## STRUCTURAL PARAMETERS IN THE FORMATION OF OMNIDIRECTIONAL HIGH REFLECTORS

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Abstract—We investigate the structural parameters for the formation of omnidirectional photonic band gap in one dimensional photonic crystal. Simple transfer matrix method is used for calculations. The effect of two parameters, namely, refractive index contrast and filling fraction on omnidirectional reflection is investigated. We find from our study that when  $n_L$ ,  $n_i$ ,  $n_s$  and d are fixed, omnidirectional bandgap increases with increasing  $n_H/n_L$  i.e., with increasing  $n_H$ . Therefore, omnidirectional bandgap can be increased by using the material of high refractive index  $n_H$  when the low index material  $n_L$  is fixed. We also find that for the considered system of Si-SiO<sub>2</sub>, omnidirectional reflection range increases with filling fraction, goes to a maximum value and finally comes to zero. The maximum value of the omnidirectional reflection range is obtained at a value of 0.29 of the filling fraction. The range for allowable values of refractive index of ambient medium  $(n_i)$  has also been estimated.

#### 1. INTRODUCTION

There are two types of reflectors (mirrors): the most common are the old age metallic mirrors and the more recent dielectric mirrors. Metallic mirrors reflect light over a wide range of frequencies incident from arbitrary angles (i.e., omnidirectional dielectric reflectance), but they are lossy because of absorption, especially at infrared and optical frequencies. For applications in which energy loss, although very small is not desirable, scientists use multilayer dielectric reflectors. Multilayer dielectric mirrors reflect a narrow range of frequencies incident from a particular angle or particular angular range. Due to low optical loss, high reflectivity and high mechanical robustness of the dielectric reflectors, they are preferred to metallic reflectors. The desired position and width of the reflection band can be obtained by the suitable choice of the layer thicknesses and refractive indices of the constituent materials.

The ability to reflect light of arbitrary angle of incidence for all dielectric structures is associated with the existence of complete photonic band gap (PBG), which can exist only in a system with a dielectric function that is periodic along three orthogonal directions. This is the case of 3D photonic crystals [1–4]. In simple dielectric mirrors i.e., one dimensional photonic crystals, there is no absolute photonic band gap owing to two factors. The first is that the edges of the directional PBGs will shift to higher frequencies with the increase in the incident angle, usually leading to the closure of the overall PBGs. The second is that at the Brewster angle the TM mode cannot be reflected. However, the absence of an absolute PBG does not mean that there is no omnidirectional total reflection. The criterion for the existence of omnidirectional total reflection is that there are no propagating modes that can couple the incident wave.

Recently, several research groups worldwide have reported that a simple-to-fabricate periodic 1D medium can have high reflectivity over a broad range of frequencies at all incident angles, i.e., an omnidirectional reflection, if the refractive indices and the thickness of the constituent dielectric layers are properly chosen [5–14]. First design criterion for omnidirectional reflection in one dimensional photonic crystal was given by Fink et al. [5] and Winn et al. [6]. They constructed a stack of nine alternating micrometer thick layers of Polystyrene and Tellurium and demonstrated omnidirectional reflection over the wavelength range from 10 to 15 micrometers. Chen et al. [7] fabricated six bilayers of  $SiO_2$  and  $TiO_2$  quarter wave films using the sol-gel method. They found an omnidirectional PBG of about 70 nm in near IR range. Chigrin et al. [8,9] fabricated a lattice consisting of 19 layers of  $Na_3AlF_6$  and ZnSe and found that omnidirectional photonic band gap exists in the spectral range 604.3-638.4 nm. Lee and Yao [10] studied theoretically and experimentally a wide range of realistic fabrication parameters for the formation of omnidirectional photonic band gaps (PBGs) in one-dimensional photonic crystals. C. J. Wu has theoretically studied the microwave transmission and reflection in a periodic superconductor/dielectric film multilater structure in the mixed state [13].

## 2. CRITERION OF OMNIDIRECTIONAL REFLECTION IN ONE DIMENSIONAL PHOTONIC CRYSTAL

According to Snell's law,  $n_i \sin \theta_i = n_H \sin \theta_H$  and  $n_H \sin \theta_H =$  $n_L \sin \theta_L$  i.e.,  $\theta_H = \sin^{-1} [n_i \sin \theta_i / n_H]$  and  $\theta_L = \sin^{-1} [n_H \sin \theta_H / n_L]$ , where  $n_H$  and  $n_L$  are the refractive indices of high index and low index media, and  $n_i$  is the refractive index of the an ambient medium (Figure 1). If the maximal refracted angle is smaller than the internal Brewster angle  $\theta_B = \tan^{-1} (n_L/n_H)$ , the incident wave from the outside cannot couple to the Brewster window, leading to the total reflection for all incident angles. Therefore the condition of the omnidirectional reflection for the ambient medium without influence of Brewster's angle  $(\theta_B)$  is  $\theta_B > \theta_H^{Max}$ , where  $\theta_H^{Max} = \sin^{-1}(n_i/n_H)$  is the maximum refractive angle of the light in high index medium, and  $\theta_B = \tan^{-1}(n_L/n_H)$  is the internal Brewster's angle on the interface between high index and low index media. The omnidirectional photonic band gap for both TE and TM polarizations is defined by the lower band edge at the normal incidence and the upper band edge at the perpendicular incidence ( $90^{\circ}$  incidence angle). The omnidirectional photonic band gap for the TM polarization is completely located within the omnidirectional PBG for TE polarization. Therefore, omnidirectional PBG for any polarization is the omnidirectional PBG for the TM polarization.

