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DISPERSION CHARACTERISTICS OF ELECTROMAGNETIC WAVES IN CIRCULARLY CORED HIGHLY BIREFRINGENT WAVEGUIDE HAVING ELLIPTICAL CLADDING

A. K. Shahi

Department of Physics
Technical University
Lucknow, (U.P.) India

V. Singh

Photonics and Optoelectronics Division
Department of Physics
Banaras Hindu University
Varanasi-221005, (U.P.) India

S. P. Ojha

C.C.S. University
Meerut, (U.P.) India.

Abstract—In this article a new type of circularly cored highly birefringent (Hi-Bi) waveguide having elliptical cladding is proposed and analyzed for the first time in our knowledge. By choosing appropriate orthogonal co-ordinates and using the boundary conditions of the considered waveguide, the eigen value equation in terms of modified Mathieu functions is derived under the weak guidance condition and is presented in this paper. Using this equation the modal dispersion curves for even and odd guided modes are obtained and plotted for different cladding ellipticity e. It is seen that the proposed (Hi-Bi) fiber supports less guided modes than standard circular fiber. Finally, the modal birefringence in the said fiber is also estimated.

1. INTRODUCTION

The well known and the simplest optical waveguide is the standard optical fiber having circular core cross-section which has been studied extensively due to its novel applications in optical telecommunication systems. Waveguides were first dealt with Lord Rayleigh [1], later the dielectric waveguide was studied theoretically by Hondras and Debye [2] and experimentally by Schriever [3]. In 1960 Clarricoats [4] studied the propagation of E.M. waves along the bounded and unbounded dielectric rods. In 1961 Snitzer [5] considered the propagation of cylindrical dielectric waveguide modes near cutoff and far from cutoff. Gloge [6] proposed a new theory of dielectric waveguide which greatly simplifies the complicated theory of dielectric waveguide proposing the weak guidance approximation where the difference between the refractive indices of core and cladding is very small. Under this condition Gloge [7] studied the propagation effects in the optical fiber having circular core. Snyder and Young [8] researched on modes of optical waveguides using weak guidance condition. Recently Ghatak and Sharma [9] have presented an overview of most of the important characteristics of cylindrically symmetric single mode fiber which are now being extensively used in fiber-optic communication systems. Very recently Vivek Singh et al. [10] have studied the modal dispersion characteristics of a standard circular fiber loaded with a conducting sheath helix winding on its core cladding interface. In this study photonic band gaps [11, 12] of recent interest have been observed in some cases.

In brief many studies [13–16] towards modal characteristics and related properties of the standard fiber have been made but the propagation characteristics of E.M. waves in a new type of circularly-cored highly birefringent (Hi-Bi) fiber surrounded by elliptical cladding [17] are not considered till now. In this article we will consider and present this waveguide with its characteristic eigen value equation, dispersion curves and its modal birefringence. Our main motivation is to study a large number of a new and unconventional structure and geometry [18–20, 25–27] and present their new and unconventional characteristics so that some researchers interested for particular property for use in engineering and technology can choose the particular waveguide with desired property from these investigations.

This paper is organized in the following way: Section 2 deals with the derivation of eigen value equation and other parameters. The results and discussions are described in Section 3. Finally conclusion is presented in Section 4.

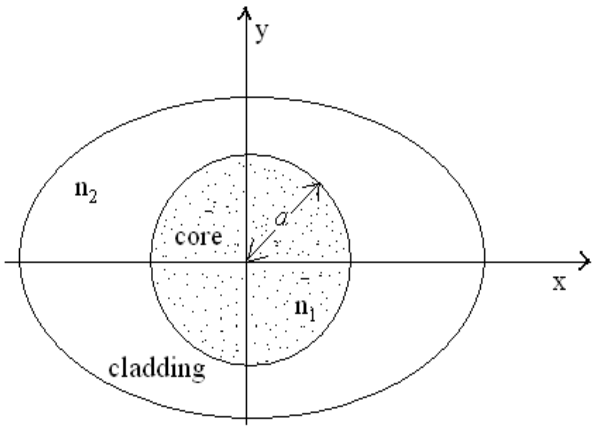

Figure 1. Cross-sectional view of a circularly-cored (Hi-Bi) fiber having elliptical cladding.

