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THE MAGNETIC-MOMENT QUADRIC AND CONDITIONS OF VANISHING MAGNETIC MOMENT FOR A ROTATIONAL CHARGED BODY

G.-Q. Zhou

Department of Physics
Wuhan University
Wuhan 430072, China

W.-J. Zhou

School of Electrical Engineering
Wuhan University
Wuhan 430072, China

Abstract—Based on the concept of charge moment tensor $\widetilde{T}$ which is different from the existent electric multiple-moment, and the concept of principal axes and principal-axis scalar charge moment, the condition of zero magnetic moment for an arbitrary rotational body with given charge distribution has been given explicitly in this paper. We find the loss of positive definiteness of $\widetilde{T}$ is its most important characteristic which forms a sharp contrast with that of its mechanic counterpart — the positive definite inertia tensor of rigid bodies. Meanwhile the relationship between the quadric distributive law of magnetic moment and the parameters of tensor $\widetilde{T}$ is discussed in detail. According to the theory of analytic geometry, we give a series of test formulae, classify and enumerate every kind of possible quadric in a table. Finally, conclusion is given that any rotation axis which passes through origin O and along with any of the asymptotic line of the quadric (hyperboloids or hyperbolic cylinders) can lead to a vanishing magnetic moment.

1. INTRODUCTION

It is a well known fact that a charged body rotating around a given axis has definite magnetic moment [1–6]. Based on a strict and delicate analogue relation given in references [1–4], references [2–4] has deduced a series of rules and given some examples about calculating the magnetic moment of a rotational charged body. Especially the interesting and useful concepts such as charge moment tensor, scalar charge moment and the principal scalar charge moment given in references [2] make it very easy for us to compute the magnetic moment of an arbitrary rotational charged body with respect to an arbitrary given axis.

On the other hand, computing $\tilde{T}$ and magnetic moment $\vec{P}_m$ of a rotational charged body is only the first step to study its dynamic and kinetic behaviors in electromagnetic fields. Researching of the magnetic effect of rotational charged bodies is necessary and significant especially in fields of astrophysics and cosmology [7], and often overlooked in the studying of optimization techniques for antenna design and radar application [8,9]. In view of space technology, it is of special meaning and extraordinary importance to make a rotating charged body have a zero magnetic moment and maintain its rotation equilibrium. In this paper, conditions of zero magnetic moment for an arbitrary rotational charged body have been given explicitly. Meanwhile, the relationship between the quadric distributive law of scalar charge moment and the parameters of tensor is discussed in detail. According to the theory of analytic geometry, we give a series of test formulae, classify and enumerate every kind of possible magnetic-moment quadric in Table 1.

2. THE FUNDAMENTAL CONCEPTS OF CHARGE MOMENT TENSOR

In an arbitrary body-coordinate system O-xyz which is rigidly linked with a charged body of definite volume (area, line or discrete) charge distribution, the So-called **charge moment tensor** with respect to the given origin O is defined as:

$$\tilde{T}(O) = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix} \tag{1}$$

Under the case of disperse charge distribution, the tensor element

$$I_{\alpha\beta(O)} = \sum_i Q_i[r_i^2\delta_{\alpha\beta} - x_{i\alpha}x_{i\beta}](\alpha \text{ and } \beta = 1,2,3) \tag{2}$$

here $\vec{r_i}$ is the position vector of point charge Q_i, $\vec{r_i} = (x_{i1}, x_{i2}, x_{i3}) = (x_i, y_i, z_i)$, and for the case of continual charge distribution such as a

Table 1. Quadric classification contingent upon magnetic moment parameters.