Three dimensional photonic crystals (PCs) with complete three dimensional photonic band gap in near-infrared region have been fabricated. It is not easy to fabricate a 3D PC with complete 3D PBG in visible region. Fabrication of 1D photonic crystal is relatively simple. Various simple planar deposition techniques such as sol-gel, r.f.-sputtering, holography etc. have been proposed [15–18]. Due to simple fabrication techniques, 1D PCs are cheap and effective to be used in various optoelectronic devices. Depending on the chosen geometry and frequency region, a lot of applications are possible. The planar geometry, for example, can be used to improve the properties of vertical-cavity surface-emitting lasers (VCSEL's) and microwave antenna, and to design transmission and energy-saving filters [19, 21]. By rolling into hollow fibers, the mirror can be used as inside walls of high-finesse waveguides and microcavities [22–25]. They can also be used in coating design, narrow band filters, laser measurements, optical sensors for high precession detection, laser beam security, photovoltaic applications and so on [26–39].

For the purpose of achieving real optical integration, Si-based materials are especially important for designing the PCs [35–39]. Silicon is a material of great interest for the fabrication of photonic devices in the near infrared region. It has a large refractive index with excellent mechanical and thermal properties and obviously it is compatible with Silicon based microelectronics.

The aim of the present work is to show the design rule of one dimensional omnidirectional photonic crystal. Simple transfer matrix method is used for calculations. The effect of two parameters, namely, refractive index contrast and filling fraction on omnidirectional reflection is investigated.

## 3. THEORETICAL BACKGROUND

We consider the propagation of electromagnetic waves through the periodic layered medium consisting of alternate layers of materials with different refractive indices. The schematic diagram of dielectric multilayer structure is as shown in the Figure 1, where  $n_i$  is the refractive index of the ambient media and  $n_s$  is the refractive index of substrate. We take x-axis in the direction normal to the layers. The indices of refraction of the system are given as,

$$n(x) = \begin{cases} n_H & d_L < x < d_H \\ n_L & 0 < x < h_L \end{cases} \quad \text{with } n(x) = n(x+d) \quad (1)$$

where  $h_1$  and  $h_2$  are the thicknesses of the layers with refractive indices  $n_L$  and  $n_H$  and  $d = d_L + d_H$  is the period of the structure.

To solve the photonic spectra of this periodic structure, we have used transfer matrix method [40, 41]. The electric field for the



**Figure 1.** Schematic diagram of a dielectric 1D PC structure.  $n_H$ ,  $d_H$  and  $n_L$ ,  $d_L$  are the refractive index and thickness of the alternating layers. A period is  $d (= d_H + d_L)$ . An ambient medium and substrate has refractive indices  $n_i$  and  $n_S$ .

Maxwell's equation can be written as

$$E = E(x)e^{i(\omega t - \beta z)},\tag{2}$$

where  $\beta$  is the z-component of the wave vector and  $\omega$  is angular frequency.

The electric field within each layer can be expressed as the sum of an incident plane wave and reflected plane wave. Coefficients in the first layer of the *n*th unit and (n-1)th unit cell are related by a matrix

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = T_n \begin{pmatrix} a_n \\ b_n \end{pmatrix}, \tag{3}$$

where  $T_n = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$  is the transfer matrix, and  $a_n$  and  $b_n$  are the coefficients of the right and left hand side propagating waves in the first layer of the *n*th unit cell respectively, and

$$T_{11} = \exp(ik_{1x}d_H) \left[\cos k_{2x}d_L + \frac{i}{2}\left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}}\right)\sin k_{2x}d_L\right]$$
(4)

$$T_{12} = \exp(-ik_{1x}d_H) \left[ \frac{i}{2} \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}d_L \right]$$
(5)

$$T_{21} = \exp(ik_{1x}d_H) \left[ -\frac{i}{2} \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}d_L \right]$$
(6)

$$T_{22} = \exp(-ik_{1x}d_H) \left[\cos k_{2x}d_L - \frac{i}{2}\left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}}\right)\sin k_{2x}d_L\right]$$
(7)

