Progress In Electromagnetics Research, PIER 64, 53–67, 2006

ELECTROMAGNETIC PULSE DIFFRACTION BY A MOVING HALF-PLANE

A. Ciarkowski

Institute of Fundamental Technological Research
Polish Academy of Sciences
Świętokrzyska 21, 00-049 Warsaw, Poland

Abstract—This paper is concerned with the scattering of an electromagnetic (EM) pulse by a perfectly conducting half-plane, moving in a free space. It is assumed that the source signal is a plane wave pulse with its envelope described by a Dirac delta function. The representation for the total field is found, and physical interpretation of the solution is given. This representation, valid for all screen velocities, is then reduced to the case of moderate and low velocities, important for practical applications.

1. INTRODUCTION

Electromagnetic wave scattering by moving objects has a long history and is interesting from both practical and theoretical point of view. Its applications can be found in telecommunication, object recognition, space science and astronomy. Of special interest are scattering objects with edges. In [1] plane wave diffraction by a moving cylinder was analyzed and such phenomena as Doppler shift of equiphase surfaces in the diffracted wave and angular shift of the location of its amplitude singularities were reported. Those phenomena were also confirmed in [2], where diffraction by a wedge in motion was considered. In more recent work [3], concerned with plane wave diffraction by a moving half-plane, similar phenomena were noticed, and also a rotation of the incident and reflected wave shadow boundaries were observed. In [4–6] different solving approaches were analyzed and effectively applied to problems with Gaussian beam excitations and moving cylinder and wedge shaped obstacles.

Most of works on wave scattering by objects with edges, including all those aforementioned, deal with time harmonic fields. In this paper we extend the results obtained in [3] to the case where the exciting field is a pulse signal, with its envelope described by a Dirac delta function. The moving object is a half-plane, which is the simplest structure possessing an edge. The problem studied herein is 2-dimensional. In the analysis we employ two frames of reference, the laboratory frame, in which the source field is given, and the primed frame, in which the half-plane is at rest. Quantities in both frames are related through the Lorentz transformation. We take advantage of Fourier and Felsen [7] integral techniques which enable representation of transient fields via time-harmonic ones. Essential in our analysis is the well known Sommerfeld solution for the diffraction of a time harmonic plane wave by a stationary half-plane. We find the total electromagnetic field in the laboratory frame, give its physical interpretation, and finally simplify it to the case of non-relativistic velocities, that is important for practical applications.

Solutions corresponding to exciting pulse signals with different envelopes can be obtained by integration of the solution here obtained with appropriate weight functions. In the model considered in this paper it is assumed that both the scatterer and the surrounding medium are frequency independent. This allows us to obtain the solution in a relatively simple analytical form. As a consequence, phenomena accounting for dispersive properties of real media are neglected. It is believed that despite this simplification the results here obtained can be of practical importance in applications. It is worth of mentioning that even in the absence of a scatterer pulse evolution in a dispersive medium is a complicated problem in itself (comp. [8–10]).

We have chosen Dirac delta function as the source signal envelope because it plays a role of a Green function. It means that solutions to related problems, with different envelope of the incident pulse, can be obtained via integration of the solution here obtained with appropriate weight functions (comp. [7]).

2. FORMULATION OF THE PROBLEM

Consider two inertial frames of reference S and S', in which space-time coordinates (x, y, z, t) and (x', y', z', t'), respectively, are introduced. The frame S is associated with a source pulse electromagnetic field, and the frame S' is used to describe a moving screen, on which the source field undergoes diffraction. As observed from the S frame, thereafter referred to as laboratory frame, the S' frame is moving with a constant velocity $v = \hat{x}v$, $\hat{x}$ being a unit vector in the x direction. We assume that the frames S and S' coincide at the moments $t = t' = 0$. The screen has the form of the perfectly conducting half-plane, defined in S' frame by $x' \leq 0$, $-\infty < y' < \infty$ and $z' = 0$. It is at rest in S', and it is moving with the velocity v in S.

