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AN ENHANCED METHOD FOR INVERSE SCATTERING PROBLEMS USING FOURIER SERIES EXPANSION IN CONJUNCTION WITH FDTD AND PSO

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Abstract—A new computationally efficient algorithm for reconstruction of lossy and inhomogeneous 1-D media by using inverse scattering method in time domain is proposed. In this algorithm, cosine Fourier series expansion is utilized in conjunction with finite difference time domain (FDTD) and particle swarm optimization (PSO) methods. The performance of the proposed algorithm is studied for several 1-D permittivity and conductivity profile reconstruction cases. Various types of regularization terms are examined and compared with each other in the presented method. It is shown that the number of unknowns in optimization routine is reduced to about 1/3 as compared with conventional methods which leads to a considerable reduction in the amount of computations, while the precision of the solutions would not be affected significantly. Another advantage of the proposed expansion method is that, since only a limited number of terms are taken in the expansion, the divergence of the algorithm is far less likely to occur. Sensitivity analysis of the suggested method to the number of expansion terms in the algorithm is studied, as well.

1. INTRODUCTION

The aim of inverse scattering problems in electromagnetics is to extract the unknown parameters of a medium from measured back scattered fields of an incident wave illuminating the target. The unknowns to be extracted could be any parameter affecting the propagation of waves in the medium.

Inverse scattering has found vast applications in different branches of science such as medical tomography, non-destructive testing, object detection, geophysics, and optics [1–7].

From a mathematical point of view, three main topics must be preliminary addressed in an inverse scattering problem; the non-uniqueness, the ill-posedness, and the intrinsic nonlinearity [8–10]. Generally speaking, the non-uniqueness and the ill-posedness of the inverse problems are due to the limited amount of information that can be collected. In fact, the amount of independent data achievable from the measurements of the scattered fields in some observation points is essentially limited. Hence, only a finite number of parameters can be accurately retrieved. Other reasons such as noisy data, unreachable observation data, and inexact measurement methods increase the ill-posedness of such problems. Also, increasing the number of unknown parameters leads to the more ill-posedness and as a consequence, the divergence is more likely to occur. To stabilize the inverse problems, based on a priori information about desired parameters, usually various kinds of regularizations are used [11–14]. For example, the solution may be required to stay close to an a priori known value or by penalizing local variations. On the other hand, due to the multiple scattering phenomena, the inverse scattering problem is nonlinear in nature. Therefore, when multiple scattering effects are not negligible, the use of nonlinear methodologies is mandatory.

Recently, inverse scattering problems are considered in global optimization-based procedures. The unknown parameters of each cell of the medium grid would be directly considered as the optimization parameters and several types of regularizations are used to overcome the ill-posedness. All of these regularization terms commonly use a priori information to confine the range of mathematically possible solutions to a physically acceptable one.

In this case, the general form of cost function for optimization routine is expressed as

$$\begin{aligned} & F\left(\varepsilon_r, \sigma, \mu, \vec{E}, \vec{H}\right) \\ = & \sum_{i=1}^{I}\sum_{j=1}^{J}\int_0^T \left(\left\|\vec{E}_{ij}^{sim} - \vec{E}_{ij}^{meas}\right\|^2 + \eta_0^2\left\|\vec{H}_{ij}^{sim} - \vec{H}_{ij}^{meas}\right\|^2\right)dt \\ & +\lambda * R\left(\varepsilon_r, \sigma, \mu\right) \end{aligned} \tag{1}$$

where $\vec{E}^{sim}$ and $\vec{H}^{sim}$ are the simulated fields obtained by the FDTD method [15] in each optimization iteration. $\vec{E}^{meas}$ and $\vec{H}^{meas}$ are measured fields, I and J are the number of transmitters and receivers, respectively and T is the total time of measurement. $R\left(\varepsilon_r, \sigma, \mu\right)$ is the regularization term and λ is the regularization factor.

In this paper, we have used and compared Tikhonov energy regularization [16], second order Sobolev regularization [17], and total variation (TV) regularization [18] to overcome the ill-posedness of the inverse problem. If we consider a permittivity profile as shown in Fig. 1, the Tikhonov energy regularization is represented as

$$R(\varepsilon_r) = \int_{x=0}^{x=a} |\varepsilon_r(x)|\, dx \tag{2}$$

which is justified by the fact that the physically acceptable solutions must have limited energy. The second order Sobolev regularization is defined by

$$R(\varepsilon_r) = \int_{x=0}^{x=a} |\varepsilon_r''(x)|\, dx \tag{3}$$

which uses the second-order derivative of permittivity and requires the object to be reasonably smooth, and finally, the TV regularization is presented by

$$R(\varepsilon_r) = \int_{x=0}^{x=a} |\nabla\varepsilon_r(x)|\, dx \tag{4}$$

where ∇ denotes the gradient operator. TV regularization is particularly useful when reconstruction of profiles with discontinuities or step gradients is considered.

Unfortunately, the conventional optimization-based methods suffer from two main drawbacks. The first is the huge number of the unknowns especially for 2-D and 3-D cases which increases not only the amount of computations, but also the degree of ill-posedness. Another disadvantage is the determination of regularization factor which is not straightforward at all. Therefore, proposing an algorithm which reduces the amount of computations along with the sensitivity of the problems to the regularization term and initial guess of the optimization routine would be desirable.

In this paper, we propose an algorithm for computationally efficient reconstruction. The general form of the problem and our proposed method in which cosine Fourier series expansion is used in conjunction with FDTD and PSO, are introduced in Section 2. In Section 3, the mathematical formulations of the algorithm are derived in detail. The results of this section help us to direct the optimization routine for having faster convergence. Three inhomogeneous and lossless or lossy test cases are considered in Section 4 and the efficiency of the method is studied for all of them. Finally in Section 5, sensitivity considerations and an important issue regarding the ill-posedness in the expansion method for inverse scattering problems, are addressed.