For a periodic structure with N period of layers with refractive indices  $n_H$  and  $n_L$  of constituent layers and thicknesses  $d_H$  and  $d_L$ , the coefficients of right and left hand side propagating states in both sides of multilayer structures are calculated by multiplying transfer matrices of each cell as shown below,

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = T_1 T_2 \dots T_N \begin{pmatrix} a_N \\ b_N \end{pmatrix}$$
(8)

The coefficient of reflection is given by

$$r_N = \left(\begin{array}{c} a_0\\ b_0 \end{array}\right)_{b_N = 0} \tag{9}$$

The reflectance is obtained by taking the absolute square of  $r_N$ ,

$$R_N = \left| r_N \right|^2 \tag{10}$$

Propagation of electromagnetic waves through periodic media is similar to the motion of electrons in crystalline solids. The electromagnetic wave in this structure is the Bloch waves and satisfies the following dispersion relation [40]

$$K(\beta,\omega) = \frac{1}{h} \cos^{-1} \left[ \frac{T_{11} + T_{22}}{2} \right],$$
(11)

where  $K(\beta, \omega)$  is the Bloch wave number,  $k_{1x}$  and  $k_{2x}$  denotes the x component of the wave vectors  $k_1$  and  $k_2$  in the first and the second constituent dielectric layer. For  $\left[\frac{T_{11}+T_{22}}{2}\right] < 1$ ,  $K(\beta, \omega)$  is the real number and the wave can transmit through the photonic crystal. However, for  $\left[\frac{T_{11}+T_{22}}{2}\right] > 1$ ,  $K(\beta, \omega)$  is an imaginary number, the wave cannot transmit through one dimensional photonic crystal and is totally reflected, which corresponds to the band gap of one dimensional PC. The critical case of  $\left[\frac{T_{11}+T_{22}}{2}\right] = 1$  corresponds to the edges of the band gap of the photonic crystal.

## 4. RESULTS AND DISCUSSION

Our one dimensional photonic crystal which is a multilayer structure, consists of two alternating layers of refractive indices  $n_H$  and  $n_L$  ( $n_H > n_L$ ) having thicknesses  $d_H$  and  $d_L$  and period  $d = d_H + d_L$ . We assume the constituent layers to be nonabsorbing, isotropic and homogeneous dielectrics. The photonic band structure of one dimensional photonic

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crystal can be calculated by using conventional TMM method for the parameters  $n_i$ ,  $n_H$ ,  $n_L$ ,  $\eta$  and d [7]. The omni-PBG depends on refractive index contrast of two materials  $(n_H/n_L)$ . It also depends on  $(n_L/n_i)$ , period d and filling factor  $\eta$  defined by the ratio of thicknesses of the two alternating layers  $(d_H/d_L)$ . Alternatively, we can define a parameter, filling fraction  $\gamma$  given by the ratio of thickness of high index medium and total thickness  $(d_H/d)$ . In this chapter, we have studied the dependence of omnidirectional reflection on two parameters; refractive index contrast and filling fraction.

# 4.1. Effect of Refractive Index Contrast on the Omnidirectional Reflection Range

In our work, we fix  $\eta$  to the wave thickness condition,  $d_H n_H = d_L n_L$ i.e.,  $\eta = \left(\frac{d_H}{d_L}\right) = \left(\frac{n_L}{n_H}\right)$ . Five system of 1D PC based on SiO<sub>2</sub> having different refractive index contrast are studied. The thickness of a period i.e., periodicity is kept constant for all the five cases,  $d = d_H + d_L = 500$  nm. We can define a dimensionless parameter, normalized omnidirectional bandwidth with respect to central wavelength of the omnidirectional reflection (ODR) band, which represents the width of the omnidirectional reflection band as  $\delta = \frac{\Delta\lambda}{\lambda_C} = \frac{2(\lambda_H - \lambda_L)}{(\lambda_H + \lambda_L)}$ , where  $\lambda_H$ and  $\lambda_L$  are the upper and lower wavelength edges for a given ODR band.

**Case I.**  $\operatorname{TiO}_2$ -SiO<sub>2</sub> System: In this case, we study 1D PC consisting of alternating layers of TiO<sub>2</sub> and SiO<sub>2</sub> with refractive indices  $n_H = 2.3$  and  $n_L = 1.5$  respectively. The refractive index contrast  $(n_H/n_L)$  is 1.533. The twelve-paired structure of TiO<sub>2</sub> and SiO<sub>2</sub> is supposed to be on a substrate of glass with refractive index  $n_S = 1.5$  and ambient medium is air  $n_i = 1.0$ . The thickness of the layers are taken according to the wave thickness condition,  $n_H d_H = n_L d_L$ , where  $d_H$  and  $d_L$  are the thicknesses of the TiO<sub>2</sub> and SiO<sub>2</sub> layers. Since the periodicity is  $d = d_H + d_L = 500$  nm. Therefore, according to wave thickness condition  $d_H = 197.37$  nm and  $d_L = 302.63$  nm. The filling factor  $\eta$  is fixed by the wave thickness condition and is equal to 0.652. For this one dimensional photonic crystal  $\theta_H^{Max} = \sin^{-1}(n_i/n_H) = 25.77$  degree and  $\theta_B = \tan^{-1}(n_L/n_H) = 33.11$  degree. Since  $\theta_B > \theta_H^{Max}$ , the effect of Brewster angle can be completely ignored.

