

**A COMPREHENSIVE PERFORMANCE STUDY OF  
CIRCULAR AND HEXAGONAL ARRAY GEOMETRIES  
IN THE LMS ALGORITHM FOR SMART ANTENNA  
APPLICATIONS**

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**Abstract**—Space division multiple access (SDMA) is a promising candidate for improving channel capacity in future wireless communication systems. Considering that discrimination performance of the user in the spatial domain depends on the array arrangement, and as a result the optimum element arrangement for SDMA should be defined, beamforming play a very important role providing fundamental theory of design procedure. However the pattern of antenna array is determined by array geometry. Two-dimensional (2-D) spatial filters that can be implemented by microstrip technology are capable of filtering the received signal in the angular domain as well as the frequency domain. This paper focuses on various geometries of eight and nine elements antenna arrays using circularly patch elements as well as hexagonal array with seven elements. The network throughput is further analyzed to determine if using a fully adaptive pattern (LMS algorithm generated pattern) results in a higher throughput with or without presence of mutual coupling effects.

## 1. INTRODUCTION

Nowadays, one of the most recent innovations to overcome the problem of increasing demand for capacity is to deploy smart antennas for wireless communications [1]. Narrower main-beam and more nulls in the pattern can resolve the Signals-Of-Interest (SOI) more accurately and allow the smart antenna system to reject more Signals-Not-Of-Interest (SNOI). Smart antenna system design is subdivided into three major areas: antenna design, adaptive beamforming algorithm and adaptive estimation of direction of arrival (DoA).

Studies on array geometries have shown that its pattern is influenced by the position and structure of antenna array. The array geometry includes mainly uniform linear array, uniform rectangular array and circular array. Linear array has excellent directivity and it can form the least main-lobe in a given direction, but it is difficult to be kept consistent in large range and do not treat equally all azimuths [2]. This paper mainly studies circular arrays and multi-ring arrays. Circular array is high side-lobe geometry so the distance of array antenna should be small and the basic symmetry of circular arrays offers a great ability to compensate for the effects of mutual coupling [3]. For ignoring high sidelobe levels multi ring arrays are utilized, moreover they have some other advantages. Furthermore a hexagonal array is presented for smart antenna applications and the results of designs are compared with each other. The optimum criterion adopted by this paper is that the geometry must make the main-lobe minimum in the case that the minimum side-lobe is less than half-power.

In adaptive beamforming the goal is to adapt the beam by adjusting the amplitude and phase (i.e., complex weight) on each antenna element such that a desirable pattern is formed. A possible scenario in smart antenna systems is to use first a DoA algorithm to resolve the angles of arrival of all signals, separate SOI and SNOI, and then use the beamformer to direct the maximum radiation of the antenna pattern toward the SOI while placing nulls toward the SNOIs. In general, there is a tradeoff between rate of adaptation and complexity among these adaptive algorithms. Therefore, in order to guide the design of beamforming algorithms for high network capacity, the network throughput is evaluated for various training packet lengths. One of the simplest algorithms and most commonly used to adapt the amplitude and phase excitation of the antenna elements is the least mean square (LMS) algorithm. The LMS algorithm updates the complex weights of the antenna elements at each iteration in order to minimize the mean square error between the received signal and a training sequence which is a priori known at the

receiver.

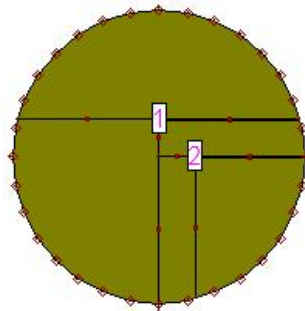
This paper analyzes the performance of array pattern based on the eight and nine elements circular arrays with one main-lobe and studies impact of array structure and radius on the pattern of the antenna array. The elements are circular microstrip patch antenna and they are designed to cover both 1.85 and 2 GHz. These ranges of frequencies are desirable in modern wireless communications. Furthermore, we provide a comprehensive analysis of the performance of the LMS adaptive beamformers by considering the effects of parameters such as number of beamformer elements and their spacing. Other parameters investigated include taking into account the mutual coupling effects on array radiation pattern with different directional of arrival of incident signals. The performance study is proven to be essential for an optimal design of smart antenna systems utilizing LMS beamformers.

