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A THEORETICAL STUDY OF ELECTROMAGNETIC TRANSIENTS IN A LARGE CONDUCTING PLATE DUE TO CURRENT IMPACT EXCITATION

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Abstract—Maxwell's equations are solved to determine transient electromagnetic fields inside as well as outside a large conducting plate of an arbitrary thickness. The plate is carrying a uniformly distributed excitation winding on its surfaces. Transient fields are produced due to sudden interruption of the d.c. current in the excitation winding. On the basis of a linear treatment of this initial value problem it is concluded that the transient fields may decay at a faster rate for plates with smaller value of relaxation time. It is also shown that the energy dissipated in the eddy current loss may exceed the energy stored in the initial magnetic field.

1. INTRODUCTION

Impact excitation of an electric system can be classified into two broad groups. (i) ***Voltage impact excitation***, where a voltage is either instantly impressed at the terminals of a circuit or it is instantly set to zero; and (ii) ***Current impact excitation***, where a current is either instantly forced into a circuit or it is instantly interrupted.

If a step d.c. voltage is applied to a coil, the resulting current in the coil is time-dependent. At any instant the coil terminal voltage must equal the resistance drop minus the induced voltage due to the growth of coil flux. This leads to an ordinary differential equation. The magnetic field in the core of the coil varies with time as well as space coordinates. Therefore, the system involves both, ordinary as well as partial differential equations.

As an example of current impact excitation, consider the magnetic flux in a transformer core at no-load, when the high-speed circuit breaker on the primary side trips. The no-load current is interrupted almost instantly. The core flux shall, however, not become instantly zero as eddy currents in the core tend to support this flux. Since the core is invariably laminated, eddy currents will be small, resulting in a fast decay of flux. In the case of fast changing fields the role of displacement currents become significant.

Many technical papers have been published on electromagnetic transients in solid blocks of steel. Weber [1], Wagner [2], Concordia and Poritsky [3], and also Pohl [4] are amongst the early contributors. A number of research papers on electromagnetic transients [5–15] appeared subsequently. Recently, research papers on transient analysis of grounding systems, transmission lines and impulsive sources appeared [16–20], indicating the importance of the study.

Barring few [14, 15], most authors in their analysis ignored the presence of displacement currents in comparison to the conduction currents. In the present treatment Maxwell's equations are solved without ignoring displacement currents, for transient fields in a large conducting plate with constant values for permeability, μ, permittivity, ϵ, and conductivity, σ. The method of separation of variables has been adopted to solve the one-dimensional initial-value problem.

2. FIELD EQUATIONS

Consider Fig. 1, showing a large conducting plate of thickness W, carrying uniform current sheets of density $\pm K_y$ on its surfaces located at $x = \pm W/2$. These current sheets simulate the excitation winding carrying d.c. currents. If these currents are instantly interrupted, say

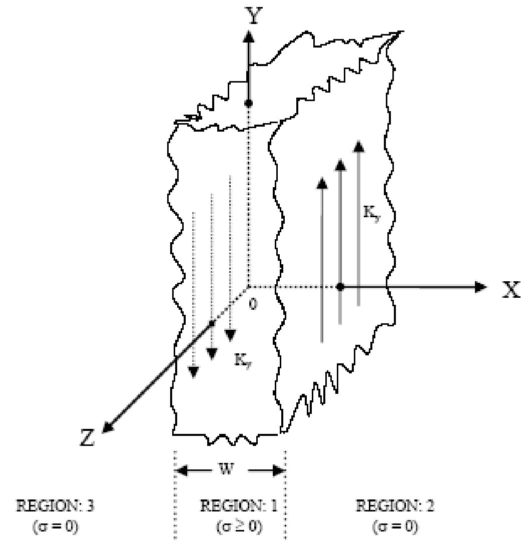

Figure 1. Large conducting plate with surface current sheets.

at $t = 0$, transient fields are caused.