Figure 2(a) show the reflectance spectra for TE and TM polarizations of a twelve pair 1D PC consisting of TiO<sub>2</sub> and SiO<sub>2</sub> multilayers. The spectra are plotted in terms of wavelength and for incident angle  $\theta_i$ . Figure 2(b) represents the conventional photonic



Figure 2. (a) Reflectance spectra of twelve-pair  $TiO_2/SiO_2$  1D PC for TE and TM polarizations and (b) their photonic band structure.

band structure which can be obtained by the projection of  $R \approx 1$  for Figure 2(a). In Figure 2(b), dark regions represent the forbidden band and white regions allowed. The area between the two horizontal lines gives the ODR band. The data for the nearly 100% reflectance is given in the Table 1. Here, the forbidden bands represent the regions with reflectivity greater than 95%. We observe that for TE polarization

Incident	TE polariz	ation	TM polarization		
angle,	Reflection range	Gap width	Reflection range	Gap width	
$\theta_i$ (degree)	(nm)	(nm)	(nm)	(nm)	
0	2110-1594	516	2110-1594	516	
30	2052-1514	538	1996-1546	450	
60	1932-1342	590	1742-1451	291	
75	1888-1273	615	1624-1422	202	
85	1878-1248	630	1486-1393	93	

Table 1. Total reflection region and gap width for  $TiO_2/SiO_2$  one dimensional photonic crystal.

at 0° incident angle, the total reflection region is from 1594 nm to 2110 nm, at 30°, high reflection range covers the region from 1514 nm to 2052 nm, at 60° incident angle, it spreads from 1342 nm to 1932 nm and at an incident angle of 85°, the region of high reflection is from 1248 nm to 1878 nm. Therefore omnidirectional reflection range for TE polarization has its range from 1594 nm to 1878 nm. For TM polarization, nearly 100% reflection region at 0° angle of incidence, is from 1546 nm to 2110 nm, at 30° incident angle, this covers the region from 1546 nm to 1996 nm, at an incident angle of 60°, high reflection range is from 1451 nm to 1742 nm and at 75° incident angle, the total reflection range is from 1422 nm to 1624 nm. For TM polarization

Therefore, omnidirectional reflection for both the polarizations (TE and TM) of the considered 1D PC is obtained only below 75° of angle of incidence and it covers the wavelength region,  $\lambda_H = 1594$  nm to  $\lambda_L = 1624$  nm, with bandwidth  $\Delta \lambda = (\lambda_H - \lambda_L) = 30$  nm. For incident angle greater than 75°, omnidirectional reflection ceases to exist. Normalized omnidirectional bandwidth is equal to 1.86 % at central wavelength  $\lambda_C = 1609$  nm.

**Case II.** ZnS-SiO<sub>2</sub> System: Twelve pairs of ZnS and SiO<sub>2</sub> with refractive indices  $n_H = 2.5$  and  $n_L = 1.5$  constitute the second 1D PC for our study. The refractive index contrast  $(n_H/n_L)$  is 1.667. Glass with refractive index  $n_S = 1.5$  is the substrate and ambient medium is air with  $n_i = 1.0$ . The thickness of the layers are taken according to the wave thickness condition,  $n_H d_H = n_L d_L$ , where  $d_H$  and  $d_L$  are the thicknesses of the ZnS and SiO<sub>2</sub> layers. As the period is  $d = d_H + d_L = 500$  nm, the thicknesses of the layers are  $d_H = 187.5$  nm and  $d_L = 312.5$  nm. The filling factor  $\eta$  is equal to



Figure 3. (a) Reflectance spectra of twelve-pair  $ZnS/SiO_2$  1D PC for TE and TM polarizations and (b) their photonic band structure.

0.60. For this one dimensional photonic crystal,  $\theta_H^{Max} = 23.58$  degree and  $\theta_B = 30.96$  degree. Since  $\theta_B > \theta_H^{Max}$ , the effect of Brewster angle can be completely ignored and omni-PBG is possible.

Reflectance spectra for TE and TM polarizations of a twelve pair 1D PC consisting of ZnS and SiO<sub>2</sub> multilayers are shown in Figure 3(a). The spectra are plotted in terms of wavelength and for incident angle

Incident	TE polariz	ation	TM polarization		
angle,	Reflection range Gap wi		Reflection range	Gap width	
$\theta_i(\text{degree})$	(nm)	(nm)	(nm)	(nm)	
0	2248-1607	641	2248-1607	641	
30	2194-1530	664	2130-1563	567	
60	2082-1362	720	1866-1472	394	
85	2031-1271	760	1733-1415	318	

Table 2. Total reflection region and gap width for  $ZnS/SiO_2$  one dimensional photonic crystal.