types	Test expression		quadric types	Quadric equations
$K\neq 0$ centric quadric	$J>0$ $IK>0$	$\mathrm{sign}(I_l)K>0$	Ellipsoid (Fig.1)	*e.g.* $I_1x^2+I_2y^2+I_3z^2=\mathrm{sign}(I_l)$ $K=I_1I_2I_3\neq 0$
		$\mathrm{sign}(I_l)K\leq 0$	Non-existent	
	$J\leq 0$ or $IK<0$	$\mathrm{sign}(I_l)K<0$	Hyperboloids of one sheet (Fig.2)	
		$\mathrm{sign}(I_l)K>0$	Hyperboloids of two sheets (Fig.3)	
K=0 Centreless quadric	$J>0$	$\mathrm{sign}(I_l)=1$	Elliptic cylinder (Fig.4)	$I_1x^2+I_2y^2=\mathrm{sign}(I_l)$ etc. $\mathrm{sign}(I_1)=\mathrm{sign}(I_2)=\mathrm{sign}(I_l)$, $I_3=0$
		$\mathrm{sign}(I_l)=-1$	Elliptic cylinder (Fig.4)	
	$J<0$	$\mathrm{sign}(I_l)=1$	Hyperbolic cylinder (Fig.5)	e.g. $I_1x^2+I_2y^2=\mathrm{sign}(I_l)$ $(I_1I_2<0, I_3=0)$,etc.
		$\mathrm{sign}(I_l)=-1$	Hyperbolic cylinder (Fig.5)	
	$J=0$	$\mathrm{sign}(I_l)I>0$	Parallel planes	$I_1x^2=\mathrm{sign}(I_l)$ $I_2=I_3=0$, $\mathrm{sign}(I_l)=\mathrm{sign}(I)$ etc.
		$\mathrm{sign}(I_l)I<0$	Non-existent	
		$I=0$	Single plane	

charged body with a volume charge density of $\rho(x_1, x_2, x_3)$,

$$I_{\alpha\beta}(O) = \int\limits_v \rho_e(x_1, x_2, x_3)[r^2\delta_{\alpha\beta} - x_\alpha x_\beta]dv \tag{3}$$

here $dv = dx_1dx_2dx_3$, $r^2 = x_1^2 + x_2^2 + x_3^2$.

Taking the disperse distribution case as an instance, we give a concrete expression of $\tilde{T}(O)$ as:

$$\tilde{T}(O) = \begin{pmatrix} \sum\limits_i Q_i(y_i^2 + z_i^2) & -\sum\limits_i Q_ix_iy_i & -\sum\limits_i Q_ix_iz_i \\ -\sum\limits_i Q_ix_iy_i & \sum\limits_i Q_i(z_i^2 + x_i^2) & -\sum\limits_i Q_iy_iz_i \\ -\sum\limits_i Q_ix_iz_i & -\sum\limits_i Q_iy_iz_i & \sum\limits_i Q_i(x_i^2 + y_i^2) \end{pmatrix} \tag{4}$$

It is similar in form to the inertia tensor of a rigid body but actually different from it because the former has lost its positive definiteness. On the other hand, charge moment tensor $\tilde{T}(O)$ introduced here doesn't belong to any of the second rank tensors listed in references [10, 11], and is also different from the existent concept such as the **electric quadruple moment** [5], i.e., $D_{ij} = \int_{V'}(3x_i'x_j' - r'^2)\rho(\vec{r}')dv'$, $(x,\ j = 1, 2, 3)$.

Then the So-called scalar charge moment I_l with respect to the same point O and arbitrary direction (provided its direction cosine is $\vec{l} = (\cos\theta_1, \cos\theta_2, \cos\theta_3)$) is

$$\begin{aligned} I_l(O, \vec{l}) &= \vec{l}\cdot\tilde{T}\cdot\vec{l} = (\cos\theta_1 \cos\theta_2 \cos\theta_3)\begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}\begin{pmatrix} \cos\theta_1 \\ \cos\theta_2 \\ \cos\theta_3 \end{pmatrix} \\ &= I_{11}\cos^2\theta_1 + I_{22}\cos^2\theta_2 + I_{33}\cos^2\theta_3 + 2I_{12}\cos\theta_1\cos\theta_2 \\ &\quad + 2I_{23}\cos\theta_2\cos\theta_3 + 2I_{31}\cos\theta_3\cos\theta_1 \end{aligned} \tag{5}$$

Then it can be immediately deduced that a rotational charged body with an angular velocity of $\vec{\omega}$ with respect the same axis $(O, \vec{l})$, must has a magnetic moment (given $\vec{\omega} = \omega\vec{l}$):

$$\vec{P}_m(O, \vec{l}) = \frac{1}{2}I_l\vec{\omega} = \frac{1}{2}\left(\frac{\vec{\omega}}{\omega}\cdot\tilde{T}\cdot\frac{\vec{\omega}}{\omega}\right)\vec{\omega} = \frac{1}{2}l_i\tilde{T}_{ij}l_j\vec{\omega} \tag{6}$$

According to Einstein's convention, here the repeated indices represent summation from 1 to 3.

