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NUMERIC CALCULATION OF INPUT IMPEDANCE FOR A GIANT VLF T-TYPE ANTENNA ARRAY

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Abstract—The input reactance and radiation resistance of a giant very low frequency (VLF) 'T type' antenna with ground screen is computed by Method of Moment (MoM). The loss resistance of a complex grounding system is calculated by numeric integral of near field from MoM solution of the antenna on the ground surface. The numeric calculation results agree well with the measured data.

1. INTRODUCTION

VLF communication (ranging from 3 to 30 kHz) is one of the most important means in applications such as time broadcasting, navigation, underground and/or submarine communication. Generally, a VLF transmitting antenna is an electrically-small antenna which operates under high voltage and current[1]. In order to enhance antenna radiation efficiency, a practical VLF antenna is always laid on a large scale ground screen.

VLF antenna system is different from both a GPR antenna system [12] and a grounding system [13, 14]. Input impedance is an important electric parameter of antennas. There are many references dealing with radiation characteristics of vertical antennas over normal ground using different method [2–4, 15, 16], but input impedance failed to get enough attention. It's very difficult to accurately calculate the input

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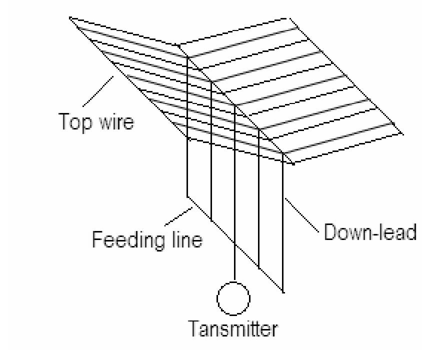

Figure 1. A VLF T type antenna curtain.

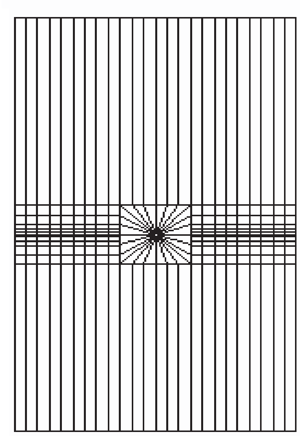

Figure 2. Ground screen of a VLF T type antenna.

impedance of a VLF antenna because of its huge size in scale and its complexity in structure. In this paper, Method of Moments (MoM) is used to compute the input impedance of a very low frequency (VLF) 'T-type' transmitting antenna array over a perfect ground [5, 6]. In order to calculate the ground loss resistance of an antenna with ground screen, a new method is developed, which incorporates numerical integration as well as near field data from MoM solution.

The T-type antenna array considered here consists of eleven mutually connected top wires and five down-leads as show in Fig. 1. The grounding system is divided into three regions. They are helix house ground screen, high-voltage feed cage ground screen and top wire ground screen as shown in Fig. 2. Helix house ground screen is a small square area located at the center of the whole ground screen. Its grounding wires are laid in radial directions. The high-voltage feed cage ground screen lies in the nearby region of the feed cage, and the wires are parallel to feed cage. Top wire ground screen is the region except for helix house ground screen and high-voltage feed cage ground screen in whole ground screen region. Its grounding wires are laid parallel to top wires. The whole ground screen area is slightly larger than the projection of the antenna top structure.

2. NUMERICAL CALCULATIONS FOR T-TYPE ANTENNA ARRAY

Although VLF T-type antenna is huge in scale, its electric dimension is small. Its electric characteristics are suitable to be analyzed by MoM [7].

