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OPTIMUM NORMALIZED-GAUSSIAN TAPERING WINDOW FOR SIDE LOBE REDUCTION IN UNIFORM CONCENTRIC CIRCULAR ARRAYS

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Abstract—In this paper, a new tapered beamforming function for side lobe reduction in the uniform concentric circular arrays (UCCA) is proposed. This technique is based on tapering the current amplitudes of the rings in the array, where all elements in an individual ring are weighted in amplitude by the same value and the weight values of different rings are determined by a function that has a normalized-gaussian probability density function variation. This novel tapering window is optimized in its parameters to have the lowest possible side lobe level that may reach 43 dB below the main lobe and these optimum weights are found to be function of the number of elements of the innermost ring and the number of rings in the array. The proposed tapering window can be modified to compensate the gain reduction due to tapering when compared with the uniform feeding case.

1. INTRODUCTION

Circular arrays has considerable interest in various applications including sonar, radar, and mobile communications [1–3]. It consists of a number of elements usually omnidirectional arranged on a circle [1] and can be used for beamforming in the azimuth plane for example at the base stations of the mobile radio communications system [3]. In two-dimensional beamforming especially at directions perpendicular to the circular array plane, the side lobe level will be high (approximately 8 dB below the main lobe) and the array is inefficient if utilized at angles near to the normal of the array. Therefore, one possible solution to reduce this higher side lobe level is to use multiple concentric circular arrays (CCA) of different number of elements and radii. Uniform

CCA (UCCA) is one of the most important configurations of the CCA [4, 5] where the inter-element spacing in individual rings and the inter-ring spacing are kept almost half of the wavelength. The side lobes in the UCCA will drop to about 17.5 dB especially at larger number of rings [4]. However, this side lobe level will be very high in some applications especially that utilizes frequency reuse as in mobile or broadband communications from high altitude platforms (HAPs) [6]. In these applications the higher side lobe levels will result in degraded carrier-to-interference ratio (CIR). Therefore, this paper is devoted for reducing the side lobe levels in UCCA using a tapered beamforming technique where the rings in the array are tapered in amplitude. The proposed tapering function has an expression like a normalized-gaussian density function of a mean value that equals 1 and standard deviation that is function of the array geometry and size. This tapering window is optimized to provide the lowest possible side lobe levels and compromise the array gain reduction due to tapering. The paper is arranged as follows; in Section 2 discusses the UCCA geometry and Section 3 introduces the proposed tapering window. In Section 4, the tapered UCCA performance is discussed in terms of beam power pattern, side lobe levels and beamwidth while Section 5 discusses the problem and possible solutions of the gain reduction due to tapering and finally Section 6 concludes the paper.

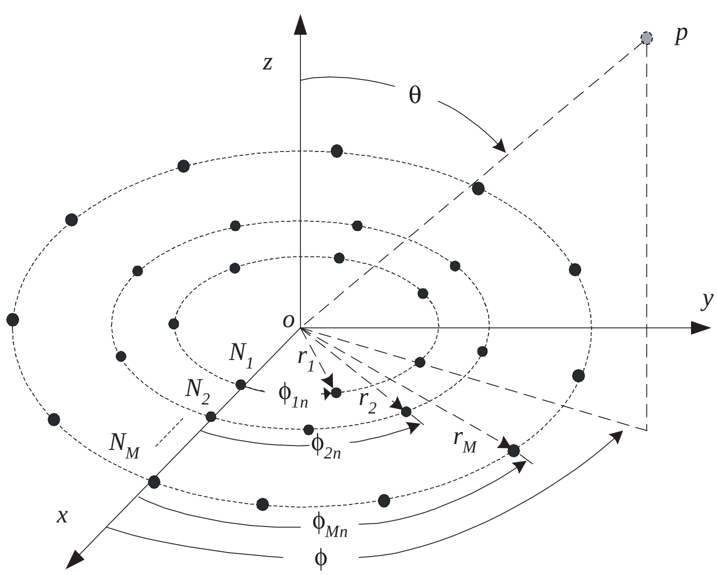

Figure 1. Concentric circular arrays (CCA).

2. GEOMETRY OF THE UCCA

Concentric circular antenna arrays has elements arranged in multiple concentric circular rings which differ in radius and number of elements as shown in Figure 1, where there are M concentric circular rings. The mth ring has a radius r_m and number of elements N_m where $m = 1, 2, \ldots, M$.