2. COSINE FOURIER SERIES EXPANSION METHOD

We consider the permittivity and conductivity profiles reconstruction of a lossy and inhomogeneous 1-D medium as shown in Fig. 1.

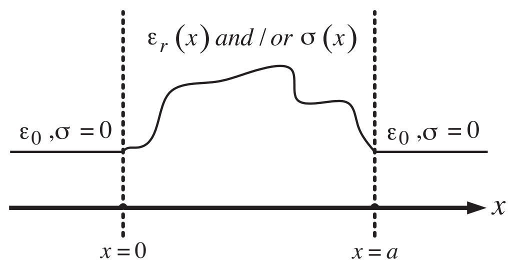


Figure 1. General form of the problem. Reconstruction of the permittivity and conductivity profiles of a lossy and inhomogeneous 1-D medium surrounded by known media (here free space) is considered.

Instead of direct optimization of the unknowns, we expand them in terms of a complete set of orthogonal basis functions and optimize the coefficients of this expansion in a global optimization routine like PSO [19–21]. In a general 3-D structure, the relative permittivity could be expressed as

$$\varepsilon_r(x, y, z) = \sum_{n=0}^{N} d_n f_n(x, y, z) \tag{5}$$

where f_n is the nth term of the complete orthogonal basis functions.

It is clear that in order to expand any profile into this set, the basis functions must be complete. On the other hand, orthogonality is favourable because with this condition, a finite series will always represent the object with the best possible accuracy and coefficients will remain unchanged while increasing the number of expansion functions [22].

Because of the straightforward relation to the measured data and its simple boundary conditions, using harmonic functions over other orthogonal sets of basis functions is preferable. On the other hand, cosine basis functions have simpler mean value relation in comparison with sine basis functions which is an important condition in our algorithm. It is known that any real and positive function can be extended into an even periodic function and this periodic function if satisfies the Dirichlet conditions, can be represented by a cosine (even) Fourier series expansion. If cosine basis functions are used in one-dimensional cases, the expansion of the permittivity profile along x

which is homogeneous along the transverse plane is

$$\varepsilon_r(x) = \sum_{n=0}^{N} d_n \cos\left(\frac{n\pi x}{a}\right) \tag{6}$$

where a is the dimension of the problem in the x direction and the coefficients, d_n, are to be optimized. In this case, the number of optimization parameters is N in comparison with conventional methods in which this number is equal to the number of grid points. This results in a considerable reduction in the amount of computations. The proposed algorithm is shown in Fig. 2.

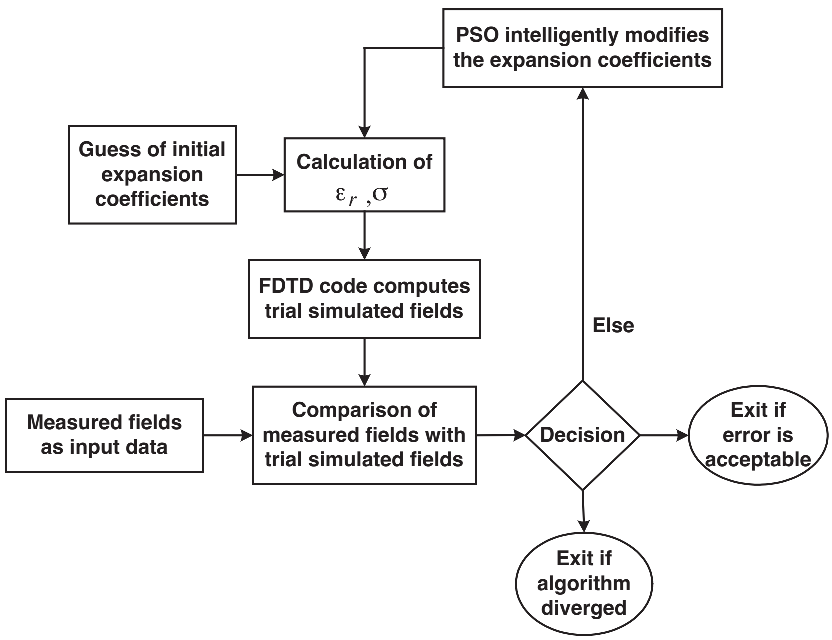


Figure 2. Proposed algorithm for reconstruction by expansion method. We use FDTD as a forward solver in time domain and PSO as a global optimization routine in inverse problem.

According to Fig. 2, based on an initial guess for a set of expansion coefficients, d_n, for a lossless case, the permittivity is calculated according to (6). Then, the FDTD code computes a trial electric and magnetic simulation fields. Here we use a Gaussian pulse for plane wave excitation and magic time-step ($\Delta x = c * 2\Delta t$) [15] for easy truncation of the problem space in FDTD simulations. Δx and Δt are the grid length and time step, respectively. In our study, because of the lack of real measurement data, the measured fields are also obtained by an FDTD simulation. At each iteration of optimization routine, the cost function is calculated according to (1). Then PSO as a global optimizer is used to minimize this cost function by changing the coefficients of permittivity and conductivity profiles expansions. The modified value of expansion coefficient, d_n, is computed for each of the $N+1$ dimensions of the unknown vector according to the following equation

$$d_n = d_n + \Delta t * v_n \tag{7}$$

where $N+1$ is the number of expansion terms in (6) and v_n is the velocity of the particle in the nth dimension which may be expressed as

$$\begin{aligned} v_n \; = \; & w * v_n + c_1 * rand() * (p_{best,n} - d_n) \\ & + c_2 * rand() * (g_{best,n} - d_n) \end{aligned} \tag{8}$$

in which w is weighting factor, c_1 and c_2 are acceleration factors, $rand()$ is the random number uniformly distributed within the interval [0,1], and $p_{best,n}$, $g_{best,n}$ are personal best position of each particle and global best position of all the particles in the nth dimension, respectively [19].