2. THEORETICAL DETAILS

Figure 1. illustrates the schematic cross sectional view of the proposed waveguide with a circular core cross section having core refractive index n_1 and elliptical cladding with refractive index n_2, such that $\frac{n_1-n_2}{n_1} \ll 1$. That is we are going to use weak guidance approximation by choosing the appropriate coordinate systems with some mathematical steps given in reference [21, 22]. We may write the longitudinal component of electric field and magnetic field for circular core region where $\xi < \xi_0$ as,

$$E_{z1} = B_0 J_\nu(\xi u)\sin(\phi) \tag{1}$$

$$H_{z1} = A_0 J_\nu(\xi u)\cos(\phi) \tag{2}$$

where (ξ, ϕ, z) are the cylindrical coordinate system, $u^2 = k_0^2 n_1^2 - \beta^2$, J_ν are the Bessel function of first kind, β is the longitudinal component of propagation vector, $k_0 = \frac{2\pi}{\lambda_0}$ and λ_0 is the wavelength of incident wave. Similarly the longitudinal component of electric field and magnetic field for elliptical cladding region where $\xi > \xi_0$ can be written as:

even mode

$$E_{z1} = B_1 Gek_\nu\left(\xi, -y_2^2\right) se\nu(\nu\eta) \tag{3}$$

$$H_{z1} = A_1 Fek_\nu\left(\xi, -y_2^2\right) ce\nu(\nu\eta) \tag{4}$$

and odd mode

$$E_{z1} = B_1 Fek_\nu\left(\xi, -y_2^2\right) se\nu(\nu\eta) \tag{5}$$

$$H_{z1} = A_1 Gek_\nu\left(\xi, -y_2^2\right) ce\nu(\nu\eta) \tag{6}$$

where (ξ, ϕ, z) are the elliptical coordinate system, $y_2^2 = \frac{\left(k_0^2 n_2^2 - \beta^2\right)}{4} q^2$, the semi focal distance is denoted by q and, $se\nu$ and $ce\nu$ are Mathieu azimuthal functions and Fek_ν, Gek_ν, are the modified Mathieu functions. The notations for the Mathieu functions are as that of Adams [22]. Here we will analyze the even mode propagation only in proposed waveguide. Similar analysis can also be made for odd mode propagation. The transverse component of the field in terms of longitudinal component is written as: for core region

$$E_{\eta 1} = \frac{i}{u^2}[\beta B_0 J_\nu(\xi u)\cos(\nu\eta)\nu - \omega\mu_0 A_0 J'_\nu(\xi u) u\cos(\nu\eta)] \tag{7}$$

$$H_{\eta 1} = \frac{i}{u^2}[-\beta A_0 J_\nu(\xi u)\sin(\nu\eta)\nu + \omega\varepsilon_0\varepsilon_1 B_0 J'_\nu(\xi u) u\sin(\nu\eta)] \tag{8}$$

$$\begin{aligned} E_{\eta 2} = \frac{-i}{w^2 q l}\Big[&\beta B_1 Gek_\nu\left(\xi, -y_2^2\right)\cos(\nu\eta)\nu \\ &- \omega\mu_0 A_1 Fek'_\nu\left(\xi, -y_2^2\right)\cos(\nu\eta)\Big] \end{aligned} \tag{9}$$

$$\begin{aligned} H_{\eta 2} = \frac{-i}{w^2 q l}\Big[&-\beta A_1 Fek_\nu\left(\xi, -y_2^2\right)\sin(\nu\eta)\nu \\ &+ \omega\varepsilon_0\varepsilon_2 B_1 Gek'_\nu\left(\xi, -y_2^2\right)\sin(\nu\eta)\Big] \end{aligned} \tag{10}$$