Because of symmetry only y-component of electric field and only z-component of magnetic field exist. The former is an odd function and the latter is an even function of x. Further, both transient fields vanish as t tends to infinity. These fields satisfy Maxwell's equations in one dimension:

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \tag{1}$$

$$\text{and} \quad -\frac{\partial H_z}{\partial x} = \sigma E_y + \epsilon \frac{\partial E_y}{\partial t} \tag{2}$$

Therefore, electromagnetic fields obey the following equations:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu\sigma \frac{\partial E_y}{\partial t} + \mu\epsilon \frac{\partial^2 E_y}{\partial t^2} \tag{3}$$

$$\text{and} \quad \frac{\partial^2 H_z}{\partial x^2} = \mu\sigma \frac{\partial H_z}{\partial t} + \mu\epsilon \frac{\partial^2 H_z}{\partial t^2} \tag{4}$$

For free space:

$$\sigma = 0, \tag{5}$$

$$\mu = \mu_{0,} \tag{6}$$

$$\text{and} \quad \epsilon = \epsilon_0 \tag{7}$$

Now, if displacement currents are ignored, Eqs. (2), (3) and (4) reduce to:

$$-\frac{\partial H_z}{\partial x} = \sigma E_y \tag{8}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu\sigma\frac{\partial E_y}{\partial t} \tag{9}$$

$$\text{and} \quad \frac{\partial^2 H_z}{\partial x^2} = \mu\sigma\frac{\partial H_z}{\partial t} \tag{10}$$

The last two equations can be identified as eddy current equations.

3. INITIAL DISTRIBUTION OF ELECTROMAGNETIC FIELDS

As shown in Fig. 1, let the region occupied by the plate, $-W/2 < x < W/2$, be indicated as region 1, while the regions $x > W/2$ and $x < -W/2$, be indicated as regions 2 and 3, respectively. In view of the symmetry, it will be sufficient to consider the field distributions in regions 1 and 2 only. At the boundary between these two regions, i.e., at $x = W/2$, we have

$$H_{1z} = H_{2z} + K_y \tag{11}$$

$$\text{and} \quad E_{1y} = E_{2y} \tag{12}$$

where suffix 1 indicates fields in region 1, and suffix 2 indicates fields in region 2. The current sheet with surface current density K_y, simulates the excitation winding carrying d.c. current. Its value becomes zero after the excitation current is interrupted, i.e., for $t > 0$.

Before the onset of transient, i.e., for $t \leq 0$, the initial fields are:

$$H_{1z} = K_y \tag{13}$$

$$E_{1y} = 0 \tag{14}$$

$$H_{2z} = 0 \tag{15}$$

$$\text{and} \quad E_{2y} = 0 \tag{16}$$

These solutions are consistent with boundary conditions defined by Eqs. (11) and (12).

4. APPROXIMATE DISTRIBUTION OF TRANSIENT FIELDS

If displacement currents are ignored, electromagnetic fields satisfy eddy current equation. Therefore field distributions inside the conducting plate can be given as:

$$H_{1z} = \sum_{n-odd}^{\infty} \left[K_y . \frac{4}{\pi} . \frac{\sin\left(\frac{n\pi}{2}\right)}{n} \right] . \cos\left(n\frac{\pi}{W}x\right) . e^{-\frac{1}{\mu\sigma}.\left(n\frac{\pi}{W}\right)^2 .t} \quad (17)$$

and $$E_{1y} = \sum_{n-odd}^{\infty} \left[K_y . \frac{4}{W\sigma} . \sin\left(\frac{n\pi}{2}\right) \right] . \sin\left(n\frac{\pi}{W}x\right) . e^{-\frac{1}{\mu\sigma}.\left(n\frac{\pi}{W}\right)^2 .t} \quad (18)$$

These solutions are consistent with initial conditions defined by Eqs. (13) and (14). Eddy current loss per unit plate surface area is

$$P_e = \int_{-W/2}^{W/2} \sigma E_{1y}^2 dx \quad (19)$$

Therefore, using the approximate expression for E_{1y} from Eq. (18), we get

$$P_e = \sum_{n-odd}^{\infty} \left[K_y^2 . \frac{8}{W\sigma} \right] . e^{-\frac{2}{\mu\sigma}.\left(n\frac{\pi}{w}\right)^2 .t} \quad (20)$$

Further, energy dissipated due to eddy current loss is given by

$$\exists_e = \int_0^{\infty} P_e dt \quad (21)$$

Thus on performing the integration and summing the resulting infinite series [21, 22], we get

$$\exists_e = K_y^2 . \frac{W}{2} . \mu \quad (22)$$

This is exactly equal to the energy stored in the magnetic field per unit plate surface area. Since displacement currents are ignored, energy radiated outside the plate is zero. Therefore outside fields remain zero during transient as well.