It is obvious that the performance of this expansion method directly depends on the value of N. Larger N, results in a more precise reconstruction at the expense of higher degree of ill-posedness. On the other hand, decreasing N leads to a less accurate solution with higher probability of convergence of the inverse algorithm. Therefore, suitable selection of N has a notable impact on the convergence speed of the algorithm.

3. MATHEMATICAL CONSIDERATIONS

As mentioned before, inverse problems are non-unique and ill-posed. Thus, a priori information must be applied for stabilizing the algorithm as much as possible which is quite straightforward in direct optimization method. In this case, all the information can be applied directly on the medium parameters which are the same as optimization parameters. In the expansion algorithm, however, the optimization parameters are the Fourier series expansion coefficients and a priori information could not be considered directly. Hence, a useful indirect routine is vital to overcome this difficulty.

There are two main assumptions about the parameters of an unknown medium. We may assume first that the relative permittivity and conductivity have limited ranges of variation, i.e.,

$$1 \le \varepsilon_r(x) \le M \tag{9}$$

and

$$0 \leq \sigma(x) \leq P \tag{10}$$

The second assumption is that the permittivity and conductivity profiles may not have severe fluctuations or oscillations. These two important conditions must be transformed in such a way to be applicable on the expansion coefficients in the initial guess and during the optimization process.

It is known that average of a function with known limited range is located within that limit, that is if

$$L_1 \leq g(x) \leq L_2, \quad a \leq x \leq b \tag{11}$$

Then

$$L_1 \leq \frac{1}{b-a}\int_a^b g(x)dx \leq L_2 \tag{12}$$

Thus, for permittivity profile expansion we have

$$1 \leq d_0 \leq M \tag{13}$$

For $x = 0$, (6) reduces to

$$\varepsilon_r(0) = \sum_{n=0}^{N} d_n \Rightarrow 1 \leq \sum_{n=0}^{N} d_n \leq M \tag{14}$$

and for $x = a$, we have

$$\varepsilon_r(a) = \sum_{n=0}^{N} (-1)^n d_n \Rightarrow 1 \leq \sum_{n=0}^{N} (-1)^n d_n \leq M \tag{15}$$

Using Parseval theorem, another relation between expansion coefficients and upper bound of permittivity may be written. Assuming that $g(x)$ is periodic with period T, we have

$$\frac{1}{T}\int_T |g(x)|^2 dx = \sum_{n=0}^{\infty} |d_n|^2 \tag{16}$$

Based on (6), (16) may be simplified to

$$1 \leq \sum_{n=0}^{N} |d_n|^2 \leq M^2 \tag{17}$$

By using (13), (14), (15), and (17) in the initial guess of the expansion coefficients and as the damping boundary condition [19] during the optimization, the routine converges in a considerable faster rate. Similar conditions can be used for conductivity profiles in lossy cases.

4. SIMULATION RESULTS

Proposed method stated above is utilized for three different cases. In each case, the direct optimization method and the proposed expansion method are compared in terms of the number of unknowns and reconstruction precision by using different types of regularization introduced in Section 1. In the simulations of all test cases, one transmitter and two receivers are used around the medium as shown in Fig. 3.

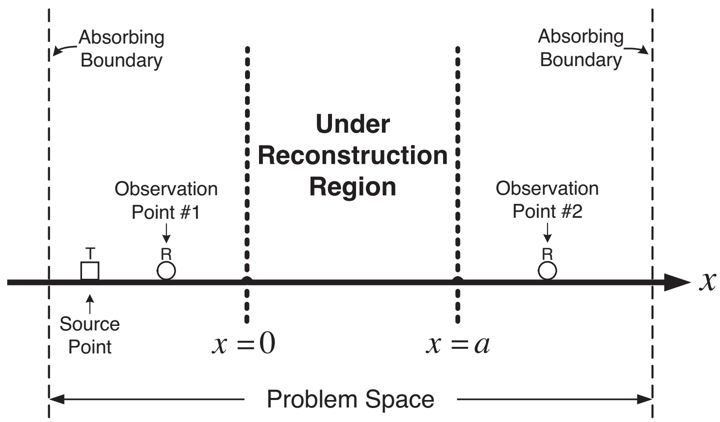


Figure 3. Geometrical configuration of the problem. One transmitter and two receivers are used in all test cases.

Test case #1: In the first sample case, we consider an inhomogeneous and lossless medium consisting 20 cells. Therefore, only the permittivity profile reconstruction is considered. The number of cells for the whole computational domain is 200. The unknown medium is between cells 90 and 110. The source of Gaussian pulse is located in cell number 5 and the receivers are placed in cells number 50 and 140. The center of Gaussian pulse is at $60\Delta t$ and the width of it is set to $11\Delta t$. In the expansion method, the number of expansion terms N, is set to 7. Hence, we have only 7 unknowns in expansion method instead of 20 unknowns in direct optimization method. It means the reduction in unknowns in about 2/3 which results in lower number of particles and necessary number of iterations in the optimization routine. The population in PSO algorithm is chosen equal to 50 and the maximum iteration is considered to be 200.

The exact profile and reconstructed profiles with both direct and expansion methods in several optimization iterations with the aid of Tikhonov energy regularization are shown in Fig. 4. It must be noted that the initial permittivity profile in the direct method is assumed to be like free space ($\varepsilon_r = 1$) in all the grid points, whereas a random initial expansion coefficients make a really drastic initial guess of permittivity profile according to Fig. 4(a).

The cost function (1) with the aid of Tikhonov energy regularization is plotted versus the iteration number in Fig. 5. According to this figure, the PSO as a powerful global optimization method reduces the cost function quickly in this 7-dimensional case. It is to be noted that due to drastic variations of cost function at the beginning iterations which cause the unrealizable changes in remaining iterations, the cost function is plotted from 30th iteration.

Table 1. Cosine Fourier series expansion coefficients for permittivity reconstruction of test case #1 by using different kinds of regularizations.