The current $I(s)$ along the wires satisfies the following electric-field integral equation (EFIE) [8–10]:

$$-\hat{\boldsymbol{s}} \cdot \boldsymbol{E}^i(\boldsymbol{r}) = \frac{-\mathrm{j}\eta}{4\pi k} \int_L I\left(s'\right) \cdot \left(k^2 \hat{\boldsymbol{s}}' \cdot \hat{\boldsymbol{s}} - \frac{\partial^2}{\partial s \partial s'}\right) g\left(\boldsymbol{r}, \boldsymbol{r}'\right) \mathrm{d}s' \qquad (1)$$

where $\boldsymbol{E}^i(\boldsymbol{r})$ is incident electric field, $g(\boldsymbol{r}, \boldsymbol{r}') = \exp(-\mathrm{j}k\,|\boldsymbol{r} - \boldsymbol{r}'|)/|\boldsymbol{r} - \boldsymbol{r}'|$, $k = \omega\sqrt{\mu_0\varepsilon_0}$, $\eta = \sqrt{\mu_0/\varepsilon_0}$, s is the distance parameter along the wire axis at $\boldsymbol{r}$,$\hat{\boldsymbol{s}}$ is its unit vector tangent to the wire axis. Since $\boldsymbol{r}$**'** is a point at s' on the wire axis and $\boldsymbol{r}$ is a point at s on the wire surface, $|\boldsymbol{r} - \boldsymbol{r}'| \geq a$ and the integrand term is bounded.

The integral equation will be solved by MoM. The antenna wires will be divided into several segments. The current on segment number$i$is expanded using segmented sinusoidal base function:

$$I_i(s) = A_i + B_i \sin k(s - s_i) + C_i \cos k(s - s_i) \qquad |s - s_i| \leq \Delta_i/2 \quad (2)$$

where s_i is the value of s at the centre of segment i and Δ_i is the length of segment i. Delta function $w_m(\boldsymbol{r})$=$\delta(\boldsymbol{r}$-$\boldsymbol{r}_m)$ is selected as test function, where $\boldsymbol{r}_m$ are vectors at the center of each segment. The following formal matrix equations may be obtained:

$$-\hat{\boldsymbol{s}} \cdot \boldsymbol{E}^i\left(\boldsymbol{r}_m\right) = \left(\frac{j\eta}{4\pi k}\right) \sum_{i=1}^{N} a_i \int_{L_i} I_i\left(s'\right) \cdot \left(k^2 \hat{\boldsymbol{s}}_i' \cdot \hat{\boldsymbol{s}}_m - \frac{\partial^2}{\partial s \partial s'}\right) g\left(\boldsymbol{r}_m, \boldsymbol{r}'\right) ds' \qquad m = 1, 2, \cdots, N \qquad (3)$$

The current expansion coefficient on wire may be obtained by solving the algebraic matrix equation. The current distribution on antenna as well as the input impedance Z_{in} can be obtained readily.

For a VLF T-type transmitting antenna erected on the ground, ground effect must be taken into consideration. Ground can be modeled as a perfect conductor when computing antenna input reactance and radiation resistance since the conductivity of the ground has little effects on these two parameters. In addition, the ground conductivity has few impact on the near field at the surface of the ground. Hence, the near field at the surface of a perfect ground is used instead of the practical near field.

An image of the antenna is used to model the effect of a ground plane. The Green's function in the upper half space over a perfect ground is the sum of the free-space Green's functions of both the source current and the image current:

$$\bar{\boldsymbol{G}}_{pg}\left(\boldsymbol{r},\boldsymbol{r}'\right)=\bar{\boldsymbol{G}}\left(\boldsymbol{r},\boldsymbol{r}'\right)+\bar{\boldsymbol{G}}_{I}\left(\boldsymbol{r},\boldsymbol{r}'\right) \tag{4}$$

where $\bar{\boldsymbol{G}}\left(\boldsymbol{r},\boldsymbol{r}'\right)$ and $\bar{\boldsymbol{G}}_{I}\left(\boldsymbol{r},\boldsymbol{r}'\right)=-\bar{\boldsymbol{I}}_{r}\cdot\bar{\boldsymbol{G}}\left(\boldsymbol{r},\bar{\boldsymbol{I}}_{r}\cdot\boldsymbol{r}'\right)$ are free-space dyadic Green's functions of the source and the image current respectively , and $\bar{\boldsymbol{I}}_{r}=\hat{\boldsymbol{x}}\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}\hat{\boldsymbol{y}}-\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}$ is the image dyadic.