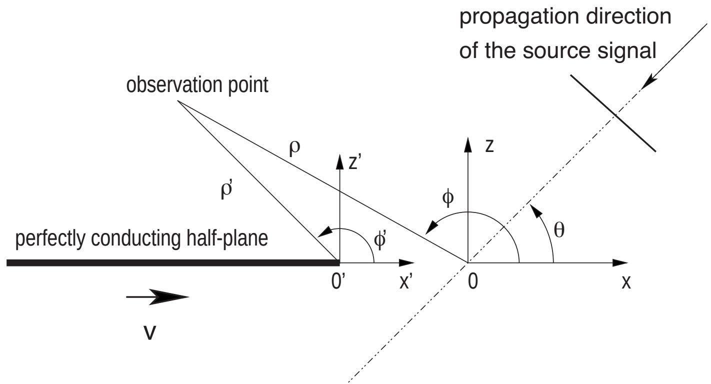

Figure 1. Geometry of the problem. Coordinates in S and S' frames of reference.

Let a plane EM pulse with its envelope described by a Dirac function be propagating perpendicularly to the screen edge. In the S frame it satisfies the Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times c\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{1}$$

where all fields involved are independent of the y-coordinate. Thus the problem is 2-dimensional and it can be decomposed into two independent problems for E-field, and H-field, respectively [11], where the particular fields are given by:
for the E-field,

$$\mathbf{E}^i = \hat{\mathbf{y}}\, u^i, \quad \mathbf{B}^i = -\int_{-\infty}^{t} \nabla \times \mathbf{E}^i \, d\tau = \hat{\mathbf{y}} \times \int_{-\infty}^{t} \nabla u^i \, d\tau, \tag{2a}$$

for the H-field,

$$c\mathbf{B}^i = \hat{\mathbf{y}}\, u^i, \quad \mathbf{E}^i = c\int_{-\infty}^{t} \nabla \times c\mathbf{B}^i \, d\tau = -\hat{\mathbf{y}} \times c\int_{-\infty}^{t} \nabla u^i \, d\tau, \tag{2b}$$

and $u^i = u^i(x, z)$. Thus to obtain the total field corresponding to an excitation field ($\mathbf{E}^i$, $\mathbf{B}^i$) with a particular polarization, one needs to single out its components parallel to the edge E_y^i and B_y^i. Then the corresponding scattering problems for E- and H-fields should be solved and their solutions added together.

In this paper we assume that

$$u^i = \delta\left(t - \frac{\mathbf{k}_0 \cdot \mathbf{r}}{c}\right), \tag{3}$$

where δ is the Dirac delta function, $\mathbf{r} = (x, y, z)$ and $\mathbf{k}_0 = (-\cos\theta, 0, -\sin\theta)$, θ being the angle measured between the x axis and the direction from which the pulse propagates. With the substitution (3) the formula (2a) reduces to

$$\mathbf{E}^i_\perp = \hat{\mathbf{y}}\, u^i, \qquad c\mathbf{B}^i_\parallel = \hat{\mathbf{x}}\, \sin\theta\, u^i, \qquad c\mathbf{B}^i_\perp = -\hat{\mathbf{z}}\, \cos\theta\, u^i \tag{4a}$$

and similarly, the formula (2b) simplifies to

$$c\mathbf{B}^i_\perp = \hat{\mathbf{y}}\, u^i, \qquad \mathbf{E}^i_\parallel = -\hat{\mathbf{x}}\, \sin\theta\, u^i, \qquad \mathbf{E}^i_\perp = \hat{\mathbf{z}}\, \cos\theta\, u^i. \tag{4b}$$

Here, $\perp$ or $\|$ symbols are traditionally used to denote field components that are perpendicular or parallel to the direction of the screen motion, respectively.

Our goal is to find the total fields in E-field, and H-field case, that satisfy the equations (1) and the boundary condition

$$\hat{\mathbf{z}} \times \mathbf{E} = 0 \tag{5}$$

of vanishing of the tangential component of the electric field on the *moving* half-plane surface.

