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MODAL DISPERSION CHARACTERISTICS OF A SINGLE MODE DIELECTRIC OPTICAL WAVEGUIDE WITH A GUIDING REGION CROSS-SECTION BOUNDED BY TWO INVOLUTED SPIRALS

P. C. Pandey and A. Mishra

Department of Physics
University Institute of Engineering & Technology
Chhatrapati Sahu Ji Maharaj University
Kanpur 228024, U.P., India

S. P. Ojha

C. C. S., University
Meerut, U.P., India

Abstract—With the use of scalar field approximation we make an analytical study of a dielectric waveguide whose core cross-section is bounded by two spirals of the form $\frac{1}{r} = \xi\theta$. This waveguide is similar to that of a distorted slab waveguide in which both a curvature and a flare are present. We derived the modal characteristic equation by analytical analysis under the weak guidance approximation. We find the modal dispersion curve, which support only single mode propagation and the same compared with the same kind of waveguide with metal claddings.

1. INTRODUCTION

The research on the conventional and unconventional shapes of the waveguides has focused much attention during the last two decades. Unconventional waveguides with core cross-sections such as elliptical, rectangular, triangular, pentagonal, annular, spiral, cardioids etc. have been studied by the several researchers [1–13]. Such unconventional waveguides play an important role in the design and operation of many integrated optical devices such as wavelength filtering, coupling and semiconductor laser technology.

New material like metallic, chiral materials, liquid crystals, magnetic crystals etc have also been introduced as constitutive materials for new kinds of unconventional waveguides in either core or cladding [14–26]. These smart structured waveguides with smart materials provide a rich variety of characteristics; several books and research papers have appeared in this area. Recently some new works have been introduced, in which a new type of annular optical waveguide whos outer cladding is made of sheath helix. The sheath helix is made up of dielectric material of lower refractive index than the core material [25, 26]. These types of waveguides provide us new ideas about the modal wave propagation with some band gaps. It is expected that the theoretical work of this kind will provide the model properties of a variety of waveguide forming a rich fund of results from which technologists and scientists working in this practical fields can choose the required characteristics and structures. In future when the necessary fabrication technology becomes available, the possibility of fabrication if not already there, is not remote in view of current advances in technology, if only the experimentalists and practical scientists are sufficiently interested or encouraged to take up this sort of work.

Dispersion in single mode fibers has been a subject of paramount interest in the field of optical telecommunication. In the recent paper Pandey et al. [12] describes the model characteristics of a waveguide with a new type of spiral geometrical cross-section. In that paper Pandey et al. [12] considered the guiding region of the involuted spiral is made of dielectric and the cladding i.e., the boundaries of the guiding region are considered to be made of metallic (highly conducting) substance. In this present paper the model dispersion characteristics of an optical waveguide with a guiding region cross-section bounded by two involuted spirals has been considered. The transverse core cross section of the proposed waveguide is shown in Fig. 1, this proposed waveguide is considered of completely dielectric i.e., both the core and cladding are made of non magnetic dielectric material. The modal dispersion curve of this case is compared with the same of the similar kind of waveguide having metallic cladding. The analysis of the proposed waveguide is done under the weak guidance approximation i.e., $(n_1-n_2)/n_1 \ll 1$, where n_1 and n_2 are the refractive indices of the core and cladding respectively. The schematic diagram of the proposed waveguide is shown in Fig. 1, in which the guiding region lies between curves.