Different formulas are used to evaluate the electric field radiated from different current components. The source current is a filament on the segment axis. The fields are evaluated with the source segment on the axis of a local cylindrical coordinate system:

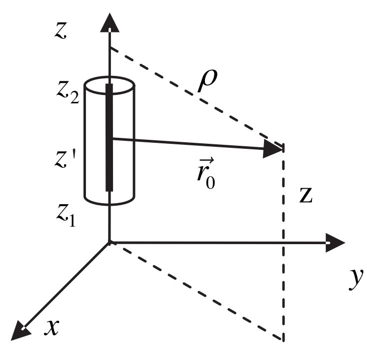

Figure 3. Geometry for the wire.

For sinusoidal current $I=I_0\begin{pmatrix}\sin kz'\\ \cos kz'\end{pmatrix}$, we have

$$\begin{aligned}E_{\rho}(\rho,z)=&\frac{-I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2\rho}G_0\left\{k(z-z')\begin{pmatrix}\cos kz'\\ \sin kz'\end{pmatrix}\right\}\Bigg|_{z_1}^{z_2}\\&-\frac{I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2\rho}G_0\left\{\left[(1-(z-z')^2\left(1+\mathrm{j}kr_0\right)\frac{1}{r_0^2}\right]\begin{pmatrix}\sin kz'\\ \cos kz'\end{pmatrix}\right\}\Bigg|_{z_1}^{z_2}\end{aligned} \tag{5}$$

$$\begin{aligned}E_{z}(\rho,z)=&\frac{I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2}G_0\left\{k\begin{pmatrix}\cos kz'\\ -\sin kz'\end{pmatrix}\right\}\Bigg|_{z_1}^{z_2}\\&-\frac{I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2}G_0\left\{(z-z')(1+\mathrm{j}kr_0)\frac{1}{r^2}\begin{pmatrix}\sin kz'\\ \cos kz'\end{pmatrix}\right\}\Bigg|_{z_1}^{z_2}\end{aligned} \tag{6}$$

For a constant current $I = I_0$, we have

$$E_\rho(\rho, z) = \frac{-I_0}{\lambda}\frac{\mathrm{j}\eta\rho}{2k^2}\left[(1+\mathrm{j}kr_0)\frac{G_0}{r_0^2}\right]_{z_1}^{z_2} \tag{7}$$

$$\begin{aligned} E_z(\rho, z) &= \frac{-I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2}\left\{\left[(z-z')(1+\mathrm{j}kr_0)\frac{G_0}{r_0^2}\right]_{z_1}^{z_2}\right\} \\ &\quad -\frac{I_0}{\lambda}\frac{\mathrm{j}\eta}{2k^2}\left\{k^2\int_{z_1}^{z_2} G_0 \mathrm{d}z'\right\} \end{aligned} \tag{8}$$

where $G_0 = \exp(-\mathrm{j}kr_0)/r_0$, $r_0 = \sqrt{\rho^2 + (z-z')^2}$.

When calculating the near field in the vicinity of the ground, the electric field in local coordinate system of antenna wire segments should be turned into the global coordinate system of the antenna array.

3. CALCULATION OF THE GROUND LOSS RESISTANCE

The antenna input resistance computed above doesn't include ground loss resistance. It is very complicated to take into account the ground loss resistance in an antenna system with ground screen.

Generally, ground losses of an antenna array with ground screen result from two sources. The first source is the loss caused by the displacement current in the distributed capacitor of the radiator(the down_lead)-ground structure. It is called electric field loss. The second source is the loss caused by the induced conductive current inside the ground flowing toward the antenna down-lead. It is called magnetic field loss. Total losses of the grounding system are sum of above two classes of loss.

