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# DESIGN, ANALYSIS AND OPTIMIZATION OF V-DIPOLE AND ITS THREE-ELEMENT YAGI-UDA ARRAY

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**Abstract**—This paper presents an optimum design technique of an asymmetric V-dipole antenna and it's a three-element Yagi-Uda array using Genetic Algorithm (GA). The optimization parameter for the V-dipole is the directivity and that for the Yagi-Uda array are the input impedance and directivity. The theoretical analysis has been done using a Moment-Method technique in a very simple step-by-step way, and subsequently the GA is applied for obtaining the optimized parameters. Comparative results are provided for 3-elements straight dipole Yagi and V-dipole Yagi array. Further, analysis for directivity with respect to included angle is given for the GA based optimization problem that gives an important aspect in the design of V-Yagi.

## 1. INTRODUCTION

The analysis of Straight dipole Yagi-Uda array [1] using Method of Moment (MoM) has been presented well before. Although the V-dipole antenna is one of the simplest antenna configurations and has been discussed for a long time [1, 2], yet closed form analytical design equations for an asymmetric V-dipole are not available to the best of our knowledge. The presence of three different physical

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parameters, viz. the two arm-lengths and the included angle, make it even more difficult. Most of the available literatures describe the design procedure for a symmetric V-dipole, and are designed for maximizing the directivity with respect to the included angle only. The work of J. H. Wang [3] only describes the shape optimization of arc V-dipoles to get maximum radiated power when the input is non-sinusoidal signal. J. C. Chiao [4, 5] in his work has explained radiation characteristic of microstrip version of V-dipole antenna and also its beam stirring nature. It was based on application rather than analysis of all the characteristics. So, this paper presents an approach of applying GA coupled with MoM for designing all the three parameters simultaneously to maximize directivity for designing asymmetric V-dipole and its Yagi array.

Genetic algorithms (GAs) [6–10] are a class of search techniques that use the mechanism of natural selection and genetics to conduct a global search of a solution space. The goal of the search is to find a good solution to the given problem. Other optimization techniques such as the *'gradient descent method'* [7], searches a solution space around the initial guess for the best local solution. For problems that have a small number of parameters, such processes perform quite well; but as the number of parameters and the solution space expands, the quality of the solution depends on the location of the initial guess. If the initial guess is poor, then the analysis reduces merely to finding the best solution among a set of poor solutions.

A considerable amount of interest has been given on optimizing the Yagi-Uda antenna since it was introduced in 1920s [1]. Because of the parasitic elements of this antenna, accurate modeling using closed form expressions is difficult. With the advent of computers, numerical methods are being used for the solution. In the design and synthesis of antennas, the goal is to find a radiating structure that meets a number of specified performance criteria, namely; gain, directivity, beam-width, input impedance, physical size, etc. For all but the simplest antenna structures, there are a large number of design variables that affect its performance, some of them drastically. Because of the coupling effect between various structures in an antenna, it is often difficult to provide a good initial guess for the design of the antenna parameters that provide expected performance. For such problems, the GA approach becomes attractive. Again when the antenna is optimized both for gain and impedance, the problem becomes harder than it is optimized for gain only. That is why GA is applied to optimize the V antenna and its array.

In this paper a Method of Moments (MoM) [1, 11, 12] analysis is done first on an asymmetric V-dipole antenna then the process is extended to get the complete analysis of V-dipole Yagi-Uda array. GA is used to optimize the elements lengths and spacing for maximum gain and impedance together.

Analysis of V-dipole is made considering two arms separately. Using MoM one can get two separate matrix equations for two arms and then these two are added as per the matrix addition rule to get the final expression. Finally, from these matrix equations current distribution and input impedance is calculated. As compared to straight dipole, the V-dipole is having three extra parameters that control the characteristics like current distribution, input impedance and radiation patterns. These parameters are included angle $\psi$, angle between upper arm length and lower arm length, $\theta_1$ and $\theta_2$, the angles made by the upper arm and lower arm with the positive vertical axis of the antenna respectively as shown in Fig. 1. For simplicity, in the analysis of the V-dipole, $\theta_1$ and $\theta_2$ are considered to be same. But $\psi$ has been changed to observe characteristic performance variations.

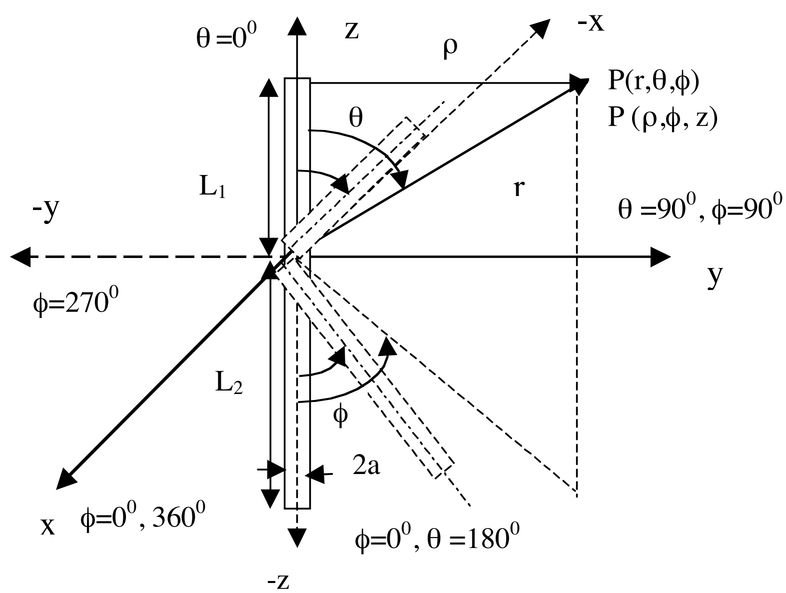


**Figure 1.** V-dipole structure with respect to straight dipole.

## 2. THEORY FOR V-ANTENNA

### 2.1. Center Fed V-dipole

The asymmetric V-dipole is considered asymmetric with its individual arm lengths lying between $0.70\lambda$ and $1.50\lambda$, and the included angle lying between 90 degrees and 120 degrees. Thus there were three design parameters: the two arm lengths of the two dipoles and the included angle.