2. THEORY

The particular geometry to be studied is depicted in the Fig. 1. We have been considered two involuted spirals, which can be treated as a

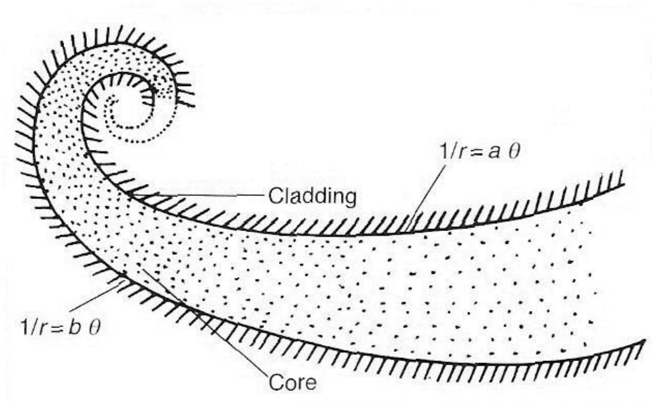

Figure 1. Schematic diagram of the transverse core cross-section of the proposed waveguide.

special case of the equation

$$r^p = \xi\theta \tag{1}$$

and the equation of its normal curves is

$$r^p = \eta e^{-p^2\theta^2/2} \tag{2}$$

where ξ & η are size parameter and p is a real number

Our structure is defined by $p = -1$. So the Equations (1) and (2) take the form

$$r^{-1} = \xi\theta \quad \text{and} \quad r^{-1} = \eta e^{-\theta^2/2} \tag{3}$$

For this study we have considered dielectric waveguide with core refractive index n_1 and n_2 the refractive index of cladding such that $(n_1 - n_2)/n_1 \ll 1$. This fulfills the condition of the weak guidance approximation. To solve this problem we have taken new co-ordinate system (ξ, η, z) as appropriate co-ordinates. For this appropriate coordinates, one uses the point of intersection of two sets of normal curves on the cross sectional plane of waveguide. The equations of the boundary and its normal curve are given in Equation (3). The direction of propagation is the z-direction, which is perpendicular to the plane of paper shown in the Fig. 1. After some straightforward steps we obtain the scale factor h_1, h_2, and h_3 for the co-ordinate ξ, η, z these are written as,

$$h_1 = \frac{r}{(p^2\theta^2 + 1)}\left(\frac{1}{p^2\eta^2} + \frac{\theta^2}{\eta^2}\right)^{\frac{1}{2}}$$

$$\begin{aligned} h_2 &= \frac{r}{(p^2\theta^2+1)}\left(\frac{p^2\theta^4}{\xi^2}+\frac{\theta^2}{\xi^2}\right)^{\frac{1}{2}} \\ h_3 &= 1 \end{aligned}$$

and for the particular case $p = -1$, i.e., for our proposed waveguide the scale factors are

$$\begin{aligned} h_1 &= \frac{r}{\eta\,(\theta^2+1)^{\frac{1}{2}}} \\ h_2 &= \frac{r\theta}{\xi\,(\theta^2+1)^{\frac{1}{2}}} \\ \text{and} \quad h_3 &= 1 \end{aligned}$$

where

$$\begin{aligned} r &= \frac{1}{\xi\left\{\frac{1}{2}\ln\left(\frac{\eta}{\xi}\right)+\frac{3}{4}\right\}} \\ \theta &= \left\{\frac{1}{2}\ln\left(\frac{\eta}{\xi}\right)+\frac{3}{4}\right\} \end{aligned}$$

Substituting the values of r and θ we have the scale factors for our case

$$\begin{aligned} h_1 &= \frac{1}{\xi\eta\left\{\frac{1}{2}\ln\left(\frac{\xi}{\eta}\right)+\frac{3}{4}\right\}\left[\left\{\frac{1}{2}\ln\left(\frac{\xi}{\eta}\right)+\frac{3}{4}\right\}^2+1\right]^{\frac{1}{2}}} \\ h_2 &= \frac{1}{\xi^2\left[\left\{\frac{1}{2}\ln\left(\frac{\xi}{\eta}\right)+\frac{3}{4}\right\}^2+1\right]^{\frac{1}{2}}} \\ \text{and} \quad h_3 &= 1 \end{aligned}$$