In array processing, it is generally assumed that all elements in the array are omnidirectional radiators or sensors, therefore the power pattern is well defined if we know the weighting and steering matrices of the array. An expression for the array steering matrix has been deduced in [4] and is given by:

$$AS(\theta,\phi) = \begin{bmatrix} e^{jkr_1\sin\theta\cos(\phi-\phi_{11})} & e^{jkr_2\sin\theta\cos(\phi-\phi_{21})} & & e^{jkr_M\sin\theta\cos(\phi-\phi_{M1})} \\ e^{jkr_1\sin\theta\cos(\phi-\phi_{12})} & e^{jkr_2\sin\theta\cos(\phi-\phi_{22})} & & e^{jkr_M\sin\theta\cos(\phi-\phi_{M2})} \\ . & . & & . \\ e^{jkr_1\sin\theta\cos(\phi-\phi_{1N_1})} & . & & . \\ 0 & e^{jkr_2\sin\theta\cos(\phi-\phi_{2N_2})} & \cdots & . \\ 0 & 0 & & . \\ 0 & 0 & & e^{jkr_M\sin\theta\cos(\phi-\phi_{MN_{M-1}})} \\ 0 & 0 & & e^{jkr_M\sin\theta\cos(\phi-\phi_{MN_M})} \end{bmatrix} \quad (1)$$

where the azimuth angle of the mnth element in the array is given by:

$$\phi_{mn} = \frac{2\pi n}{N_m}, \qquad n = 1, 2, 3, \ldots, N_m \quad (2)$$

Each column in the array steering matrix represents the corresponding ring steering vector which in general for the mth ring is given by:

$$S_m(\theta,\phi) = \Big[e^{jkr_m\sin\theta\cos(\phi-\phi_{m1})} e^{jkr_m\sin\theta\cos(\phi-\phi_{m2})} \ldots e^{jkr_m\sin\theta\cos(\phi-\phi_{mn})} \ldots e^{jkr_m\sin\theta\cos(\phi-\phi_{mN_m})}\Big]^T \quad (3)$$

Therefore, the array steering matrix may be rewritten as:

$$AS(\theta,\phi) = [S_1(\theta,\phi)S_2(\theta,\phi)\ldots\ldots S_m(\theta,\phi)\ldots\ldots S_M(\theta,\phi)] \quad (4)$$

It is usually for the ring steering vectors to have different lengths as they have different number of elements, therefore we append each column with zeros for lower length vectors in the array steering matrix.

If the interelement spacing in any array exceeds half of the wavelength, the resulted radiation pattern will have grating lobes of higher levels. On the other hand, if this distance is smaller than half of the wavelength, the pattern will have a wider main lobe and the mutual coupling between elements will increase. Therefore, the uniform concentric circular array (UCCA) configuration that has almost half of the wavelength separating distance either between neighboring elements in a ring or between any two neighboring rings is needed to have a reasonable radiation pattern and this can be obtained if the number of elements is incremented by 6 elements [4] or:

$$N_{m+1} = N_m + 6 \tag{5}$$

This gives inter-ring separation distance of 0.4775λ (which is the nearest possible value to half of the wavelength) which occurs only if for any ring the distance between any two neighboring elements is set half of the wavelength or:

$$\frac{2\pi r_m}{N_m} = 0.5\lambda \tag{6}$$

We can control the radiation pattern of the array by controlling the magnitudes and phases of the exciting currents, therefore the array factor or gain will be determined by the following equation

$$G(\theta,\phi) = SUM\left\{W(\theta,\phi)^H AS(\theta,\phi)\right\} \tag{7}$$

where the SUM operator is the summation of the elements of the resulted matrix and $W(\theta,\phi)$ is the weight matrix that controls the amplitudes and phases of the input currents. To have a delay-and-sum beamformer, we can form the main lobe in the direction (θ_0,ϕ_0) by setting the weight matrix to equal the array steering matrix at the same direction or

$$W(\theta,\phi) = AS(\theta_0,\phi_0) \tag{8}$$

and therefore, the normalized array gain is given by

$$G_n(\theta,\phi) = \frac{1}{\sum_{i=1}^{M} N_i} SUM\left\{AS(\theta_0,\phi_0)^H AS(\theta,\phi)\right\} \tag{9}$$

Figure 2 shows a typical radiation pattern of UCCA that has $N_1 = 5$ elements and $M = 10$ rings. The pattern is determined at three planes of the azimuth angle where $\phi = 0, 45, 90$ degrees. All the patterns are identical which is a property of the UCCA even if at any value of N_1 or M.