By solving the eigenvalue equation of the charge moment tensor $\tilde{T}(O)$

$$\sum_{\beta=1}^{3}(I_{\alpha\beta} - \lambda\delta_{\alpha\beta})e_\beta = 0, (\alpha = 1, 2, 3) \tag{7}$$

we get the three eigenvectors $e_{\beta_i} = (e_1^i, e_2^i, e_3^i), (i = 1, 2, 3)$ called the **principal axes**. The corresponding eigenvalues $\lambda_i = I_i$, (i=1,2,3), are called the **principal scalar charge moments**.

Collect and arrange e_β's in order, we thus construct an orthogonal transformation matrix R [12, 13] which can make the charge moment tensor $\tilde{T}(O)$ diagonal.

$$R = \begin{pmatrix} e_1^1 & e_1^2 & e_1^3 \\ e_2^1 & e_2^2 & e_2^3 \\ e_3^1 & e_3^2 & e_3^3 \end{pmatrix} \tag{8}$$

In the new Cartesian coordinate body system O-xyz spanned with above three eigenvectors $e_3^{(i)}(i = 1, 2, 3)$, the charge moment tensor can be expressed as a diagonal form

$$\tilde{T}(O) = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \tag{9}$$

Based on the concepts of principal axes and the three principal-axis scalar charge moments, The scalar charge moment I_l with respect to an arbitrary direction $\vec{l} = (\cos\theta_1, \cos\theta_2, \cos\theta_3)$ and the given point O is

$$I_l = \vec{l} \cdot \tilde{T} \cdot \vec{l} = I_1 \cos^2\theta_1 + I_2 \cos^2\theta_2 + I_3 \cos^2\theta_3 \tag{10}$$

Then according to Equation (6),

$$\vec{P}_m(O, \vec{l}) = \frac{1}{2}(I_1 \cos^2\theta_1 + I_2 \cos^2\theta_2 + I_3 \cos^2\theta_3)\vec{\omega} \tag{11}$$

3. CLASSIFICATION OF MAGNETIC MOMENT QUADRIC FOR A ROTATIONAL CHARGED BODY

When $\vec{P}_m(O, \vec{l}) \neq 0$, provided $\vec{R}_l = (x, y, z)$, and $\vec{R} \| \vec{l}$, $|\overline{OP}| = |\vec{R}| = \frac{1}{\sqrt{|I_l|}}$ then

$$I_l = \mathrm{sign}(I_l)\frac{1}{|\vec{R}_l|^2} = \mathrm{sign}(I_l)\frac{1}{|\overline{OP}|^2} \tag{12}$$

and

$$\vec{P}_m(O, \vec{l}) = \frac{1}{2} I_l \vec{\omega} = \frac{1}{2}\mathrm{sign}(I_l)\frac{1}{\overline{OP}^2}\vec{\omega} \tag{12'}$$

where $\mathrm{sign}(I_l) = \begin{cases} 1, & \text{for } I_l > 0 \\ -1, & \text{for } I_l < 0 \end{cases}$.

Then $\vec{l} = (\cos\theta_1, \cos\theta_2, \cos\theta_3) = \frac{\vec{R}_l}{|\vec{R}_l|} = (x\sqrt{|I_l|}, y\sqrt{|I_l|}, z\sqrt{|I_l|})$. Multiplying the two sides of Equation (5) with factor $|\vec{R}_l|^2 = \frac{1}{|I_l|}$, we have

$$I_{11}x^2 + I_{22}y^2 + I_{33}z^2 + 2I_{12}xy + 2I_{23}yz + 2I_{31}zx = \frac{I_l}{|I_l|} = \mathrm{sign}(I_l) \tag{13}$$

This is just the physical meaning of the so-called magnetic-moment quadric and the importance to introduce the concept of charge moment tensor. And Equation (13) depicts the spatial distribution law of the scalar charge moment $I_l(O,\vec{l})$ as well as the corresponding magnetic-moment $\vec{P}_m(O,\vec{l})$ with respect to given point O and direction $\vec{l}$. Quadric expressed by Equation (13) might not be definitely a ellipsoid which is the characteristic of the inertia tensor for a rigid body, in addition to ellipsoid, those quadric such as the hyperboloid (with one or two sheets), the cylinders (such as the elliptic cylinder, the hyperbolic cylinder), and even the cones, and son on, are also the possible candidate, due to the loss of positive definiteness of tensor $\tilde{T}(O)$.