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
Tikhonov Regularization	0.2467	0.2046	0.0763	0.0577	0.027	0.0279	0.0107
Sobolev Regularization	0.249	0.2044	0.0627	0.0489	0.0157	0.0161	0.002
TV Regularization	0.2446	0.1996	0.0736	0.052	0.0206	0.0133	0.0021

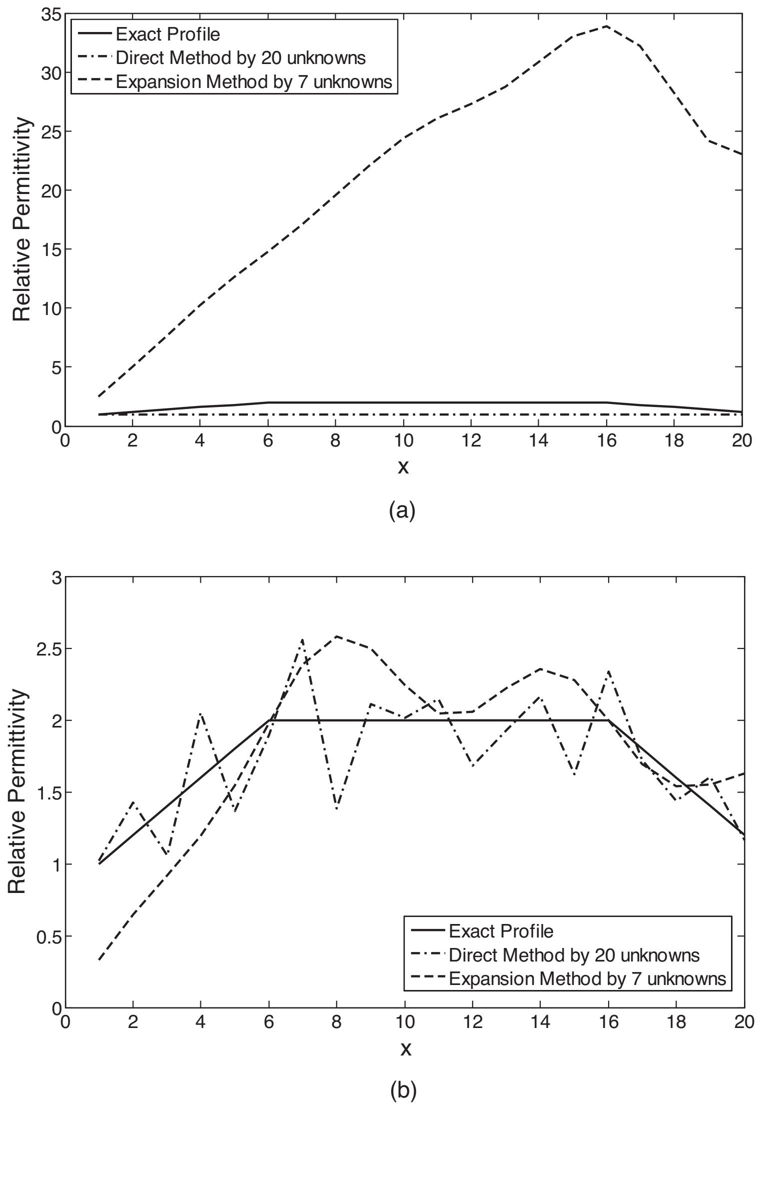
Any other regularization terms could be used for this reconstruction. Fig. 6 shows the reconstruction of this sample case using second order Sobolev and TV regularizations after 200 iterations with the number of particles set to 50. The optimized Fourier series expansion coefficients for reconstructions by using the above regularization terms are depicted in Table 1 for comparison. It is seen that the coefficients of expansion with the help of all three kinds of regularizations are very close to each other especially in lower frequencies.

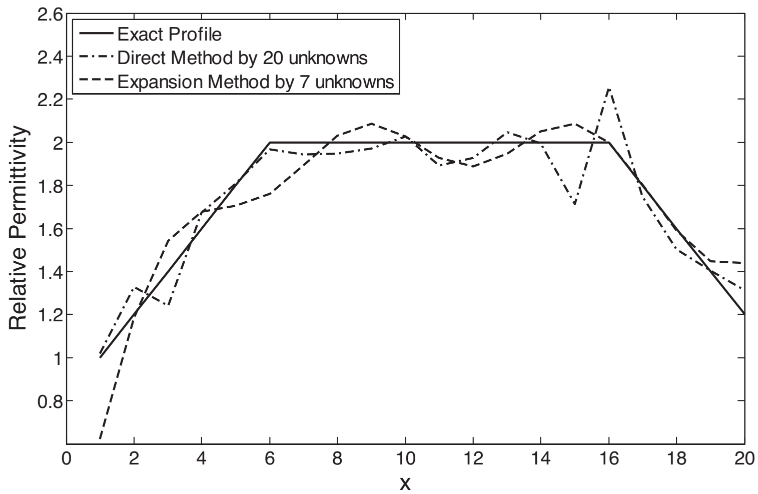
It is noted from the above reconstruction results that in spite of a valuable reduction in the amount of computations in the proposed expansion method, the reconstructed profiles by using all the introduced regularizations have quite acceptable precisions.