 $\theta_i$ . Figure 3(b) represents the conventional photonic band structure obtained by the projection of  $R \approx 1$  for Figure 3(a). In Figure 3(b), dark regions represent the forbidden band and white regions allowed. The area between the two horizontal lines gives the ODR band. The results for the nearly 100% reflectance is given in Table 2. Here the forbidden bands represent the regions with reflectivity greater than 95%. From the reflectance data as given in Table 2 for TE and TM polarizations of this 1D PC, we observe that for TE polarization, the omnidirectional reflection range is from 1607 nm to 2031 nm. For TM polarization the omnidirectional reflection is obtained in the range 1607 nm-1733 nm.

Thus, the total omnidirectional reflection for both the polarizations (TE & TM) of the considered 1D PC covers the wavelength region,  $\lambda_H = 1607 \,\mathrm{nm}$  to  $\lambda_L = 1733 \,\mathrm{nm}$ , with bandwidth  $\Delta \lambda = (\lambda_H - \lambda_L) = 126 \,\mathrm{nm}$ . Normalized omnidirectional band width is  $\frac{\Delta \lambda}{\lambda_C} = 7.54\%$  at central wavelength  $\lambda_C = 1670 \,\mathrm{nm}$ .

**Case III. Si-SiO**<sub>2</sub> **System**: Si and SiO<sub>2</sub> with refractive indices  $n_H = 3.7$  and  $n_L = 1.5$  are the constituents of third 1D PC of our study. The refractive index contrast is 2.467. The twelve-paired structure of Si and SiO<sub>2</sub> is supposed to be on a substrate of glass with refractive index  $n_S = 1.5$  and ambient medium is air  $n_i = 1.0$ . The thickness of the layers are  $d_H = 144.23$  nm and  $d_L = 355.77$  nm, which are taken according to the condition,  $n_H d_H = n_L d_L$ , where  $d_H$  and  $d_L$  are the thicknesses of the Si and SiO<sub>2</sub> layers and period is  $d = d_H + d_L = 500$  nm. The filling factor  $\eta$  is equal to 0.405. For this one dimensional photonic crystal  $\theta_H^{Max} = 15.68$  degree and  $\theta_B = 22.07$  degree. Since  $\theta_B > \theta_H^{Max}$ , the effect of Brewster angle can be completely ignored.

The reflectance spectra for TE and TM polarizations of a twelve pair 1D PC consisting of Si and SiO<sub>2</sub> multilayers are plotted in terms of wavelength and for incident angle  $\theta_i$  (Figure 4(a)). Figure 4(b) represents the conventional photonic band structures which can be obtained by the projection of  $R \approx 1$  for Figure 4(a). In Figure 4(b), dark regions represent the forbidden band and white regions allowed. The area between the two horizontal lines gives the total omnidirectional reflection band. Here the forbidden bands represent the regions with reflectivity greater than 95%. The data for total reflection is summarized in Table 3. From the data in Table 3, we infer that TE polarization has its omnidirectional reflection range from 1663 nm to 2823 nm and for TM polarization, the wavelength range 1663 nm-2309 nm gives the omnidirectional reflection range.

Therefore, for Si/SiO<sub>2</sub> 1D PC, total ODR bandwidth is  $\Delta \lambda = 636 \text{ nm}$  with upper band edge  $\lambda_H = 2309 \text{ nm}$  and lower band edge  $\lambda_L = 1663 \text{ nm}$ . Normalized omnidirectional reflection bandwidth is  $\frac{\Delta \lambda}{\lambda_C} = 32.53\%$  at  $\lambda_C = 1986 \text{ nm}$ .

Table 3. Total reflection region and gap width for  $Si/SiO_2$  one dimensional photonic crystal.

Incident	TE polariz	ation	TM polarization		
angle,	Reflection range	Gap width	Reflection range	Gap width	
$\theta_i$ (degree)	(nm)	(nm)	(nm)	(nm)	
0	2981-1663	1318	2981-1663	1318	
30	2941-1591	1350	2825-1626	1199	
60	2859-1439	1420	2484-1554	930	
85	2823-1361	1462	2309-1514	795	

