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WIDEBAND OR MULTIBAND COMPLEX IMPEDANCE MATCHING USING MICROSTRIP NONUNIFORM TRANSMISSION LINES

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Abstract—A novel method is introduced to synthesize microstrip Nonuniform Transmission Lines (NTLs) for matching between two arbitrary complex frequency dependent impedances in a wideband or multi-band frequency range. First, strip width or the characteristic impedance function of the microstrip NTL is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method is verified using some examples.

1. INTRODUCTION

Impedance matching is a very important concept in RF and Microwave engineering. There is a significant interest to design matchers for efficient matching between two real or complex frequency dependent impedances in a wideband or multi-band frequency range. For example, to design wideband amplifiers we need to transform the impedances of a region of the Smith chart to another region [1, 2]. Using uniform transmission lines and stubs is the most straightforward approach for matching between two complex impedances but in a narrow frequency band [3, 4]. Also, using multi-section quarter-wave or tapered transmission lines are the most straightforward methods for matching in a wide frequency band [3, 4]. However, the synthesis of tapered transmission lines is usually introduced only for matching between two real and constant impedances and only for a highpass frequency band. In this paper, we introduce a novel method to synthesize microstrip Nonuniform Transmission Lines (NTLs, general case of tapered TLs) for matching between two complex frequency dependent impedances in a wideband or multi-band frequency range.

First, strip width or the characteristic impedance function of the microstrip NTL is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. Utilizing truncated Fourier series expansion does not create any discontinuity in the resulted NTL. The usefulness of the proposed method is verified using some examples.

2. ANALYSIS OF NTLS

In this section the analysis of NTLs is reviewed. Figure 1 depicts the longitudinal view and the cross section of a microstrip NTL, utilized as an impedance matcher between two complex and frequency dependent impedances $Z_L(\omega)$ and $Z_S(\omega)$. The width of strip is $w(z)$ and also the relative electric permittivity and the thickness of the substrate are ε_r and h, respectively. The characteristic impedance of the NTL will be a nonuniform function $Z_c(z)$.

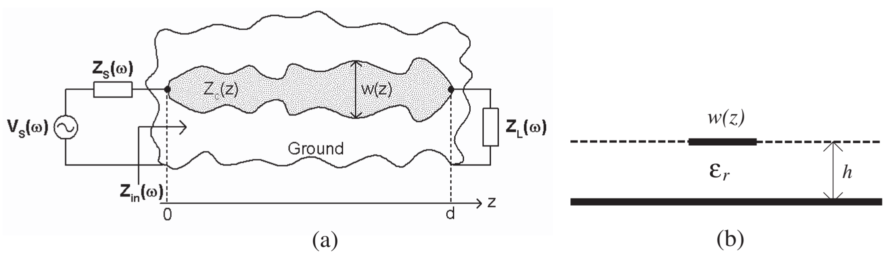

Figure 1. (a) The longitudinal view of a microstrip NTL as a matcher (b) The cross section of the matcher at point z.

There are some methods to analyze the NTLs such as finite difference [5], Taylor's series expansion [6] and Fourier series expansion [7]. Of course, the most straightforward method is subdividing NTLs into K uniform electrically short segments with length

$$\Delta z = d/K \ll \lambda_{\min} = \frac{c}{f_{\max}\sqrt{\varepsilon_r}} \tag{1}$$

in which c is the velocity of the light and $f_{\max}$ is the maximum frequency of the analysis. Then the $ABCD$ parameters of the whole of NTL is obtained from those of the segments as follows

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{2}$$

where the $ABCD$ parameters of the k-th segment are as follows

$$A_k = D_k = \cos(2\pi\Delta\theta) \tag{3}$$
$$B_k = Z_c^2((k-0.5)\Delta z)\, C_k = jZ_c((k-0.5)\Delta z)\sin(2\pi\Delta\theta) \tag{4}$$

where

$$\Delta\theta = 2\pi\frac{\Delta z}{\lambda} = \frac{2\pi}{c}\Delta z\sqrt{\varepsilon_{re}}f \tag{5}$$

is the electrical length of each segment. In (5), ε_{re} is the effective relative electric permittivity of the middle of the k-th segment. Finally, the input impedance of the matcher is determined as follows

$$Z_{in}(\omega) = \frac{AZ_L(\omega)+B}{CZ_L(\omega)+D} \tag{6}$$

3. SYNTHESIS OF IMPEDANCE MATCHERS

In this section a general method is proposed to design optimally the impedance matchers. Firstly, we consider one of the following truncated Fourier series expansion for the normalized characteristic impedance $\overline{Z}_c(z)$ or the normalized width $w(z)/h$.