Test case #2: In the second example, another lossless and inhomogeneous medium with 30 cell length is considered. The number of cells for the whole computational domain is 100. The unknown medium is between cells 40 and 70. The source of Gaussian pulse is located in cell number 5 and the receivers are placed in cells number 30 and 90. The center of Gaussian pulse is at $60\Delta t$ and the width of it is set to $11\Delta t$. In the expansion method, N is chosen equal to

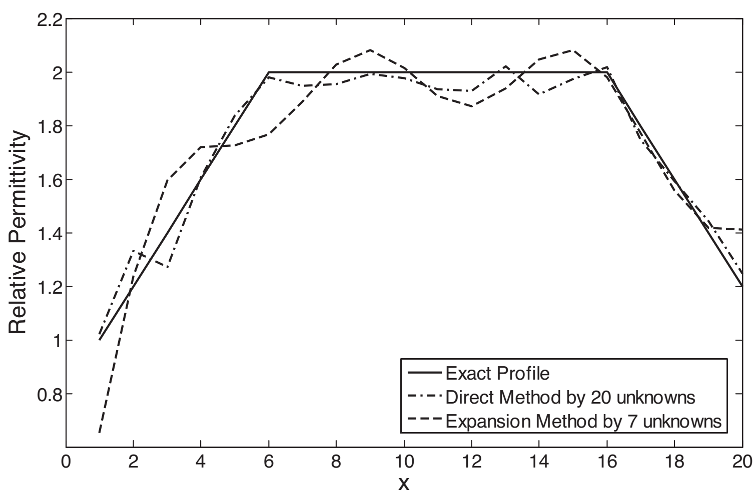
10. Therefore the number of unknowns is again reduced to one-third. The population in PSO algorithm is considered equal to 50 and the maximum iteration of optimization is set to 200. The reconstructed profiles by using different types of regularizations are shown in Fig. 7. The Fourier expansion coefficients for these cases are represented in Table 2.

Test case #3: In this case, a lossy and inhomogeneous medium





(c)



(d)

Figure 4. Comparison of permittivity profile reconstructions of test case #1 by using direct and expansion methods with the help of Tikhonov energy regularization. (a) Initial guess, (b) Reconstruction after 30 iterations, (c) Reconstruction after 100 iterations, and (d) Reconstruction after 200 iterations.

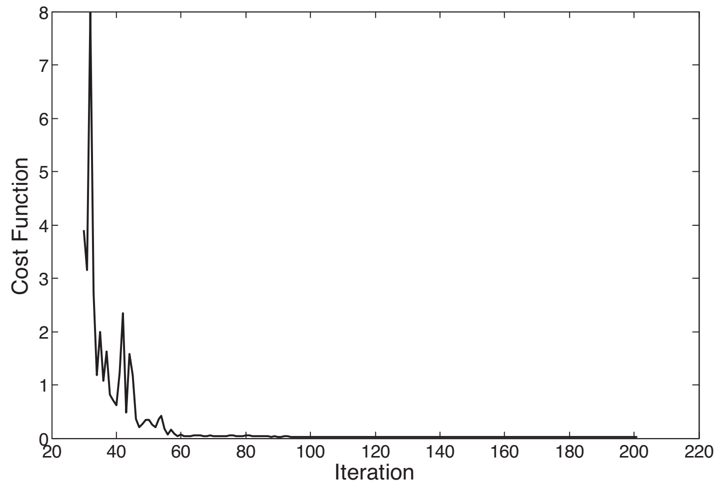


Figure 5. Cost function of test case #1 with the help of Tikhonov energy regularization. PSO as a global optimizer reduces the cost function during the optimization routine. 7 expansion terms are considered for reconstruction and number of particles and iterations are equal to 50 and 200, respectively.

with 12 cell length is considered. The number of cells for the whole computational domain is 200. The unknown medium is between cells 95 and 107. The source of Gaussian pulse is located in cell number 5 and the receivers are placed in cells number 50 and 150. The center of Gaussian pulse is at $40\Delta t$ and the width of it is set to $7\Delta t$. In the expansion method for both permittivity and conductivity profiles expansion, N is chosen equal to 6. The population in PSO algorithm is considered equal to 50 and the maximum iteration of optimization is set to 200. The reconstructed profiles of permittivity and conductivity by using Tikhonov energy regularization are shown in Fig. 8. The Fourier series expansion coefficients for this case for both permittivity and conductivity profiles expansions are represented in Table 3.

The results for all three cases which are generally inhomogeneous and lossy or lossless media show that the proposed expansion method can tolerably reconstruct the unknown media with a considerable reduction in the amount of computations as compared to the direct optimization of the unknowns.