## 2. CIRCULAR PATCH ANTENNA DESIGN FOR WIRELESS COMMUNICATION

In this section, a circular patch antenna used in wireless communications is characterized in detail. We consider design formulas of the circular patch antenna studied in [4, 5] and list the antenna parameters. The antenna geometry is shown in Figure 1. The antenna parameters are listed below: (in millimeters)

$$a = 30, \quad h = 1.27, \quad (X_{f1}, Y_{f1}) = (7.3, 0), \quad (X_{f2}, Y_{f2}) = (0, 7.3)$$

where  $a$  is radius of the antenna,  $h$  is the substrate thickness in mm and the dielectric constant (permittivity) is:  $\epsilon_r = 2.33$ . Subsequently, the antenna is fed by a coaxial probe with the feed location at:  $(X_{f1}, Y_{f1})$ . However, a tow-probe-feed circular patch has been designed to make an antenna array with circular polarization. The feed locations



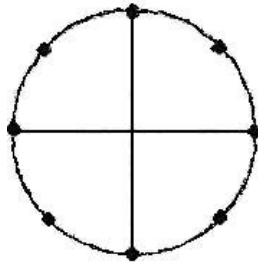
**Figure 1.** Array element.

for the circular elements of all arrays are:  $(X_{f1}, Y_{f1})$  and  $(X_{f2}, Y_{f2})$ . The  $S_{11}$  is calculated by HP-HFSS software and measured on an HP-8510 network analyzer. Some experimental results prove the validity of this design.

### 3. ARRAY GEOMETRIES

Figure 2 shows the eight elements circular array with radius  $a$  as the array elements uniformly distributed in the circumference of circular antenna array. The angular position of the  $n$ th element on the array is given by:

$$\Phi_n = 2\pi \left( \frac{n}{N} \right), \quad n = 1, 2, 3, \dots, N \quad (1)$$



**Figure 2.** Eight elements circular array geometry.

Assuming that the wave front passes through the origin at the time instant  $t = 0$ , it impinges on the  $n$ th element of the array at the time:

$$\tau_n = -\frac{a}{c} \sin \theta \cos (\Phi - \Phi_n), \quad n = 1, 2, 3, \dots, N \quad (2)$$

where  $c$  is the propagation speed of light in free space. Assuming that  $\theta$  is the angle between incident direction and the position vector of the first element of the antenna array. The phase deference  $\varphi$  between the array element and the center of circle is:

$$\varphi = \frac{2\pi a}{\lambda} \cos \left( \frac{2\pi}{N} n - \theta \right) \quad (3)$$

where:  $a$  is radius of array,  $N$  is the sum of array elements and  $\lambda$  is wavelength of signal.

### 3.1. The Eight and Nine Elements Circular Arrays

Figure 3 shows the pattern of the eight elements circular array, with MML as maximum mainlobe width and MMS as minimum side-lobe width. When  $a = 0.5\lambda$ ,  $MML = 40^\circ$  and the maximum side-lobe corresponds to the angle  $\alpha = 45^\circ$ , and the power of maximum side-lobe is just less than halfpower. Then the performance of the antenna array is optimum. From Figure 4, when  $a = 0.7\lambda$ ,  $MML = 30^\circ$ , with increasing of the circle radius, the value of MML decreases simultaneously and the number of side-lobe increases. The signal out of main-lobe cannot be efficiently restrained. Therefore the width of minimum main-lobe is  $30^\circ$  in the eight elements circular array. But the performance is not ideal. In order to improve the performance characteristic of the antenna array, the multi-layer circular array is

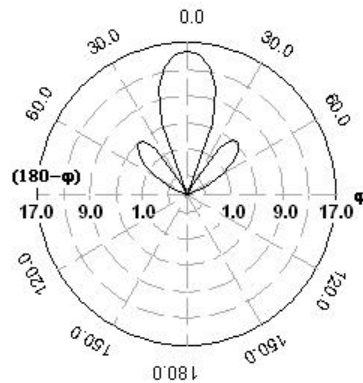


Figure 3. Eight elements array pattern.