Especially in the new body coordinate system O-xyz constructed with three orthogonal principal axes $e^{(i)}, (i = 1, 2, 3)$, Equation (13) is then reduced to be a quadric camber as follow

$$I_1x^2 + I_2y^2 + I_3z^2 = \mathrm{sign}(I_l) \tag{13'}$$

In contrast to the general form of quadric equation [14]:

$$Ax^2+By^2+Cz^2+2Dxy+2Exz+2Fyz+2G_0x+2H_0y+2I_0z+J = 0 \tag{14}$$

Equation (13) is just the quadric depicted by (14) with following special parameters

$$\begin{aligned} G_0 &= H_0 = I_0 = 0, \quad J = -\mathrm{sign}(I_l); \quad A = I_{11}, \quad B = I_{22}, \\ C &= I_{33}; \qquad D = I_{12}, \qquad E = I_{13}, \qquad F = I_{23}. \end{aligned}$$

On the other hand, we know from theory of analytic geometry [14] that the candidate kinds of quadric depicted by (13) and the quadric parameters have following one to one correspondence relation enumerated in Table 1. We also list the corresponding quadric figures. In addition to the important conclusions enumerated in the following classification Table 1, there are some interesting and useful orthogonal invariants under an arbitrary orthogonal coordinate transformation R [12, 13].

$$\begin{aligned} &\text{I,} && I_1 + I_2 + I_3 = I_{11} + I_{22} + I_{33} \equiv I \ \text{(invariant)} \\ &\text{II,} && I_1I_2+I_2I_3+I_3I_1 = I_{11}I_{22}+I_{22}I_{33}+I_{33}I_{11}-I_{12}^2-I_{23}^2-I_{31}^2 \equiv J \\ &&& \text{(invariant)} \\ &\text{III,} && I_1I_2I_3 = \begin{vmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{vmatrix} \equiv K \ \text{(invariant)} \end{aligned} \tag{15}$$

4. THE CONDITION OF ZERO MAGNETIC MOMENT FOR A ROTATIONAL CHARGED BODY

Under some circumstances it is needed to have a zero magnetic moment for a rotational charged body in order to maintain its steady state of movement or its rotation equilibrium, because a external magnetic field will impose an magnetic torque upon a Rotational charged body with nonzero magnetic moment.

$$\vec{M} = \vec{P}_m \times \vec{B} \tag{16}$$

Due to the existence of orthogonal invariants I, J, K, and the invariant of the scalar charge moment I_l, under a rotation transformation R between two body coordinate systems which are rigidly connected to the charged body (e.g., the principal-axis body coordinate system and an arbitrary body coordinate system), the following equation holds:

$$I'_l = \frac{\vec{\omega}'}{\omega'} \cdot \tilde{T}' \cdot \frac{\vec{\omega}'}{\omega'} = \frac{\vec{\omega}}{\omega} \cdot \tilde{T} \cdot \frac{\vec{\omega}}{\omega} \cdot = I_l$$

Or

$$\begin{aligned} &I'_{11}\cos^2\theta'_1 + I'_{22}\cos^2\theta'_2 + I'_{33}\cos^2\theta'_3 + 2I'_{12}\cos\theta'_1\cos\theta'_2 \\ &+2I'_{23}\cos\theta'_2\cos\theta'_3 + 2I_{31}\cos\theta'_1\cos\theta'_3 \\ &= I_1\cos^2\theta_1 + I_2\cos^2\theta_2 + I_3\cos^2\theta_3 \end{aligned} \tag{17}$$

The best approach is to treat the problem in the principal- axis body coordinate system.