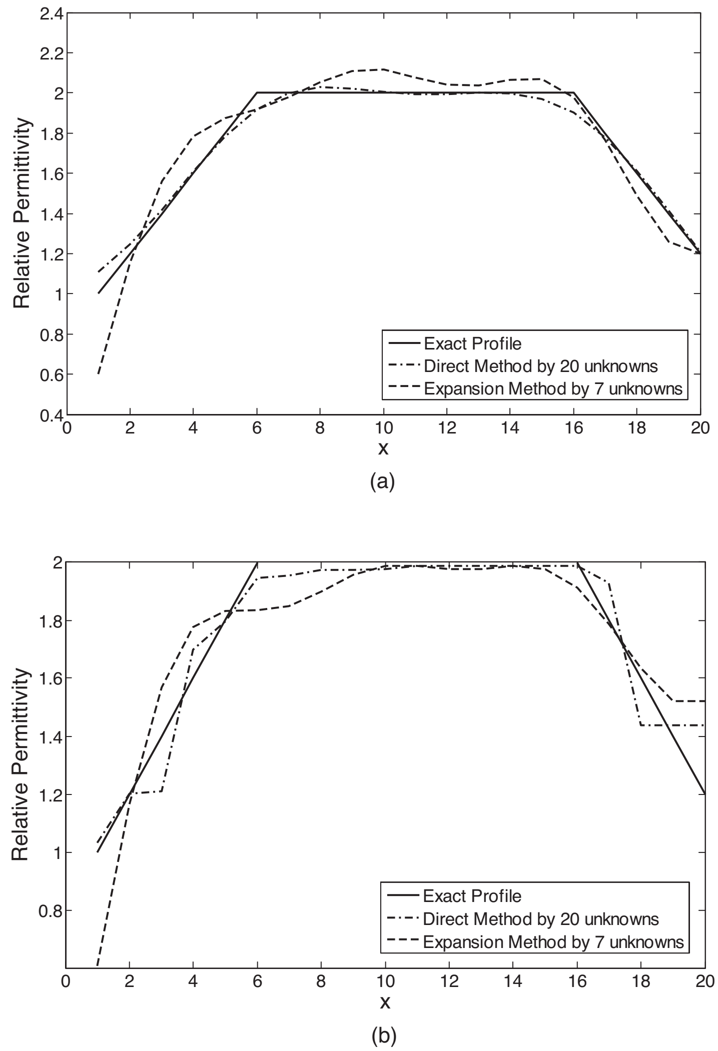
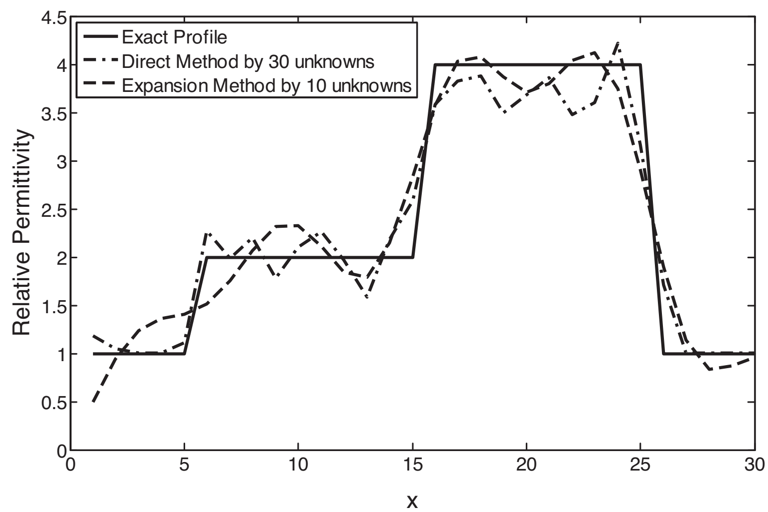


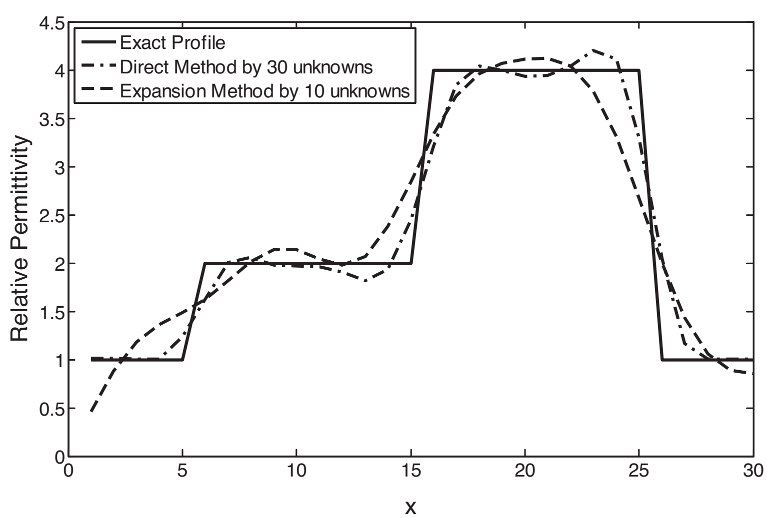
Figure 6. Comparison of permittivity profile reconstructions of test case #1 by using direct and expansion methods after 200 iterations with the aid of (a) second order Sobolev regularization and (b) TV regularization.

5. SENSITIVITY CONSIDERATIONS

Regarding sensitivity of the algorithm, there are two important issues we would like to address. First, how the optimum number of expansion terms, N, should be chosen. Small values of N reduce the accuracy of the reconstructed result, while large values for N cause oscillatory



(a)



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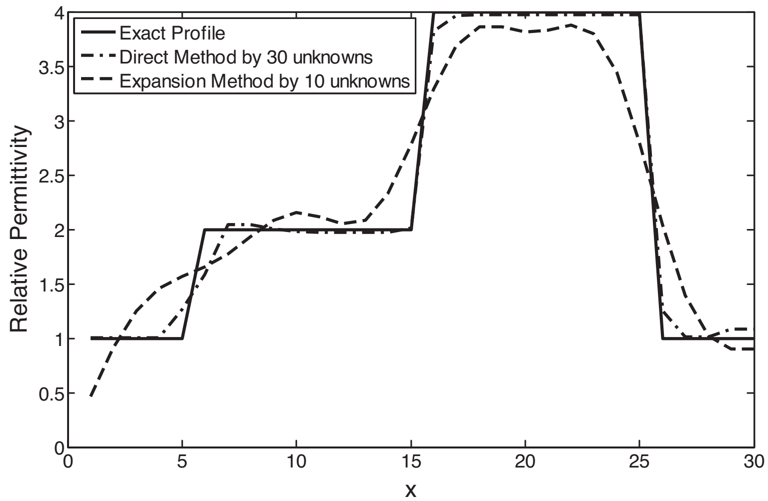


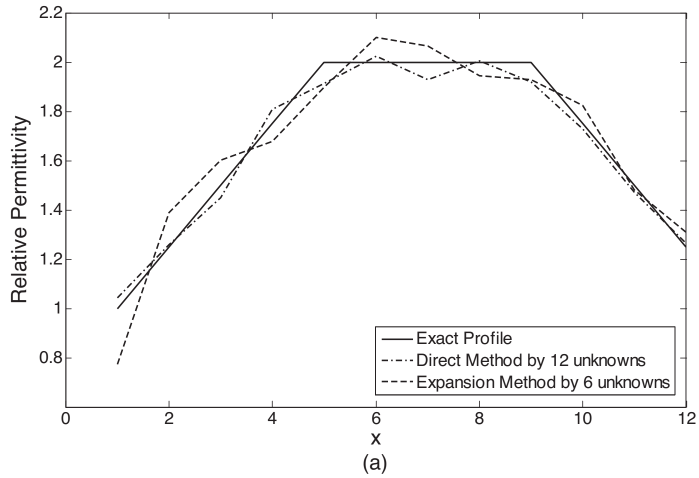
(c)

Figure 7. Comparison of permittivity profile reconstructions of test case #2 by using direct and expansion methods with the help of (a) Tikhonov energy regularization, (b) second order Sobolev regularization, and (c) TV regularization.