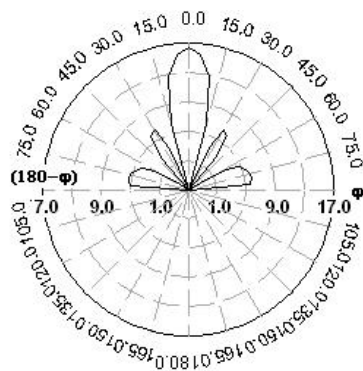
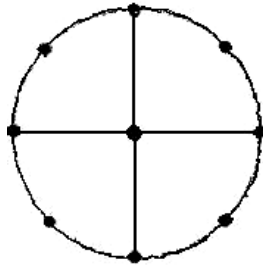


Figure 4. Eight elements array patterns.

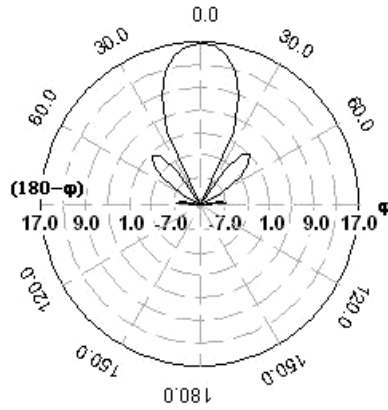
used. The pattern function of concentric circular array is:

$$F(\theta) = \left| \frac{1}{M \times N} \left( 1 + \sum_{i=0}^{N-1} e^{j \frac{2\pi R_1}{\lambda} [\cos(\frac{\pi}{N} i - \theta_m) - \cos(\frac{\pi}{N} i - \theta)]} + \dots + \sum_{i=0}^{N-1} e^{j \frac{2\pi R_m}{\lambda} [\cos(\frac{\pi}{N} i - \theta_m) - \cos(\frac{\pi}{N} i - \theta)]} \right) \right|^2 \quad (4)$$

Figure 5 shows the nine elements circular array with radius  $a = 0.6\lambda$ . In this case  $MML = 50^\circ$  and the maximum side-lobe corresponds to the angle  $\alpha = 45$ , but they are considerably weak. Figures 6, 7 show the patterns of the nine elements circular array. When  $a = 0.8\lambda$ ,  $MW = 30^\circ$ , the number of side-lobes are increased but they are still weak.



**Figure 5.** Nine elements circular array.



**Figure 6.** Nine elements array pattern.

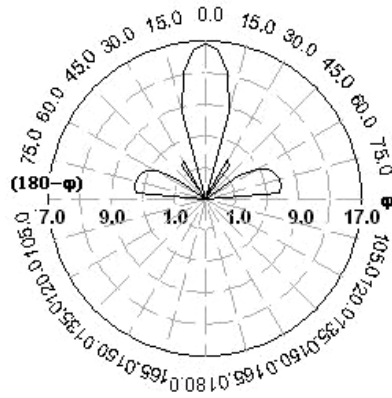


Figure 7. Nine elements array pattern.

### 3.2. 4+4 Circular Array

Figure 8 illustrates the structure of 4+4 two-ring array. This multi-ring array consists of two circular arrays uniformly distributed in the circumference of antenna array. The included angle of the first element of two rings connected with the center of circle is  $\beta$ . During the searching, the outer ring is fixed and the location and direction of the inner ring with respect to outer ring is adjusted. It assumed that the radius of outside circle is  $a_1$  and the radius of inside circle is  $a_2$ . Figure 9 illustrates the pattern when  $a_1 = 0.9\lambda$ ,  $a_2 = 0.45\lambda$ ,  $\beta = 45^\circ$  main-lobe is large. Figure 10 shows the best performance when  $\beta = 45^\circ$ ,  $a_1 = \lambda$ ,  $a_2 = 0.7\lambda$ .