The expression for vector-potential is given as follows:

$$A(r) = \int_V J(r)\{\exp(-jk|r-r'|)/(4\pi|r-r'|)\}dv' \tag{1}$$

From which the $H$ and $E$ field components can be found out as follows:

$$H(r) = (1/\mu)\text{curl}(A(r)) \tag{2}$$

and,

$$E(r) = (1/j\omega\mu)[k^2 \cdot A(r) + \nabla^2 A(r)] \tag{3}$$

Consider the center fed dipole of half length $L_1$ and radius '$a$' as shown in Fig. 1. Assumptions are made that the surface current $J_{s1}$ is the only current on the perfectly conducting dipole, and that $J_{s1}$ only has $\varphi_1$ and $z_1$ components in the cylindrical co-ordinate system. Let us assume $L_1 \gg a$, and $a \ll \lambda$. This is known as 'thin-wire approximation'.

Thin wire approximation allows us to assume that $\varphi$ component of $J_{s1}$ is small. Then

$$J_{s1} = Js(z_1')\mathbf{z_1} = I(z_1')/(2\pi a)\mathbf{z_1} \tag{4}$$

Using the boundary conditions for the $E$-field on the surface of the dipole leads to the following condition:

At $\rho_1 = a$, the condition for $E\varphi$ to be 0 is, $E_{z1} = -E_g \cos\theta_1$.

Substituting this expression into (3) results

$$\left(k^2 + \partial^2/\partial z_1^2\right)\int_0^{2\pi}\int_0^{L_1} \mathbf{z}\{I(z_1')/(2\pi a)\}\{\exp(-jkR)/(4\pi R)\}a dz_1' d\varphi$$
$$= j\omega\varepsilon_0 E_g \cos\theta_1 \tag{5}$$

This is called *Pocklington's integral equation.*

*2.1.1. Applied Field*

$$\begin{aligned} E_g &= V_g/b \quad \text{for}|z| < b/2 \\ &= 0 \qquad\quad \text{elsewhere} \end{aligned} \tag{6}$$

Let the gap-size approaches zero, i.e., $b \to 0$. Then

$$Eg = Vg\delta(z_1) \tag{7}$$

where, $\delta(z_1)$ is the *Dirac Delta Function.* This is called 'delta-gap feeding model' [1, 13].

*2.1.2. Hallen's Integral Equation*

Pocklington's integral equation may be written in a form that is more suitable for numerical computation. This version is called *'Hallen's Integral Equation'.* This is achieved by writing Equation (2) in the form:

$$(k^2 + \partial^2/\partial z_1^2)A(z_1) = -j\omega\varepsilon_0 V_g \delta(z)\cos\theta_1 \tag{8}$$

where $A(z_1) = \mu_0 \int\limits_0^{L_1} I(z_1')\{\exp(-jkR)/4\pi R\}dz_1'$.

Solving the homogeneous equation and applying the boundary conditions of continuous derivatives of $A(z_1)$ at $z_1 = 0$, and using the fact that the current is an even function of $z_1'$, ultimately Hallen's integral equation can be expressed in the form:

$$\int\limits_0^{L_1} I(z_1')G(z_1, z_1')dz_1' = -(j/2\eta)\sin k|z_1| + A_1 \cos kz_1 \tag{9}$$

where, $A_1$ is unknown and

$$H = 377\,\Omega$$

$$G(z_1, z_1') = (1/4\pi)\{\exp(-jkR_1)/R_1\}dz_1, \text{ Green's function [12, 14]} \tag{10}$$

$R_1 = [a^2 + (z_1 - z_1')^2]^{1/2}$, and both $z_1$ and $z_1'$ are constrained to $(0, L_1)$.

Equation (4) is the starting point for the method of moments [3, 4].

For the V dipole, the current is expanded in terms of pulse functions as:

$$\begin{aligned} I(z_1') &= \sum_{n=1}^{N} I_n = 1,\ (n-1)\Delta < z1 < n\Delta \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{11}$$

where, $\Delta$ is the segment length, i.e., $\Delta = z_n - z_{n-1}$.

