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MODAL ANALYSIS AND DISPERSION CURVES OF A NEW UNCONVENTIONAL BRAGG WAVEGUIDE USING A VERY SIMPLE METHOD

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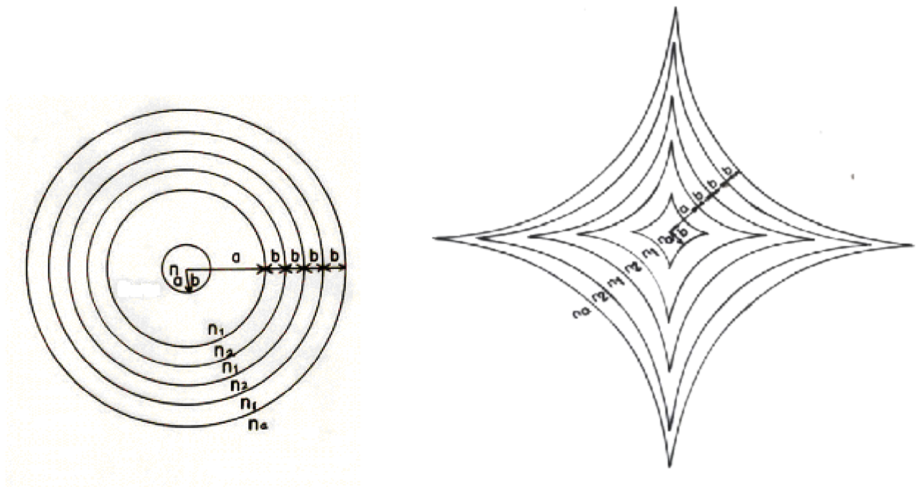
Abstract—A theoretical modal dispersion study of a new unconventional Bragg waveguide having hypocycloidal core cross-section and surrounded by Bragg cladding layers is presented using a very simple boundary matching technique [1]. An attempt has been made to determine how the modal characteristics of a standard Bragg fiber change as its circular shape is changed to the hypocycloidal shape. It is seen that in the case of a hypocycloidal Bragg waveguide single mode guidance is possible when $V \leq 10.0$ where V is the normalized frequency parameter.

1. INTRODUCTION

In the modern age of information technology, the Optical fiber and waveguides, which transmit information in the form of short optical pulses over long distances at exceptionally high speeds, have become an integral part of human life and culture. In conventional optical waveguides, light is confined and guided by total internal reflection, which requires that core has a slightly higher refractive index than the cladding. Total internal reflection is perfect in that it causes no loss other than the intrinsic absorptive and scattering losses of the materials themselves. Such losses can be reduced much in the case of a Bragg waveguide where guidance of light takes place via Bragg reflection and central core has lower refractive index (air) than that of surrounding cladding regions [1, 2]. In this way Bragg waveguides are much superior to conventional optical fibers in many ways namely: i) the air core guidance of light in Bragg fibers leads to lower absorption loss and reduce the threshold for nonlinear effects. ii) for Bragg fibers truly single guided mode is possible but for conventional fibers the fundamental double degenerated modes are always possible. In this way Bragg fibers can be employed as a mode filters [2]. iii) many undesirable polarization dependent effects can be completely eliminated in Bragg fibers. iv) Bragg fiber also offers some other possibilities such as atom guiding by optical waves [3]. v) the periodic cladding layers of alternating high and low refractive indices that surround the core of a Bragg fiber gives rise to photonic band gap of recent interest [4–7]. Therefore, these fibers can also be referred to as photonic band gap Bragg fibers [7, 8].

Several researchers have investigated the modal characteristics and other properties of Bragg fibers. A rigorous mathematical analysis was presented by Yeh et al. [2] and optimization method was used to design and optimize Bragg fiber. Since then considerable progress has been made in the theory and application of Bragg fibers [9–16]. An experimental demonstration of wave guiding in Bragg fibers was firstly demonstrated by Fink et al. [9]. Recently Xu et al. [10, 11] have proposed an asymptotic matrix theory to calculate modal dispersion, field distribution and radiation losses of any cylindrically symmetric dielectric geometries surrounded by Bragg cladding layers. Very recently Singh et al. [1] have studied the Bragg fiber using a very simple matrix method and it was shown that confined guided mode exist in Bragg fiber that consists of a low index central core including air surrounded by a suitable designed alternating cladding of high and low refractive index media Fig. 1(a). It was also shown that by using only a small number of cladding layers a Bragg waveguide is as almost as good



(a) (b)