3. FIELDS IN THE MOVING FRAME

The first step in solving this scattering problem is to Lorentz transform the source field from the S frame to the S' frame. In our case the transformation formulas take the following form for the fields

$$\begin{aligned} \mathbf{E}'_\parallel &= \mathbf{E}_\parallel, \qquad c\mathbf{B}'_\parallel = c\mathbf{B}_\parallel, \\ \mathbf{E}'_\perp &= \gamma(\mathbf{E}_\perp + \boldsymbol{\beta} \times c\mathbf{B}_\perp), \quad c\mathbf{B}'_\perp = \gamma(c\mathbf{B}_\perp - \boldsymbol{\beta} \times \mathbf{E}_\perp), \end{aligned} \tag{6}$$

and for the coordinates

$$x' = \gamma(x - \beta ct), \quad y' = y, \quad z' = z, \quad ct' = \gamma(ct - \beta x). \tag{7}$$

Here,

$$\boldsymbol{\beta} = \hat{\mathbf{x}}\, \beta, \qquad \beta = v/c, \qquad \gamma = 1/\sqrt{1 - \beta^2}. \tag{8}$$

Upon this transformation the source field in S' frame becomes: for the E-field,

$$\mathbf{E}_{\perp}^{i'} = \hat{\mathbf{y}}'\ u^{i'}, \quad c\mathbf{B}_{\parallel}^{i'} = \hat{\mathbf{x}}'\ \sin\theta'\ u^{i'}, \quad c\mathbf{B}_{\perp}^{i'} = -\hat{\mathbf{z}}'\ \cos\theta'\ u^{i'}, \tag{9a}$$

and for the H-field,

$$c\mathbf{B}_{\perp}^{i'} = \hat{\mathbf{y}}'\ u^{i'}, \quad \mathbf{E}_{\parallel}^{i'} = -\hat{\mathbf{x}}'\ \sin\theta'\ u^{i'}, \quad \mathbf{E}_{\perp}^{i'} = \hat{\mathbf{z}}'\ \cos\theta'\ u^{i'}, \tag{9b}$$

where,

$$u^{i'} = \delta\left(t' - \frac{\mathbf{k}_0' \cdot \mathbf{r}'}{c}\right), \tag{10}$$

and

$$k_0' = (-\cos\theta', 0, -\sin\theta'),\ \cos\theta' = \frac{\beta + \cos\theta}{1 + \beta\cos\theta},\ \sin\theta' = \frac{\sin\theta}{\gamma(1 + \beta\cos\theta)}. \tag{11}$$

Here, we took advantage of $\delta(ax) = \delta(x)/a$ for $a > 0$.

Comparison of (9a) through (10) with (4a), (4b) and (3) shows that the source field in the primed frame has a similar form as in the laboratory frame, except that the arguments in functions that describe the field are suitably modified.

In the second step we Fourier transform the source field in the S' frame to the frequency domain, and thus obtain for the E-field,

$$\tilde{\mathbf{E}}^{i'}(r', \omega') = \int_{-\infty}^{\infty} \mathbf{E}^{i'}(r, \tau) e^{i\omega\tau} d\tau = \hat{\mathbf{y}} e^{-ik' \cdot r'}, \tag{12a}$$

and for the H-field,

$$\tilde{\mathbf{B}}^{i'}(r', \omega') = \hat{\mathbf{y}} e^{-i\mathbf{k}' \cdot \mathbf{r}'}. \tag{12b}$$

Here,

$$\mathbf{k}' = \mathbf{k}_0' k', \qquad k' = \frac{\omega'}{c}. \tag{13}$$

The fields $\mathbf{E}^{i'}$ and $\mathbf{B}^{i'}$ satisfy the wave equation, the corresponding fields $\tilde{\mathbf{E}}^{i'}$ and $\tilde{\mathbf{B}}^{i'}$ satisfy the Helmholtz equation

$$\nabla^2 \left\{ \begin{array}{c} \tilde{\mathbf{E}}' \\ \tilde{\mathbf{B}}' \end{array} \right\} + k'^2 \left\{ \begin{array}{c} \tilde{\mathbf{E}}' \\ \tilde{\mathbf{B}}' \end{array} \right\} = 0. \tag{14}$$

Thus the original scattering problem in the S' frame is reduced to corresponding problem for the Helmholtz equation, and since the problem is 2D, it can be formulated in terms of $\tilde{E}_{y'}$ in the case of E-field, and in terms of $\tilde{B}_{y'}$ in the case of H-field. The remaining components of the EM field can be found from the Maxwell equations. Moreover, in the case of E-field the boundary condition (5) on the half-plane simplifies to the Dirichlet condition for $\tilde{E}_{y'}$, in the case of H-field it reduces to the Neuman condition for $\tilde{B}_{y'}$.