$$\ln\left(\overline{Z}_c(z)\right) = \ln(Z_c(z)/Z_0) = C_0 + \sum_{n=1}^{N}\left(C_n\cos(2\pi nz/d) + S_n\sin(2\pi nz/d)\right) \tag{7}$$
$$\ln(w(z)/h) = C_0 + \sum_{n=1}^{N}\left(C_n\cos(2\pi nz/d). + S_n\sin(2\pi nz/d)\right) \tag{8}$$

where Z_0 is a reference impedance such as $50\,\Omega$. Of course, choosing each of the expansions in (7)–(8), the other function can be obtained from the known formulas related to the microstrip concept [8]. The truncated Fourier series expansion has been considered for $w(z)/h$ as in (7), in this paper. An optimum designed matcher has to have the input reflection coefficient as small as possible in a defined large frequency range. Therefore, the optimum values of the coefficients C_n and S_n in (7) or (8) can be obtained through minimizing the following error function related to M frequencies $f_1 < f_2 < \ldots < f_M$ inside the desired matching bandwidth.

$$\text{Error} = \sqrt{\frac{1}{M}\sum_{m=1}^{M}|\Gamma_{in}(f_m)|^2} \tag{9}$$

where

$$\Gamma_{in}(f) = \frac{Z_{in}(f) - Z_S^*(f)}{Z_{in}(f) + Z_S(f)} \tag{10}$$

is the input reflection coefficient at frequency f. Moreover, defined error function should be restricted by some constraints such as mismatching at some undesired frequencies or easy fabrication. The latter one may be as follows

$$(w/h)_{\min} \leq w(z)/h \leq (w/h)_{\max} \tag{11}$$

where $(w/h)_{\min}$ and $(w/h)_{\max}$ are the minimum and maximum values of $w(z)/h$, respectively, in the fabrication step. It is worth to note that one can add some other terms such as "kz/d" to the expansions (7)–(8) to prepare the possibility of having unequal values at $z = 0$ and $z = d$.

4. EXAMPLES AND RESULTS

Consider a microstrip NTL with $\varepsilon_r = 3.5$ with assumptions of $(w/h)_{\min} = 0.1$ and $(w/h)_{\max} = 10$. We would like to design this NTL as an impedance matcher in a frequency range of 2.0 to 4.0 GHz (an octave bandwidth). We consider four cases as following for the load and source impedances.

Case 1: $Z_L = 100\,\Omega$ resistor and $Z_S = 50\,\Omega$ resistor.

Figures 2–4 illustrate $w(z), Z_c(z)$ and also $|\Gamma_{in}(f)|$, respectively for $N = 1, 2, 3$ and 4, considering $d = 5$ cm. It is observed that the solutions converge rapidly with increasing N and the converged solution ($N = 4$) have yielded a good impedance matching. It is seen that the converged function $Z_c(z)$ alternates around the known linear taper. Figures 5–6 illustrate the effect of the length of matcher, considering $N = 4$. It is seen that as the length of matcher is chosen larger its efficiency is increased. To show the possibility of designing multi-band matcher, Figures 7–8 illustrate $w(z)$ and $Z_c(z)$ and also $|\Gamma_{in}(f)|$, respectively for an impedance matcher designed only for two distinct frequencies 2.0 and 4.0 GHz, considering $N = 4$ and $d = 5$ cm.

Case 2: $Z_L = (100\,\Omega$ resistor parallel with 0.53 pF capacitor) and $Z_S = 50\,\Omega$ resistor.

Case 3: $Z_L = 100\,\Omega$ resistor and $Z_S = (50\,\Omega$ resistor series with 1.06 pF capacitor).

Case 4: $Z_L = (100\,\Omega$ resistor parallel with 0.53 pF capacitor) and $Z_S = (50\,\Omega$ resistor series with 1.06 pF capacitor).