where $l = (\cosh^2\xi - \cos^2\eta)^{1/2}$ and the notation Fek'_ν etc. have been used as abbreviation for $\frac{d}{d\xi} Fek_\nu(\xi, -y_2^2)$. Here B_0, B_1, A_0, A_1 are unknown constant. The boundary conditions are,

$$Ez_1\Big|_{\xi=\xi_0} = Ez_2\Big|_{\xi=\xi_0} \qquad E\eta_1\Big|_{\xi=\xi_0} = E\eta_2\Big|_{\xi=\xi_0} \tag{11}$$

$$Hz_1\Big|_{\xi=\xi_0} = Hz_2\Big|_{\xi=\xi_0} \qquad H\eta_1\Big|_{\xi=\xi_0} = H\eta_2\Big|_{\xi=\xi_0} \tag{12}$$

In this way we get a set of equations having four unknown constants. The nontrivial solution will exists only when the determinant formed by the coefficients of the unknown constant is equal to zero, calling this 4×4 determinant we get,

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix} = 0 \tag{13}$$

where,

$$A_{11} = J_\nu(\xi_0 u), \quad A_{12} = 0, \quad A_{13} = -Gek_\nu\left(\xi_0, -y_2^2\right), \quad A_{14} = 0,$$

$$A_{21} = \frac{\nu\beta}{u^2}J_\nu(\xi_0 u), \quad A_{22} = -\frac{\omega\mu_0}{u}J'_\nu(\xi_0 u), \quad A_{23}\frac{\nu\beta}{y_2^2}Gek_\nu\left(\xi_0, -y_2^2\right),$$

$$A_{24} = -\frac{\omega\mu_0}{y_2^2}Fek'_\nu\left(\xi_0, -y_2^2\right), \quad A_{31} = 0, \quad A_{32} = J_\nu(\xi_0 u), \quad A_{33} = 0,$$

$$A_{34} = -Fek_\nu\left(\xi_0, -y_2^2\right), \quad A_{41} = \frac{\omega\varepsilon_0\varepsilon_1}{u}J'_\nu(\xi_0 u), \quad A_{42} = -\frac{\nu\beta}{u^2}J_\nu(\xi_0 u),$$

$$A_{43} = \frac{\omega\varepsilon_0\varepsilon_2}{y_2^2}Gek'_\nu\left(\xi_0, -y_2^2\right), \quad A_{44} = -\frac{\nu\beta}{y_2^2}Fek_\nu\left(\xi_0, -y_2^2\right)$$

By solving above equation (13) we get the characteristic eigen value equation for considered waveguide as,

$$\left[\frac{Fek'_\nu\left(\xi_0, -y_2^2\right)}{y_2^2 Fek_\nu\left(\xi_0, -y_2^2\right)} + \frac{1}{u}\frac{J'_\nu(\xi_0 u)}{J_\nu(\xi_0 u)}\right]\left[\frac{\varepsilon_2 Gek'_\nu\left(\xi_0, -y_2^2\right)}{y_2^2 Gek_\nu\left(\xi_0, -y_2^2\right)} + \frac{\varepsilon_1}{u}\frac{J'_\nu(\xi_0 u)}{J_\nu(\xi_0 u)}\right] = \frac{\nu^2\beta^2}{k_0^2\varepsilon_2}\left[\frac{1}{U^2} + \frac{1}{W^2}\right]^2 \quad (14)$$

For the special case $\nu = 1$, under the weak guidance condition the eigen value equations (14) can be written as

$$\frac{1}{u}\frac{J'_1(\xi_0 u)}{J_1(\xi_0 u)} + \frac{Fek'_1\left(\xi_0, -y_2^2\right)}{y_2^2 Fek_1\left(\xi_0, -y_2^2\right)} = 0 \quad (15)$$

$$\frac{\varepsilon_1}{u}\frac{J'_1(\xi_0 u)}{J_1(\xi_0 u)} + \frac{\varepsilon_2 Gek'_1\left(\xi_0, -y_2^2\right)}{y_2^2 Gek_1\left(\xi_0, -y_2^2\right)} = 0 \quad (16)$$

