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A COMPARISON BETWEEN CIRCULAR AND HEXAGONAL ARRAY GEOMETRIES FOR SMART ANTENNA SYSTEMS USING PARTICLE SWARM OPTIMIZATION ALGORITHM

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Abstract—In this paper, circular and hexagonal array geometries for smart antenna applications are compared. Uniform circular (UCA) and hexagonal arrays (UHA) with 18 half-wave dipole elements are examined; also planar (2 concentric rings of radiators) uniform circular (PUCA) and hexagonal arrays (PUHA) are considered. The effect of rotating the outer ring of the PUCA is studied. In our analysis, the method of moments is used to compute the response of the uniform circular and hexagonal dipole arrays in a mutual coupling environment. The particle swarm optimization (PSO) algorithm is used to optimize the complex excitations, amplitudes and phases, of the adaptive arrays elements for beamforming.

1. INTRODUCTION

Smart antennas refer to a group of antenna technologies that increase the system capacity by reducing the co-channel interference and increase the quality by reducing the fading effects [1]. A smart antenna array containing M identical elements can steer a directional beam to maximize the signal from desired users, signals of interest (SOI), while nullifying the signals from other directions, signals not of interest (SNOI) [2]. The array geometries that have been studied include mainly uniform linear arrays, uniform rectangular (URA), and circular arrays (UCA). A linear array has excellent directivity and it can form the narrowest main-lobe in a given direction, but it does not work equally well in all azimuthal directions. A major disadvantage of the URA is that an additional major lobe of the same intensity appears on the opposite side. An obvious advantage results from the symmetry of the circular array structure. Since a circular array does not have edge elements, directional patterns synthesized with a circular array can be electronically rotated in the plane of the array without a significant change of the beam shape [3]. Concerning the two geometries, the URA and the planar uniform circular array (PUCA) with similar areas, slightly greater directivity was obtained with the use of the PUCA [4]. On the other hand, a circular array is a high side-lobe geometry. If the distance of array elements is decreased to reduce the sidelobes, the mutual coupling influence becomes more significant. For mitigating high side-lobe levels multi-ring arrays are utilized, which have some other advantages as well. Furthermore, a hexagonal array is presented for smart antenna applications to overcome the problem of high side-lobes [5]. A structure consisting of six circular patches non-uniformly distributed along the perimeter of an antenna array and one antenna in the center has been previously examined and compared with a circular structure. It was found that it was possible to configure the array to obtain directional patterns with high gain and directivity for both (circular and hexagonal) arrays. Also, best steerablity was obtained using a uniform array of seven patch antennas, configured as a hexagon with a central element [6].

The techniques of placing nulls in the antenna patterns to suppress interference and maximizing their gain in the direction of desired signal have received considerable attention in the past and are still of great interest using evolutionary algorithms such as genetic algorithms (GA) [7, 8] or the sequential quadratic programming (SQP) algorithm [9]. It is recognized that the PSO algorithm is a practical and powerful optimization tool for a variety of electromagnetic and antenna design problems [10–15]. Compared with other evolutionary algorithms such as the GA and simulated annealing (SA), the PSO algorithm is much easier to understand and implement and requires minimum mathematical processing. In recent years, various versions of the PSO algorithm have been successfully used in linear [16, 17] and circular antenna array synthesis problems [18, 19]. Many of the attempts on antenna array synthesis assume that the elements of the array are represented by isotropic point sensors isolated from each other or the element pattern may be modeled by a cosine function. However, in practice, the elements of antenna arrays have finite physical dimensions and specific radiation characteristics. Since most of the beamforming algorithms ignore the effects of mutual coupling, the predicted system performances may not be accurate, especially in closely spaced antenna elements. Therefore, to evaluate accurately the resulting system performance of practical antenna arrays, the electromagnetic influence among the elements must be carefully considered. More recently, much attention has been paid to the effects of mutual coupling [20–22], and there have been studies integrating the genetic algorithm (GA) with the method of moments [23].