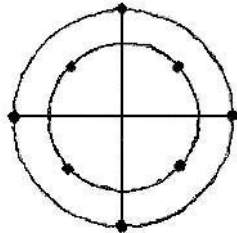


Figure 8. 4 + 4 two ring circular array.

From the analysis of the classic theory, the power of side-lobe can be reduced 20 dB or more by using two concentric circle arrays. From computer simulation, the geometry of antenna array can improve the pattern performance of antenna array, as the total elements of antenna array are fixed.

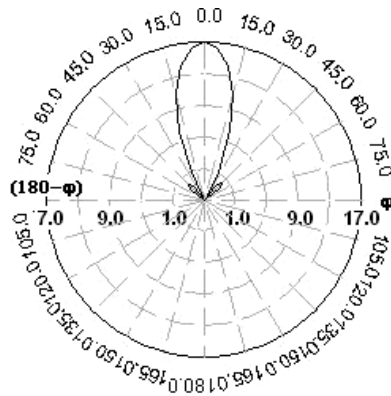


Figure 9. 4 + 4 two-ring array pattern.

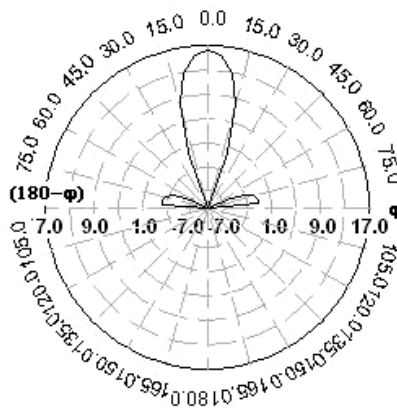


Figure 10. 4 + 4 two ring array pattern.

### 3.3. 1+3+4 Circular Array

The antenna geometry is shown in Figure 11. In this array  $a_1$ ,  $a_2$  and  $\beta$  can be adjusted. Searching under the optimum criterion by increasing  $a$ , it shows the best performance when  $\beta = 45^\circ$ ,  $a_1 = 1.3\lambda$ ,  $a_2 = \lambda$  and side-lobes are just less than half-power and they are wide. In this case  $MML = 24^\circ$ . Best performance is shown in Figure 12.

1 + 8 and 1 + 7 circular array geometries have been designed and investigated, but their patterns show just slight differences from previously presented geometries.



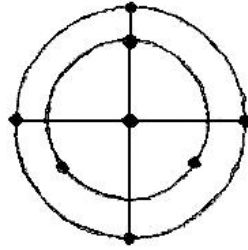


Figure 11. 1 + 3 + 4 elements circular array.

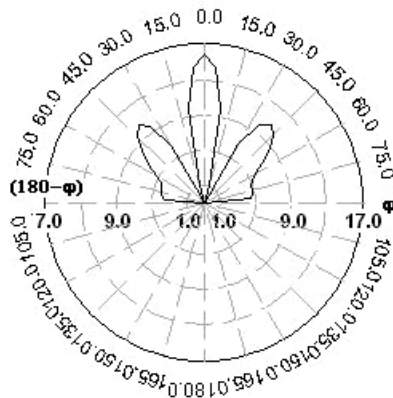
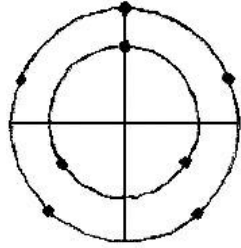


Figure 12. Array pattern.