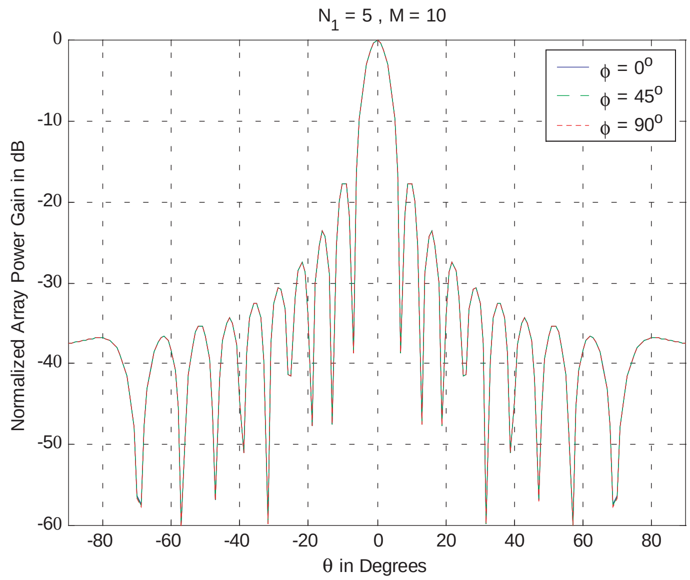

Figure 2. Radiation pattern of typical uniformly-fed UCCA.

3. THE PROPOSED TAPERING WINDOW

In this section, an amplitude tapering window is proposed that likes a normalized-gaussian probability density function that has a mean value of 1 and standard deviation set as the number of rings, M, divided by some parameter δ denoted by the correlation constant. Therefore the normalized-gaussian window has weighting amplitudes given by:

$$\alpha_m = e^{-(m-1)^2/2\sigma^2}, \qquad m = 1, 2, 3, \ldots, M \tag{10}$$

where σ is given by:

$$\sigma = \frac{M}{\delta} \tag{11}$$

the correlation constant δ controls the convergence between the weight values; therefore for lower values of δ, σ will increase which means that the weights change slowly from the innermost value (i.e., α_1).

Rewriting Eq. (10) in terms of M and δ gives:

$$\alpha_m = e^{-(m-1)^2\delta^2/2M^2} \tag{12}$$

and the complete weight matrix will be given by:

$$\begin{aligned} W(\theta, \phi) \quad = \quad & [\alpha_1 S_1(\theta_0, \phi_0)\alpha_2 S_2(\theta_0, \phi_0) \ldots\ldots \\ & \alpha_m S_m(\theta_0, \phi_0) \ldots\ldots \alpha_M S_M(\theta_0, \phi_0)] \end{aligned} \tag{13}$$

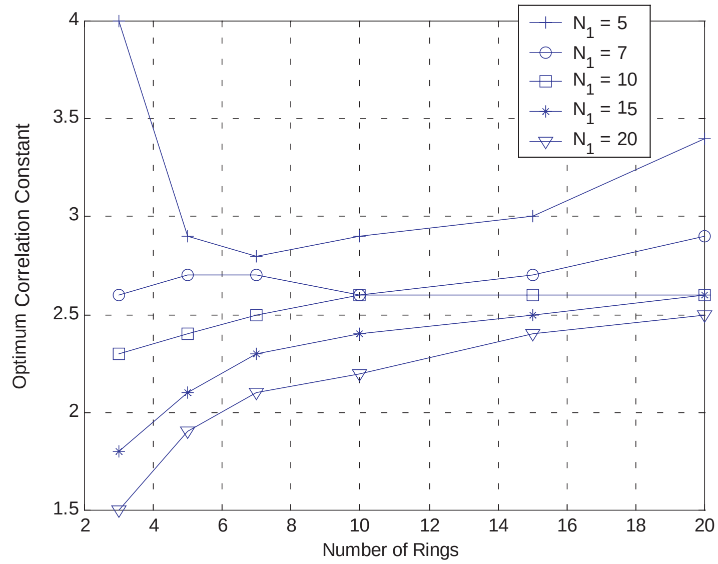

Figure 3. Optimum correlation constant variation with the number of rings at different innermost ring size.