**Case IV. Ge-SiO**<sub>2</sub> **System**: In this case, twelve pairs of alternating layers of Ge and SiO<sub>2</sub> with refractive indices  $n_H = 4.2$  and  $n_L = 1.5$  constitute the 1D PC. The refractive index contrast is 2.8. The structure is supposed to be synthesized on a substrate of glass with refractive index  $n_S = 1.5$  and ambient medium is air  $n_i = 1.0$ . The thickness of the layers which are taken according to the wave thickness condition,  $n_H d_H = n_L d_L$ , are  $d_H = 131.58$  nm and  $d_L = 368.42$  nm with period is  $d = d_H + d_L = 500$  nm where  $d_H$  and  $d_L$  are the thicknesses of the Ge and SiO<sub>2</sub> layers. The filling factor  $\eta$  is equal to 0.357. We can see that for this 1D PC  $\theta_H^{Max} = 13.77$  degree and  $\theta_B = 19.65$  degree. Since  $\theta_B > \theta_H^{Max}$ , the effect of Brewster angle can be completely ignored and omnidirectional reflection can be



**Figure 4.** (a) Reflectance spectra of a twelve-pair  $Si/SiO_2$  1D PC for TE and TM polarizations and (b) their photonic band structure.

obtained.

The reflectance spectra of a 1D PC consisting of twelve pairs of Ge and SiO<sub>2</sub> multilayers for both TE and TM polarizations is shown in Figure 5(a). The spectra are plotted in terms of wavelength and for incident angle  $\theta_i$ . Figure 5(b) represent the conventional photonic



**Figure 5.** (a) Reflectance spectra of a twelve-pair  $Ge/SiO_2$  1D PC for TE and TM polarizations and (b) their photonic band structure.

band structures which can be obtained by the projection of  $R \approx 1$ for Figure 5(a). Here the forbidden bands represent the regions with reflectivity greater than 95%. In Figure 5(b), dark regions represent the forbidden band and white regions allowed. The area between the two horizontal lines gives the total omnidirectional reflection band. The data corresponding to nearly 100% reflectance is summarized in Table 4. We observe from the data that TE polarization has its omnidirectional reflection range from 1675 nm to 3107 nm and the wavelength range from 1675 nm to 2512 nm gives the omnidirectional reflection for TM polarization.

Incident	TE polariz	ation	TM polarization		
angle,	Reflection range Gap width		Reflection range	Gap width	
$\theta_i(\text{degree})$	(nm)	(nm)	(nm)	(nm)	
0	3250-1675	1575	3250-1675	1575	
30	3213-1605	1608	3079-1640	1439	
60	3139-1458	1681	2705-1571	1134	
85	3107-1383	1724	2512-1534	978	

Table 4. Total reflection region and gap width for  $Ge/SiO_2$  one dimensional photonic crystal.

Therefore, total omnidirectional reflection (for both TE & TM polarizations) of Ge/SiO<sub>2</sub> 1D PC, has the bandwidth  $\Delta \lambda = (\lambda_H - \lambda_L) = 837 \,\mathrm{nm}$ . The upper wavelength edge of ODR band is  $\lambda_H = 2512 \,\mathrm{nm}$  and the lower wavelength edge of ODR is  $\lambda_L = 1675 \,\mathrm{nm}$ . Normalized omnidirectional band width is  $\frac{\Delta \lambda}{\lambda_C} = 39.98\%$  at  $\lambda_C = 2094 \,\mathrm{nm}$ .

**Case V. Te-SiO**<sub>2</sub> **System**: Te and SiO<sub>2</sub> with refractive indices  $n_H = 4.6$  and  $n_L = 1.5$  are the constituents of this 1D PC system. The refractive index contrast is 3.067. The system which consists of twelve periods of Ge and SiO<sub>2</sub> is supposed to be on a substrate of glass with refractive index  $n_S = 1.5$  and ambient medium is air  $n_i = 1.0$ . The thickness of the layers are  $d_H = 122.95$  nm and  $d_L = 377.05$  nm which are taken according to condition  $n_H d_H = n_L d_L$ , where  $d_H$  and  $d_L$  are the thicknesses of the Ge and SiO<sub>2</sub> layers and the period is  $d = d_H + d_L = 500$  nm. The filling factor  $\eta$  is equal to 0.326. For this 1D PC  $\theta_H^{Max} = 12.56$  degree and  $\theta_B = 18.06$  degree. Since  $\theta_B > \theta_H^{Max}$ , the effect of Brewster angle can be completely ignored and omnidirectional reflection can be obtained.

The reflectance spectra (both TE and TM polarizations) are plotted in terms of wavelength and for incident angle  $\theta_i$  (Figure 6(a)). Conventional photonic band structure which is obtained by the projection of  $R \approx 1$  for Figure 6(a) are shown in Figure 6(b). Here the forbidden bands represent the regions with reflectivity greater than 95%. In Figure 6(b), the dark regions represent the forbidden band and white regions represent the allowed. The area between the two



Figure 6. (a) Reflectance spectra of a twelve-pair  $Te/SiO_2$  1D PC for TE and TM polarizations and (b) their photonic band structure.

horizontal lines gives the total omnidirectional reflection band. The data corresponding to total reflection are given in Table 5. We observe from the data that for TE polarization ODR covers the range from 1683 nm to 3320 nm and for TM polarization ODR range is 1683 nm-2664 nm.