The complex load or source impedances in the above cases have quality factor equal to one at the center frequency. Figures 9–11

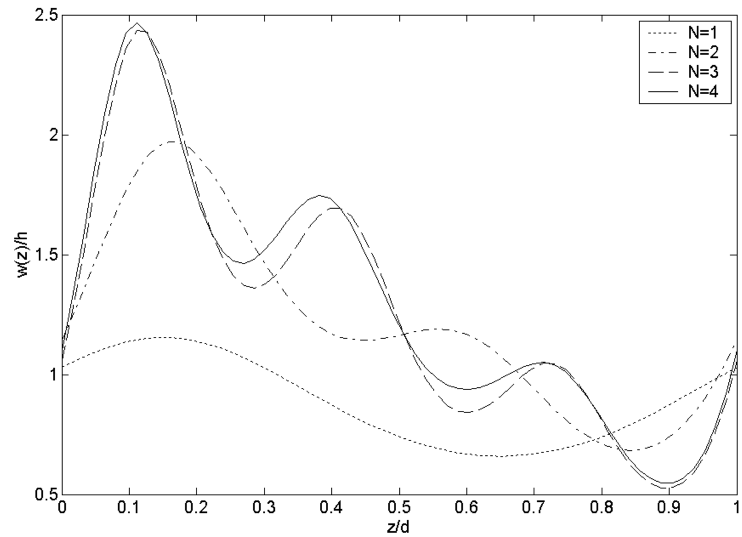

Figure 2. The width function $w(z)$ for impedance matcher in case 1, considering $d = 5\,\text{cm}$.

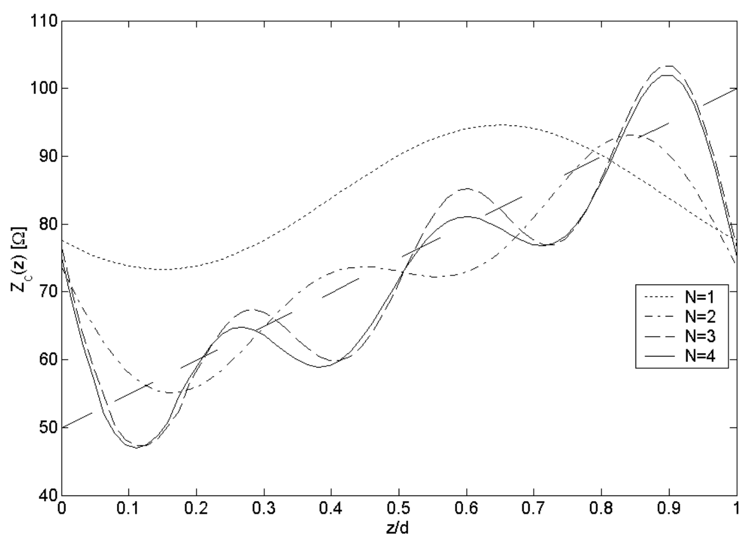

Figure 3. The characteristic impedance $Z_c(z)$ for impedance matcher in case 1, considering $d = 5\,\text{cm}$.

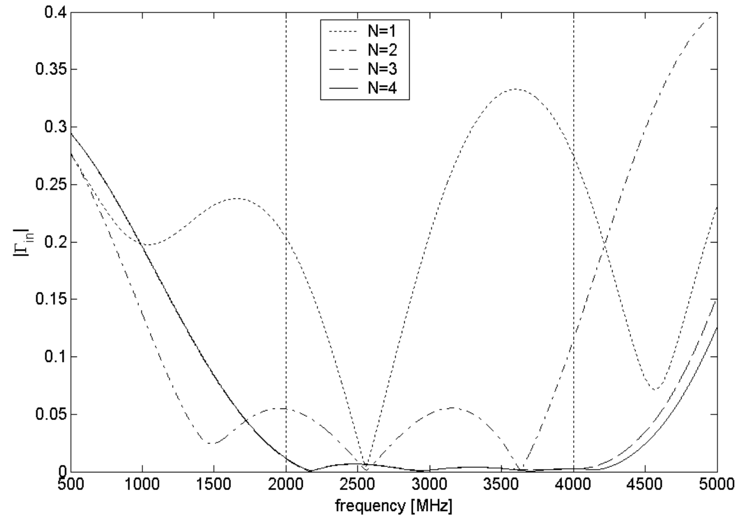

Figure 4. The input reflection coefficient $|\Gamma_{in}|$ for impedance matcher in case 1, considering $d = 5\,\mathrm{cm}$.