Substituting (11) into (9) and evaluating at the points:

$Z_{1m} = (m - 0.5)\Delta$, $m = 1, 2, 3 \ldots N$, and invoking the condition that $I(L_1) = 0$ (to evaluate the unknown constant $A_1$) we get, for the upper half-arm :

$$\sum_{n=1}^{N-1} I(n) \sum_{m=1}^{N} G_{m,n} - \sum_{m=1}^{N-1} \cos kz_{1,m} I(n) = \sum_{m=1}^{N} (-j/2\eta) \cos\theta_1 \sin k|z_1, n| \tag{12}$$

Similarly, for the lower half arm, it can be written as

$$\sum_{n=1}^{N-1} I(n) \sum_{m=1}^{N} G_{m,n} - \sum_{m=1}^{n-1} \cos kz_{1,m} I(n) = \sum_{m=1}^{N} (-j/2\eta) \cos(\theta_1 + \psi) \sin k|z_1, n| \tag{13}$$

### *2.1.3. Input Impedance*

The input current is numerically equal to the input admittance. In the pulse formulation the current is approximated by $I(1)$ and

$$Z_{in} = 1/I(1) \tag{14}$$

### *2.1.4. Far-field Patterns*

The far field expressions for E field are:

$$\begin{aligned} \mathbf{E}(r, \theta, \varphi) &= (1/4\pi)[\{\exp(-jkR_1)/R_1\} F_1(\theta, \varphi) \\ &\quad + \{\exp(-jkR2)/R2\} F_2(\theta, \varphi) \end{aligned} \tag{15}$$

where

$$F_1(\theta, \varphi) = (jk\eta/4\pi) \int_0^{L_1} I(z_1') \exp(jkz_1' \cos\theta_1) dz_1' \tag{16}$$

and

$$F_2(\theta,\varphi) = (jk\eta/4\pi)\int\limits_0^{L_2} I(z_2')\exp(jkz_2'\cos\theta_2)dz_2' \tag{17}$$

where, $F_1(\theta,\varphi)$ and $F_2(\theta,\varphi)$ are called the far-field patterns for upper half and lower half respectively. Finding the current distribution by MoM, it is easy to calculate the far field patterns from Equations (16) and (17). Using simple basis functions as the pulse, integration may be carried out analytically.

$$F_1(\theta,\varphi) = (jk\eta/4\pi)\sum_{n=1}^{N} I(n)\exp(jkz_1')\cos\theta 1)dz_1' \tag{18}$$

$$F_2(\theta,\varphi) = (jk\eta/4\pi)\sum_{n=1}^{N} I(n)\exp(jkz_2')\cos\theta 2)dz_2' \tag{19}$$

Hence the radiated power,

$$W = \int\limits_{\theta=0}^{\pi}\int\limits_{\varphi=0}^{2\pi} \mathrm{P_r} r^2 \sin\theta d\varphi d\theta \tag{20}$$

Where

$$P_r = 1/2\left\lfloor |E_\theta|^2 + |E_\varphi|^2\right\rfloor \tag{21}$$

and the directivity,

$$D = (r^2 P_r)_{\max} \times 4\pi/W \tag{22}$$

### 2.2. The Three-element Yagi-Uda Array

Figure 2 shows the three elements V-dipole Yagi array in rectangular co-ordinate system. Considering the total electric field generated by an electric current source radiating in an unbounded free space, the analytical expressions for $N$ element V Yagi array is derived. The analysis proceeds along the same line as with the V-dipole.

The current on the $n$th element V-dipole Yagi-Uda is given by,

$$\ln(z') = \sum_{m=1}^{M} I_{nm}\cos\left[(2m-1)\frac{\pi(z'/\cos\theta_1)}{\lambda_n}\right] \tag{23}$$

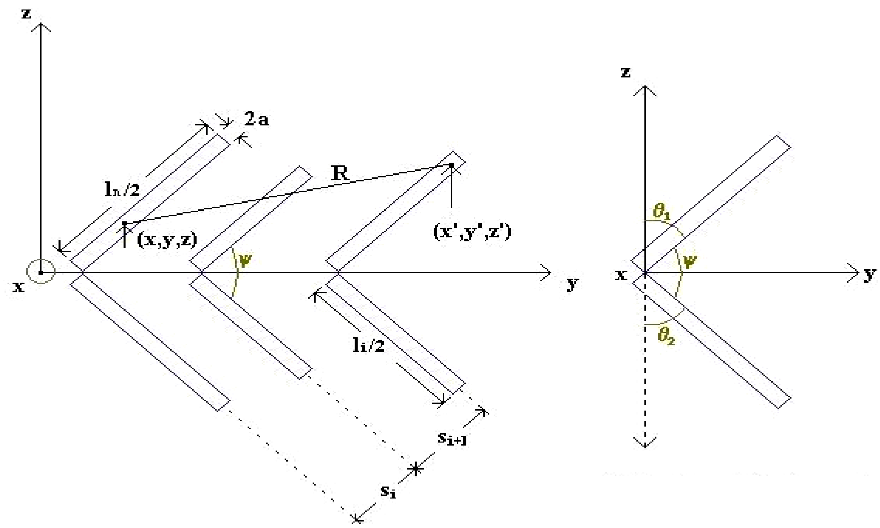


**Figure 2.** The three-element Yagi-Uda array.

For simplicity, the coupling between upper half and lower half of the same dipole has been neglected. The final equation with the relation between current at different segment and electric field for the V-dipole Yagi-Uda is given by (24),

$$\sum_{m=1}^{M} I_{nm}\left\{(-1)^{m+1}\frac{(2m-1)\pi}{\lambda_n\times\cos\theta_1}G_2(x,x',y,y'/z,\lambda_n/2)+\left[k^2-\frac{(2m-1)^2\pi^2}{\lambda_n^2\times\cos^2\theta_1}\right]\right.$$
$$\left.\times\int_0^{\lambda_n/2} G_2\left[x,x',y,y'/z,(z_n'/\cos\theta_1)\right]\cos\left[\frac{(2m-1)\pi(z_n'/\cos\theta_1)}{\lambda_n}\right]dz_n'\right\}$$
$$=j4\pi\omega\varepsilon_0 E_s^{\uparrow}\times\cos\theta_1 \tag{24}$$

where,

$$G_2(x,x',y,y'/z,z_n')=\frac{e^{-jkR_-}}{R_-}+\frac{e^{-jkr_+}}{R_+} \tag{25}$$

$$R_\pm=\left[(x-x')^2+(y-y')^2+(z\pm z')^2\right]^{1/2} \tag{26}$$

$N$ = total number of elements and $n = 1, 2, 3 \dots N$.
Here $(x, y, z)$ is the observation point and $(x', y', z')$ is the point on the antenna element. $R_\pm$ is the distance between $(x, y, z)$ and $(x', y', z')$ as shown in Fig. 2.