5. EXACT DISTRIBUTION OF TRANSIENT FIELDS

5.1. Fields inside the Plate

Consider the electromagnetic fields inside the plate. These fields must satisfy the initial conditions given by Eqs. (13) and (14), for

$-W/2 < x < W/2$, and must vanish as t tends to infinity. Therefore, field expressions satisfying Eqs. (1)–(4), are

$$H_{1z} = K_y.e^{-\frac{\sigma}{\epsilon}t} + K_y.\left(\frac{\sigma}{\epsilon}\right).\theta.\frac{\cos\left(\theta.\frac{2x}{W}\right)}{\sin(\theta)}.t.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}t} - \sum_{m=1}^{\infty} a_m.\cos\left(m\frac{2\pi}{W}x\right).\left[e^{-\alpha_m t} - e^{-\beta_m t}\right] \tag{23}$$

and,

$$E_{1y} = K_y.\mu\frac{\sigma}{\epsilon}.x.e^{-\frac{\sigma}{\epsilon}t} - K_y.\mu\frac{\sigma}{\epsilon}.\frac{W}{2}.\frac{\sin\left(\theta\frac{2x}{W}\right)}{\sin(\theta)}.\left[1-\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).t\right]e^{-\frac{1}{2}\frac{\sigma}{\epsilon}t} - \sum_{m=1}^{\infty}\frac{b_m}{(\alpha_m-\beta_m)}\sin\left(m\frac{2\pi}{W}x\right).\left[\alpha_m.e^{-\alpha_m t} - \beta_m.e^{-\beta_m t}\right] \tag{24}$$

where,

$$\theta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}.\frac{W}{2} \tag{25}$$

$$\alpha_m, \beta_m = \frac{1}{2}\frac{\sigma}{\epsilon} \pm \frac{1}{\sqrt{\mu\epsilon}}.\frac{2\pi}{W}.\sqrt{\left(\frac{\theta}{\pi}\right)^2 - m^2} \tag{26}$$

$$a_m = b_m.\frac{1}{\mu}.\frac{\left(m\frac{2\pi}{W}\right)}{(\alpha_m-\beta_m)} \tag{27a}$$

$$\text{and} \quad b_m = K_y.\mu\frac{\sigma}{\epsilon}.\frac{W}{\pi}.\frac{\cos(m\pi)}{m}.\frac{\left(\frac{\theta}{\pi}\right)^2}{\left[m^2 - \left(\frac{\theta}{\pi}\right)^2\right]} \tag{27b}$$

5.2. Fields outside the Plate

Both magnetic- and electric-field, as shown by Eqs. (15) and (16) are zero till the current sheets on the plate surfaces are suddenly switched off at the instant $t = 0$. For $t > 0$, the magnetic field is continuous on the plate surfaces. Since both, eddy currents and displacement currents oppose any sudden change of magnetic flux in the plate, H_{1z} cannot change instantly. Consequently, on the plate surface in region-2, the magnetic field H_{2z} suddenly changes from its original zero value to the value of H_{1z} at $t = 0$, i.e., k_y.

The effect of sudden disappearance of the current sheets, at a distant point, is also an abrupt change in the magnitude of electromagnetic field at that point. This change, however, takes place at a later instant of time as the electromagnetic disturbance propagates in free space at a finite velocity c.

The sudden interruptions of current sheets induce traveling waves in free space that vanish as t tends to infinity. Wave-front of each wave moves away from the plate surfaces with the velocity c.