Using the Helmholtz scalar-wave equation for the z-component of the electric field E_z under the weak guidance condition, the wave equation can be written as

$$\nabla^2 E_z + \omega^2\mu_0\varepsilon E_z = 0 \tag{4}$$

∇^2 is the scalar Laplacian operator, and its form in terms of scale factors is given by

$$\nabla^2 = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial\xi}\left(\frac{h_2h_3}{h_1}\frac{\partial}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\frac{h_1h_3}{h_2}\frac{\partial}{\partial\eta}\right)+\frac{\partial}{\partial z}\left(\frac{h_1h_2}{h_3}\frac{\partial}{\partial z}\right)\right]$$

Since the dielectric material is considered as non magnetic so the permeability of the medium is taken to be μ_0 and ε is the permittivity of the dielectric medium and ω is the angular frequency of the electromagnetic wave. In the scalar field approximation the Equation (4) can be solved by using the method of separation of variables. This is possible only if we assume that $\eta \to \xi$. In this approximation the modified differential equation is given by

$$\frac{3}{4}\frac{\xi^2\eta}{d^2}\left[\begin{array}{l}\frac{3}{4}\frac{\eta}{\xi}\frac{\partial^2 E_z}{\partial \xi^2} - \frac{3}{4}\frac{\eta}{\xi^2}\frac{\partial E_z}{\partial \xi} + \frac{4}{3}\frac{\xi}{\eta}\frac{\partial^2 E_z}{\partial \eta^2} - \\ \frac{4}{3}\frac{\xi}{\eta^2}\frac{\partial E_z}{\partial \eta} + \frac{4}{3}\frac{d^2}{\xi^3\eta}\frac{\partial^2 E_z}{\partial z^2}\end{array}\right] + \omega^2\mu\varepsilon E_z = 0 \qquad (5)$$

where $d = \text{constant} = 0.8$.

For the method of separation of variable for the solution of Equation (5), we assumed that the solution E_z is a function of ξ and η. That is

$$E_z = E_1(\xi)E_2(\eta)\exp[j(\omega t - \beta z)] \qquad (6)$$

where β is the propagation constant along the z-direction. Now the above Equation (5) takes the form

$$\xi^2\frac{\partial^2 E_1(\xi)}{\partial \xi^2} - \xi^3\frac{\partial E_1(\xi)}{\partial \xi} + \frac{16}{9}d^2U^2E_1(\xi) = 0 \qquad (7)$$

$$\eta^2\frac{\partial^2 E_2(\eta)}{\partial \eta^2} - \eta^3\frac{\partial E_2(\eta)}{\partial \eta} - \frac{16}{9}d^2W^2E_2(\eta) = 0 \qquad (8)$$

where $U = \sqrt{(k_0^2n_1^2 - \beta^2)}$ and $W = \sqrt{(\beta^2 - k_0^2n_2^2)}$ also $k_0 = 2\pi/\lambda_0$.

Equation (7) is valid for the guiding region and Equation (8) for the cladding region of the proposed waveguide. To obtain the field in core and cladding we have to solve these equations.

For convenience, we now use new symbols, such that

$$E_1(\xi) \equiv y$$

and

$$\xi \equiv x$$

and $\frac{16d^2}{9}U^2 = P$, then Equation (7) reduces to

$$x^4\frac{d^2y}{dx^2} - x^3\frac{dy}{dx} + Py = o \qquad (9)$$

Similarly for the cladding region we have

$$x^4\frac{d^2y}{dx^2} - x^3\frac{dy}{dx} - P'y = o \qquad (10)$$

where $\frac{16d^2}{9}W^2 = P'$.

We choose the substitution

$$\ell = Px^{-2}$$

Then the Equation (9) becomes

$$\frac{d^2y}{d\ell^2} + \frac{2}{\ell}\frac{dy}{d\ell} + \frac{1}{4\ell}y = 0 \tag{11}$$

Similarly the Equation (10) becomes

$$\frac{d^2y}{dm^2} + \frac{2}{m}\frac{dy}{d\ell} - \frac{1}{4m}y = 0 \tag{12}$$

for the substitution of $m = P'x^{-2}$.