The methods of beam pattern synthesis generally based on controlling the complex weights (the amplitude and phase), the excitation amplitude only, the phase only, and the element position only have been extensively considered in the literature [2, 7, 15, 24]. The most important method is based on controlling the complex weights. This technique fully exploits the degrees of freedom for the solution space. Furthermore, the sidelobe level (SLL) and the main beam characteristics can be controlled. On the other hand, it is also the most expensive technique considering the cost of both a phase shifter and a variable attenuator for each array element. Furthermore, when the number of elements in the antenna array increases, the computational time to find the values of element amplitudes and phases will also increase. This results in a trade-off between the quality of the constrained pattern and the complexity of the electronic control. PSO is used to optimize the weights of phase shift and amplitude of the excitation of each element of the array for beamforming synthesis. In this paper, the PSO algorithm program was implemented using MATLAB-software version 7.0.4 and linked to a FORTRAN code program to simulate the antenna arrays using Microsoft Developer Studio 97.

This paper is primarily a comparative study of several different array geometries using 18 half-wave dipole elements in free space. The first comparison will be between a uniform circular (UCA) and a uniform hexagonal array (UHA); then a comparison between a planar uniform circular (PUCA) and a planar uniform hexagonal array

(PUHA) will be considered. Also, the effect of rotating the outer ring of the PUCA on the radiation pattern will be studied. The performance of the designed arrays is assessed using a full EM analysis based on the method of moments (MoMs) [25]. In this analysis, the mutual coupling effects between the array elements are fully taken into account. As noted before, the array consists of center-fed half-wave dipoles. The dipoles are identical and oriented perpendicular to the plane of the array.

2. PARTICLES SWARM OPTIMIZATION ALGORITHM

Particle swarm optimization (PSO) is a population-based, self-adaptive search optimization technique first introduced by Kennedy and Eberhart [26] in 1995. A new inertia weight parameter was incorporated into the original PSO algorithms by Shi and Eberhart [27]. The PSO method is becoming very popular due to its simplicity of implementation and ability to quickly converge to a reasonably good solution. In the particle swarm algorithm, the trajectory of each individual in the search space is adjusted by dynamically altering the velocity of each particle, according to its own flying experience and the flying experience of the other particles in the search space.

The position vector and the velocity vector of the ith particle in the D-dimensional search space can be represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ and $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$ respectively. To update the velocity matrix at each iteration k, every particle should know its personal best and the global best position vectors in addition to the neighbor best position. The personal best position vector defines the position at which each particle attained its best fitness value up to the present iteration. The personal best position of the ith particle is represented as $Pbest_i = (pbest_{i1}, pbest_{i2}, \ldots, pbest_{iD})$. The global best position vector defines the position in the solution space at which the best fitness value was achieved by all particles, and is defined by $Gbest = (gbest_1, gbest_2, \ldots, gbest_D)$. In a new version of PSO algorithm [28], each particle is attracted towards the best previous positions visited by its neighbors. In this case, we take into account two neighbors on each side using a circular neighborhood topology [29]. The best neighbor position discovered by the whole population is represented as $Nbest = (nbest_1, nbest_2, \ldots, nbest_D)$. The particles are manipulated according to the following equations:

$$\begin{aligned} v_{id}^{k+1} &= \omega v_{id}^{k} + c_1 rand_1 (pbest_{id} - x_{id}^{k}) \\ &\quad + c_2 rand_2 (gbest_d - x_{id}^{k}) + c_3 rand_3 (nbest_d - x_{id}^{k}) \qquad (1) \\ x_{id}^{k+1} &= x_{id}^{k} + v_{id}^{k+1} \Delta t \qquad (2) \end{aligned}$$

where c_1, c_2, and c_3 are the acceleration constants, which represent the weighting of stochastic acceleration terms that pull each particle towards *pbest, gbest*, and *nbest* positions. $rand_1, rand_2$, and $rand_3$ are three random numbers in the range [0,1], ω is the inertia weight introduced to balance between the global and local search abilities, and Δt is taken as a unit time step.

In our PSO algorithm, the terminology of soft and hard boundary conditions is applied to describe the way in which particles are enforced to stay inside the desired domain of interest [30]. The soft boundary conditions rely on a velocity clipping technique to prevent particles from explosion, where if $|v_{id}|$ exceeds a positive constant value specified by the user, then the velocity of that dimension is assigned to be $\text{sign}(v_{id})\ V_d^{\max}$, i.e., the velocity along each dimension is clamped to a maximum magnitude $V_d^{\max}$. This is done to help keep the swarm under control. The maximum velocity was set to the upper limit of the dynamic range of the search ($V_d^{\max} = X_d^{\max}$). The hard boundary condition is any boundary that uses a position-clipping criterion, where if x_{id} exceeds $X_d^{\max}$, then it is assigned to be $X_d^{\max}$, and also if x_{id} falls below $X_d^{\min}$, then it is assigned to be $X_d^{\min}$.