The solution to the boundary value problem for the Helmholtz equation is well known and was first obtained by Sommerfeld. For the geometry used in this paper the resulting scattered field is given by:

$$\tilde{u}' = \tilde{E}_{y'} = -\frac{\sqrt{k' + \xi_0}}{i2\pi} \int_C \frac{e^{i(-\xi x' + \Gamma|z'|)}}{\sqrt{k' + \xi}\,(\xi - \xi_0)}\, d\xi \tag{15a}$$

for E-field, and

$$\tilde{u}' = \tilde{B}_{y'} = \mathrm{sign}(z') \frac{\sqrt{k' - \xi_0}}{i2\pi} \int_C \frac{e^{i(-\xi x' + \Gamma|z'|)}}{\sqrt{k' - \xi}\,(\xi - \xi_0)}\, d\xi \tag{15b}$$

for H-field. Here, $\Gamma = \sqrt{k'^2 - \xi^2}$, $\mathrm{Im}\Gamma > 0$, $\xi_0 = k' \cos\theta'$, and the contour C in the complex ξ-plane is running along the line $\mathrm{Im}\,\xi = 0$ with indentations above $\xi = -k'$ and below $\xi = k'$ and $\xi = \xi_0$. The representation (15b) is given in [12], and the representation (15a) can be derived by following the technique described in [12].

Notice, that for $x' < 0$ and $z' = 0$ the integration contour C can be closed in the upper ξ-half-plane by a semi-circle of a radius tending to infinity. For E-field it follows from (15a) and from the residue theorem that $\tilde{u}' = -\exp(-ik'\xi_0 x)$, and thus the sum of the scattered (15a) and source (12a) electric field vanishes on the screen surface. For H-field, $du'/dz' = i\sqrt{k'^2 - \xi_0{}^2} = ik' \sin\theta'$ on $x' < 0$ and $z' = 0$, where u' is defined by (15b). This cancels out the result of differentiation of (12b) with respect to z'. Thus for H-field the tangential total electric field also vanishes on the screen surface, as it was expected.

The fields (15a) and (15b) should now be converted to the time domain. Their direct inverse Fourier transformation to the time domain is not an easy task. Therefore we shall resort to a technique offered by Felsen [7].

The outcome of the Felsen technique can be formulated as follows: assume the field in the frequency domain is expressed as the contour integral

$$G(r, r^s; \omega) = \int_P Q(r, r^s; w) e^{i\frac{\omega}{c}\gamma(r, r^s; w)} dw, \tag{16}$$

where P is the integration path from $-a+i\infty$ to $a-i\infty$, $0<a<\pi/2$, symmetrical about $w=0$ and $r=r^s$ are the source coordinates. The functions Q and γ are independent of ω, $iQ(r,r^s;w)$ is real for real w, and $\gamma(r,r^s;w)$ is an even function of w and it is real for real w.

Then the transient field due to Dirac delta excitation is given by

$$G(r,r^s;t) = -H\left(t-\frac{D}{c}\right)2c\frac{\mathrm{Re}\;[iQ(r,r^s;-i\beta)\}}{(d/d\beta)\gamma(r,r^s;-i\beta)}, \tag{17}$$

where $D=\gamma(r,r^s;0)$ and β is defined implicitly by

$$ct = \gamma(r,r^s;-i\beta). \tag{18}$$

To facilitate the analysis we introduce the cylindrical coordinates (ρ',ϕ',y'), which are related to the Cartesian coordinates via

$$\rho' = \sqrt{x'^2+z'^2},\ \tan\phi' = z'/x',\ \text{and}\ \eta = \mathrm{sign}(z')\pi-\phi',\ -\pi<\eta<\pi.$$