*2.2.1. Far Field Pattern for V-dipole Yagi Arrays*

The total field of the Yagi-Uda array is obtained by summing the contribution from each. For a V-dipole Yagi-uda array there will be two far-field electric components $E_\theta$ and $E_\varphi$, because of the orientation of the arms. The far field electric components are,

$$E_\theta = \sum_{n=1}^{N} E_{\theta n} = \frac{jk\eta e^{-jkr}}{4\pi r}\sin\theta\cos\theta_1 \sum_{n=1}^{N}\left\{e^{jk(x_n\sin\theta\cos\varphi+y_n\sin\theta\sin\varphi)} \times \sum_{m=1}^{M} I_{nm}\left[\frac{\sin(z^+)}{(z^+)}+\frac{\sin(z^-)}{(z^-)}\right]\right\} \quad (27)$$

$$E_\varphi = \sum_{n=1}^{N} E_{\varphi n} = \frac{jk\eta e^{-jkr}}{4\pi r}\cos\varphi\sin\theta_1 \sum_{n=1}^{N}\left\{e^{jk(x_n\sin\theta\cos\varphi+y_n\sin\theta\sin\varphi)} \times \sum_{m=1}^{M} I_{nm}\left[\frac{\sin(z^+)}{(z^+)}+\frac{\sin(z^-)}{(z^-)}\right]\right\} \quad (28)$$

where,

$$z^\pm = \left[\frac{(2m-1)\pi}{\lambda_n} \pm k\cos\theta\cos\theta_1\right]\frac{\lambda_n}{2}\times\cos\theta_1 \quad (29)$$

The process of simulation for the V-Yagi-Uda array was similar to that of the V-dipole antenna, only with added complexities.

## 3. GENETIC ALGORITHMS: IMPLEMENTATION IN ANTENNA DESIGN

We have applied GA to the optimization of two types of antennas. The first one is relatively simple, the asymmetric V-dipole antenna which has been designed to maximize its directivity. The second design is relatively complex, a three-element V Yagi-Uda array to optimize its directivity and to match the real part of its input impedance as close to 50 Ω as possible.

This is a very famous technique to conduct global search for the required parameters from a large solution domain by the process of natural selection of genetics. The process of GA used in this paper is similar to that used for its application to any other optimization problem [16–18]. Since it is a very well known method of optimization, the details are not given in this paper.

In the natural world, crossover and mutation takes place in a chromosome to form new chromosomes. These new chromosomes propagate new properties. Here also in the optimization problem the same phenomenon is simulated. Fig. 3 shows a typical process of crossover. Mutation means the random changing of a bit in the chromosome length from one to zero or from zero to one. This is done to maintain the global nature of the solution space and to maintain the diversity in future generations.

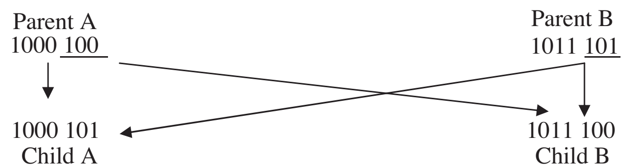


**Figure 3.** Process of crossover.

Two main characteristic features of the algorithm are:

### 3.1. Objective Function

The function, which is used in the program to find out the result, is known as the '*objective function*'. If this function does not give negative value at any time it can be treated as a fitness function of the GA. If it is not true, some separate fitness function is to be used. For this, the objective function should be scaled properly to create the proper fitness function.

### 3.2. Fitness Function

The goal of the design process is to develop an antenna that meets or exceeds some desired performance characteristics. The quality of a design is expressed mathematically by an objective function. The fitness function compares the present result obtained from the objective function with the chosen maximum value. If the difference lies below some pre-defined value, then the particular chromosome is chosen for the next operation. For example, the following can be a fitness function

$$F(x) = R(\max) - |O(x)| \tag{30}$$

Where, $R(\max)$ = maximum value of fitness function
$O(x)$ = value of objective function
$F(x)$ = fitness function

When $O(x)$ is maximum, $F(x)$ is minimum. However, $F(x)$ should be non-negative.

In our design, we have formulated the fitness function as

$$F(x) = A|P(1) - P(1)_{\text{opt}}| + B|P(2) - P(2)_{\text{opt}}| \tag{31}$$

Where $A$, $B$ are two positive constants, and $P(1)$ and $P(2)$ are two parameters to be optimized by global search, $P(1)_{\text{opt}}$ and $P(2)_{\text{opt}}$ being the two optimum values of the parameters. We rank the designed antennas according to the descending values of the fitness function, rather than the ascending values. Choice of the values of $A$ and $B$ depends on the relative importance to be assigned to the parameters being considered, and the matching of whose value with the optimum value is more desirable.