In this case, a time-varying maximum velocity is used, where v_{id} is changed linearly from $V_d^{\max}$ to 0.1 $V_d^{\max}$, because as the particles approach to the optimal result it is preferred to have them move with lower velocities [31]. The concept of time-varying acceleration coefficients (TVAC) c_1 and c_2 is taken into account in addition to the time-varying inertia weight (TVIW) factor, to effectively control the global search and convergence to the global best solution. An improved optimum solution was observed when changing c_1 from 2.5 to 0.5, changing c_2 from 0.5 to 2.5, and fixing c_3 to 1.0, over the full range of the search. The weighting function, ω, can be calculated from the following equation:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times iter \tag{3}$$

where, ω_{max} is the initial weight (0.9), $\omega_{\min}$ is the final weight (0.4), $iter_{\max}$ is the maximum iteration number, and $iter$ is the current iteration number.

3. OBJECTIVE FUNCTION

The objective function provides the interface between the physical problem and the optimization algorithm. In general this could be the antenna gain, directivity, SLL, peak cross-polarization, and weight, or some kind of weighted sum of all these factors. So the quality of an antenna beamforming is expressed mathematically by an objective function. The following objective function rewards the antenna array for maximizing the output power toward the desired signal at φ_i and minimizing the total output power in the direction of the interfering signals at φ_j.

$$Objectiv\_function = \sum_{i=1}^{N} a_i G(\varphi_i) - \sum_{j=1}^{M} b_j G(\varphi_j) \tag{4}$$

where G is the antenna array gain and the constants a_i and b_j are the weights that control the contribution from each term to the overall objective function. The constant N represents the number of desired users, and M represents the number of interferers. In our analysis, we take $N = 1$ and $M = 2$. The weights a_i and b_j are considered to be ($a_1 = 20$), ($b_1 = 1$), and ($b_2 = 1$) to give a higher priority to maximizing the output power toward the desired signal while minimizing the total output power in the direction of the interfering signals.

4. ARRAY GEOMETRIES

Different shapes of circular and hexagonal arrays using half-wave dipole elements are presented and compared with each other. The first array considered is a UCA. The geometry of the array is shown in Figure 1a. This geometry consists of 18 elements uniformly distributed with a ring radius $r = (9/2\pi)\,\lambda$. Also the geometry of a planar uniform circular array (PUCA) is considered: it consists of two concentric arrays with uniformly distributed antenna elements as shown in Figure 1b. The inner ring of radius $r_1 = (3/2\pi)\,\lambda$ consists of 6-elements and the outer ring of radius $r_2 = (6/2\pi)\,\lambda$ consists of 12-elements.

The second type of arrays is a UHA. The geometry consists of an 18-element hexagonal array with radius $r = (9/2\pi)\,\lambda$ as shown in Figure 2a. Also a planar uniform hexagonal array (PUHA) is considered. The geometry of a PUHA consists of two concentric hexagonal arrays with uniformly distributed antenna elements as shown in Figure 2b. The inner hexagon of radius $r_1 = (3/2\pi)\,\lambda$ consists of 6-elements, and the outer hexagon of radius $r_2 = (6/2\pi)\,\lambda$ consists of 12-elements.

The antenna elements in all cases consist of vertical (z-directed) half-wave dipole elements equally spaced in the x-y plane along a circular ring, where the distance between adjacent elements is $d_c = 0.5\,\lambda$ and the dipole wire radius is $a = 0.003369\,\lambda$.

