$</td><td>$-\hat{z}$</td><td>$\hat{z}$</td><td>$\hat{z}$</td></tr>
</table>

vector and propagation are the same, and the surface wave is a forward wave. Based on Eq. (22), we can see that if $\varepsilon_1\varepsilon(\varepsilon\varepsilon_0 - \varepsilon_1^2) < 0$, i.e., $(\varepsilon\varepsilon_0 - \varepsilon_1^2) > 0$, the total energy flow is in the opposite direction of propagation. If $|\varepsilon_1| > \varepsilon_0$, there must be $(\varepsilon\varepsilon_0 - \varepsilon_1^2) < 0$ no matter what the frequency is. That means the total energy flow is in the same direction of propagation. If $|\varepsilon_1| < \varepsilon_0$, only when $\omega > \omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2}$, is $(\varepsilon\varepsilon_0 - \varepsilon_1^2)$ positive, otherwise it is negative. So, we can see from Table 2 that in most cases of (ii) and (iv), the total energy flow is in the propagation direction, but it is opposite to that of propagation when region 1 is a Left-Handed Material, which has the parameter $|\varepsilon_1| < \varepsilon_0$, and the frequency larger than $\omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2}$ is needed.

5. CONCLUSION

This paper gives an investigation on how surface TM modes response in different frequency domains at the interface between an isotropic medium and a uniaxial plasma which has dispersion characteristics. We consider a special case of uniaxial medium: an electron plasma applied with an external infinitely strong magnetic filed. Four cases for the isotropic medium, including normal, LHM, magnetic, and metallic media, are considered. The conditions for the existence of surface modes in each case are analyzed, showing that the existence is determined by the parameter of media, working frequency, and the direction of the external magnetic field. The Poynting vector in the propagating direction is also calculated and we have found that if region 1 is normal or magnetic medium, i.e., $\varepsilon_1 > 0$, the surface waves in both regions are forward waves and the power flow along the propagation direction; if region 1 is metallic medium or LHM, the surface mode in region 1 is backward wave but forward in region 2, and the power mostly flow in the propagation direction except the case when region 1 is a Left-Handed Material which has the parameter $|\varepsilon_1| < \varepsilon_0$, and the frequency larger than $\omega_p\varepsilon_0/\sqrt{\varepsilon_0^2 - \varepsilon_1^2}$ is needed.

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