### 3.3. Typical Flow-Chart

The following is a typical flow chart (Fig. 4) of the step by step procedure adopted to optimize the antenna parameters:

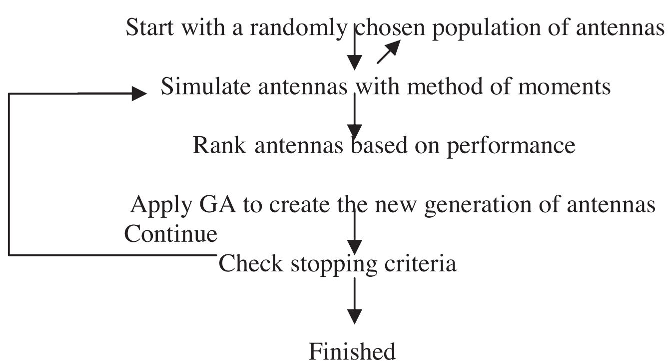


**Figure 4.** Flow chart for GA.

## 4. SIMULATION RESULTS

The analytical procedure was simulated in a **MATLAB** environment, using the method of moments. Two separate programs were written — one for the center-fed V-dipole and the other for the three-element V Yagi-Uda array.

**Table 1.** Optimized design parameters with respect to directivity of V-dipole.

| Serial No. | DMAX of 1st population in dB | DMAX of 2nd Population in dB | DMAX of Crossed population in dB | Input Impedance of Crossed Polpulation Zin Ω | $L_1/\lambda$ | $L_2/\lambda$ | ψ in degree |
|---|---|---|---|---|---|---|---|
| 1. | 3.6362 | 5.8437 | 6.1380 | 6.8403e1 - j 1.2089e4 | 1.4500 | 0.8125 | 96. 5625 |
| 2. | 4.8596 | 4.8603 | 5.4911 | 1.7651e3 - j 2.7279e4 | 1.3000 | 0.0075 | 60. 7031 |
| 3. | 3.9651 | 5.1820 | 5.2636 | 2.1081e3 - j 2.3446e4 | 0.9750 | 1.1875 | 62. 8125 |
| 4. | 4.8006 | 5.0763 | 6.4487 | 1.5790 e3 - j 4.2791e4 | 1.4125 | 0.7250 | 60. 9375 |
| 5. | 3.7999 | 5.4060 | 5.5439 | 1.4233e3 - j 1.4988e4 | 0.9125 | 1.3000 | 62. 8125 |
| 6. | 4.4777 | 4.4340 | 6.0433 | 5.9089e2 - j 2.5610e4 | 1.4650 | 0.7125 | 72. 8906 |
| 7. | 3.9497 | 4.2977 | 4.5350 | 1.2987e4 - j 4.2913e4 | 0.8375 | 1.2375 | 62. 3438 |
| 8. | 4.0132 | 3.9647 | 5.6763 | 1.3712e3 - j 1.7656e4 | 0.9500 | 1.2500 | 61. 4063 |

### 4.1. Results for the V-dipole

The length of each dipole was taken between $0.70\lambda$ and $1.50\lambda$ and the included angle was taken between 90 degrees and 120degrees for the search procedure. The radius of each arm of the dipole was chosen to be $0.0001\lambda$. During each simulation 20 members of a randomly generated population (1st population) were crossed with 20 other members of another randomly generated population (2nd population) using a relatively simple algorithm of symmetrically bisecting each gene and then combining the gene pieces from the two populations. The fitness-function was defined as:

$$F = |D(\text{opt}) - D_{\text{dB}}| \tag{32}$$

Where $D_{\text{dB}}$ is the directivity in dB and $D(\text{opt})$ is the optimum directivity chosen here to be 6.50 dB. Thus in case of V-dipole, we only consider the directivity as the parameter of interest.

### 4.2. Three-Element V-Yagi-Uda Array

The three element V Yagi-Uda array antenna was simulated in a MATLAB environment. Each arm of each dipole was divided into 101 sections. The search — space consists six parameters: the lengths of the three dipoles ($L_1$, $L_2$ and $L_3$), the distance of separations between the dipoles ($d_1$ and $d_2$), and the angle of inclination between the arms ($\psi$). The goal of the design is to simultaneously optimize two parameters: to maximize the directivity and to keep the real part of the input impedance as close to 50 Ω. The fitness-function was chosen

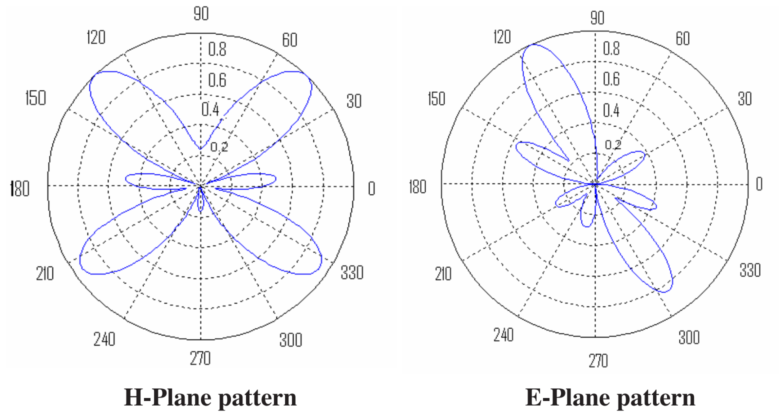


**Figure 5.** Field-patterns of a V-dipole ($L_1 = 1.4500\lambda$, $L_2 = 0.8125\lambda$ and $\psi = 96.6525$ degrees).

in the form

$$F = 0.1|\text{real}(Z_{in}) - 50| + Abs(D_{\text{opt}} - D) \tag{33}$$

Where $D$ is the directivity in dB and $D_{\text{opt}}$ was chosen to be 10 dB.