The electromagnetic fields in region-2 satisfy wave equation, thus

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E_y}{\partial t^2} \tag{28}$$

$$\text{and} \quad \frac{\partial^2 H_z}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 H_z}{\partial t^2} \tag{29}$$

Further, we have

$$H_z\left(t \pm \sqrt{\mu_0\epsilon_0}.x'\right) = \mp\sqrt{\frac{\epsilon_0}{\mu_0}}.E_y\left(t \pm \sqrt{\mu_0\epsilon_0}.x'\right) \tag{30}$$

where,

$$x' = x - W/2 \tag{31}$$

Therefore, in view of boundary conditions, vide Eqs. (11) and (12), the expressions for electric and magnetic fields in region 2 are found as follows:

$$\begin{aligned} E_{2y} =& \frac{1}{2}K_y\mu W.\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).\left[e^{-\frac{\sigma}{\epsilon}.t_-} - \left\{1-\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).t_-\right\}.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_-}\right] \\ &+\frac{1}{2}K_y\mu W.\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).\left[e^{-\frac{\sigma}{\epsilon}.t_+} - \left\{1-\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).t_+\right\}.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_+}\right] \\ &+\sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}.K_y.\left[e^{-\frac{\sigma}{\epsilon}.t_-} - e^{-\frac{\sigma}{\epsilon}.t_+}\right] \\ &+\sqrt{\frac{\mu_0}{\epsilon_0}}.\frac{1}{2}\sum_{m=1}^{\infty}a_m\cos(m\pi).\left[\left(e^{-\alpha_m t_-} - e^{-\alpha_m t_+}\right) - \left(e^{-\beta_m t_-} - e^{-\beta_m t_+}\right)\right] \\ &+\sqrt{\frac{\mu_0}{\epsilon_0}}.K_y.\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).\theta\cot(\theta).\left[t_-.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_-} - t_+.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_+}\right] \end{aligned} \tag{32}$$

and

$$
\begin{aligned}
H_{2z} &= \frac{1}{2}K_y.\left[e^{-\frac{\sigma}{\epsilon}.t_-} + .e^{-\frac{\sigma}{\epsilon}.t_+}\right] \\
&+\frac{1}{2}\sum_{m=1}^{\infty} a_m \cos(m\pi).\left[\left(e^{-\alpha_m t_-} + e^{-\alpha_m t_+}\right) - \left(e^{-\beta_m t_-} + e^{-\beta_m t_+}\right)\right] \\
&+K_y.\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).\theta\cot(\theta).\left[t_-.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_-} + t_+.e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_+}\right] \\
&+\sqrt{\frac{\epsilon_0}{\mu_0}}.\frac{1}{2}.K_y.\mu W\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right)\left[e^{-\frac{\sigma}{\epsilon}.t_-} - \left\{1-\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).t_-\right\}e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_-}\right] \\
&-\sqrt{\frac{\epsilon_0}{\mu_0}}.\frac{1}{2}.K_y.\mu W\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right)\left[e^{-\frac{\sigma}{\epsilon}.t_+} - \left\{1-\left(\frac{1}{2}\frac{\sigma}{\epsilon}\right).t_+\right\}e^{-\frac{1}{2}\frac{\sigma}{\epsilon}.t_+}\right]
\end{aligned} \tag{33}
$$

where, the retarded time t_-, and the accelerated time t_+ are defined as:

$$t_- = t - \sqrt{\mu_0\epsilon_0}.x' \tag{34}$$

$$\text{and} \quad t_+ = t + \sqrt{\mu_0\epsilon_0}.x' \tag{35}$$

In order to satisfy the initial conditions, vide Eqs. (15) and (16), we multiply the R.H.S. of Eqs. (32) and (33) by the unit step function: $u(t_-)$. Field expressions thus modified are consistent with the observation made at the beginning of this section.