Thus according to Equation (11), under the case of given charge distribution, in order to make $\vec{P}_m$ vanish, we need to impose some constraints upon the directions of rotation axes $\vec{l} = (\cos\theta_1, \cos\theta_2, \cos\theta_3)$. The condition of zero magnetic moment for a rotational charged body is

$$\vec{P}_m(O,\vec{l}) = \frac{1}{2}\left(I_1\cos^2\theta_1 + I_2\cos^2\theta_2 + I_3\cos^2\theta_3\right)\vec{\omega} = 0 \tag{18}$$

in addition to a constraint on the direction cosines:

$$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1 \tag{19}$$

Equation (18) and (19) give the zero magnetic moment condition as follows. Given I_1, I_2, I_3=constant, (i.e., given charge distribution of rotational body) and suppose $\cos^2\theta_1 = X$, $\cos^2\theta_2 = Y$, $\cos^2\theta_3 = Z$,

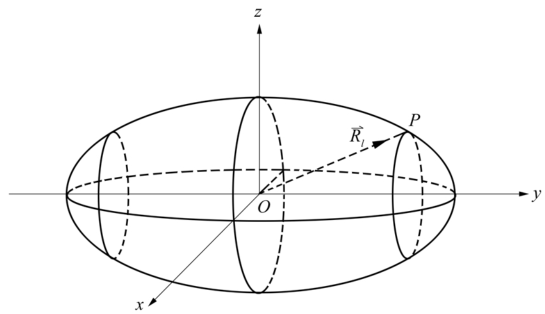

Figure 1. Ellipsoid.

then worked in coordinate system of O-XYZ, the wanted directions range of the rotation axes is given by the intersection line segment of two planes expressed by following equations:

$$\text{Line segment } MN \begin{cases} \text{plane } \pi_1 : I_1X + I_2Y + I_3Z = 0 & (20) \\ \text{plane } \pi_2 \text{ (i.e., } ABC) : X + Y + Z = 1 & (21) \\ 0 \le X \le 1,\ 0 \le Y \le 1,\ 0 \le Z \le 1 & (22) \end{cases}$$

Then following conclusions can immediately be drawn from Equations (20)–(22).

Once the intersection segment MN between plane π_1 and π_2 lies in the inner part of positive triangle ABC, every point $P'(X, Y, Z) = P'$ $(\cos^2\theta_1, \cos^2\theta_2, \cos^2\theta_3)$ on MN will correspond to several wanted directions $\vec{l_0} = (\pm\cos\theta_1, \pm\cos\theta_2, \pm\cos\theta_3)$ (c.f. Equation (20)–(22)). Along these directions (i.e., $\vec{\omega} = \omega\vec{l_0}$), the rotational charged body will generate a zero magnetic moment, just as shown in Fig. 6.

Without loss of generality, we suppose $I_3 \neq 0$. In fact, eliminating Z from Equations (20)–(22), we can express the equation of line segment MN more explicitly as

$$\left(1 - \frac{I_1}{I_3}\right) X + \left(1 - \frac{I_2}{I_3}\right) Y = 1 \tag{23}$$

$$Z = 1 - X - Y, \quad 0 \le X \le 1, \quad 0 \le Y \le 1, \quad 0 \le Z \le 1 \tag{24}$$

As a matter of fact, in terms of quadric parameters and by means of pure analytic geometry in the **principal-axes body-coordinate system**O-xyz, there is another approach to determine the wanted directions of rotation axes with zero magnetic moment.

Generally speaking, from Equation (12) and (12'), we know that, when point P on any of above listed quadric tends to infinite, then $|OP| \to \infty$, and $I_l \to 0$. Thus every case as shown in Figs. 2, 3, 4, 5 (except for case as shown in Fig. 1) can generate a zero magnetic moment.–that is to say, any rotation axis passing through origin O and along with any of the **asymptotic line** of the quadric shown in Figs. 2, 3, 5 can satisfy Equation (18) and lead to a vanishing magnetic moment.

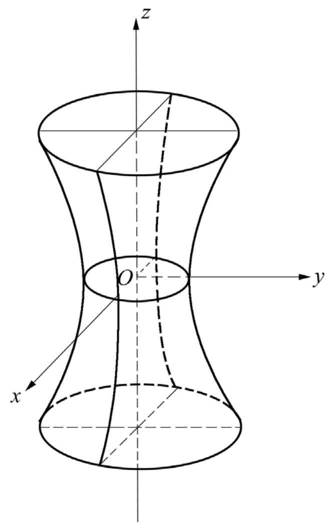

Figure 2. Hyperboloids of one sheet.