With the change of integration variable defined by $\xi = k'\cos\alpha$ and $\Gamma(\xi) = \mathrm{sign}(z')k'\sin\alpha$ (comp. [13]) the integrals simplify to:

$$\tilde{u}' = \frac{i}{4\pi}\int_{P'_z}\left(\frac{1}{\sin\frac{\alpha+\theta'}{2}}+\frac{1}{\sin\frac{\alpha-\theta'}{2}}\right)e^{ik'\rho'\cos(\alpha-\eta)}\,d\alpha, \tag{19a}$$

for E-field, and

$$\tilde{u}' = \frac{i}{4\pi}\int_{P'_z}\left(\frac{1}{\sin\frac{\alpha+\theta'}{2}}-\frac{1}{\sin\frac{\alpha-\theta'}{2}}\right)e^{ik'\rho'\cos(\alpha-\eta)}\,d\alpha, \tag{19b}$$

for H-field, where, depending on whether $z'>0$ or $z'<0$, the contour P'_z in the α-complex plane is running from $\pi-i\infty$ to π, then from π to 0 with an indentation above the pole at θ', and from 0 to $i\infty$, or from $-\pi+i\infty$ to $-\pi$, then from $-\pi$ to 0 with an indentation below the pole at θ', and from 0 to $-i\infty$, respectively.

By making a proper account of the residue at $\alpha=\theta'$ the integral representation (19a) can be equivalently written down as

$$\begin{aligned}\tilde{u}' = &-\mathrm{sign}(z')\frac{i}{4\pi}\int_{P}\left(\frac{1}{\sin\frac{\alpha+\eta+\theta'}{2}}+\frac{1}{\sin\frac{\alpha+\eta-\theta'}{2}}\right)e^{ik'\rho'\cos\alpha}\,d\alpha\\ &-H(\theta'-|\eta|)\,e^{ik'\rho'\cos(\theta'-|\eta|)},\end{aligned} \tag{20a}$$

and similarly for the representation (19b),

$$\tilde{u}' = -\mathrm{sign}(z')\frac{i}{4\pi}\int_P \left(\frac{1}{\sin\frac{\alpha+\eta+\theta'}{2}} - \frac{1}{\sin\frac{\alpha+\eta-\theta'}{2}}\right) e^{ik'\rho'\cos\alpha}\, d\alpha$$
$$- \mathrm{sign}(z')H(\theta' - |\eta|)\, e^{ik'\rho'\cos(\theta'-|\eta|)}, \tag{20b}$$

where the contour P is the same as in (16).

These representations are now back transformed to the time domain. The inverse Fourier transform of the residue contributions is

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega' t'} e^{-ik'\rho'\cos(\phi'\pm\theta')}\, d\omega' = \delta\left[t' + \frac{\rho'}{c}\cos(\phi'\pm\theta')\right]. \tag{21}$$

The integrals in (20a) and (20b) have the form of (16) with $w = \alpha$, $\gamma = \rho'\cos\alpha$ and

$$Q = \mathrm{sign}(z')\frac{i}{4\pi}\left(\frac{1}{\sin\frac{\alpha+\eta+\theta'}{2}} \pm \frac{1}{\sin\frac{\alpha+\eta-\theta'}{2}}\right) e^{ik'\rho'\cos\alpha},$$

and hence the Felsen method can be applied to them. Simple calculation shows that the contribution from the integrals to the transient scattered field is

$$u^{d'\pm}(\rho', \phi', t') =$$
$$-\frac{H\left(t' - \frac{\rho'}{c}\right)\sqrt{\frac{\rho'}{c}}}{\sqrt{2}\pi\sqrt{t' - \frac{\rho'}{c}}}\left(\frac{\cos\frac{\phi'+\theta'}{2}}{t' + \frac{\rho'}{c}\cos(\phi'+\theta')} \pm \frac{\cos\frac{\phi'-\theta'}{2}}{t' + \frac{\rho'}{c}\cos(\phi'-\theta')}\right). \tag{22}$$