6. POWER DISSIPATION

The expression for instantaneous eddy current loss per unit plate area, found using Eqs. (19) and (24), is as follows:

$$P_e = P_0 \sum_{n=1}^{6} \wp_{en}(t) \tag{36}$$

where P_0 is the initial energy stored in the magnetic field per unit plate area, divided by the relaxation time of the plate material, i.e.,

$$P_0 = \frac{\left(\frac{1}{2}K_y^2\mu W\right)}{\tau} \triangleq \frac{\exists_0}{\tau} \tag{37}$$

where the relaxation time,

$$\tau = \epsilon/\sigma \tag{38}$$

while,

$$\wp_{e1}(t) = \frac{8}{3}.\theta^2.e^{-2t/\tau} \tag{39a}$$

$$\wp_{e2}(t) = 4.\left\{\theta^2.\cos ec^2(\theta) - \theta.\cot(\theta)\right\}.\left\{1 - \frac{1}{2}\frac{t}{\tau}\right\}^2.e^{-t/\tau} \tag{39b}$$

$$\wp_{e3}(t) = 16.\{\theta.\cot(\theta) - 1\}.\left\{1 - \frac{1}{2}\frac{t}{\tau}\right\}.e^{-\frac{3}{2}t/\tau} \tag{39c}$$

$$\wp_{e4}(t) = 16\sum_{m=1}^{\infty}\frac{(\theta/\pi)^6}{m^2.\left\{m^2 - (\theta/\pi)^2\right\}^2}.\gamma_m^2(t) \tag{39d}$$

$$\wp_{e5}(t) = -32\sum_{m=1}^{\infty}\frac{(\theta/\pi)^4}{\left\{m^2 - (\theta/\pi)^2\right\}^2}.\gamma_m(t).\left\{1 - \frac{1}{2}\frac{t}{\tau}\right\}e^{-\frac{1}{2}t/\tau} \tag{39e}$$

$$\wp_{e6}(t) = 32\sum_{m=1}^{\infty}\frac{(\theta/\pi)^4}{m^2.\left\{m^2 - (\theta/\pi)^2\right\}}.\gamma_m(t).e^{-t/\tau} \tag{39f}$$

and

$$\gamma_m(t) = \frac{\alpha_m.e^{-\alpha_m t} - \beta_m.e^{-\beta_m t}}{\alpha_m - \beta_m} \tag{40}$$

7. ENERGY DISSIPATION

Energy dissipated in the eddy current loss, found in view of Eqs. (21) and (36), can be given as:

$$\exists_e = \exists_0\sum_{n=1}^{7}\exists_{en} \tag{41}$$

where,

$$\exists_0 = \tau.P_0 = \frac{1}{2}K_y^2\mu W \tag{42}$$

$$\exists_{e1} = \frac{4}{3}\theta^2 \tag{43a}$$

$$\exists_{e2} = 2\left[\theta^2.\cos ec^2(\theta) - \theta.\cot(\theta)\right] \tag{43b}$$

$$\exists_{e3} = \frac{64}{9}\left[\theta.\cot(\theta) - 1\right] \tag{43c}$$

$$\exists_{e4} = 8\sum_{m=1}^{\infty}\frac{(\theta/\pi)^6}{m^2.\left[m^2-(\theta/\pi)^2\right]^2} \tag{43d}$$

$$\exists_{e5} = -64\sum_{m=1}^{\infty}\frac{(\theta/\pi)^6}{\left[m^2-(\theta/\pi)^2\right]^2.\left[m^2+\left(\sqrt{3}\theta/\pi\right)^2\right]} \tag{43e}$$

$$\exists_{e6} = -64\sum_{m=1}^{\infty}\frac{(\theta/\pi)^6}{\left[m^2-(\theta/\pi)^2\right].\left[m^2+\left(\sqrt{3}\theta/\pi\right)^2\right]^2} \tag{43f}$$

and $$\exists_{e7} = 128\sum_{m=1}^{\infty}\frac{(\theta/\pi)^6}{m^2.\left[m^2-(\theta/\pi)^2\right].\left[m^2+\left(2\sqrt{2}\theta/\pi\right)^2\right]} \tag{43g}$$