Therefore, total ODR for Te/SiO<sub>2</sub> 1D PC, for both TE & TM

Incident	TE polariz	ation	TM polarization		
angle,	Reflection range Gap wi		Reflection range	Gap width	
$\theta_i(\text{degree})$	(nm)	(nm)	(nm)	(nm)	
0	3453-1683	1770	3453-1683	1770	
30	3418-1613	1805	3271-1648	1623	
60	3349-1470	1879	2872-1581	1291	
85	3320-1397	1923	2664-1545	1119	

Table 5. Total reflection region and gap width for  $Te/SiO_2$  one dimensional photonic crystal.

**Table 6.** Omnidirectional bandwidth of studied one dimensionalphotonic crystals.

Photonic crystal	$\lambda_H$ (nm)	$\lambda_L$ (nm)	$\lambda_C$ (nm)	$\begin{aligned} \Delta \lambda \\ = (\lambda_H - \lambda_L) \\ (\text{nm}) \end{aligned}$	$\frac{\Delta\lambda}{\lambda_C}(\%)$
${\rm TiO_2/SiO_2}$	1624	1594	1609	30	1.86
$ZnS/SiO_2$	1733	1607	1670	126	7.54
$\rm Si/SiO_2$	2309	1663	1986	646	32.53
$Ge/SiO_2$	2512	1675	2094	837	39.98
${\rm Te/SiO_2}$	2664	1683	2174	981	45.13

polarizations exists between upper band edge  $\lambda_H = 2664 \text{ nm}$  and lower band edge  $\lambda_L = 1683 \text{ nm}$ , with bandwidth  $\Delta \lambda = (\lambda_H - \lambda_L) = 981 \text{ nm}$ . Normalized omnidirectional band width is  $\frac{\Delta \lambda}{\lambda_C} = 45.13\%$  at  $\lambda_C = 2174 \text{ nm}$ .

The results of omnidirectional reflection corresponding to all five photonic crystals, investigated are given in Table 6. The variation of upper band edge  $\lambda_H$ , lower band edge  $\lambda_L$  and central wavelength  $\lambda_C$  of the omnidirectional band gap as a function of refractive index contrast ratio  $n_H/n_L$  are shown in Figure 7(a). There is a slow increase in  $\lambda_L$ , but  $\lambda_H$  increases sharply with increasing  $n_H/n_L$ . We can see that for the lowest value of the  $n_H/n_L$ , 1.533, which is for the system TiO<sub>2</sub>/SiO<sub>2</sub>,  $\lambda_L$  and  $\lambda_H$  are nearly same i.e., there is no omnidirectional reflection band for all angles. As shown in the Figure 7(b), omnidirectional bandwidth increases sharply with increasing  $n_H/n_L$  when  $n_L/n_i$  is fixed. Similar trend is also observed in the Figure 7(c) which gives the normalized omnidirectional bandwidth vs.  $n_H/n_L$ .

$d_H$ (nm)	$d_L$ (nm)	Filling fraction	$\lambda_H$ (nm)	$\lambda_L$ (nm)	$\lambda_C$ (nm)	$\begin{aligned} \Delta \lambda \\ = (\lambda_H - \lambda_L) \\ (nm) \end{aligned}$	$\frac{\Delta\lambda}{\lambda_C}(\%)$
20	380	0.050	0	0	0	0	0
30	370	0.075	1215	1202	1208.5	13	1.076
40	360	0.100	1320	1195	1257.5	125	9.94
50	350	0.125	1410	1197	1303.5	213	16.341
60	340	0.15	1492	1204	1348.0	288	21.365
70	330	0.175	1566	1215	1390.5	351	25.243
80	320	0.200	1635	1230	1432.5	405	28.272
90	310	0.225	1698	1251	1474.5	447	30.315
100	300	0.250	1757	1276	1516.5	481	31.718
110	290	0.275	1810	1307	1558.5	503	32.275
115.4	284.6	0.289	1837	1325	1581.0	512	32.385
120	280	0.300	1858	1342	1600.0	516	32.25
130	270	0.325	1900	1383	1641.5	517	31.496
140	260	0.350	1934	1428	1681.0	506	30.101
150	250	0.375	1954	1476	1715.0	478	27.872
160	240	0.400	1942	1528	1735.0	414	23.862
170	230	0.425	1918	1583	1750.5	335	19.137
180	220	0.450	1933	1640	1786.5	293	16.401
190	210	0.475	1959	1700	1829.5	259	14.157
200	200	0.500	1986	1761	1873.5	225	12.01
210	190	0.525	2012	1825	1918.5	187	9.747
220	180	0.550	2033	1890	1961.5	143	7.29
230	170	0.575	2041	1957	1999	84	4.202
240	160	0.600	0	0	0	0	0

**Table 7.** Omnidirectional bandwidth of  $Si-SiO_2$  one dimensional photonic crystals for different filling fractions.