Several methods [23, 24] have been used to calculate the modal birefringence of the optical waveguides. Among these, the method used by Wong and Chiang [24] is very convenient and simple. Using this method the modal birefringence B may be written as:

$$B = b'_{TE} - b'_{TN} \quad (17)$$

where b' is the normalized propagation constant defined as $b' = \frac{\left(\frac{\beta}{k_0}\right)^2 - n_2^2}{n_1^2 - n_2^2}$ and the normalized frequency parameter $V = \frac{2\pi a}{\lambda_0}(n_1^2 - n_2^2)^{\frac{1}{2}}$. Here $a = \xi_0$ at the interface of guide and the matching condition are valid only for small eccentricity e.

3. NUMERICAL RESULTS AND DISCUSSION

The modal eigen value equation (14) comprises the dispersion relation which is one of the main results of the present communication. In this article, equation (14) contains modified Mathieu functions which have been expressed in terms of Bessel functions considering references [21, 22] for $\nu = 1$. Now equation (15) is solved numerically to study the dispersion characteristics of the proposed waveguide for even and odd guided modes .For this purpose we choose some parameters like $n_1 = 1.5$, $n_2 = 1.3$, $\lambda_0 = 1.55\,\mu$m. For dispersion curves, it is very convenient to plot normalized propagation constant b' versus normalized frequency parameter V defined respectively as $b' = \frac{\beta^2 - k_0^2 n_2^2}{n_1^2 - n_2^2}$ and $V = \frac{2\pi a}{\lambda_0}(n_1^2 - n_2^2)^{1/2}$. Next the left hand side of equation (15) is evaluated for many admissible β values, lying between $k_0 n_1$ and $k_0 n_2$. Now the left hand side of equation (15) is plotted against the β-values for a fixed value of a and the zero crossings are noted. Each zero crossing corresponds to a particular sustained mode. Several such curves are plotted for different values of a and from these graphs one can find out how β varies with a for a given mode (zero crossing). From β we can find out b' and from a we can find out V using above equations. Thus the b' Vs V curves (dispersion curves) can be plotted for each mode Fig. 2, Fig. 3 and Fig. 4 illustrate the dispersion curves for even guided modes for the proposed fiber with cladding ellipticity $e = 0.4$, $e = 0.6$ and $e = 0.8$ respectively. Similarly Fig. 5, Fig. 6 and Fig. 7 illustrate the dispersion curves for odd guided modes for

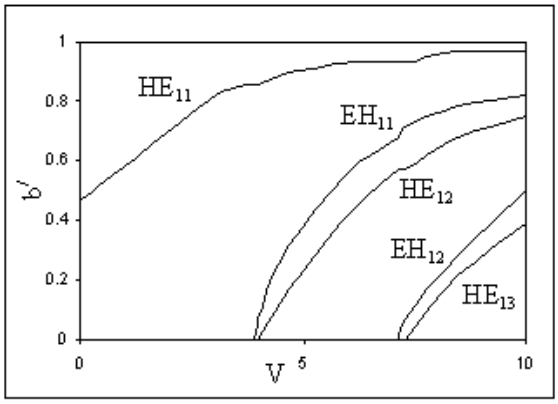

Figure 2. Dispersion curves (b' Vs V graphs) for even guided modes when $e = 0.4$.

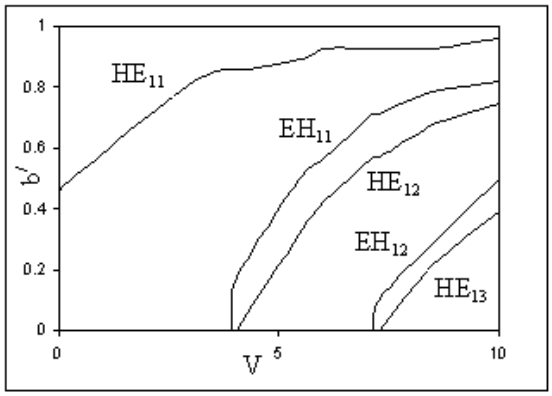

Figure 3. Dispersion curves (b' Vs V graphs) for even guided modes when $e = 0.6$.