Various infinite series involved in Eqs. (43d)–(43g) can be summed up [21, 22], resulting:

$$\exists_{e4}=\frac{4}{3}\theta^2+2\theta^2.\cos ec^2(\theta)+6\theta.\cot(\theta)-8 \tag{44a}$$

$$\exists_{e5}=-4\theta^2.\cos ec^2(\theta)-6\theta.\cot(\theta)-\frac{2}{\sqrt{3}}\theta.\coth\left(\sqrt{3}\theta\right)+\frac{32}{3} \tag{44b}$$

$$\exists_{e6}=\frac{4}{3}\theta^2.\cos ech^2\left(\sqrt{3}\theta\right)+\frac{10}{3\sqrt{3}}.\theta.\coth\left(\sqrt{3}\theta\right)+2\theta.\cot(\theta)-\frac{32}{9} \tag{44c}$$

and $$\exists_{e7}=-\frac{8}{3}\theta^2-\frac{64}{9}\theta.\cot(\theta)+\frac{2\sqrt{2}}{9}\theta.\coth\left(2\sqrt{2}\theta\right)+7 \tag{44d}$$

Therefore, in view of Eq. (41), we have

$$\exists_e/\exists_0 = \frac{4}{3}\theta^2.\cos ech^2\left(\sqrt{3}\theta\right)+\frac{4\sqrt{3}}{9}\theta.\coth\left(\sqrt{3}\theta\right) + \frac{2\sqrt{2}}{9}\theta\coth\left(2\sqrt{2}\theta\right)-1 \tag{45}$$

Thus, for zero conductivity:

$$\exists_e/\exists_0 = 0 \tag{45a}$$

8. POWER RADIATION

The expression for instantaneous power radiated per unit area from the two surfaces of the plate, found using Poynting theorem and Eqs. (23)

and (24), is as follows:

$$P_r = P_0 \sum_{n=1}^{9} \wp_{rn}(t) \tag{46}$$

where,

$$\wp_{r1} = 2e^{-2t/\tau} \tag{46a}$$
$$\wp_{r2} = -2e^{-\frac{3}{2}t/\tau} \tag{46b}$$
$$\wp_{r3} = (t/\tau).e^{-\frac{3}{2}t/\tau} \tag{46c}$$
$$\wp_{r4} = 2\theta.\cot(\theta).(t/\tau).e^{-\frac{3}{2}t/\tau} \tag{46d}$$
$$\wp_{r5} = -2\theta.\cot(\theta).(t/\tau).e^{-t/\tau} \tag{46e}$$
$$\wp_{r6} = \theta.\cot(\theta).(t/\tau)^2.e^{-t/\tau} \tag{46f}$$

$$\wp_{r7} = -4\sum_{m=1}^{\infty} \frac{(\theta/\pi)^2}{\left\{m^2-(\theta/\pi)^2\right\}}.\left[\frac{e^{-\left(\alpha_m+\frac{1}{\tau}\right)t} - e^{-\left(\beta_m+\frac{1}{\tau}\right)t}}{(\alpha_m-\beta_m).\tau}\right] \tag{46g}$$
$$\wp_{r8} = 4\sum_{m=1}^{\infty} \frac{(\theta/\pi)^2}{\left\{m^2-(\theta/\pi)^2\right\}}.\left[\frac{e^{-\left(\alpha_m+\frac{1}{2\tau}\right)t} - e^{-\left(\beta_m+\frac{1}{2\tau}\right)t}}{(\alpha_m-\beta_m).\tau}\right] \tag{46h}$$
$$\wp_{r9} = -2\sum_{m=1}^{\infty} \frac{(\theta/\pi)^2}{\left\{m^2-(\theta/\pi)^2\right\}}.\left[(t/\tau).\frac{e^{-\left(\alpha_m+\frac{1}{2\tau}\right)t} - e^{-\left(\beta_m+\frac{1}{2\tau}\right)t}}{(\alpha_m-\beta_m).\tau}\right] \tag{46i}$$