For practical considerations, it is popular to the weight amplitudes to have little spread or low σ to minimize the radiation pattern distortion due to amplitude variations or errors, while lower spread decreases the amount of side lobe reduction. In the normalized-gaussian UCCA, it is found that there are some optimum values of the weight amplitudes which give the lowest possible side lobe level and occur at optimum values of the correlation constant δ.

Figure 3 depicts the optimum values of the correlation constant δ as a function of the number of the array number of rings, M, at different values of the innermost ring size (i.e., N_1). This figure shows that most of the optimum values lies in the range between 2 and 3 for most array geometries and almost has a mean of 2.5. If the number of elements in the innermost ring increases, the optimum value of δ will decrease while it increases for larger arrays (i.e., for higher values of M). Figure 4 displays the amplitude values of the window function, α_m, as a function of the ring number at different values of the correlation constant for a typical UCCA of 10 rings. In this figure, the range of most optimum values of the weights lies between two curves corresponding to $\delta = 2$ and 3.

As the correlation constant increases, the weight amplitude values will drop rapidly and the outermost rings will have very low weight

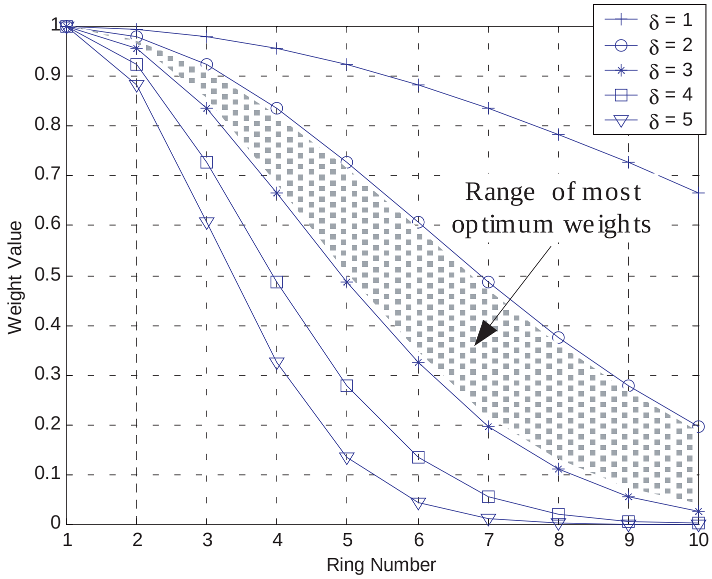

Figure 4. The variation of tapering weights at different values of the correlation constant for a typical 10 ring UCCA.

values, while for lower values of δ, the weights has little spread and drop slowly.

4. POWER PATTERN, SIDE LOBE LEVEL AND BEAMWIDTH PERFORMANCE

Figure 5 displays a typical normalized power gain pattern of a tapered UCCA at $\delta = 2.5$ where $N_1 = 5$ elements and $M = 10$ rings and shows a reduction in side lobe level to less than -40 dB. Generally, the side lobe level at different innermost ring sizes and at $\delta = 2.5$ is shown in Figure 6 where it will decrease if we increase the number of rings or decreasing the innermost ring size, for example it drops to -43 dB at $N_1 = 5$ and $M = 20$ that was -17.5 dB without tapering indicating a reduction in the side lobe level by 25.5 dB. The price paid here is obtaining wider beam as shown in Figure 7, which has a beamwidth of 3.7 degrees that was 2.82 degrees in the uniform feeding case (i.e., without tapered beamforming). This problem can be alleviated by increasing the number of elements in the tapered array to have the same beamwidth without tapering. The increase in beamwidth will be smaller for UCCA of larger sizes than for smaller ones, therefore the