3.1. Magnetic Field Loss Resistance

Magnetic field loss power resulting from electric current inside the ground is:

$$P_H = \mathrm{Re}\left(\int_A \boldsymbol{J}_t \cdot \boldsymbol{E}_t \mathrm{d}A\right) = \mathrm{Re}\left(\int_A Z_e \left|\boldsymbol{H}_t\right|^2 \mathrm{d}A\right) = \int_A R_e \left|\boldsymbol{H}_t\right|^2 \mathrm{d}A \tag{9}$$

where $\boldsymbol{H}_t$ and $\boldsymbol{E}_t$ are the tangential components of magnetic and electric field on the surface of the ground respectively, $Z_e$is the surface impedance of the ground, R_e=Re(z_e) is the surface resistance of the ground, and A is the area including all near field zone around antenna.

The surface impedance Z_e is the ratio of tangential electric field intensity $\boldsymbol{E}_t$ to tangential magnetic field intensity $\boldsymbol{H}_t$ on the surface of the ground, and represents the ground impedance per unit area. If the ground is a good conductor, then

$$Z_e \approx (1 + \mathrm{j})\sqrt{\frac{\mathrm{j}\omega\mu}{2\sigma}} = R_e + \mathrm{j}X_e \tag{10}$$

where ω is angular frequency, σ is earth conductivity, μ is earth permeability, and X_e is the ground surface reactance.

When a screen is laid on the surface of a ground, the ground can be modeled as a ground with an equivalent surface impedance Z_g but without a screen. When distance between two grounding wires is smaller than the penetration depth, the equivalent surface impedance Z_g equals to the pure surface impedance Z_e in parallel with impedance Z_s of a perfect conductive screen per unit area in free space [11].

The impedance of perfect conductive screen per unit area in free space is

$$Z_s \approx \mathrm{j}\eta_0 \frac{t}{\lambda} \ln\left(\frac{t}{\pi d}\right) \tag{11}$$

where η_0=377Ω is wave impedance in free space, t is the distance between two grounding wires, d is the diameter of a grounding wire, and λ is wavelength. For radial grounding wires in helix house's ground screen, t is the average value of the interval between adjacent grounding wires.

For buried screen, expression (11) is also correct as the distance between adjacent buried wires is less than the wavelength in the ground. Adjacent wire separation must be less than the skin depth in a conductive medium.

When $\sigma/\omega\varepsilon \gg 1$, the ground equivalent surface resistance with a screen is obtained from (10) and (11) as:

$$R'_H = Re(Z_g) = \frac{xy^2/\sqrt{2}}{x^2 + y^2 + \sqrt{2}xy} \tag{12}$$

where $x = (\mu_0 2\pi f/\sigma)^{\frac{1}{2}}$,$y = \mu_0 f \sin\left(\frac{t}{\pi d}\right)$.

Magnetic field loss resistance is

$$R_H = \frac{P_H}{I_0^2} = \frac{1}{I_0^2} \int_A R'_H \left|\boldsymbol{H}_t\right|^2 \mathrm{d}A \tag{13}$$

where I_0 is the current at antenna root.

3.2. Electric Field Loss Resistance

Assuming that the displacement current density $\boldsymbol{J}_d{=}\omega\varepsilon_0 E_z$, where E_z is vertical electric field component, is perpendicular to the ground and equal to the electric current density $\boldsymbol{J}_z$ flowing vertically underground at any given position on ground screen. The electric field loss power can be computed using the following formula:

$$P_E = \int_A |\boldsymbol{J}_z|^2 R'_E \mathrm{d}A = \int_A J_d^2 R'_E \mathrm{d}A = \int_A R'_E (\omega\varepsilon_0)^2 E_z^2 \mathrm{d}A \qquad (14)$$

where R'_E is effective serial resistance per unit area.

The surface impedance of the ground per unit area between the screen of a buried depth h' and the horizontal top wires of a height h over screen (equivalent to a conductive plane) is expressed as

$$Z_c = \frac{\bar{h}}{\sigma + \mathrm{j}\omega\varepsilon} + \frac{h}{\mathrm{j}\omega\varepsilon_0} \qquad (15)$$

where $\bar{h}$ is equivalent depth of grounding wires, σ is ground conductivity, $\varepsilon_0$andε are dielectric constants in free space and the ground respectively.