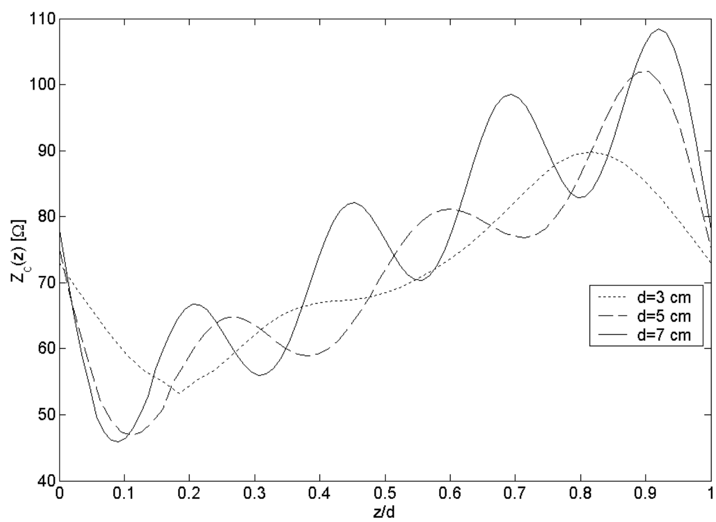

Figure 5. The characteristic impedance $Z_c(z)$ for impedance matcher in case 1, considering $N = 4$.

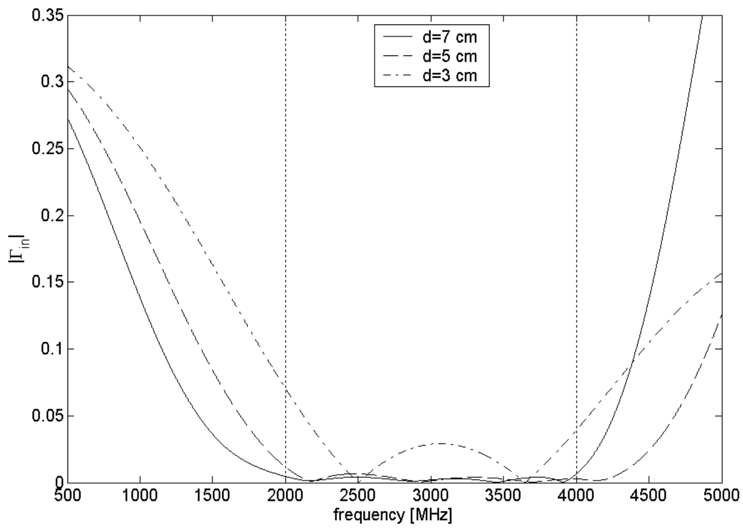

Figure 6. The input reflection coefficient $|\Gamma_{in}|$ for impedance matcher in case 1, considering $N = 4$.

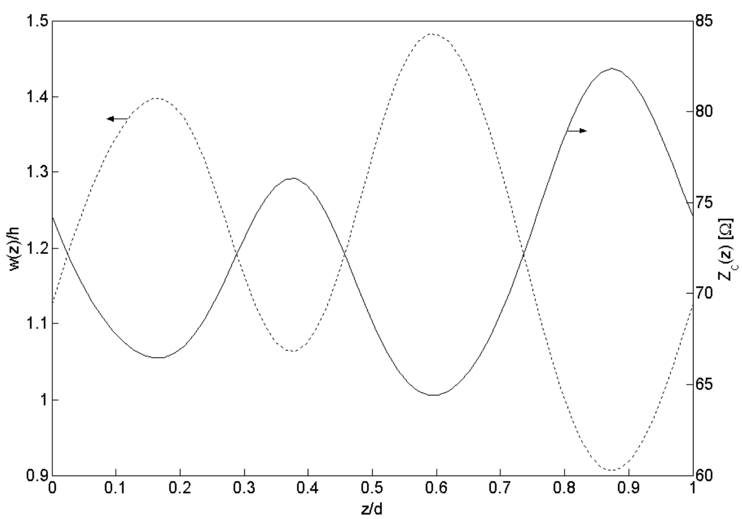

Figure 7. The width and characteristic impedance for impedance matcher in case 1 (Double-Band) , considering $d = 5\,\text{cm}$ and $N = 4$.