By applying power solution method, we have four independent solutions for core and clad region are given by,

$$y_{core}(x) = c_1y_1(x) + c_2y_2(x)$$

$$y_{clad}(x) = c_3y_3(x) + c_4y_4(x)$$

where

$$y_1(x) = a_0\left[1 + \tau\frac{(P/x^2)}{1.2} + \tau^2\frac{(P/x^2)^2}{1.2^2.3} + \tau^3\frac{(P/x^2)^3}{1.2^2.3^2.4} + \ldots\right]$$

$$y_2(x) = \frac{1}{a_0}\left[-\frac{1}{(P/x^2)^2} - \frac{\tau}{2} + \frac{\tau^2}{2}(P/x^2) - \frac{41}{144}\tau^2(P/x^2)^2 - \frac{12}{144}\tau^4(P/x^2)^3 - \ldots \right.$$
$$\left. -\tau\log\left(P/x^2\right)\left\{1 + \tau\frac{(P/x^2)}{1.2} + \tau^2\frac{(P/x^2)^2}{1.2^2.3} + \tau^3\frac{(P/x^2)^3}{1.2^2.3^2.4} + \ldots\right\}\right]$$

$$y_3(x) = c_0\left[1 + \sigma\frac{(P'/x^2)}{1.2} + \sigma^2\frac{(P'/x^2)^2}{1.2^2.3} + \sigma^3\frac{(P'/x^2)^3}{1.2^2.3^3.4} + \ldots\right]$$

$$y_4(x) = b_0\left[\frac{1}{(P'/x^2)} - \sigma - \frac{5}{4}\sigma^2(P'/x^2) - \frac{5}{18}\sigma^3(P'/x^2)^2 - \frac{47}{48\times36}\sigma^4(P'/x^2)^3 - \ldots \right.$$
$$\left. +\sigma\log(P'/x^2)\left\{1 + \sigma\frac{(P'/x^2)}{1.2} + \sigma^2\frac{(P'/x^2)^2}{1.2^2.3} + \sigma^3\frac{(P'/x^2)^3}{1.2^2.3^2.4} + \ldots\right\}\right]$$

where $\tau = -\frac{1}{4}$ and $\sigma = \frac{1}{4}$, and c_1, c_2, c_3 and c_4 are constant, which can be determined by applying the boundary condition for the scalar fields and a_0, c_0, and b_0 are arbitrary constants. These are

$$\begin{aligned} y_{core}(x)|_{x=a} &= y_{clad}(x)|_{x=a} \\ y_{core}(x)|_{x=b} &= y_{clad}(x)|_{x=b} \\ y'_{core}(x)|_{x=a} &= y'_{clad}(x)|_{x=a} \\ y'_{core}(x)|_{x=b} &= y'_{clad}(x)|_{x=b} \end{aligned}$$

Using these boundary conditions and for non trivial solution we have

$$\begin{vmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{vmatrix} = 0$$

where

$$y_{11} = a_0\left[1 + \tau\frac{(P/a^2)}{1.2} + (\tau)^2\frac{(P/a^2)^2}{1.2^2.3} + (\tau)^3\frac{(P/a^2)^3}{1.2^2.3^2.4} + \dots\right]$$

$$\begin{aligned} y_{12} = \frac{1}{a_0}\Bigg[&-\frac{1}{(P/a^2)} - \frac{(\tau)}{2} + \frac{(\tau)^2}{2}\left(P/a^2\right) + \frac{41(\tau)^3}{144}\left(P/a^2\right)^2 - \frac{12(\tau)^4}{144}\left(P/a^2\right)^3 + \dots \\ &-(\tau)\log(P/a^2)\left\{1 + (\tau)\frac{(P/a^2)}{1.2} + (\tau)^2\frac{(P/a^2)^2}{1.2^2.3} + (\tau)^3\frac{(P/a^2)^3}{1.2^2.3^2.4} + \dots\right\}\Bigg] \end{aligned}$$

$$y_{13} = c_0\left[1 + (\sigma)\frac{(P'/a^2)}{1.2} + (\sigma)^2\frac{(P'/a^2)^2}{1.2.3} + (\sigma)^3\frac{(P'/a^2)^3}{1.2^2.3^2.4} + \dots\right]$$