When $h \gg s$ and the separation of adjacent grounding wires is much greater than buried depth, i.e. $s \gg 2h' + d$, the equivalent depth of grounding wires can be expressed as

$$\bar{h} = h' + 0.366s \left\{ \lg\left(\frac{s}{\pi d}\right) + R'_0 \lg\left(\frac{s}{2\pi\,(2h' + d/2)}\right) \right\} \qquad (16)$$

where

$$R'_0 = \frac{\sigma^2 + \omega^2(\varepsilon^2 - \varepsilon_0^2) - 2\omega^2\varepsilon\varepsilon_0}{\sigma^2 + \omega^2(\varepsilon + \varepsilon_0)^2} \qquad (17)$$

Electric field loss resistance is

$$R_E = \frac{P_E}{I_0^2} = \frac{1}{I_0^2}\int_A R'_E (\omega\varepsilon_0)^2 E_z^2 \mathrm{d}A \qquad (18)$$

where$R'_E = Re(Z_c)$.

4. COMPUTATION EXAMPLES

As an example, a VLF T-type transmitting antenna array is modeled and simulated. The length of top wires of the antenna curtain is about 1500 m. The interval between adjacent top wires is 50 m. The length of down-leads is 170 m and the average height of top wires is 200 m.

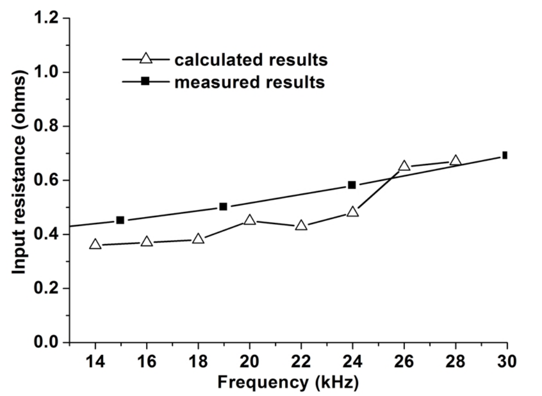

Figure 4. Input resistance of the T-type antenna array.

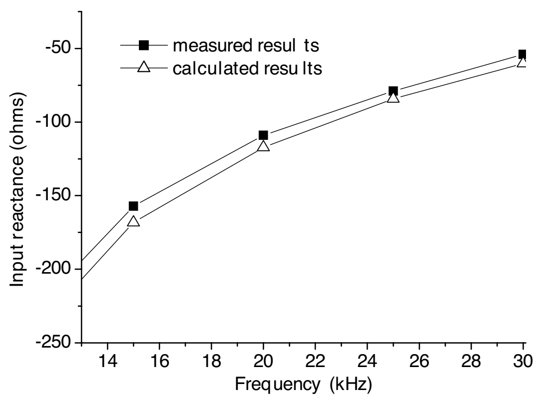

Figure 5. Input reactance of the T-type antenna array.

The helix house ground screen is a 100m×100 m square area and the separation of radial grounding wires is 5°. The length of the top wires ground screen region is about 1800m with an interval of 10 m. The total width of the high voltage feed cage ground screen region is 100 m, and the grounding wires are laid with increased intervals 1m, 1.5 m, 2 m, 2.5 m, 3 m, 4 m and 5m, starting from the middle, and are parallel to the high voltage feed cage.

After the current distribution as well as the voltage U_0 and current I_0 at the feed point of the antenna on perfect ground is obtained by MoM, the antenna radiation resistance R_r and input reactance X_{in} can be represented as:

$$R_r + X_{in} = U_0/I_0 \tag{19}$$

The resistance computed above doesn't include ground loss resistance because of the assumption that the ground surface is a perfect conductor. Ground loss resistance is obtained by (13) and (18). Antenna input resistance is:

$$R_{in} = R_r + R_H + R_E \tag{20}$$

Calculated and measured results of input resistance and reactance for the T-type antenna array are plotted in Fig. 4 and Fig. 5 respectively. The good agreement verifies the correctness of the computational method presented in this paper. Calculated input resistance doesn't consider the ground loss beneath the high voltage feed lines and copper loss in wires since the grounding wires beneath high voltage feed lines are densely laid and both the top wires and down-leads themselves are large in amount and thick in diameter. They have much smaller loss resistance and make little contribution to input resistance and thus be neglected.