Table 2. Cosine Fourier series expansion coefficients for permittivity reconstruction of test case #2 by using different kinds of regularizations.

	d_0	d_1	d_2	d_3	d_4	d_5
Tikhonov Regularization	0.2039	0.1232	0.0142	0.0771	0.0073	0.0108
Sobolev Regularization	0.2024	0.1194	0.0104	0.0735	0.0056	0.0052
TV Regularization	0.2033	0.1257	0.0175	0.0742	0.0092	0.0104

	d_6	d_7	d_8	d_9
Tikhonov Regularization	0.0134	0.0281	0.0242	-0.0081
Sobolev Regularization	0.0107	0.015	0.0129	0.0004
TV Regularization	0.009	0.0143	0.01	-0.0054



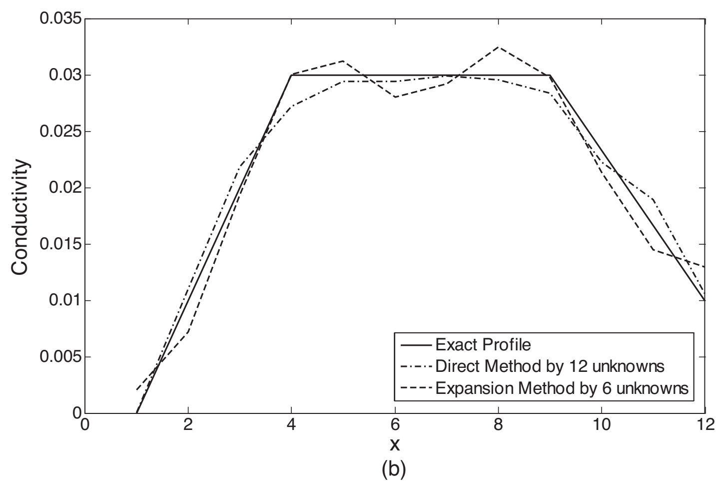


Figure 8. Permittivity and conductivity profiles reconstructions of test case #3 by using both direct and expansion methods with the help of Tikhonov energy regularization. (a) Permittivity profile and (b) Conductivity profile.

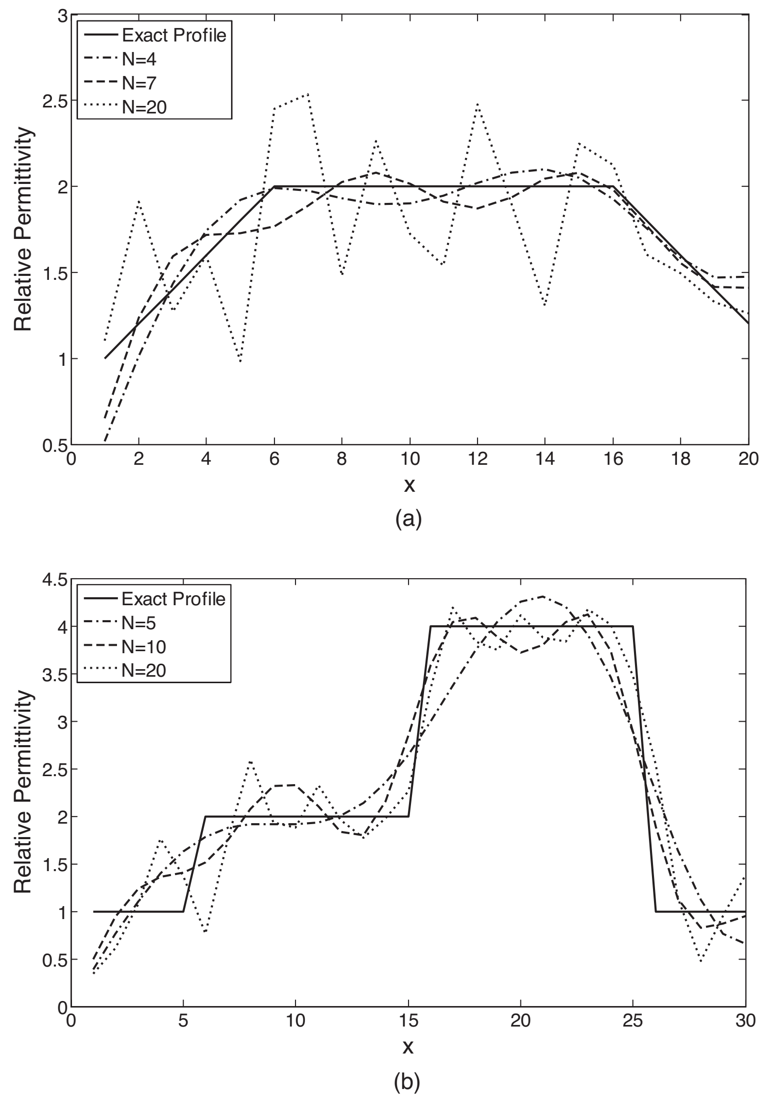


Figure 9. Comparison of reconstructions by the expansion method with different number of cosine Fourier series expansion terms. (a) test case #1 and (b) test case #2.