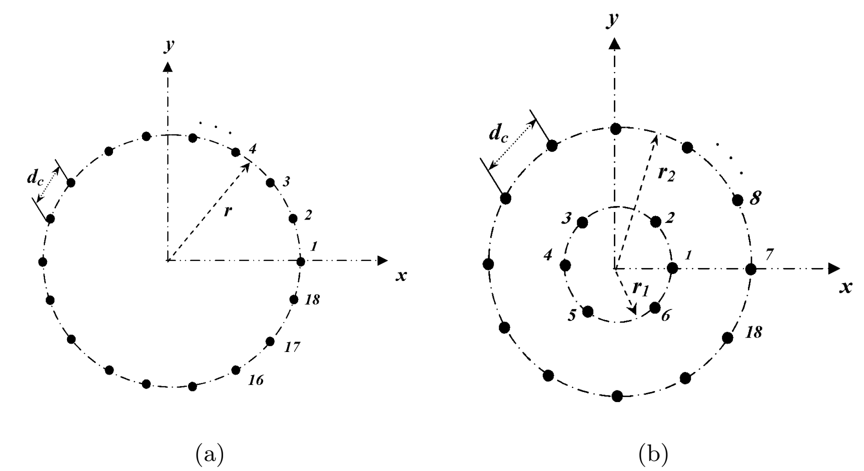

Figure 1. Circular array geometries (a) Uniform circular array (UCA). (b) Planar uniform circular array (PUCA).

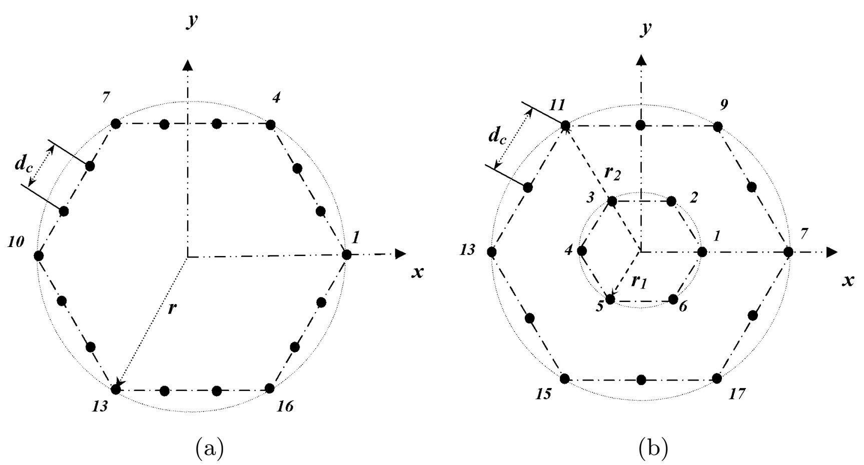

Figure 2. Hexagonal array geometries (a) Uniform hexagonal array (UHA). (b) Planar uniform hexagonal array (PUHA).

4.1. All Elements Excited with Equal Amplitude and Phase

To illustrate the difference between circular and hexagonal arrays geometries, the normalized radiation pattern for each case is plotted first, assuming all elements are excited with the same amplitude and phase $(1, 0)$. Figure 3a shows a comparison between the normalized radiation pattern in the H-plane for the UCA and the UHA. Also

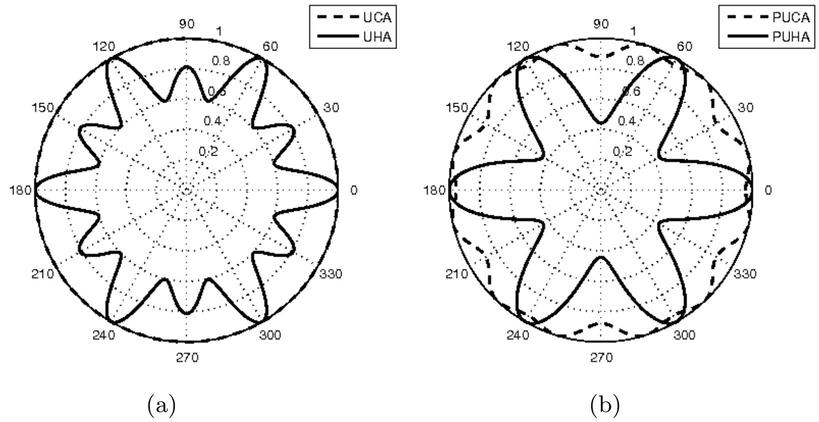

Figure 3. Normalized radiation pattern comparison between (a) UCA and UHA. (b) PUCA and PUHA.

Figure 3b shows the normalized radiation pattern comparison between PUCA and PUHA in the same plane.

It can be observed from these plots that, the hexagonal arrangement can give better steerablity and higher gain characteristics than circular arrangements for both uniform and planar uniform arrays. Also, the planar uniform array is more steerable than the uniform array.