5. CONCLUSIONS

Although the structure of the VLF T-type antenna array is very complex, its electrical size is small. In order to effectively calculate the ground loss, a method involving the integral of near field from MoM solution is developed. Calculated and measured results show good agreement for a real VLF T-type antenna array with complex antenna structure and grounding system. This numerical computational method may serve as a reference for engineering design of a VLF antenna.

REFERENCES

1. Hurdsman, D. E., P. M. Hansen, and J. W. Rockway, "LF and VLF antenna modeling," *IEEE APS*, Vol. 4, 811–814, 2003.
2. King, R. W. P., "On the radiation efficiency and the electromagnetic field of a vertical electric dipole in the air above a dielectric or conducting half-space," *Progress In Electromagnetics Research*, PIER 04, 1–43, 1991.
3. Rafi, Gh. Z., R. Moini-Mazandaran, and R. Faraji-Dana, "A new time domain approach for analysis of vertical magnetic dipole radiation in front of lossy half-space," *Progress In Electromagnetics Research*, PIER 29, 57–68, 2000.
4. Nikoskinen, K. I., "Time-domain study of half-space transmission problem with vertical and horizontal dipoles," *IEEE Trans. Antennas Propagat.,* Vol. 41, 1399–1407, Oct. 1993.

5. Rachuram, R., R. L. Smith, and T. F. Bell, "VLF antarctic antenna: impedance and efficiency," *IEEE Transactions on Antennas and Propagation*, Vol. 22, No. 2, 318–322, 1974.
6. Cui, T. J. and W. C. Chew, "Accurate analysis of wire structures from very-low frequency to microwave frequency," *IEEE Transactions on Antennas and Propagation*, Vol. 50, No. 3, 301–307, March 2002.
7. Trueman, C. W. and S. J. Kubina, "Verifying wire-grid model integrity with program 'Check'," *ACES Winter*, Vol. 5, No. 2, 1990.
8. Burke, G. J. and A. J. Poggio, "Numerical Electromagnetics Code (NEC) — method of moments," Rep. UCID 18834, Lawrence Livermore Laboratory, CA, 1981.
9. Miller, E. K. and F. J. Dearick, *Some Computational Aspects of Thin-wire Modeling. Numerical and Asymptotic Techniques in Elecctromagnetics*, 89–127, Springer-Verlag, New York, 1975.
10. Taylor, D. and P. Loschialpo, "Imaging of helical surface wave modes in the near field," *Journal of Electromagnetic Waves and Appl.*, Vol. 17, No. 11, 1593–1604, 2003.
11. Watt, A. D., *VLF Radio Engineering*, Pergamon Press, London, 1967.
12. Uduwawala, D., M. Norgren, P. Fuks, and A. Gunawardena, "A complete FDTD simulation of a real GPR antenna system operating above lossy and dispersive grounds," *Progress In Electromagnetics Research*, PIER 50, 209–229, 2005.
13. Poljak, D. and V. Doric, "Wire antenna model for transient analysis of simple grounding systems, Part I: The vertical grounding electrode," *Progress In Electromagnetics Research*, PIER 64, 149–166, 2006.
14. Poljak, D. and V. Doric, "Wire antenna model for transient analysis of simple grounding systems, Part II: The horizontal grounding electrode," *Progress In Electromagnetics Research*, PIER 64, 167–189, 2006.
15. Lu, Y.-Q. and J. Y. Li, "Optimization of broadband top-load antenna using micro-genetic algorithm," *Journal of Electromagnetic Waves and Appl.*, Vol. 20, No. 6, 793–801, 2006.
16. Riddolls, R. J., "Near-field response in lossy media with exponential conductivity inhomogeneity," *Journal of Electromagnetic Waves and Appl.*, Vol. 20, No. 11, 1551–1558, 2006.