Figure 1. (a) The cross-sectional view of standard Bragg waveguide [1]. (b) The cross-sectional view of the hypocycloidal Bragg waveguide.

as a conventional standard fiber under the weak guide condition with an additional advantage that there is a very small absorption loss. In the present article we have chosen an unconventional Bragg waveguide namely a hypocycloid Bragg waveguide Fig. 1(b). Using a simple matrix method [1] and replacing the boundary condition by matrix equation, the modal eigen value equation of the proposed waveguide has been obtained analytically. Computed results are shown in the form of dispersion curves and are compared with dispersion curves of standard Bragg fiber. It is seen that in the case of hypocycloidal Bragg waveguide $V \leq 10.0$ a single mode guide is possible [17] where as this is not found in the case of a standard Bragg waveguide]. In addition, we hope that analytical work of this kind will provide the modal characteristics of a variety of waveguides [18] forming a rich fund of results from which researchers and engineers working in practical fields can choose the required characteristics and structures in future when the necessary fabrication technology becomes available. The feasibility of fabrication, if not already there, is not remote in view of modern advances in nano-technology if only the experimentalists and practical engineers are sufficiently interested or encouraged to take up this sort of work.

2. THEORETICAL BACKGROUND

We consider the standard Bragg waveguide having circular core and a new unconventional Bragg waveguide, having hypocycloidal core cross section shown in Fig. 1(a)and Fig. 1(b) respectively. The modal characteristics of the standard Bragg waveguide have already been investigated in our recent paper [1]. Its diagram and computed results (in Table 1) are given here only for the comparison purposes. We now consider the new hypocycloidal Bragg waveguide in the following section.

3. THE CHARACTERISTICS EIGEN VALUE EQUATION FOR HYPOCYCLOIDAL BRAGG WAVEGUIDE

Here we will use a simple matrix method to compute the modal characteristics of a hypocycloidal Bragg waveguide. The basic idea is to replace the boundary condition by a matrix equation. The cross-sectional view of six layered Bragg waveguide is shown in Fig. 1(b). It has low refractive index (n_a) in central region and higher refractive indices n_1 and n_2 ($n_1 > n_2$) in the cladding regions around it. Thereby we have suitably designed alternating claddings of high and low refractive indices. The index profile is then given by

$$n(r) = \begin{pmatrix} n_a & ; & 0 < r < b \\ n_1 & ; & b < r < a \\ n_2 & ; & a < r < a+b \\ n_1 & ; & a+b < r < a+2b \\ n_2 & ; & a+2b < r < a+3b \\ n_1 & ; & a+3b < r < a+4b \\ n_a & ; & r > a+4b \end{pmatrix} \tag{1}$$

The general equation for hypocycloidal curves can be written as [19, 20]

$$x^N + y^N = a^N \tag{2}$$

where $N = \frac{2}{3}$ and 'a' is a parameter related to the size of the hypocycloid. Since the curve is symmetrical both with respect to the x and the y-axis, it is sufficient to consider only one quadrant. We choose new coordinates (ξ, η, z) appropriate for proposed geometry and assume that electromagnetic wave propagates along the z-axis. Using the new coordinates and Maxwell equations we can obtain the expressions for the field E and H in terms of the new coordinates. The details of this procedure are given in a previous paper by the authors [19–21]. Our assumption $\xi \gg \eta$ as given in reference [20] is valid in all

Table 1. Cutoff frequencies (V-values)for some modes in standard Bragg fiber [1] for three different thicknesses of the cladding strips.

Mode No.	Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=0.01μm. 0<V<16			Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=0.10μm. 0<V<16			Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=1.00μm. 0<V<16		
LP_{1m}	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered
LP_{11}	4.55	4.71	4 .98	3.02	3.93	4 .77	-	-	4.57
LP_{12}	7.80	7.98	8 .06	6.05	6.97	7 .86	-	3.09	8.10
LP_{13}	10.89	11.06	11 .21	9.24	10.10	10.94	0.54	6.51	11.18
LP_{14}	13.92	14.11	1 4.00	12.31	13.22	13.98	3.18	8.87	14.18
LP_{15}	- -	-		15.33	- -		6.50	11.96	-
LP_{16}	- -	-		-	- -		9.22	13.99	-
LP_{17}	- -	-		-	- -		13.01	-	-

the four conical regions of the waveguide. That is it holds best when one is in the middle region of the hypocycloidal annulus, far from the corner regions. The longitudinal components of the fields for the even modes, as a first approximation, can be written as