4.2. Beamforming of Smart Antenna Arrays

Now, as an example of adaptive beamforming with all arrays geometries, we considered the desired user at $\varphi = 180°$ while the other two users are at $\varphi = 60°$ and $\varphi = 240°$, which are considered as interferers.

Figure 4a shows a comparison between the resulting normalized gain pattern for the UCA and UHA using half-wave dipole elements. Also, a comparison between the resulting normalized gain pattern for the PUCA and PUHA is depicted in Figure 4b.

The PSO algorithm shows good performance in directing the maximum towards the direction of the SOI while placing deep nulls towards the angles of SNOIs even when the mutual coupling between elements is fully taken into account. The PSO algorithm is employed with a swarm size of 100 and 200 iterations. It required about 16 min on a 32-bit Dell Precision Workstation 690 (Dual Core Intel (tm) Xeon (tm) Processor 5050 at 3.0 GHz) to get the result. This time can be reduced very significantly by calculating the Z-matrix of the

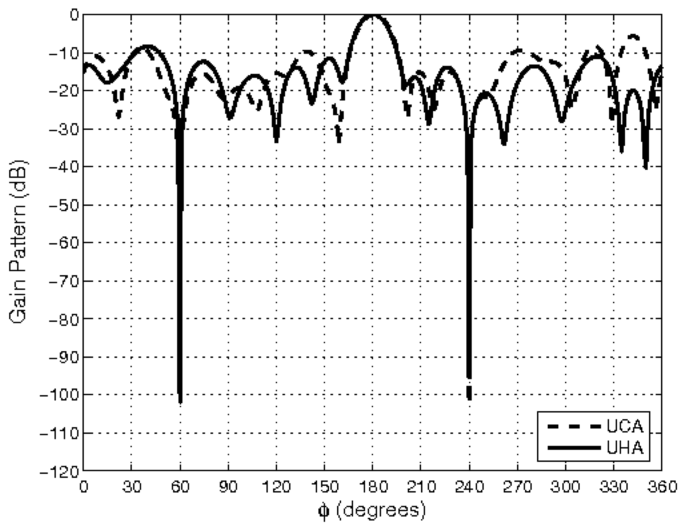

Figure 4a. Normalized gain pattern comparison between UCA and UHA.

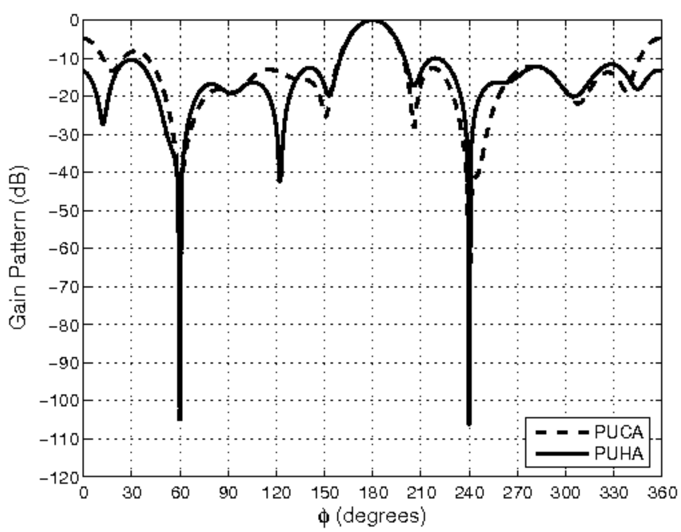

Figure 4b. Normalized gain pattern comparison between PUCA and PUHA.

antenna array, which takes most of the time only once and calling it when needed instead of calculating it each time $(100 * 200)$. For example, the time required is only 24 sec when the same array is implemented using isotropic elements (where the Z-matrix calculation is not required). Also, it is not necessary to wait until the end of all iterations; the program can be ended when acceptable results are obtained. For beamforming synthesis, the amplitude was allowed to vary between 1.0 and 3.0 and the phase is allowed to vary between $-\pi$ and π.

The required amplitude and phase excitations of each element to obtain the beampatterns in Figures 4a, 4b as will as the directivity comparison between the different types of array are shown in Table 1. It is noted that, the hexagonal array geometries achieves slightly deeper nulls towards the angles of interfering signals with an increase in gain around 0.9 dB compared with the circular array geometries. Also, it is noted that the planar uniform circular and hexagonal arrays achieve deeper nulls towards the angles of interfering signals compared with the uniform circular and hexagonal arrays. Also, as a comparison between circular and hexagonal array geometries, it is noted that the area of the circular array is $(3.14r^2)$ but for hexagonal array geometry the area is $(2.16r^2)$, which is less than the area of the circular array.