The length of the reflector dipole was varied between $0.49\lambda$ and $0.60\lambda$; that of the driven element was between $0.48\lambda$ and $0.50\lambda$ and for the director element it was between $0.30\lambda$ and $0.50\lambda$. The distance between the reflector and the driven element ($d_1$) and that between the director and the driven element ($d_2$) were both varied within the range $0.10\lambda$ and $0.40\lambda$. The included angle between the two arms of the dipole was varied between 90 degrees and 100 degrees.

From Tables 3 and 4, it can be observed that use of long V-antenna array can provide larger gain compared to straight dipole array where impedance matching is not at all a problem. The lengths, which have been taken here, are referred as long V-dipole cases. The real and imaginary parts of impedances are obtained as shown in the Table 3. For simplicity, the included angles for all the V-dipoles have been kept at 90° close to the optimized included angle obtained in Table 2. It is observed that for matching the input impedance, the gain of the antenna reduces. Impedance matching is achieved at the cost of gain as clear from Tables 2 and 3. From all these observations it is clear that few specific cases of V-dipole Yagi array are really useful and interesting to study so far as input impedance and directivity are concerned. Making the search space very narrow in the second phase of optimization for directivity, from Table 4, it is seen that directivity is improved much when the included angle is near about 80° that again

**Table 2.** GA based optimized design parameters considering both directivity and input impedance of 3-element V-Yagi array antenna.

| Serial No. | $D_{opt}$ for 1st pop. in dB | $D_{opt}$ for 2nd pop. in dB | $D_{opt}$ for Crossed pop. in dB | $Z_{in}\,\Omega$ for crossed population | $D_{max}$ in dB | $L_1/\lambda$ | $L_2/\lambda$ | $L_3/\lambda$ | $d_1/\lambda$ | $d_2/\lambda$ | $\psi$ in degree |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 4.4104 | 3.6097 | 3.4372 | 50.1260 + j 140.0310 | 6.5754 | 0.2630 | 0.2430 | 0.1688 | 0.1281 | 0.2266 | 94.8438 |
| 2. | 3.6338 | 3.6920 | 3.2207 | 30.1010 - j 259.6010 | 8.7693 | 0.2587 | 0.2402 | 0.2469 | 0.2500 | 0.1141 | 91.7188 |
| 3. | 4.2099 | 4.4376 | 3.4851 | 48.5642 + j 4.9702 | 6.6585 | 0.2579 | 0.2409 | 0.1563 | 0.2641 | 0.1609 | 99.2188 |
| 4. | 3.9711 | 3.6722 | 3.4463 | 48.2226 + j 63.3777 | 6.7315 | 0. 2845 | 0.2416 | 0.1641 | 0.1844 | 0.1516 | 92.8125 |
| 5. | 4.5803 | 3.9712 | 3.8983 | 44.2331 + j 79.7370 | 6.6784 | 0.2639 | 0.2463 | 0.1609 | 0.1958 | 0.1797 | 90.7813 |
| 6. | 4.4687 | 3.3193 | 2.6750 | 57.131 - j 320.3411 | 8.0381 | 0.2587 | 0.2402 | 0.2484 | 0.2500 | 0.1094 | 90.3125 |

**Table 3.** GA based optimized design parameter for long length 3 elements V-dipole Yagi array without considering impedance optimization [$\psi = 90°$ for all except the * cases].

| $L_1/\lambda$ | $L_2/\lambda$ | $L_3/\lambda$ | $d_1/\lambda$ | $d_2/\lambda$ | Zin in Ωs | D in dB |
|---|---|---|---|---|---|---|
| 0.6390 | 0.5704 | 1.4820 | 0.2781 | 0.1844 | 9.364+j290 | 12.0813 |
| 1.4176 | 0.7958 | 0.7454 | 0 .2641 | 0 .3578 | 2244+j498.5 | 13.3353 |
| 1.1690 | 0.7154 | 0.8890 | 0 .3625 | 0 .3203 | 1270+j1265 | 15.2241 |
| 1.3050 | 0.6188 | 0.9430 | 0.3109 | 0.1984 | 278+j869.01 | 16.7939 |
| 1.3182 | 0.5382 | 1.1946 | 0.3484 | 0.1891 | 138+j349.4 | 18.2475 |
| 0.4900 | 1.2946 | 1.1586 | 0.2781 | 0.3109 | 223.9–j494.1 | 19.1434 |
| 0.8312 | 1.4500 | 1.5626 | 0.3438 | 0.3063 | 151.6–j110.6 | 21.5739 |
| 1.3346 | 0.5382 | 0.9250 | 0.3250 | 0.3109 | 187.7+j365.8 | 21.6782 |
| *0.4562 | 1.5062 | 0.9812 | 0.1938 | 0.1938 | 166.3–j3.05 | 25.2206 |
| *0.4562 | 1.5062 | 0.9812 | 0.1938 | 0.1813 | 160.1–j3.461 | 27.1427 |

‘*’ - The included angle is taken as $80.2^0$

can have explanation from Figure 8. Directivity becomes very sensitive with respect to the distance $d_2$.