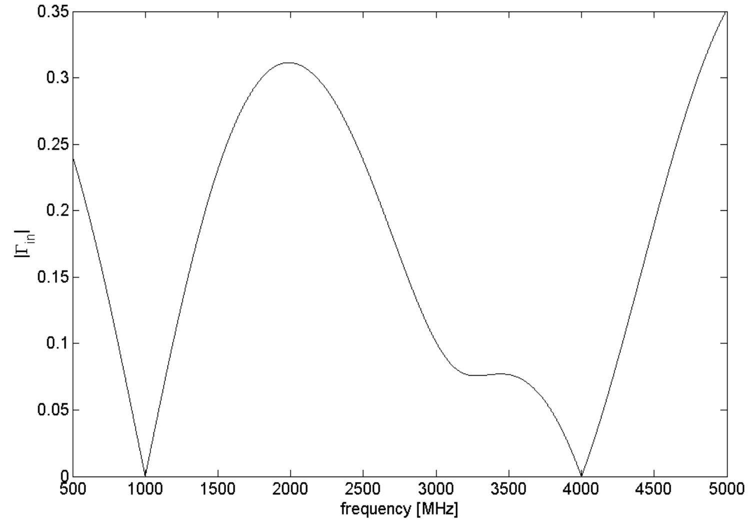

Figure 8. The input reflection coefficient $|\Gamma_{in}|$ for impedance matcher in case 1 (Double-Band), considering $d = 5\,\text{cm}$ and $N = 4$.

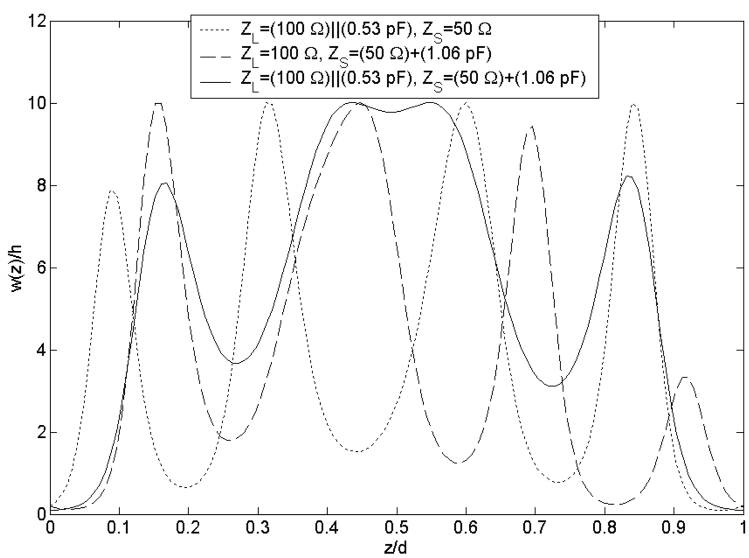

Figure 9. The width function $w(z)$ for impedance matcher in case 2, considering $d = 5\,\text{cm}$ and $N = 5$.

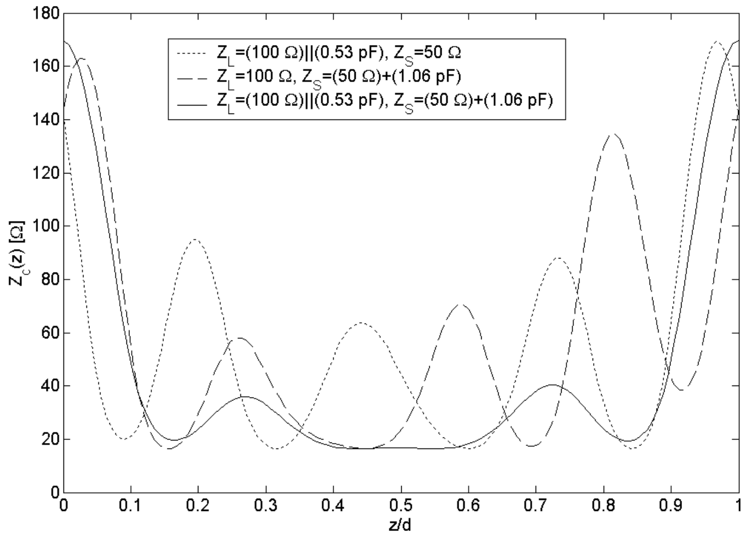

Figure 10. The characteristic impedance $Z_c(z)$ for impedance matcher in case 2, considering $d = 5\,\mathrm{cm}$ and $N = 5$.