$$E_{z1} = (u_i\xi)^{\frac{2}{3}} \left[AJ_{\frac{2}{3}}(u_i\xi d) + BY_{\frac{2}{3}}(u_i\xi d)\right] E_z(\eta)\exp(j(\omega t - \beta z)) \quad \text{(3a)}$$

$$H_{z1} = (u_i\xi)^{\frac{2}{3}} \left[CJ_{\frac{2}{3}}(u_i\xi d) + DY_{\frac{2}{3}}(u_i\xi d)\right] E_z(\eta)\exp(j(\omega t - \beta z)) \quad \text{(3b)}$$

where $u_i^2 = k^2 n_i^2 - \beta^2,\ i = 1, 2$ correspond to refractive index n_1 and n_2. The solution of central region and the outermost region can be written as

$$E_{z2} = (w\xi)^{\frac{2}{3}} \left[E' I_{\frac{2}{3}}(w\xi d) + FK_{\frac{2}{3}}(w\xi d)\right] E_z(\eta)\exp(j(\omega t - \beta z)) \quad \text{(3c)}$$

$$H_{z2} = (w\xi)^{\frac{2}{3}} \left[GI_{\frac{2}{3}}(w\xi d) + H' K_{\frac{2}{3}}(w\xi d)\right] E_z(\eta)\exp(j(\omega t - \beta z)) \quad \text{(3d)}$$

Where $w = \sqrt{\beta^2 - k^2 n_a^2}$, n_a being the common refractive index of these regions. In the above equations $J_{\frac{2}{3}}$ and $Y_{\frac{2}{3}}$ are the Bessel functions of first and second kind while $I_{\frac{2}{3}}$ and $K_{\frac{2}{3}}$ are the modified Bessel functions respectively. Here β is the axial component of propagation vector, ω is the wave frequency, μ is the permeability of non-magnetic medium, ε_1 and ε_2 are the permittivity of the core and the cladding region respectively. Also d is a number $\frac{1}{2}$ which emerges in the analysis because of the peculiarity of the geometrical shape. Also A, B, C, D, E', F, G and H' all are unknown constants.

The boundary conditions can be written as

$$\left.E_{z1}\right|_r = \left.E_{z2}\right|_r$$
$$\left.\frac{\partial E_{z1}}{\partial \xi}\right|_r = \left.\frac{\partial E_{z2}}{\partial \xi}\right|_r$$

Thus we get a set of equations having twenty two unknown constants. The nontrivial solution will exist only when the determinant formed by the coefficients of the unknown constants is equal to zero. Calling this 12×12 determinant Δ, we have

$$\Delta = 0 \tag{4}$$

The element in the rows and columns of this determinant can be identified readily. We also define (see Fig. 1) that

$$\Delta n = n_1 - n_2$$
$$\Delta n' = n_1 - n_a$$
$$V = k_0(a-b)\left(n_1^2 - n_a^2\right)^{\frac{1}{2}} = k_0(a-b)[2n(\Delta n + \Delta n')]^{\frac{1}{2}} \tag{5}$$

where k_0 is vacuum wavenumber. We define the usual normalized propagation parameter

$$b' = \frac{\beta^2 - k_0^2 n_a^2}{k_0^2\left(n_1^2 - n_a^2\right)} \approx \frac{\beta - k_0 n_a}{k_0(\Delta n + \Delta n')} \quad \text{(weakly guidance case)} \tag{6}$$

The dimensionless V-parameter is introduced to incorporate the parameters n_a, n_1, n_2, a, b and k_0 which may possibly have an effect on the propagation. One may choose other alternative ways to define the quantities V and b', but as an illustrative case, the present definitions are adequate.

4. NUMERICAL RESULT AND DISCUSSION

The characteristic equation (4) has all of the information that we can obtain from our modal analysis and it gives the central results of this investigation. We now proceed to some numerical computation in order to have the modal dispersion curves for the proposed Bragg waveguide. It is convenient to plot the normalized propagation constant $b' \approx \frac{\beta - k_0 n_a}{k_0(\Delta n + \Delta n')}$ against the V-parameter defined by $V = k_0(a-b)[2n(\Delta n + \Delta n')]^{\frac{1}{2}}$. Now we choose $n_a = 1.0002$, $n_1 = 1.45$, $n_2 = 1.50$, $b = 0.01\mu\text{m}$, $0.1\mu\text{m}$, $1.0\mu\text{m}$, an operating wavelength