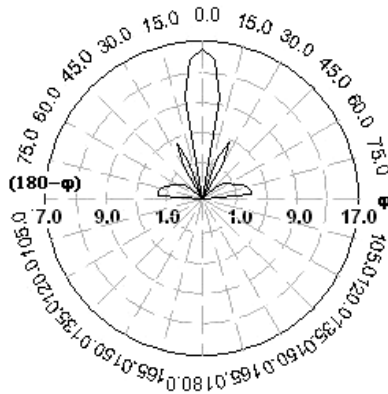
### 3.4. 3+5 Circular Array

The outer circular has five antenna elements and the number of array elements on the inner ring is three. Figure 13 shows the structure, with the radius of the outer ring as  $a_1$  and the radius of the inner ring as  $a_2$ . The outer ring is fixed. Figure 14 indicates the pattern of array when  $\beta = 45^\circ$ ,  $a_1 = 1.3\lambda$ ,  $a_2 = \lambda$ , and side-lobes are also much stronger. Searching under the optimum criterion, it shows the best performance when and side-lobe less than half-power.

From comparing Figure 14 with other array patterns that were illustrated before, we can observe the later design has the narrowest main-lobe about  $20^\circ$  degrees and four side-lobes which are excessive for a good design. By comprising some arrays, it is obvious when the numbers of elements of outer ring are odd the performance is better than the case of even elements, and the conclusion can be generalized through simulation by computer.



**Figure 13.** 3 + 5 elements circular array geometry.



**Figure 14.** Array pattern.

### 3.5. Hexagonal Array

Unfortunately Circle array is high side-lobe geometry so the distance of array antenna should be small [3]. To overcome this problem we consider utilizing a new geometry for smart antenna applications. This geometry concept with seven patch elements in a hexagonal arrangement. With both of these structures (circular and hexagonal) it is possible to configure the radiation pattern to obtain directional patterns with high gain and directivity. However narrower main-beam and lower level side-lobes in the antenna pattern can resolve the Signals-Of-Interest more accurately. This structure has two side-lobes with lower power level but the previous design has four side-lobes with low power. Both of these geometries have the same main-beams.

The element geometry is shown in Figure 1 and Figure 15 indicates the structure of hexagonal antenna array. This structure consists of six circular patches non-uniformly distributed in the circumference of antenna array and one in the center. It assumed that the distance

between outer elements and the center element is  $a_1$ . Figure 16 indicates the pattern when  $a_1 = \lambda$ . It shows the best performance for this structure.

#### 4. LMS ALGORITHM

Research on adaptive beamforming algorithms has been carried out and various algorithms including blind and non-blind algorithms have been investigated. Most conventional beamforming algorithms require the DoA's to be determined first by using prior knowledge of the array manifold. As a result, their performance is strongly dependent on reliability of knowledge of the manifold. In many applications such as digital mobile communications, the array manifold is poorly defined because of a highly variable propagation environment [6]. An alternative way for array beamforming that does not require DoA's

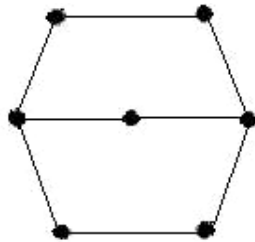


Figure 15. Hexagonal array with one element in the center.

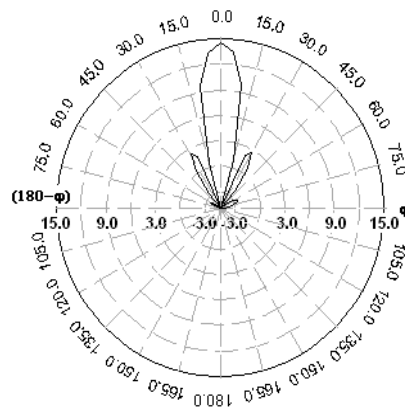


Figure 16. Pattern of the hexagonal array.

is to use training sequences. Nonetheless, this approach results in bandwidth inefficiency. Recently, different blind algorithms that exploit the temporal rather than the spatial structure of signals have been developed. Blind methods therefore do not require training signals for the demodulation process. Hence, bandwidth efficiency can be improved.

Beamforming is the process by which the information, obtained from the signals incident on an antenna array, are used to generate an optimum pattern which maximizes the radiated power towards the intended users and minimizes it, in the form of radiation nulls, towards interferers. Estimating the direction of the desired user is achieved by using high resolution direction finding techniques [6, 7]. By knowing the direction of the desired signal, a set of weights can be adjusted to optimize the radiation pattern. The application of the LMS algorithm to estimate the optimum weights of an antenna array is widespread and its study has been of considerable interest [8].