Table 3. Cosine Fourier series expansion coefficients for permittivity and conductivity reconstructions of test case #3 by using Tikhonov energy regularization.

	d_0	d_1	d_2	d_3	d_4	d_5
Permittivity	0.3532	0.2477	0.0579	0.0584	0.0294	0.0254
Conductivity	0.0041	0.0018	-0.0014	-0.0008	-0.0012	-0.0005

response or even divergence of the algorithm. The reconstructed profiles for different values of N are shown in Figs. 9(a), (b) for test case #1 and #2, respectively. Our experiences in studying various permittivity and conductivity profiles reconstructions show that choosing N between 7 and 10 may be suitable for most of the practical 1-D reconstruction problems. However, this is not a limitation of the algorithm and generally speaking, one can choose N arbitrarily based on the considerations discussed in this section. The results shown in Fig. 9, confirm this idea for our two sample cases.

The second important issue is that although the degree of ill-posedness is reduced by limiting the number of expansion terms, using the regularization terms is mandatory. Without utilizing the regularization, the algorithm may not converge.

6. CONCLUSION

A novel inverse scattering method in time domain based on combination of the cosine Fourier series expansion, the FDTD and the PSO algorithms has been proposed. The mathematical formulations of the method have been derived completely and the algorithm has been examined for reconstruction of several inhomogeneous lossless and lossy one-dimensional cases with the aid of various regularization terms. With a considerable decrease in the number of the unknowns (in about 2/3) and consequently the amount of computations as compared with conventional inverse scattering methods, the relative permittivity and conductivity profiles of three 1-D media have been reconstructed successfully. Finally, it has been shown by sensitivity analysis that for obtaining accurate reconstruction and well-posedness of the algorithm simultaneously, the number of expansion terms must be chosen intelligently.

REFERENCES

1. Bindu, G., A. Lonappan, V. Thomas, C. K. Aanandan, and K. T. Mathew, "Active microwave imaging for breast cancer detection," *Progress In Electromagnetics Research*, PIER 58, 149–169, 2006.
2. Colton, D. and P. B. Monk, "Target identification of coated objects," *IEEE Transactions on Antennas and Propagation,* Vol. 54, No. 4, 1232–1242, 2006.
3. Semenov, S. Y., V. G. Posukh, A. E. Bulyshev, Y. E. Sizov, and P. N. Repin, "Microwave tomographic imaging of the heart in intact swine," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 7, 874–890, 2006.
4. Rosenthal, A. and M. Horowitz, "Inverse scattering algorithm for reconstructing strongly reflecting fiber bragg gratings," *IEEE Journal of Quantum Electronics*, Vol. 39, No. 8, 1018–1026, 2003.
5. Chen, X. and K. Huang, "Microwave imaging of buried inhomogeneous objects using parallel genetic algorithm combined with FDTD method," *Progress In Electromagnetics Research*, PIER 53, 283–298, 2005.
6. Popovic, M. and A. Taflove, "Two-dimensional FDTD inverse-scattering scheme for determination of near-surface material properties at microwave frequencies," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 2, 2366–2373, 2004.
7. Huang, C. H., Y. F. Chen, and C. C. Chiu, "Permittivity distribution reconstruction of dielectric objects by a cascaded method," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 2, 145–159, 2007.
8. Colton, D. and L. Paivarinta, "The uniqueness of a solution to an inverse scattering problem for electromagnetic waves," *Arc. Ration. Mech. Anal.*, Vol. 119, 59–70, 1992.
9. Isakov, V., "Uniqueness and stability in multidimensional inverse problems," *Inverse Problems*, Vol. 9, 579–621, 1993.
10. Sheen, D. and D. Shepelsky, "Uniqueness in the simultaneous reconstruction of multiparameters of a transmission line," *Progress In Electromagnetics Research*, PIER 21, 153–172, 1999.
11. Abubakar, A., P. M. Van Den Berg, T. M. Habashy, and H. Braunisch, "A multiplicative regularization approach for deblurring problems," *IEEE Transactions on Image Processing*, Vol. 13, No. 11, 1524–1532, 2004.
12. Chung, Y. S., C. Cheon, and S. Y. Hahn, "Reconstruction of dielectric cylinders using FDTD and topology optimization

technique," *IEEE Transactions on Magnetics*, Vol. 36, No. 4, 956–959, 2000.

13. Rekanos, I. T. and A. Raisanen, "Microwave imaging in the time domain of buried multiple scatterers by using an FDTD-based optimization technique," *IEEE Transactions on Magnetics*, Vol. 39, No. 3, 1381–1384, 2003.
14. Abubakar, A., T. M. Habashy, and P. M. Van Den Berg, "Nonlinear inversion of multi-frequency microwave Fresnel data using the multiplicative regularized contrast source inversion," *Progress In Electromagnetics Research*, PIER 62, 193–201, 2006.
15. Taflove, A. and S. C. Hagness, *Computational Electrodynamics: The Finite-difference Time-domain Method*, 3rd edition, Artech House, 2005.
16. Tikhonov, A. N. and V. Arsenine, *Solutions of Ill-Posed Problem*, Winston, New York, 1977.
17. Hansen, P. C., *Rank Deficient and discrete Ill-posed Problems: Numerical Aspects of Linear Inversion*, SIAM, Philadelphia, 1998.
18. Abubakar, A. and P. M. Van Den Berg, "Total variation as a multiplicative constraint for solving inverse problems," *IEEE Transactions on Image Processing*, Vol. 10, No. 9, 1384–1392, 2001.
19. Robinson, J. and Y. Rahmat-Samii, "Particle swarm optimization in electromagnetics," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 2, 397–407, 2004.
20. Lee, K. C. and J. Y. Jhang, "Application of particle swarm algorithm to the optimization of unequally spaced antenna arrays," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 2001–2012, 2006.
21. Chen, T. B., Y. L. Dong, Y. C. Jiao, and F. S. Zhang, "Synthesis of circular antenna array using crossed particle swarm optimization algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 13, 1785–1795, 2006.
22. Meyer, T., A. Jostingmeier, and A. S. Omar, "Microwave imaging using a novel regularization scheme," *Proceedings of the Antennas and Propagation Society International Symposium*, Vol. 3, 175–178, 2003.