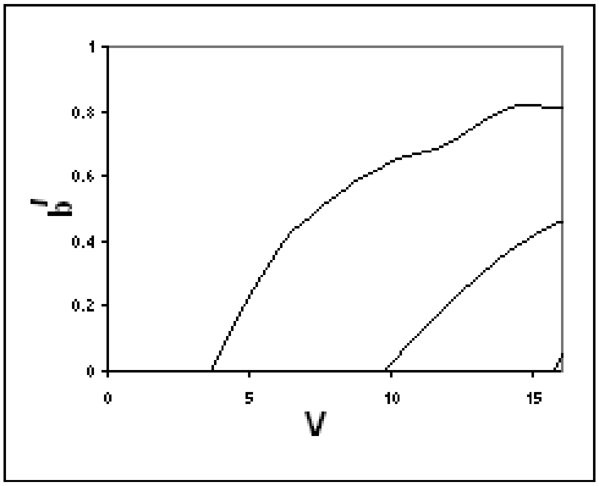


Figure 2. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 0.01\mu\text{m}$ (hypocycloidal Bragg waveguide).

$\lambda_0 = 1.55\mu\text{m}$ and various values of dimensional parameter a in a regular increasing order. For each value of a we obtain the V-parameter and also compute the values of β from the characteristic equation (4) by graphical method. It means that the left hand side of characteristic equation is plotted against β for the assumed value of a and the zero crossing of the graph with the β axis are noted. These values are the solutions of the characteristic equation for the different modes. From the β values of the guided modes we can obtain b' and then plot the b' versus V graphs.

These curves are shown in Fig. 2 to Fig. 10 for different number of cladding layers having different thicknesses. The cutoff values and their dependence on the thickness of the cladding strip and on different number of cladding layers can be seen for the standard Bragg waveguide and the hypocycloidal Bragg waveguide in Table 1 and Table 2 respectively. It is to be noted that Table 1 is taken directly from our previous paper [1] for the compression purpose only. Several interesting results can be seen in the tables and figures showing dispersion curves. We first consider Table 1 and Table 2, we note particularly that the dependence of the cutoff V-values on the thickness is such that as thickness b is increased from $b = 0.01\mu\text{m}$ to $b = 1.00\,\mu\text{m}$ the cutoff values decreases for all the mode in the hypocycloidal case and also for the circular case. The decrease is, however, considerably

Table 2. Cutoff frequencies (V-values)for some modes in hypocycloidal Bragg fiber for three different thicknesses of the cladding strips.

Mode No.	Cut off frequencies of various modes in hypocycloidal Bragg fiber with thickness of cladding strip b=0.01μm. 0<V<16			Cut off frequencies of various modes in hypocycloidal Bragg fiber with thickness of cladding strip b=0.10μm. 0<V<16			Cut off frequencies of various modes in hypocycloidal Bragg fiber with thickness of cladding strip b=1.00μm. 0<V<16		
LP_{1m}	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered
LP_{11}	3.607	4.01	4.04	1.62	2.43	3.242	-	-	1.62
LP_{12}	9.68	10.93	10.96	7.701	8.512	9.323	-	0.0	7.29
LP_{13}	15.70	15.74	15.78	13.7	14.9	15.4	-	4.86	13.7
LP_{14}	-	-	-	-	-	-	3.24	11.3	-
LP_{15}	-	-	-	-	-	-	9.32	-	-

larger in the case of circular case. We also observe from tables that as the number of cladding layers decreases the cutoff value increases both the Bragg waveguides. We know that the greater the cutoff values the fewer will be number of modes sustained.

Coming now the dispersion curves (Fig. 2 to Fig. 4), we find the interesting feature that for thickness $b = 0.01\mu$m and for $V \leq 10.0$ the hypocycloidal Bragg waveguide sustains only a single mode for different cladding layers taken whereas in the same conditions, the standard Bragg waveguide sustains two modes for all cases considered. This shows that hypocycloidal Bragg waveguide with small number of cladding layers shows comparable or even better performance than the standard Bragg fiber in respect of single mode guidance in addition to the obvious advantage that there will be very little absorption of energy in these Bragg waveguides. Similarly considering Fig. 5, Fig. 6 and Fig. 7, it is clear that the hypocycloidal Bragg waveguide allows only three modes in all three cases whereas the standard Bragg waveguide sustains four modes. Thus it is clear form Fig. 2 to Fig. 7 that hypocycloidal Bragg waveguide strict the number of guided modes compared to the standard Bragg waveguide. Thus hypocycloidal Bragg waveguide may be used as a mode filters.