The main requirement of the adaptive filtering process is the existence of a reference signal. This signal can be a spatial reference, if DoA are known or can be estimated. This technique does not perform well when the number of users in the systems is large compared to the number of antenna elements, because DoA estimation methods generate ambiguous results [9, 10]. The other possible reference signal is a known temporal signal, unique for each user and correlated with other's users reference signals. In GSM, for example, this signal is a known training sequence for each user. In the case of W-CDMA, in both forward and reverse links, a known sequence of pilot bits is transmitted. These pilot bits can be the same for several users, but can be distinguished through the spreading and scrambling codes.

## 5. RESULTS AND DISCUSSION

In this section the effect of mutual coupling dependence to  $\lambda$  on the performance of LMS Algorithm is also investigated. The effect of mutual coupling between array elements on the performance of smart antenna systems has been subject of study for more than twenty years.

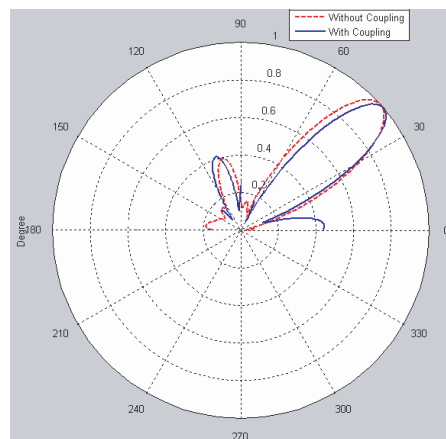
This effect has been investigated mainly in two areas: beamforming and direction of arrival estimation [1, 2]. In what follows we compare two different cases. In the first case no mutual coupling is presented on the input signal. In the second case this effect has been taken to account on the input signal. The LMS algorithm is used to evaluate the effect of mutual coupling over designed array.

Consider the hexagonal planar array with seven circular patch elements introduced in this paper. The element radial spacing between

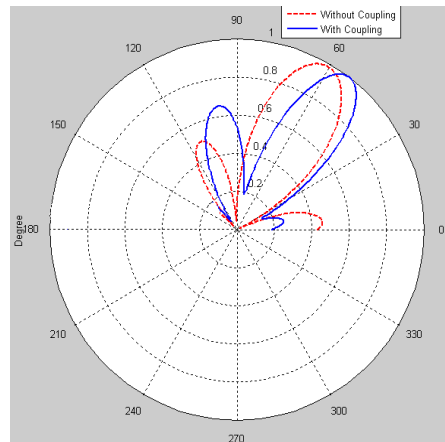
the circular patches is  $\lambda$ , while the signal arrival directional is  $45^\circ$ . Figure 17 (all figures illustrated in this subsection have been depicted with no Logarithmic scale) shows the array performance in two case mentioned above, the former case shows radiation patterns in solid blue lines, while the later case shows radiation patterns under the influence of mutual coupling in dashed red line. We can observe mutual coupling has faint effects on the radiation pattern. Clearly in this design, mutual coupling doesn't stir away the main-beam and side-lobes pattern; furthermore sidelobe level changes are dispensable. From Figure 18 when the element radial spacing is reduced to  $0.7\lambda$ , when the array geometry is not optimized, we can observe mutual coupling has some unpleasant effects on array performance. Main-beam slipped away around  $6^\circ$  while the correct angle of arrival is  $60^\circ$ , side-lobe level is increased about 20% and stir away near  $10^\circ$ .

The same procedure is used for investigating the mutual coupling effect for  $3 + 5$  circular array with the particular geometry specified in this work. With this method we could partially observe the effect of mutual coupling on LMS algorithm. The errors are sketched for two element spacing.