Consequently, the expressions in the primed frame for appropriate *scattered* field y'-components can be written down as

$$\begin{aligned} E_{y'} &= u'(\rho', \phi', t') \\ &= -H(\theta' + \phi' - \pi)\delta[t' + \tilde{t}\cos(\phi' + \theta')] \\ &\quad -H(\theta' - \phi' - \pi)\delta[t' + \tilde{t}\cos(\phi' - \theta')] \\ &\quad -\frac{H(t' - \tilde{t})\sqrt{\tilde{t}}}{\sqrt{2}\pi\sqrt{t' - \tilde{t}}}\left(\frac{\cos\frac{\phi'+\theta'}{2}}{t' + \tilde{t}\cos(\phi' + \theta')} + \frac{\cos\frac{\phi'-\theta'}{2}}{t' + \tilde{t}\cos(\phi' - \theta')}\right) \end{aligned} \tag{23a}$$

in the case of E-field, and

$$\begin{aligned} cB_{y'} &= u'(\rho', \phi', t') \\ &= H(\theta' + \phi' - \pi)\delta[t' + \tilde{t}\cos(\phi' + \theta')] \\ &\quad - H(\theta' - \phi' - \pi)\delta[t' + \tilde{t}\cos(\phi' - \theta')] \\ &\quad - \frac{H(t'-\tilde{t})\sqrt{\tilde{t}}}{\sqrt{2}\pi\sqrt{t'-\tilde{t}}}\left(\frac{\cos\frac{\phi'+\theta'}{2}}{t'+\tilde{t}\cos(\phi'+\theta')} - \frac{\cos\frac{\phi'-\theta'}{2}}{t'+\tilde{t}\cos(\phi'-\theta')}\right) \end{aligned} \tag{23b}$$

in the case of H-field. Here, $\tilde{t} = \rho'/c$.

The first term on the right hand side of (23a) and (23b) is interpreted as the signal reflected from the screen, the second term cancels out the source signal in its shadow region, and the third term accounts for the diffraction of the source signal by the edge of the screen.

In order to be able to perform the Lorentz transformation of the scattered field from the moving to the laboratory frame we must find all the remaining components of this field.

In the case of E-field, the magnetic induction is given by

$$\mathbf{B}' = \hat{y}' \times \int_{-\infty}^{t'} \nabla u' \, d\tau = \hat{\mathbf{x}}' B'_{\parallel} + \hat{\mathbf{z}}' B'_{\perp}, \tag{24}$$

where

$$B'_{\parallel} = \int_{-\infty}^{t'} \frac{\partial u'}{\partial z'} \, d\tau, \qquad B'_{\perp} = -\int_{-\infty}^{t'} \frac{\partial u'}{\partial x'} \, d\tau. \tag{25}$$

If we consider the function

$$v'(\rho', \phi', t') = \frac{H(t' - \tilde{t})\sqrt{\tilde{t}}}{\sqrt{2}\pi\sqrt{t' - \tilde{t}}} \frac{\cos\frac{\phi' \pm \theta'}{2}}{t' + \tilde{t}\cos(\phi' \pm \theta')}, \tag{26}$$

then straightforward calculation of $\nabla v'$ shows that while both derivatives as a whole are integrable functions, they contain components that are non-integrable in arbitrarily small vicinity of $t' = \tilde{t}$. To overcome this difficulty we express $\partial v'/\partial\tilde{t}$ by $\partial v'/\partial t'$:

$$\frac{\partial v'}{\partial \tilde{t}} = -\frac{\partial v'}{\partial t'} + v' \frac{t' - 2\tilde{t} - \tilde{t}\cos(\phi' \pm \theta')}{2\tilde{t}[t' + \tilde{t}\cos(\phi' \pm \theta')]}, \tag{27}$$

and after some calculations obtain

$$c\int_{-\infty}^{t'} \frac{\partial v'}{\partial z'} \, d\tau = -v' \sin\phi' + \frac{H(t' - \tilde{t})}{\sqrt{2\tilde{t}}\pi} \frac{\sqrt{t' - \tilde{t}} \, \sin\frac{\phi' \mp \theta'}{2}}{t' + \tilde{t}\cos(\phi' \pm \theta')} \tag{28}$$

and

$$c\int_{-\infty}^{t'} \frac{\partial v'}{\partial x'}\, d\tau = -v' \cos\phi' + \frac{H(t'-\tilde{t})}{\sqrt{2\tilde{t}}\pi}\ \frac{\sqrt{t'-\tilde{t}}\ \cos\frac{\phi' \mp \theta'}{2}}{t' + \tilde{t}\cos(\phi' \pm \theta')}. \tag{29}$$