#### *4.2.1. Three Elements V-Yagi-Uda Array: Performance Evaluation in Respect of Included Angle*

The directivity and the real part of the input impedance for a three element Yagi-Uda array have been plotted against the included angle between the two arms of a dipole with the following parameters: $L_1 = 0.2630$, $L_2 = 0.2430$, $L_3 = 0.1688$, $d_1 = 0.1281$, $d_2 = 0.2266$.

**Table 4.** GA based optimized parameter for long length 3 elements straight dipole Yagi array without impedance optimization.

| $L_1$ in λ | $L_2$ in λ | $L_3$ in λ | $d_1$ in λ | $d_2$ in λ | Zin in Ωs | D in dB |
|---|---|---|---|---|---|---|
| 1.2939 | 1.2863 | 1.0328 | 0.3719 | 0.2828 | 96.7–j306.4 | 7.2129 |
| 1.2447 | 0.9287 | 0.7453 | 0.2078 | 0.1047 | 1062–j1279 | 8.4358 |
| 0.9166 | 0.5225 | 0.4937 | 0.1563 | 0.3438 | 145.8+j121.6 | 11.1757 |
| 1.2119 | 0.5550 | 0.6195 | 0.2500 | 0.2969 | 187.4+j123.8 | 12.9494 |
| 1.2283 | 1.5137 | 0.5297 | 0.2594 | 0.1750 | 114.1+j50.77 | 15.8085 |
| 0.7033 | 1.0425 | 0.4039 | 0.1563 | 0.3016 | 338.9–j853.4 | 15.2831 |
| 0.7033 | 1.0425 | 0.4039 | 0.1656 | 0.3250 | 326.2–j821 | 16.5503 |
| 0.7033 | 1.0425 | 0.4039 | 0.1563 | 0.3250 | 325.1–j835.6 | 16.8833 |
| 0.7197 | 1.0750 | 0.3500 | 0.1891 | 0.2828 | 294.7–j703.5 | 18.5867 |
| 0.7197 | 1.0750 | 0.3500 | 0.1844 | 0.3578 | 275.8–j693.4 | 20.3905 |

**Table 5.** Variation of directivity vs. included angle of a 3-element Yagi-Uda array antenna.

| Included Angle (ψ) | Directivity (in dB) | Re(Zin) (in Ω ) | Included Angle(ψ) | Directivity (in dB) | Re(Zin) (in Ω) |
|---|---|---|---|---|---|
| 60 | 6.6776 | 174.63 | 125 | 6.8864 | 54.51 |
| 65 | 6.7209 | 582.48 | 130 | 6.9067 | 55.12 |
| 70 | 6.5118 | 1783.20 | 135 | 6.9249 | 5.49 |
| 75 | 6.9795 | 15.639 | 140 | 6.9411 | 55.71 |
| 80 | 6.7846 | 130.02 | 145 | 6.9511 | 55.84 |
| 85 | 6.7638 | 72.37 | 150 | 6.9670 | 55.96 |
| 90 | 6.7452 | 57.31 | 155 | 6.9766 | 56.10 |
| 95 | 6.7434 | 56.23 | 160 | 6.9841 | 56.31 |
| 100 | 6.7663 | 50.43 | 165 | 6.9895 | 56.60 |
| 105 | 6.7909 | 49.92 | 170 | 6.9928 | 56.99 |
| 110 | 6.8160 | 51.00 | 175 | 6.9940 | 57.48 |
| 115 | 6.8407 | 52.38 | 180 | 6.9931 | 58.09 |
| 120 | 6.8643 | 53.60 | | | |

## 5. DISCUSSIONS

The use of Genetic Algorithms in the design of V-dipole arrays can lead to better designs where the directivity might improve by as much as 30% over the parent population.

The procedure brings into view the sensitivity of directivity on the included angle ($\psi$), which otherwise is very difficult to visualize. The procedure described in this paper was also extended to the optimum design with respect to directivity and the real part of the input impedance (e.g., 50 ohm) of a 3-element V-dipole. The V-dipole

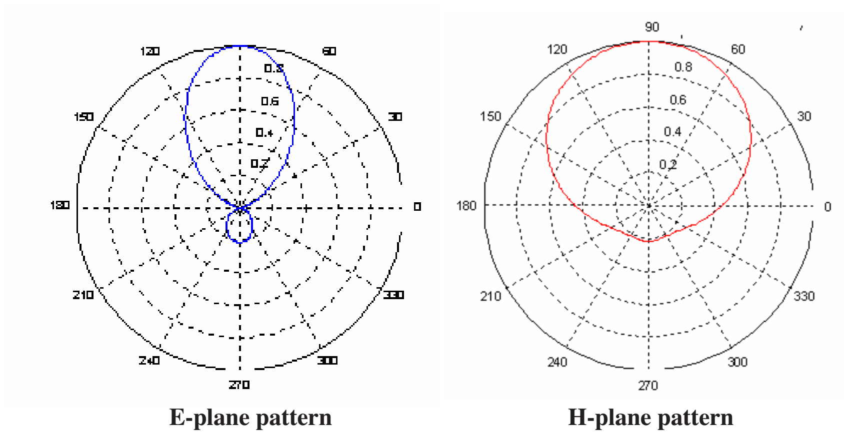


**Figure 6.** Field-patterns of a 3-element V-Yagi-Uda array antenna (Parameters same as sl.no.2 of Table 2).

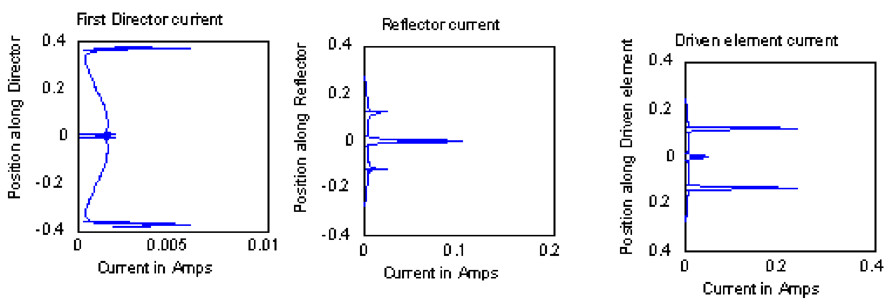


**Figure 7.** Variation of currents in the different elements of a 3-element V-Yagi-Uda array antenna (Parameters same as that of sl.no.2 of Table 2).