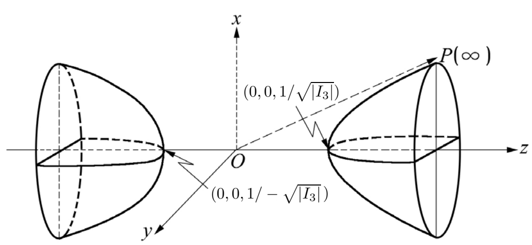

Figure 3. Hyperboloids of two sheets.

Let us concretely give the mathematical expressions for the axes of zero magnetic moment.

For case of hyperboloid of one sheet as shown in Fig. 2, the equation of **inner asymptotic cone surface** is

$$|I_1|x^2 + |I_2|y^2 - |I_3|z^2 = 0 \tag{25}$$

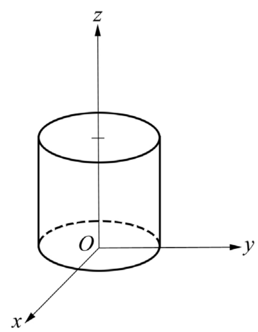

Figure 4. Elliptic cylinder.

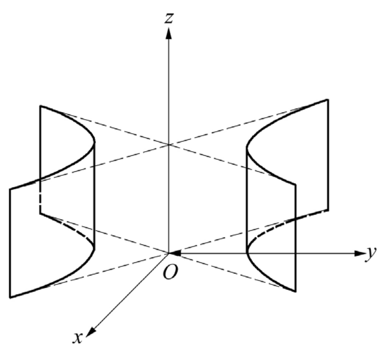

Figure 5. Hyperbolic cylinder.

then any of **generatrix lines** on the cone surface represents a rotation axis which leads to a vanishing magnetic moment.

For case of hyperboloid of two sheets as shown in Fig. 3, the equation of **outer asymptotic cone surface** is also (25). The **generatrix lines** on the cone surface also represent rotation axes which lead to a vanishing magnetic moment, too.

For case of hyperbolic cylinder as shown in Fig. 5, the equations of its **asymptotic planes** are

$$\sqrt{|I_1|}x \pm \sqrt{|I_1|}y = 0 \quad \text{(coordinate } z \text{ is arbitrary)} \tag{26}$$

Then any **asymptotic line** which is in above **asymptotic planes** and passes through origin O will represent a wanted rotational axis around which the rotational charged body will generate a zero magnetic moment.

For case of elliptic cylinder as shown in Fig. 4, the z-axis is just

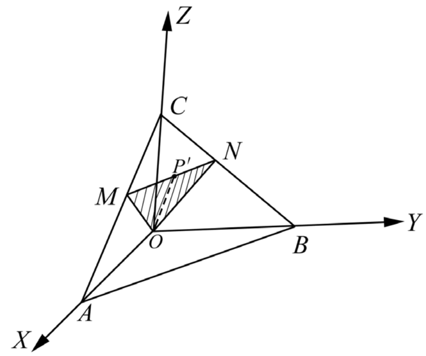

Figure 6. The zero magnetic moment condition in terms of square direction cosine (X, Y, Z).

the wanted rotation direction for the charged body which leads to a vanishing magnetic moment.

5. CONCLUSIONS AND SUMMARY

In view of space technology, it is of special meaning and extraordinary importance to make a rotational charged body have zero magnetic moment and maintain its rotation equilibrium. The concepts of charge moment tensor $\widetilde{T}$, principal axes and the corresponding principal-axis scalar charge moment I_l supply us perfect tools and a systematic method (5) and (6) to calculate the magnetic moment of rotational charged bodies around an arbitrary axes. We emphasize that the loss of positive definiteness of $\widetilde{T}$ is its most important characteristic which forms a sharp contrast with that of its mechanic counterpart — the positive definite inertia tensor of rigid bodies. On the other hand, the quadric distribution law of magnetic moment makes these abstract concepts more concrete, figurative and understandable. Based on these useful concepts, the conditions of zero magnetic moment for an arbitrary rotational charged body have been given explicitly in two forms. Finally, conclusion is given that any rotation axis which passes through origin O and along with any of the asymptotic line of the quadric (hyperboloids or hyperbolic cylinders) can lead to a vanishing magnetic moment. In a subsequent paper the classical dynamic and kinetic equation for a rotational charged body in a uniform magnetic field will be given and discussed in detail.

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