Denote for short

$$u^{x'\pm}(\rho',\phi',t') = -H(t'-\tilde{t})\frac{\sqrt{t'-\tilde{t}}}{\sqrt{2\tilde{t}}\pi}\left[\frac{\sin\frac{\phi'-\theta'}{2}}{t'+\tilde{t}\cos(\phi'+\theta')} \pm \frac{\sin\frac{\phi'+\theta'}{2}}{t'+\tilde{t}\cos(\phi'-\theta')}\right], \tag{30}$$

$$u^{z'\pm}(\rho',\phi',t') = -H(t'-\tilde{t})\frac{\sqrt{t'-\tilde{t}}}{\sqrt{2\tilde{t}}\pi}\left[\frac{\cos\frac{\phi'-\theta'}{2}}{t'+\tilde{t}\cos(\phi'+\theta')} \pm \frac{\cos\frac{\phi'+\theta'}{2}}{t'+\tilde{t}\cos(\phi'-\theta')}\right] \tag{31}$$

and

$$\begin{aligned} u^{p'\pm}(\rho',\phi',t') \;=\; & H(\theta'+\phi'-\pi)\delta[t'+\tilde{t}\cos(\phi'+\theta')] \\ & \pm H(\theta'-\phi'-\pi)\delta[t'+\tilde{t}\cos(\phi'-\theta')]. \end{aligned} \tag{32}$$

Then on virtue of (23a), (25), (28) and (29) the magnetic induction components in the case of the E-field are found to be

$$c\mathbf{B}'_{\|} = \hat{\mathbf{x}}'(-\sin\phi'\cdot u^{d'+} - u^{x'+} + \sin\theta'\cdot u^{p'-}) \tag{33}$$

and

$$c\mathbf{B}'_{\perp} = \hat{\mathbf{z}}'(\cos\phi'\cdot u^{d'+} + u^{z'+} + \cos\theta'\cdot u^{p'+}). \tag{34}$$

In a similar manner we obtain for the H-field

$$\mathbf{E}'_{\|} = \hat{\mathbf{x}}'(\sin\phi'\cdot u^{d'-} + u^{x'-} + \sin\theta'\cdot u^{p'+}) \tag{35}$$

and

$$\mathbf{E}'_{\perp} = \hat{\mathbf{z}}'(-\cos\phi'\cdot u^{d'+} - u^{z'-} + \cos\theta'\cdot u^{p'-}). \tag{36}$$

4. FIELDS IN THE LABORATORY FRAME

Transformation of the fields from S' frame to S frame is given by

$$\begin{aligned} \mathbf{E}_{\|} \;=\; & \mathbf{E}'_{\|}, \qquad c\mathbf{B}_{\|} = c\mathbf{B}'_{\|}, \\ \mathbf{E}_{\perp} \;=\; & \gamma(\mathbf{E}'_{\perp} - \boldsymbol{\beta}\times c\mathbf{B}'_{\perp}), \quad c\mathbf{B}_{\perp} = \gamma(c\mathbf{B}'_{\perp} + \boldsymbol{\beta}\times\mathbf{E}'_{\perp}). \end{aligned} \tag{37}$$

As a result, in the case of E-field, perpendicular to the direction of the screen motion field components in the laboratory frame are

$$\mathbf{E}_{\perp} = \hat{\mathbf{y}}\gamma[(1 + \beta\cos\phi') \cdot u^{d'+} + \beta \cdot u^{z'+} - (1 - \beta\cos\theta') \cdot u^{p'+}], \quad (38)$$

$$c\mathbf{B}_{\perp} = \hat{\mathbf{z}}\gamma[(\beta + \cos\phi') \cdot u^{d'+} + u^{z'+} + (\cos\theta' - \beta) \cdot u^{p'+}], \quad (39)$$

and in the case of H-field

$$c\mathbf{B}_{\perp} = \hat{\mathbf{y}}\gamma[(1 + \beta\cos\phi') \cdot u^{d'-} + \beta \cdot u^{z'-} + (1 - \beta\cos\theta') \cdot u^{p'-}], \quad (40)$$

$$\mathbf{E}_{\perp} = \hat{\mathbf{z}}\gamma[-(\beta + \cos\phi') \cdot u^{d'-} - u^{z'-} + (\cos\theta' - \beta) \cdot u^{p'-}]. \quad (41)$$

These formulas, together with (33) and (35), represent the full scattered field in the laboratory frame. To keep them in a compact form they are expressed in primed coordinates. The change to unprimed ones can be accomplished with the help of (7) and (11).