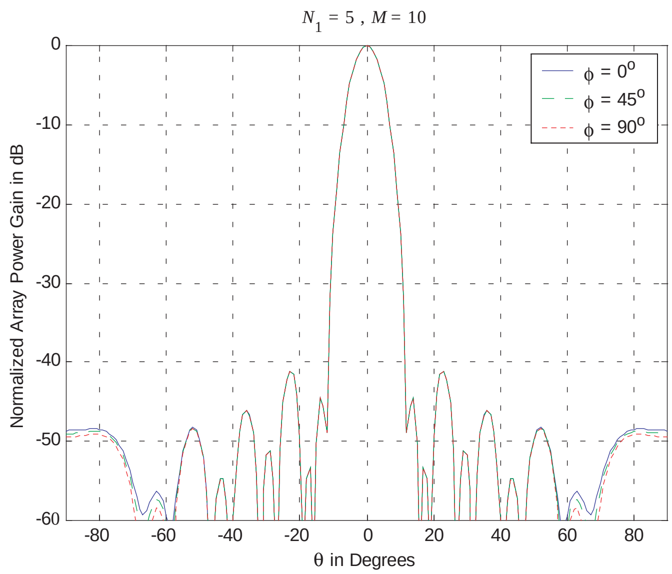

Figure 5. Radiation pattern of typical tapered UCCA at $\delta = 2.5$.

tapering is more efficient for larger arrays. On the other hand if we decrease the number of elements in the innermost ring, the beamwidth will increase at a specific number of rings while the side lobe level decreases.

5. ARRAY GAIN DEGRADATION: PROBLEM AND SOLUTION

Another cost paid is the reduced array gain compared with the uniform feeding because in tapering, the outer rings has a little contribution than the inner ones. This reduction can be determined considering the amplitude of the weight value, α_m, and the number of elements, N_m, corresponding to each ring. The maximum array gain in the case of uniform feeding will be:

$$G_{\max} = \sum_{i=1}^{M} N_i \tag{14}$$

while in the case of tapered beamforming it will be given by:

$$G_{\max} = \sum_{i=1}^{M} \alpha_i N_i \tag{15}$$

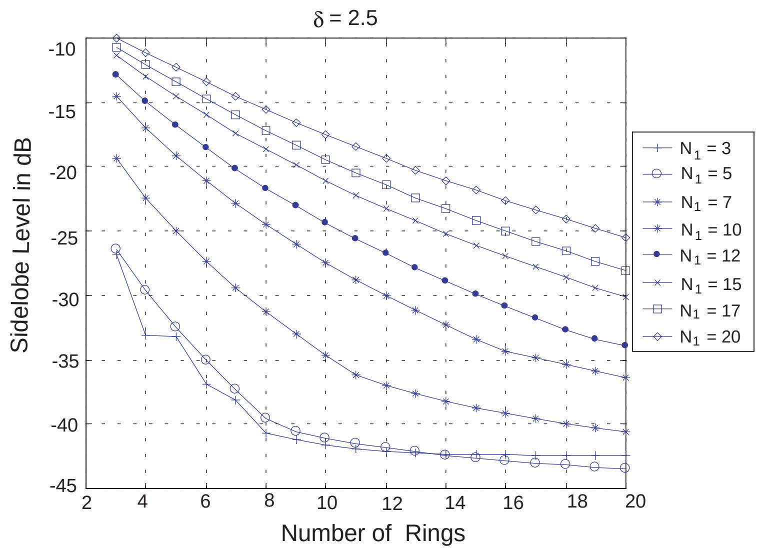

Figure 6. Variation of the side lobe level with the number of rings at different innermost ring size and at $\delta = 2.5$.