9. RADIATED ENERGY

Energy radiated per unit area from the two plate surfaces can be given as

$$\exists_r = \int_0^{\infty} P_r dt \tag{47}$$

Therefore, using Eqs. (46)–(46i), one obtains

$$\exists_r = \exists_0 \sum_{n=1}^{4} \exists_{rn} \tag{48}$$

where,

$$\exists_{r1} = \frac{1}{9} \tag{48a}$$

$$\exists_{r2} = \frac{8}{9}\theta.\cot(\theta) \tag{48b}$$

$$\exists_{r3} = 16\sum_{m=1}^{\infty}\frac{(\theta/\pi)^4}{\left[m^2-(\theta/\pi)^2\right].\left[m^2+\left(2\sqrt{2}\theta/\pi\right)^2\right]} \tag{48c}$$

$$\text{and}\quad \exists_{r4} = -16\sum_{m=1}^{\infty}\frac{(\theta/\pi)^4}{\left[m^2+\left(\sqrt{3}\theta/\pi\right)^2\right]^2} \tag{48d}$$

Infinite series involved in the last two equations can be summed up [21, 22]. Thus, in view of Eq. (48)

$$\exists_r/\exists_0 = 2-\frac{2\sqrt{2}}{9}\theta.\coth\left(2\sqrt{2}\theta\right)-\frac{4\sqrt{3}}{9}\theta.\coth\left(\sqrt{3}\theta\right)-\frac{4}{3}\theta^2.\cos ech^2\left(\sqrt{3}\theta\right) \tag{49}$$

Therefore, for zero energy radiation:

$$\theta = 1\cdot 809316 \tag{50}$$

while for zero conductivity:

$$\exists_r/\exists_0 = 1 \tag{50a}$$

10. CONCLUSION

On the basis of the approximate solution of Maxwell's equations, obtained by ignoring displacement currents, one may conclude that:

(i) It may be seen from Eqs. (17) and (18) that larger the conductivity of the plate, slower is the decay of the electromagnetic fields in it. Smaller is the value of the power dissipated due to eddy current loss for plates with large values of conductivities, as shown in Eq. (20).

(ii) The energy dissipated due to eddy current loss in the plate is independent of its conductivity. Even with zero plate conductivity, this energy is equal to the energy stored in the magnetic field caused by d.c. excitation, vide Eq. (22).

The exact solution of Maxwell's equations, however, presents an altogether different picture. Therefore, modified conclusions are:

(i) Electromagnetic fields in a plate with larger value of conductivity (or smaller value of relaxation time) decay at a faster rate, vide Eqs. (23) and (24).

(ii) Energy dissipated due to eddy current loss in the plate is a function of the plate parameter, θ, vide Eq. (25). For zero conductivity the energy dissipated is zero, vide Eq. (45a). This energy increases as the conductivity is increased.

(iii) There is a radiation of energy outside the plate, vide Eq. (49). This energy also depends on the plate parameter, θ. For zero plate conductivity, the energy radiated is equal to the energy stored in the d.c. field inside the plate.

(iv) The sum of the energy dissipated due to eddy current loss and the energy radiated outside the plate is equal to the energy stored in the d.c. magnetic field. If the plate parameter, θ, is more than $1{\cdot}809316$, the energy dissipated due to eddy current loss will be more than the energy stored in the d.c. magnetic field. In this case the plate receives energy across its surfaces as the Poynting vector is reversed.

The treatment considers a hypothetical situation involving a plate with infinite surface area and uniformly distributed current sheets on its surfaces. In a practical situation, it may be concluded that for the study of current impact excitation, displacement currents may not be ignored. Further, it may be possible to design a system with a minimum radiation. In the case of zero radiation, the energy stored in the d.c. magnetic field is completely dissipated in the eddy current loss. It appears that under certain conditions it is possible that at the instant the excitation current is switched off, the d.c. source radiates some energy that is received by the conducting plate. After all, it is well known that if the current flowing in a series R-L circuit is suddenly interrupted there is a sparking at the switch. On the basis of the treatment presented, it may be inferred that a part of this radiated energy accounts for the energy stored in the inductor minus its core loss and the rest comes from the source.

The present work may be extended for conducting plates made of left-handed materials with simultaneously negative permittivity and permeability [23–25].

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