$$\begin{aligned} y_{14} = b_0\Bigg[&\frac{1}{(P'/a^2)} - \sigma - \left(\frac{5}{4}\right)(\sigma)^2(P'/a^2) + \frac{5}{18}(\sigma)^3(P'/a^2)^2 \\ &-\frac{47}{48\times 36}(\sigma)^4(P'/a^2)^3 + \dots + \sigma\log(P'/a^2) \\ &\times\left[1 + (\sigma)\frac{(P'/a^2)}{1.2} + (\sigma)^2\frac{(P'/a^2)^2}{1.2^2.3} + (\sigma)^3\frac{(P'/a^2)^3}{1.2^2.3^2.4} + \dots\right]\Bigg] \end{aligned}$$

$$y_{21} = a_0\left[1 + (\tau)\frac{(P/b^2)}{1.2} + (\tau)^2\frac{(P/b^2)^2}{1.2^2.3} + (\tau)^3\frac{(P/b^2)^3}{1.2^2.3^2.4} + \dots\right]$$

$$\begin{aligned} y_{22} = \frac{1}{a_0}\Bigg[&-\frac{1}{(P/b^2)^2} - \frac{1}{2}(\tau) + \frac{1}{2}(\tau)^2\left(P/b^2\right) - \frac{41}{144}(\tau)^3\left(P/b^2\right)^2 \\ &-\frac{12}{144}(\tau)^4\left(P/b^2\right)^3 + \dots - (\tau)\log\left(P/b^2\right) \\ &\times\left\{1 + (\tau)\frac{P/b^2}{1.2} + (\tau)^2\frac{(P/b^2)^2}{1.2^2.3} + (\tau)^3\frac{(P/b^2)^3}{1.2^2.3^2.4} + \dots\right\}\Bigg] \end{aligned}$$

$$y_{23} = c_0\left[1 + (\sigma)\frac{(P'/b^2)}{1.2} + (\sigma)^2\frac{(P'/b^2)^2}{1.2^2.3} + (\sigma)^3\frac{(P'/b^2)^3}{1.2^2.3^2.4} + \dots\right]$$

$$y_{24} = b_0\Bigg[\frac{1}{(P'/b^2)} - \sigma - \left(\frac{5}{4}\right)\sigma^2(P'/b^2) + \frac{5}{18}\sigma^3(P'/b^2)^2 - \frac{47}{48\times 36}\sigma^4(P'/b^2)^3 + \dots$$
$$+\sigma\log(P'/b^2)\left[1 + \sigma\frac{(P'/b^2)}{1.2} + \sigma^2\frac{(P'/b^2)^2}{1.2^2.3} + \sigma^3\frac{(P'/b^2)^3}{1.2^2.3^2.4} + \dots\right]\Bigg]$$

$$y_{31} = a_0\left[-\tau\frac{2\,(P/a^2)}{1.2.a} - \tau^2\frac{4\,(P/a^2)^2}{1.2^2.3.a} - \tau^3\frac{6\,(P/a^2)^3.6}{1.2^2.3^2.4.a} - \cdots\right]$$

$$y_{32} = \frac{1}{a_0}\Bigg[-\frac{2}{a^3(P/a^2)^2} - \tau^2(P/a^2) - \frac{164\tau^3(P/a^2)^2}{144.a} - \frac{72\tau^4(P/a^2)^3}{144.a} - \dots$$
$$-\tau\log\left(P/a^2\right)\left\{-\tau\frac{2(P/a^2)}{1.2.a} - \frac{4\tau^2(P/a^2)^2}{1.2^2.3.a} - 6\tau^3\frac{(P/a^2)^3}{1.2^2.3^2.4.a} - \cdots\right\}$$
$$+\frac{2\tau}{a}\left\{1 + \tau\frac{(P/a^2)}{1.2} + \tau^2\frac{(P/a^2)^2}{1.2^2.3} + \tau^3\frac{(P/a^2)^3}{1.2^2.3^3.4} + \dots\right\}\Bigg]$$