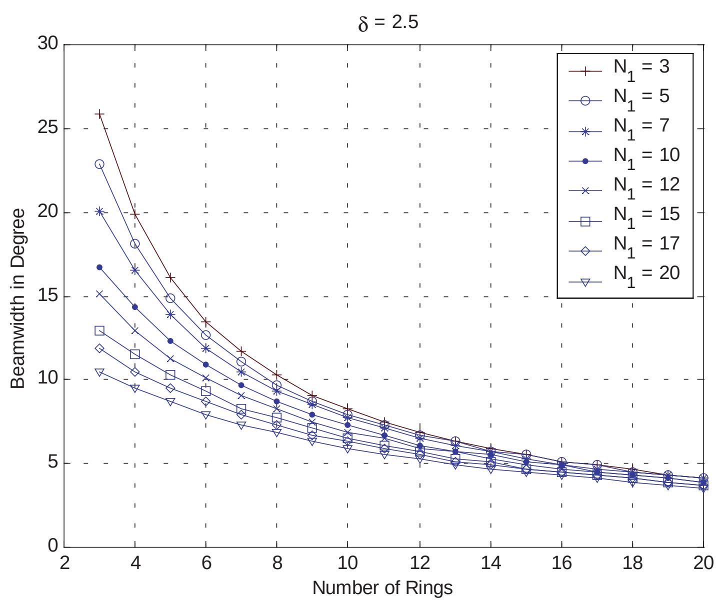

Figure 7. Variation of the beamwidth with the number of rings at different innermost ring size and at $\delta = 2.5$.

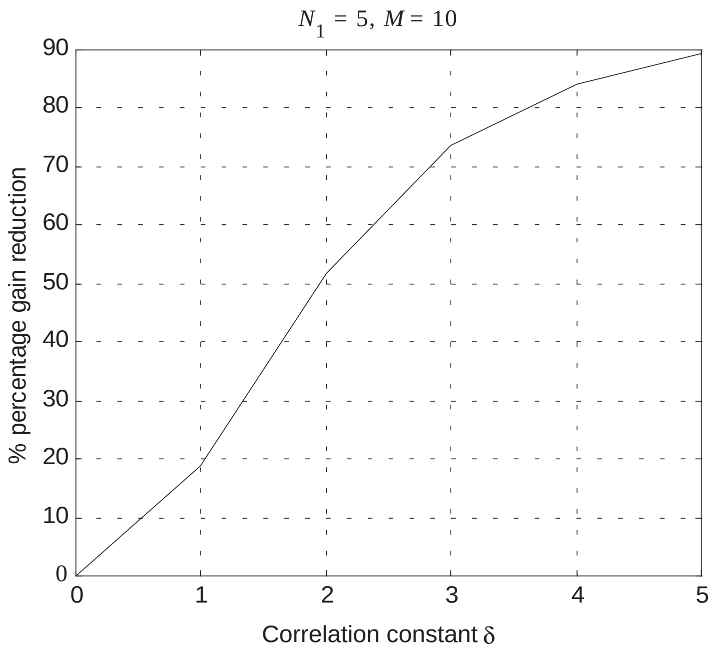

Figure 8. Percentage gain reduction due to tapering as a function of the correlation constant.

therefore the reduction in the maximum array gain due to tapering will be:

$$\Delta G = \sum_{i=1}^{M} N_i - \sum_{i=1}^{M} \alpha_i N_i \tag{16}$$

or

$$\Delta G = \sum_{i=1}^{M} (1 - \alpha_i) N_i \tag{17}$$

and it will be advantageous to express the amount of reduction as a percentage of the

$$\%\Delta G = \frac{\sum_{i=1}^{M} (1 - \alpha_i) N_i}{\sum_{i=1}^{M} N_i} \times 100\% \tag{18}$$

Figure 8 depicts the percentage gain reduction due to tapering as a function of the correlation constant δ where the reduction may reach

90% at $\delta = 5$, while there is no reduction for $\delta = 0$ (i.e., in the case of uniform array feeding).

The solution to gain reduction may be either normalizing the array gain to have unit gain in the mainlobe direction for the two cases or increasing the amplitude values of the tapering window by the ratio of the maximum gain for uniform feeding to that for tapered feeding. The second solution may be formulated in the weights of the tapering window as follows:

$$\alpha_m = \left(\frac{\sum_{i=1}^{M} N_i}{\sum_{i=1}^{M} N_i e^{-(i-1)^2\delta^2/2M^2}} \right) e^{-(m-1)^2\delta^2/2M^2} \qquad (19)$$

6. CONCLUSION

Uniform concentric circular arrays (UCCA) has considerable interest in various applications where two dimensional beamforming is possible. The array performance can be improved in terms of side lobe levels reduction by tapered beamforming where the array elements are weighted by a tapering window to reduce these levels. A novel tapering window is proposed and the array performance is discussed showing the reduction in side lobe levels. The tapering function has optimum values which are function of the array geometry such as the number of rings and the number of elements of the innermost ring. This beamforming technique is also found to be more efficient for arrays of larger number of rings where the reduction in sidelobe levels will be more than 40 dB at lower cost of increased beamwidth.

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