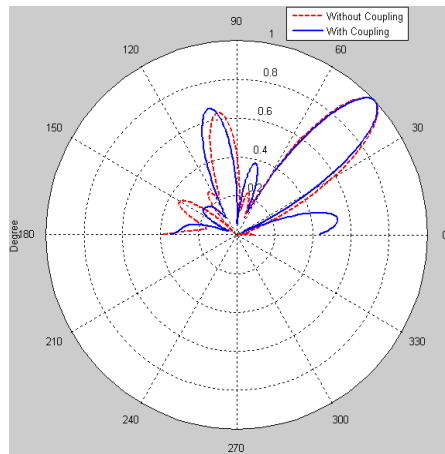
The first case of  $\beta = 45^\circ$ ,  $a_1 = 1.3\lambda$ ,  $a_2 = \lambda$ , produce a radiation pattern illustrated in Figure 14 while the signal directional of arrival is  $45^\circ$ . As it is observed from Figure 19 the mutual coupling between array elements reaches its minimum for this case and it doesn't stir away the main-beam pattern. However the side-lobe is deviated near  $5^\circ$  from its original direction. Figure 20 indicates the pattern of array



**Figure 17.** Hexagonal array radiation pattern with and without mutual coupling effect on the LMS.

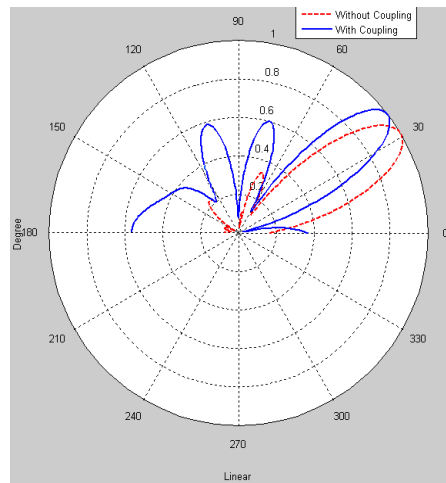


**Figure 18.** Effect of mutual coupling presence on the LMS beamformer. (hexagonal array).



**Figure 19.** 3 + 5 array radiation pattern with and without mutual coupling effect on the LMS.

with and without mutual coupling effect when  $\beta = 45^\circ$ ,  $a_1 = 0.9\lambda$ ,  $a_2 = 0.6\lambda$ , while the incident wave directional of arrival is  $30^\circ$ . This spacing reduction has the following undesirable effects on radiation pattern. Main-beam stirs away about  $10^\circ$  and the side-lobes level changes are Considerable; they are increased about 100%. In other words the mutual coupling at this elevation angle has smaller non-diagonal terms compared with hexagonal array previously designed.



**Figure 20.** Effect of mutual coupling presence on the LMS beamformer. (3 + 5 circular array).

## 6. CONCLUSIONS

It is very important to search for the best pattern based on a fixed number of elements, by changing the geometry of antenna array. In this paper some structures of eight elements antenna arrays and it concluded that: 1-The radius of circle array mainly influences main-lobe of pattern; Larger radius makes narrower main-lobes and vice versa. 2-In order to keep performance consistent in total range of 360, the elements should be distributed uniformly around the ring. 3-In circle array, the center element is important to improve the performance of pattern. Especially, as the outer elements are odd, but it can not be consider as a general role.

A hexagonal antenna array had been also presented; in this case, simulations show the array performance is more desirable than the circular two rings arrays.

The arrays discussed by this paper are regular. In fact, the operation precision is influenced by a lot of factors such as gross of operation or so, but the conclusion of this paper is general. For multi-ring array, increasing the number of rings might improve performance of pattern. However the numbers of rings are restricted on a given antenna. Most of the time increasing ring number needs more elements and thus is more expensive. In fact, the optimum performance of pattern can be obtained by adjusting the number of rings.

In this paper we apply the exact modeling for analyzing the effect

of mutual coupling on a digital beamformer algorithm using the LMS algorithm. An effective design for eliminating the effect of mutual coupling is also proposed and examined. In Section 5 the effect of mutual coupling on the LMS algorithm is considered thus the ability of eradicating the effect of mutual coupling by the presented arrays has been also investigated.

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