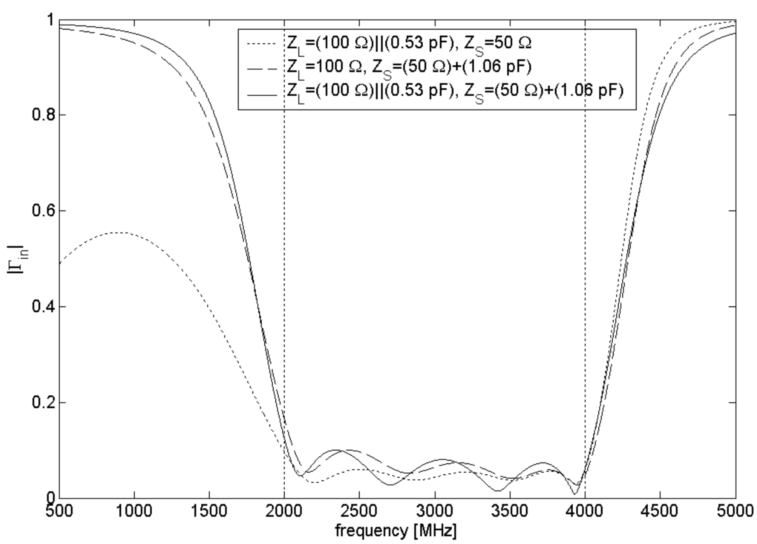

Figure 11. The input reflection coefficient $|\Gamma_{in}|$ for impedance matcher in case 2, considering $d = 5\,\mathrm{cm}$ and $N = 5$.

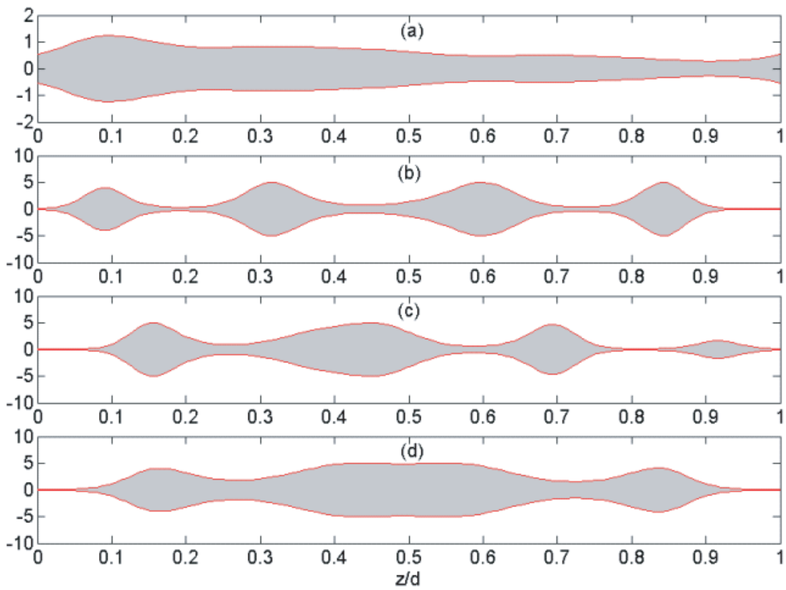

Figure 12. The top view of the resulted microstrip NTL in the cases 1-4, considering $d = 5\,\text{cm}$ and $N = 5$.

illustrate $w(z)$, $Z_c(z)$ and also $|\Gamma_{in}(f)|$, respectively for three Cases 2–4 simultaneously considering $N = 5$ and $d = 5\,\text{cm}$. Again, it is observed that the solutions have yielded a good impedance matching. However, the resulted efficiency in these three cases is less than that of Case 1, which is due to the Bode-Fano criteria [3]. Finally, Figure 12 depicts top view of the resulted microstrip NTL in the Cases 1–4 considering $N = 5$ and $d = 5\,\text{cm}$.

5. CONCLUSION

A novel method was introduced to synthesize microstrip Nonuniform Transmission Lines (NTLs) for matching between two arbitrary complex frequency dependent impedances in a wideband or multi-band frequency range. First, strip width or the characteristic impedance function of the microstrip NTL is expanded in a truncated Fourier series. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. Utilizing truncated Fourier series expansion does not create any discontinuity in the resulted NTL. The usefulness of the proposed method was verified using some examples (Wideband and Double-band matching between two resistors along with and without capacitors). It is observed that the solutions yield a good impedance matching and as the length of matcher is chosen larger its efficiency is increased.

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