$$y_{33} = c_0\left[-\sigma\frac{(P'/a^2)}{1.2.a} - \sigma^2\frac{4(P'/a^2)^2}{1.2^2.3.a} - \sigma^3\frac{6(P'/a^2)^3}{1.2^2.3^2.4.a} - \sigma^4\frac{8(P'/a^2)^4}{1.2^2.3^24^25.a} - \dots\right]$$

$$y_{34} = b_0\Bigg[\frac{1}{(P'/a^2)}\cdot\frac{1}{a} + \frac{5}{2}\sigma\left(P'/a^2\right)\cdot\frac{1}{a} + \frac{5\times 4}{18}\sigma^3\left(P'/a^2\right)^2\frac{1}{a}$$
$$+\frac{47\times 6}{48\times 36}\sigma^4\left(P'/a^2\right)^3\cdot\frac{1}{a} + \dots + \sigma\log\left(P'/a^2\right)$$
$$\left\{-2\sigma\cdot\frac{(P'/a^2)}{1.2.a} - \frac{4\sigma^2\cdot(P'/a^2)^2}{1^2.2^2.3.a} - \frac{6\sigma^3\cdot(P'/a^2)^3}{1^2.2^2.3^2.4.a} + \dots\right\}\Bigg]$$

$$y_{41} = a_0\left[-\sigma\frac{2\cdot(P/b^2)}{1.2..b} - \sigma^2\frac{4\cdot(P/b^2)^2}{1.2^2.3.b} - \sigma^3\frac{6\cdot(P/b^2)^3}{1.2^2.3^2.4.b} - \dots\right]$$

$$y_{42} = \frac{1}{a_0}\Bigg[-\frac{2}{b^3\cdot(P/b^2)^2} - \tau^2\frac{(P/b^2)}{b} - \frac{164}{144}\tau^3\frac{(P^2/b^2)^2}{b} - \frac{72}{144}\tau^4\frac{(P/b^2)^3}{b} - \cdots$$
$$-\tau\log(P/b^2)\left\{-\tau\frac{2\cdot(P/b^2)}{1.2.b} - \tau^2\frac{4\cdot(P/b^2)^2}{1.2^2.3.b} - \tau^3\frac{6(P/b^2)^3}{1.2^2.3^2.4.b} - \dots\right\}\Bigg]$$

$$y_{43} = c_0\left[-\sigma\frac{(P'/b^2)}{1.2..b} - \sigma^2\frac{4\cdot(P'/b^2)^2}{1.2^2.3.b} - \sigma^3\frac{6\cdot(P'/b^2)^3}{1.2^2.3^2.4.b} - \cdots\right]$$

$$y_{44} = b_0\Bigg[\frac{1}{(P'/b^2)}\cdot\frac{1}{b} + \frac{5}{2}\sigma(P'/b^2)\cdot\frac{1}{b} + \frac{5\times 4}{18}\sigma^3(P'/b^2)^2\frac{1}{b}$$

$$+\frac{47\times 6}{48\times 36}\sigma^4\left(P'/b^2\right)^3\cdot\frac{1}{b}+\ldots+\sigma\log\left(P'/b^2\right)$$
$$\left\{-2\sigma\cdot\frac{2\cdot(P'/b^2)}{1.2}\cdot\frac{1}{b}-\sigma^2\frac{4\cdot(P'/b^2)^2}{1.2^2.3}\cdot\frac{1}{b}-\sigma^3\frac{6\cdot(P'/b^2)^3}{1.2^2.3^2.4}\cdot\frac{1}{b}+\ldots\right\}\Bigg]$$

Equation (12) can be written as

$$|y_{ij}|=0 \tag{13}$$

where indices i, j each may take integral value from 1 to 4.

Equation (13) is the characteristic equation of the waveguide under consideration. With the help of this characteristic equation the modal properties of the wave propagation through this proposed waveguide under the weak guidance condition can be obtained.