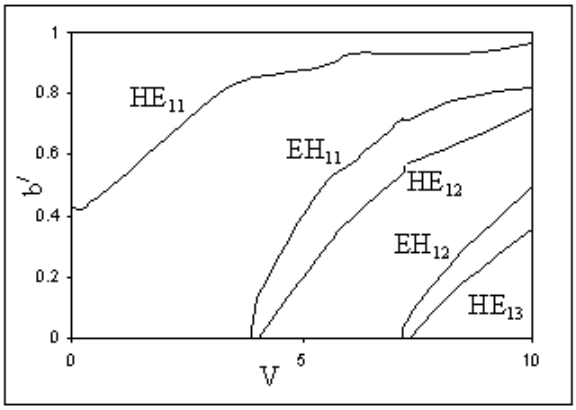

Figure 4. Dispersion curves (b' Vs V graphs) for even guided modes when $e = 0.8$.

the said fiber having cladding ellipticity $e = 0.4,\ e = 0.6$ and $e = 0.8$ respectively. We observe that all dispersion curves are of the expected standard shape except for the curve corresponding to the lowest order mode. This is the fundamental mode having no cutoff value for all cases except in one case Fig. 7 where we have no fundamental mode. Several interesting points are noteworthy here.

First, the introduction of the dielectric elliptical cladding around the circular core changes the nature of the dispersion curves for all modes compared to the dispersion curves of a standard circular fiber.

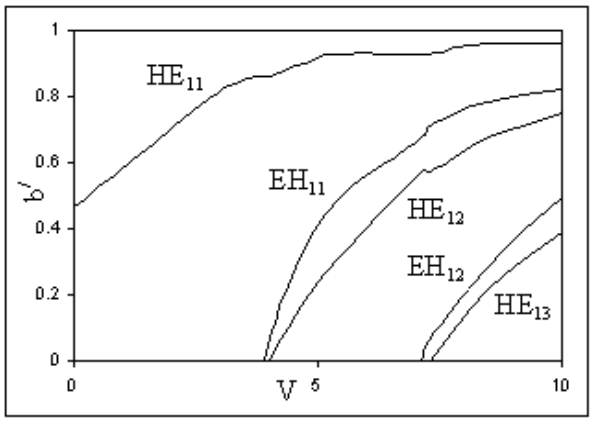

Figure 5. Dispersion curves (b' Vs V graphs) for odd guided modes when $e = 0.4$.

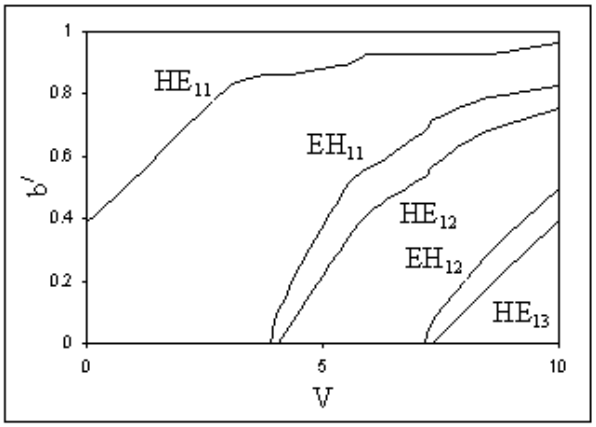

Figure 6. Dispersion curves (b' Vs V graphs) for odd guided modes when $e = 0.6$.