5. THE EFFECT OF ROTATING THE OUTER RING OF A PUCA

Because of the symmetry of the circular array structure, the synthesized directional patterns can be electronically rotated without a significant change of the beam shape. Now, we will study the effect of rotating the outer ring of a PUCA with a certain degree to improve the null depth. As shown in Figure 5 the inner ring is fixed and the direction of the outer ring is rotated by an angle φ. It is found that, when the outer ring is rotated by $15°$, the radiation pattern is enhanced and becomes most like the planar uniform hexagonal array radiation pattern as shown in Figure 6, which also indicates that the null depth is improved compared with the precious PUCA without outer ring rotation.

Now, the PSO algorithm will be used for adaptive beamforming for this case, with the SOI and SNOI in the same directions as the previous examples. A comparison between the resulting normalized gain pattern for the rotated PUCA and the unrotated PUHA is shown in Figure 7.

It is found that, by adjusting the rotation of the outer ring of the PUCA a better performance is obtained where the null depth is

Table 1. Amplitude and phase excitation for each element and Directivity comparison for different array types.

Element No.	Circular Arrays		Hexagonal Arrays	
	UCA	PUCA	UHA	PUHA
1	$3.00\angle 45.82^o$	$3.00\angle 180^o$	$3.00\angle -110.12^o$	$3.00\angle -179.2^o$
2	$3.00\angle 75.91^o$	$2.38\angle -85.02^o$	$2.81\angle 12.96^o$	$3.00\angle -58.02^o$
3	$2.79\angle 179.73^o$	$2.92\angle 45.25^o$	$3.00\angle 91.64^o$	$2.81\angle 180^o$
4	$3.00\angle -28.28^o$	$2.34\angle 177.97^o$	$3.00\angle 180^o$	$1.67\angle 180^o$
5	$2.89\angle -180^o$	$3.00\angle 66.93^o$	$3.00\angle -8.92^o$	$2.95\angle 147.35^o$
6	$2.90\angle -2.01^o$	$1.00\angle -76.46^o$	$3.00\angle 180^o$	$2.99\angle -32.57^o$
7	$2.95\angle 180^o$	$3.00\angle -22.97^o$	$2.68\angle -69.84^o$	$2.99\angle -25.34^o$
8	$1.76\angle -33.26^o$	$2.66\angle 52.68^o$	$3.00\angle 114.88^o$	$3.00\angle 91.33^o$
9	$2.04\angle -105.46^o$	$3.00\angle 152.8^o$	$3.00\angle 179.64^o$	$2.87\angle 178.27^o$
10	$2.81\angle 18.24^o$	$1.40\angle -8.35^o$	$3.00\angle 180^o$	$3.00\angle -6.04^o$
11	$2.89\angle -2.73^o$	$3.00\angle 180^o$	$1.00\angle 173.17^o$	$2.97\angle -180^o$
12	$2.51\angle 180^o$	$1.00\angle -8.26^o$	$3.00\angle 105.07^o$	$1.14\angle -66.59^o$
13	$1.60\angle 44.49^o$	$3.00\angle -76.5^o$	$2.88\angle -3.85^o$	$1.00\angle 180^o$
14	$2.80\angle -9.09^o$	$2.87\angle -3.53^o$	$3.00\angle -176.87^o$	$2.99\angle -53.37^o$
15	$1.13\angle -180^o$	$3.00\angle 179.9^o$	$3.00\angle -6.67^o$	$3.00\angle 180^o$
16	$3.00\angle -6.95^o$	$2.80\angle -37.92^o$	$2.97\angle -180^o$	$3.00\angle -8.46^o$
17	$2.57\angle 180^o$	$2.47\angle 180^o$	$3.00\angle 79.25^o$	$3.00\angle 180^o$
18	$2.32\angle 49.93^o$	$2.99\angle 61.43^o$	$3.00\angle -6.58^o$	$3.00\angle 91.81^o$
Directivity	12.13 dB	12.17 dB	13.17 dB	12.97dB