**Figure 7.** Change in (a) $\lambda_H$ ,  $\lambda_L$  and  $\lambda_C$ , (b)  $\Delta\lambda$  and (c)  $\Delta\lambda/\lambda_C$  as a function of  $n_H/n_L$ .

#### 4.2. Effect of Filling Fraction on the Omnidirectional Reflection Range

In this part of the paper, the dependence of the omnidirectional reflection range on the filling fraction  $d_H/d$  is investigated. One dimensional photonic crystal consisting of alternate layers of Si and  $SiO_2$  with refractive indices  $n_H = 3.7$  and  $n_L = 1.5$  respectively, The period of the structure is d =is taken for our study. 400 nm. Omnidirectional reflection (ODR) ranges are calculated taking different values of the  $d_H/d$ , with d = 400 nm. The ODR range for different values of  $d_H$ , starting from 20 nm to 240 nm in steps of 10 nm are measured. The edges of omnidirectional reflection bands are noted. The ODR band width, central wavelength of ODR bands and normalized omnidirectional band width are calculated. The data corresponding to these calculations are given in Table 7. Figure 8(a)gives the variation of ODR band edges and central wavelength of ODR bands versus filling fraction  $d_H/d$ . Figure 8(b) gives the ODR bandgap in terms of filling fraction. The normalized omnidirectional bandwidth, which is the measure of ODR band, is plotted in Figure 8(c) in terms of filling fraction. From the figures, it is observed that initially ODR bandwidth is zero below a certain value of filling fraction, then it increases with increasing filling fraction and to a maximum value and finally it again comes to zero at some value of filling fraction above which no ODR bandwidth is obtained. The maximum normalized ODR bandwidth is obtained at the value of filling fraction of 0.29. This is the value of wave thickness condition. For this value of filling fraction, the thickness of Si and SiO<sub>2</sub> layers are 115.4 nm and 284.6 nm respectively. The reflectance spectra for this photonic crystal are shown in the Figures 9(a) and (b). Figure 9(c) gives photonic band structure of this 1D PC. The upper and lower edges of ODR band have the values  $\lambda_H = 1837 \,\mathrm{nm}$  and  $\lambda_L = 1325 \,\mathrm{nm}$ , and ODR bandwidth  $\Delta\lambda = 512\,\mathrm{nm}$  . The normalized ODR bandwidth is 32.385% at the central wavelength of  $\lambda_H = 1581 \,\mathrm{nm}$ .

We can also calculate the allowable range for refractive index of ambient medium for getting omnidirectional reflection. The condition for the omnidirectional reflection is given by  $\theta_B > \theta_H^{Max}$ , where  $\theta_H^{Max} = \sin^{-1}(n_i/n_H)$  is the maximum refractive angle of the light in high index medium, and  $\theta_B = \tan^{-1}(n_L/n_H)$  is the internal Brewster's angle on the interface between high index and low index media. In the case of Si-SiO<sub>2</sub> system,  $n_H = 3.7$ , and  $n_i = 1.5$ . So, the allowable range of  $n_i$  for this 1D PC is  $1 < n_i < 1.39$ , i.e., the maximum allowable value of  $n_i$  is 1.39.



**Figure 8.** (a) $\lambda_H$ ,  $\lambda_L$  and  $\lambda_C$ , (b)  $\Delta\lambda$  as a function of  $d_H/d$  and (c)  $\Delta\lambda/\lambda_C$  as a function of  $d_H/d$ .



**Figure 9.** (a) and (b) Reflectance spectra and (c) photonic band structure of Si-SiO<sub>2</sub> 1D PC. The thicknesses of Si and SiO<sub>2</sub> layers are 115.4 nm and 284.6 nm respectively.

#### 5. CONCLUSION

We have investigated the structural parameters for the formation of omnidirectional photonic band gap in one dimensional photonic crystal. We can say from our study that when  $n_L$ ,  $n_i$ ,  $n_s$  and dare fixed, omnidirectional bandgap increases with increasing  $n_H/n_L$ i.e., with increasing  $n_H$ . Therefore, omnidirectional bandgap can be increased by using the material of high refractive index  $n_H$  when the low index material  $n_L$  is fixed. We have also demonstrated the effect of filling fraction on omnidirectional reflection range. For the considered system of Si-SiO<sub>2</sub>, omnidirectional reflection range increases with filling fraction, goes to a maximum value and finally comes to zero. The maximum value of the omnidirectional reflection range is obtained at a value of 0.29 of the filling fraction. The range for allowable values of refractive index of ambient medium  $(n_i)$  has also been estimated.

## ACKNOWLEDGMENT

Mr. Sudesh Kumar Singh is thankful to CSIR, INDIA and Khem B. Thapa is thankful to UGC, INDIA for financial assistance. Janardan P. Pandey acknowledges UGC, INDIA for assistance under FIP scheme.

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