3. NUMERICAL COMPUTATIONS, RESULTS AND DISCUSSION

In order to understand the content of the characteristic Equation (13), in which is central result exists of this paper, it is necessary to make some numerical computations by choosing physically realizable values of the parameters n_1, n_2, a^{-1}, b^{-1} and λ_0. We choose the values $n_1 = 1.48$, $n_2 = 1.46$, $a^{-1} = 2\,\mu$m, $b^{-1} = 4\,\mu$m and $\lambda_0 = 1.55\,\mu$m. We next choose a set of regularly spaced β values and for each β, compute the L.H.S. of Equation (13). In our case, as we plot the L.H.S. Equation (13) against β, we find only one point of intersection of the graph with β axis. This indicates that the only one mode can be sustained. Keeping the values a^{-1} fixed, we now repeat the computations for different increasing values of b^{-1}, and make a list of the β values intersecting with the β axis. We now plot these β or equivalently b' i.e., normalized propagation constant values against the normalized frequency parameter V.

$$b'=\frac{\left(\frac{\beta^2}{k^2}\right)-n_2^2}{n_1^2-n_2^2}$$

and $V=\frac{2\pi}{\lambda}\left(b^{-1}-a^{-1}\right)\left(n_1^2-n_2^2\right)^{\frac{1}{2}}$.

We thus get the dispersion curve as shown in Fig. 2. The single dispersion curve shows that our structure is a single mode waveguide. The modal cutoff occurs at $V = 16$, below which not even a single mode is sustained; for all value of $V > 16$ we get only a single mode and the mode attains a saturation values of b' slowly as V reaches the relatively large value, $V = 40$.

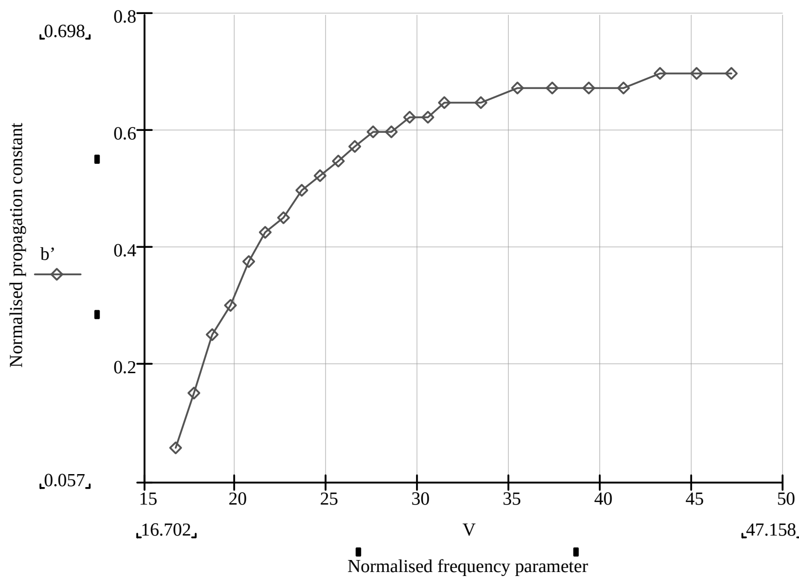

Figure 2. Modal dispersion characteristic curve of the proposed waveguide.

The similar kind of dispersion curve has also been obtained when the same kind of waveguide [12] was considered with perfectly conducting (metallic) claddings. Only the cut off in this case was appeared at $V = 4$ and saturation of the dispersion curve was also relatively low value of V compared to the dielectric claddings.

The greater the cut off value for the single mode propagation is required for the convenience of manufacturing of the single mode waveguide, so this type of the waveguide fulfills the desire of the scientific field at some extent. Although it is difficult to visualize the cross-section at the narrow end, at the wider end the cross-section is similar to that of a distorted slab waveguide in which both a curvature and flare are present so this kind of waveguide can be used to study the tolerance of slab waveguide which is deformed at on end.

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