Further we observe that the cutoff values are somewhat smaller for hypocycloidal Bragg waveguide than for the circular Bragg waveguide. For example in Fig. 2, Fig. 3 and Fig. 4, the cutoff V-values for the lowest order mode for hypocycloidal Bragg waveguide at $V = 3.60$, $V = 4.01$ and $V = 4.04$ respectively, are much smaller than the

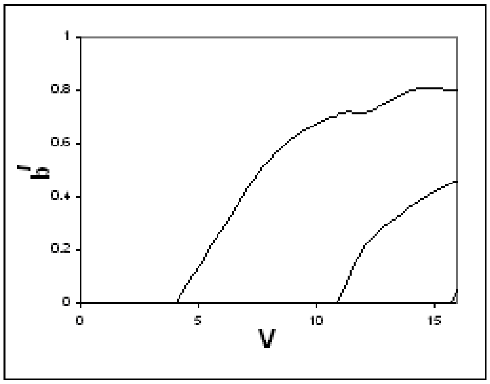


Figure 3. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 0.01\mu$m (hypocycloidal Bragg waveguide).

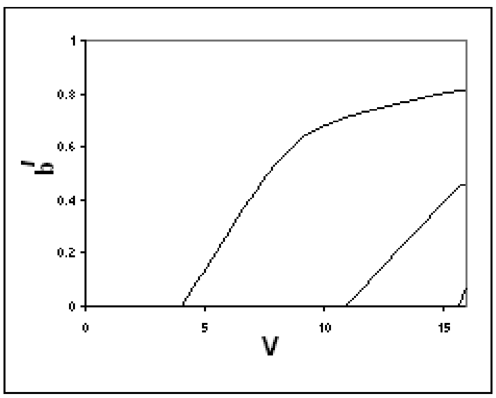


Figure 4. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the two layered cladding regions with its thickness $b = 0.01\mu$m (hypocycloidal Bragg waveguide).

cutoff V values for standard Bragg waveguide which are given in our earlier paper [1] as $V = 4.55$, $V = 4.71$ and $V = 4.98$ respectively. This feature is also expected. We know that when a square is compressed from outside we get a hypocycloidal structure and when a hypocycloid is opened from inner side we get a circle. It is well known that square waveguide has larger modal cutoff values than circular waveguide.

An anomalous feature in the dispersion curves is observable for

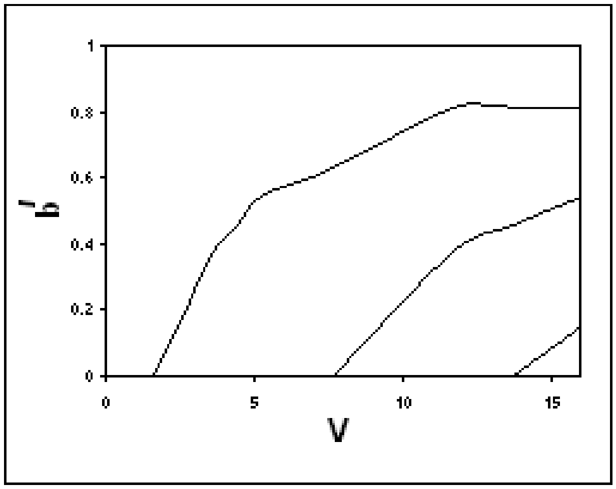


Figure 5. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 0.10\mu$m (hypocycloidal Bragg waveguide).

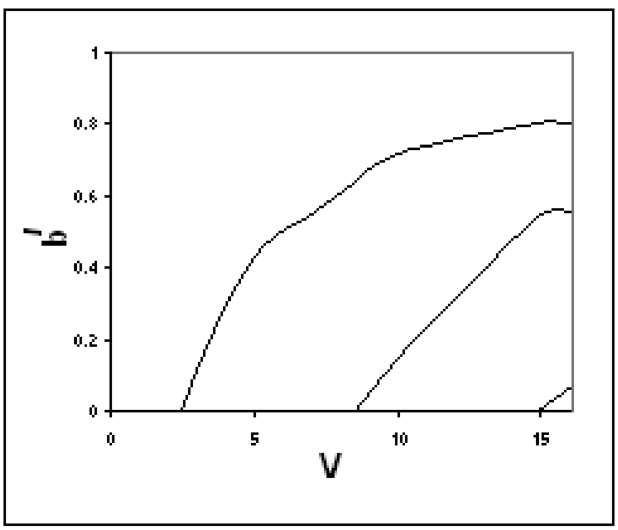


Figure 6. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 0.10\mu$m (hypocycloidal Bragg waveguide).