antenna is an effective option where space constraint limits the use of straight dipoles, which might not always offer sufficient directivity. Another advantage with V-dipoles is that it offers an extra design parameter viz. the included angle, which offers larger number of degrees of freedom in the design and thus makes it possible to design more directive antennae. The Yagi-Uda array, where the six parameters constituted the search space was the three element lengths, the two inter-separations and the included angle. From the plots it is clear that although the directivity does not vary too much with the included angle for such a simple form of Yagi- Uda array, yet the input impedance varies tremendously with the included angle in the order of $10^3$. This

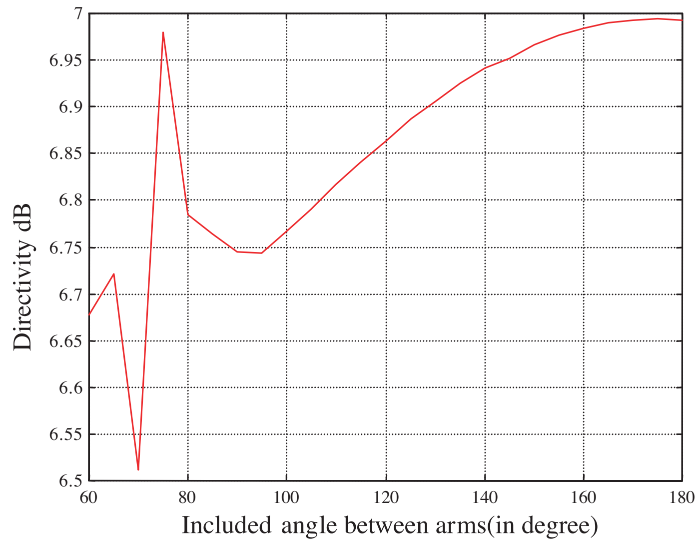


**Figure 8.** Variation of directivity vs. included angle in a 3-element V-Yagi-Uda array.

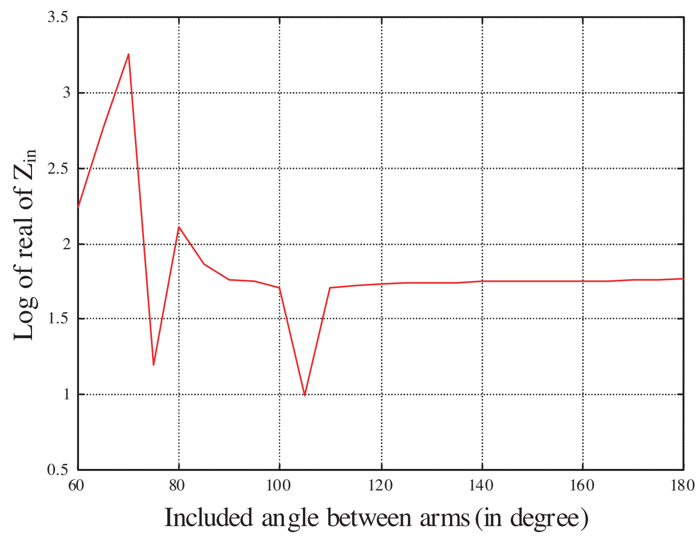


**Figure 9.** Variation of log (Real (Zin)) vs. included angle in a 3-element V-Yagi-Uda array.

effect has been given due consideration in the optimization process. Although we see that the directivity is reasonable at an angle of about 75 degrees and also increases with the included angle after $\psi$ exceeds 100 degrees, yet we have to keep in mind the large variation of the antenna resistance before choosing the final design. Otherwise, the large impedance mismatch with the feeding line will render the design ineffective. Another interesting feature is that more we try to increase the directivity the worse is the matching. Thus the increase in directivity comes at the cost of poorer impedance matching.

As application areas are concerned, it can be used to propagate the electromagnetic signal to longer distances using the directional nature for long distance direction finding of the object.

## 6. CONCLUSION

From all these observations and discussion it is clear that a few specific cases of V-dipole Yagi-Uda array are really useful so far as input impedance matching and directivity are concerned. As have been shown by Jones and Joines [10], by increasing the number of elements of the array to 15, we may obtain directivity close to 15.4 dB and the impedance matching also was quite good (50.01–j.05 $\Omega$). Compared with straight dipole arrays, these designs have the advantage of being able to be operated in situations of space constraint. Besides, some particular designs may provide much improved directivity and impedance matching when compared with the straight dipole arrays of the same dimensions. The V-dipole antenna is an effective option where space constraint limits the use of straight dipoles, which might not always offer sufficient directivity. Another advantage with V-dipoles is that it offers an extra design parameter viz. the included angle, which offers larger number of degrees of freedom in the design and thus makes it possible to design more directive antennae.

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