Second interesting feature is that for both type of guided modes (even and odd), there is a tendency of the adjacent EH and HE modes to have common cutoff values. It appears therefore, that the presence of the dielectric cladding around the circular core has the effect of splitting a mode into two modes, removing the degeneracy of the mode. However, there is exception for example Fig. 7. Third point to be noted is that as the value of cladding ellipticity e increases from 0.4 to 0.8, the number of guided modes remains unchanged. Lastly we observe

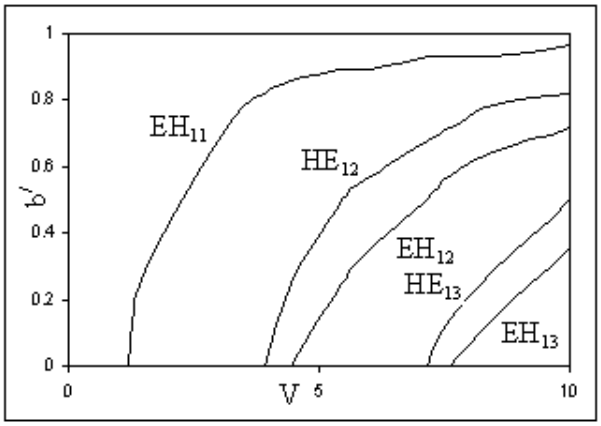

Figure 7. Dispersion curves (b' Vs V graphs) for odd guided modes when $e = 0.8$.

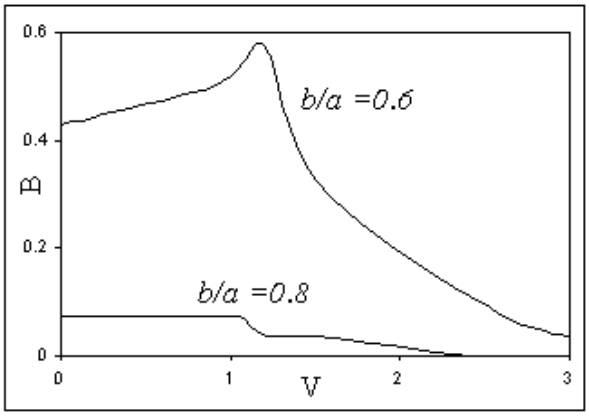

Figure 8. The dependence of the normalized modal birefringence B on the normalized frequency parameters V for $\frac{b}{a} = 0.6$ and for $\frac{b}{a} = 0.8$.

that the number of guided modes for the proposed waveguide is less than those for the standard fiber. Finally we have obtained the modal birefringence B of considered (Hi-Bi) fiber for HE11 modes following the method of Wong and Chiang [24]. Fig. 8 illustrates the variation of the normalized modal birefringence B with respect to the normalized frequency parameters V for $e = 0.6$ and $e = 0.8$. We observe from Fig. 8 that maximum birefringence occurs at $V = 1.21$ which is very close to the cutoff value of lowest order mode in elliptical cladding fiber

and hence this will be the preferred operating point for single mode fiber.

When we compare our estimated results with those as reported by Adams [22], we see that in each case nature of the curve is similar.

4. CONCLUSION

In this article an analysis of the eigen modes of a new optical waveguide having circular core and elliptical cladding is presented. An eigen value equation in terms of modified Matheiu functions is derived. This equation comprises the dispersion relation which is one of main results of this paper. The characteristics eigen value equation (15) is solved numerically to study the dispersion characteristics of the proposed waveguide for even and odd guided modes. Observing the obtained dispersion curves, some important features are noted: First, the introduction of dielectric elliptical cladding around the circular core changes the nature of dispersion curves for all modes compared to the nature of dispersion curves of the standard circular fiber. Second interesting feature is that for both type of guided modes there is a tendency for the adjacent E.H. and H.E. modes to have common cutoff values. Third point to be noted is that as the value of cladding ellipticity increases from 0.4 to 0.8, the number of guided modes remains unchanged. Lastly we notice that the number of guided modes for the proposed waveguide is less than those for the standard fiber. Finally we note from Fig. 8 that the maximum birefringence occurs at $V = 1.21$ which is very close to the cutoff value of the lowest order mode for the proposed fiber and hence this will be the preferred operating point for a single mode fiber.

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