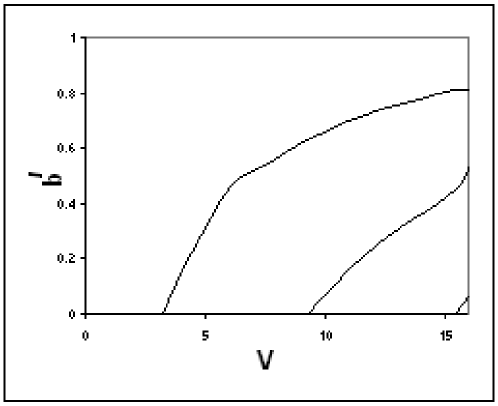


Figure 7. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the two layered cladding regions with its thickness $b = 0.10\mu$m (hypocycloidal Bragg waveguide).

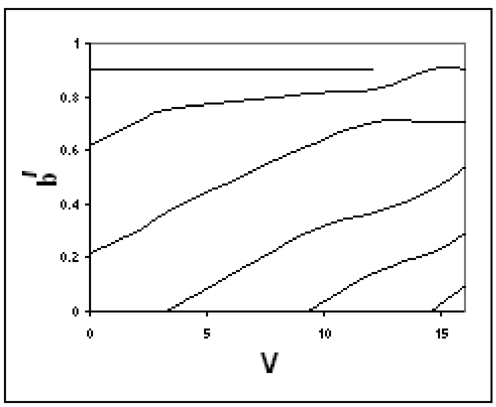


Figure 8. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 1.0\mu$m (hypocycloidal Bragg waveguide).

thickness $b = 1.0\mu$m for both type of lightguide. In the case of Hypocycloidal Bragg waveguide the curves in Fig. 8 and Fig. 9 have no resemblance with standard dispersion curves. Similarly in the case of standard Bragg fiber some curves are not in standard shape. This means that for greater thickness having large number of cladding layers the structure becomes complicated and a simple physical explanation of this feature is not possible.

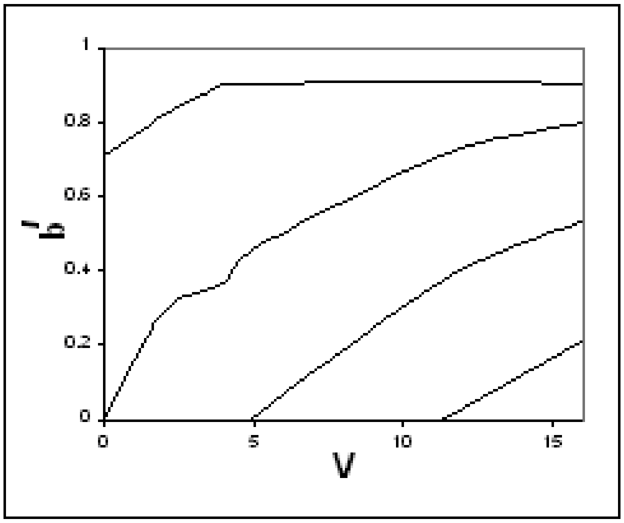


Figure 9. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 1.0\mu$m (hypocycloidal Bragg waveguide).

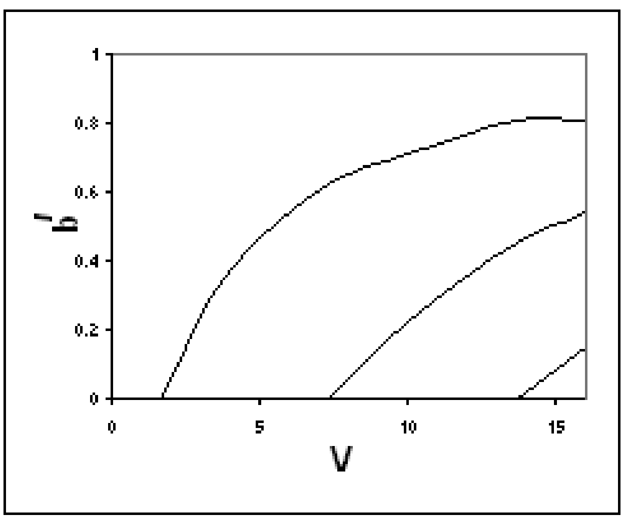


Figure 10. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the two layered cladding regions with its thickness $b = 1.0\mu$m (hypocycloidal Bragg waveguide).

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