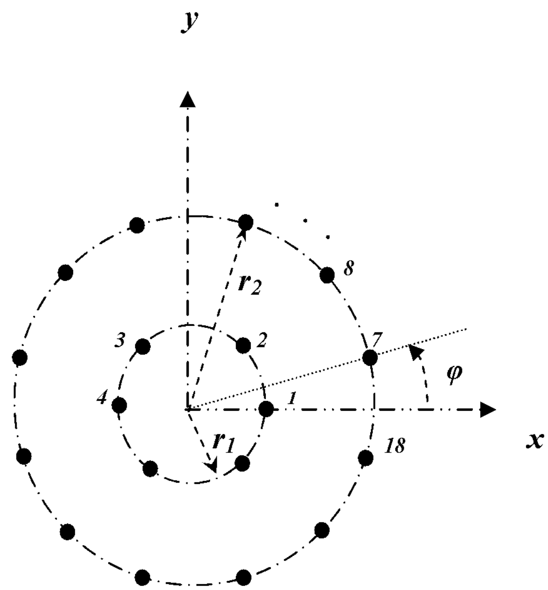

Figure 5. Planar uniform circular array (PUCA) geometry its outer ring rotated by 15°.

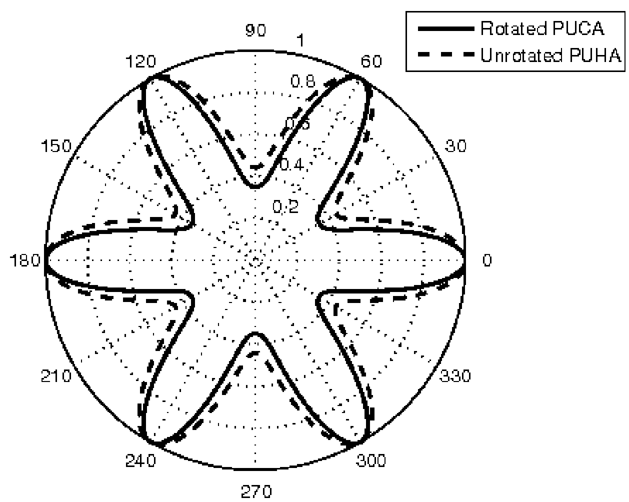

Figure 6. Normalized radiation pattern comparison between the rotated PUCA and PUHA.

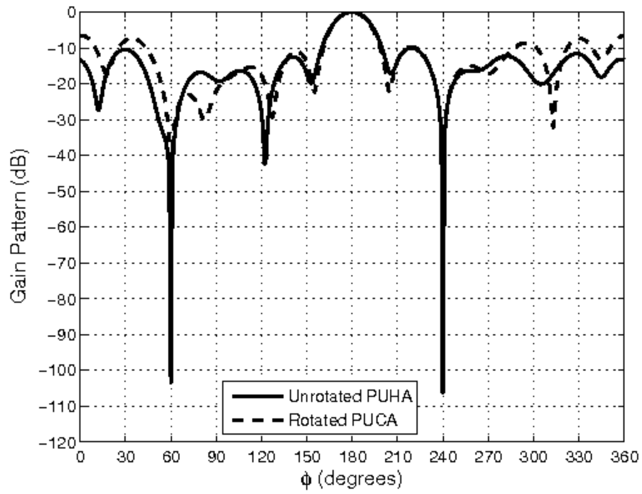

Figure 7. Normalized gain pattern comparison between the rotated PUCA and unrotated PUHA.

slightly increased and the gain is also increased by 0.23 dB compared with the unrotated PUCA.

6. CONCLUSION

In this paper, smart adaptive arrays such as UCA, PUCA, UHA, and PUHA are considered. Each array consists of center-fed half-wave dipoles and the mutual coupling effects between the array elements are fully taken into account. By integrating the particle swarm optimization algorithm with the method of moments, the amplitudes and phases of the antennas are calculated. The comparison between circular array and hexagonal array shows that hexagonal array geometries give slightly deeper nulls, a higher gain by approximately 0.9 dB, and a smaller overall size, with the same beamwidth as circular array geometries. The planar arrays achieve deeper nulls towards the angles of interfering signals compared with the 1-D arrays. Also, it is found that, the rotation of the outer ring of PUCA is very effective, where by rotating the outer ring by 15° the gain and the null depth are increased compared with the unrotated PUCA and approach the performance of the planar uniform hexagonal array PUHA.

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