The total field can be characterized as follows:

(i) As in the stationary case this field consists of three wave species: the source, the reflected and the diffracted field. The incident and reflected fields are plane waves appearing only in their illuminated regions and being nonzero only on their wave fronts. The diffracted signal does not vanish behind its wave front. This front has the form of an expanding circular cylinder surface $ct = \rho$, centered at the point $x = 0$, $z = 0$, where the front of the source field has hit the screen edge. The fact that the diffracted signal is nonzero behind its front is interpreted physically ([14]) as a result of field contributions from distant parts of the edge, and is characteristic of 2D problems. On its front the diffracted wave has an algebraic singularity.

(ii) The boundary condition on the screen is satisfied separately by the sum of the incident and reflected signal, and by the diffracted signal itself.

(iii) Unlike the stationary case, the shadow boundaries of the source and the reflected pulses are not parallel to the directions of their propagation.These boundaries are given by (comp. [3])

$$\frac{z}{x - vt} = \pm\tan\theta \, \frac{1}{1 + \frac{\beta}{\cos\theta}}. \quad (42)$$

The factor multiplying $\tan\theta$ introduces the rotation of a shadow boundary towards the negative x half-axis. As a result, the illuminated and shadow regions are modified as compared to the stationary case (see Fig. 2).

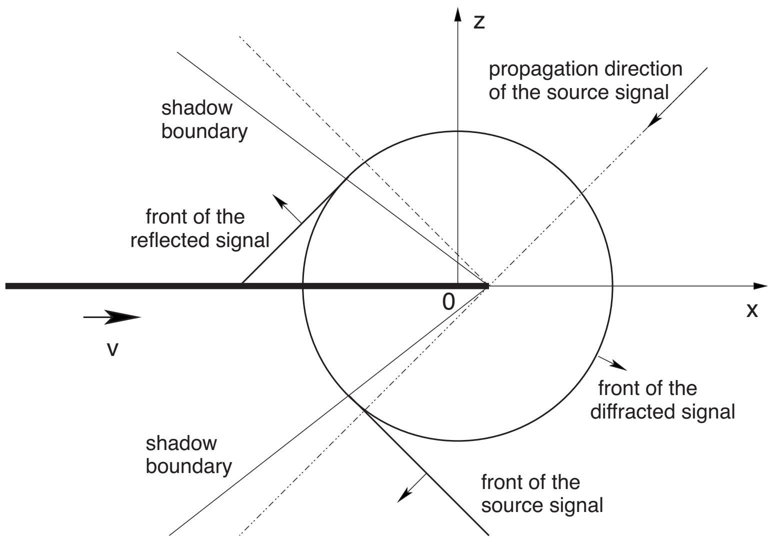

Figure 2. Wavefronts of incident, reflected and diffracted pulses due to scattering by the moving half-plane.

5. THE CASE OF MODERATE AND SMALL VELOCITY V

In most, if not in all applications, the velocity of a scattering object is much smaller than the velocity of light, i.e. $\beta << 1$. Under this assumption our field description in the laboratory frame can be significantly simplified by expanding appropriate formulas in powers of β and retaining only linear part of the expansion. In this procedure one should keep in mind that $\beta ct = vt$ appearing in x' need not be small as compared to x.

Thus we find

$$x' = x - vt + O(\beta^2), \quad t' = t - \beta\frac{x}{c} + O(\beta^2), \quad \rho' = \bar{\rho} + O(\beta^2), \quad (43)$$

where $\bar{\rho} = \sqrt{(x - vt)^2 + z^2}$,

$$\cos\theta' = \cos\theta + \beta\sin^2\theta + O(\beta^2), \quad \sin\theta' = \sin\theta - \beta\sin\theta\cos\theta + O(\beta^2), \quad (44)$$

and

$$\cos\phi' = (x - vt)/\bar{\rho} + O(\beta^2), \quad \sin\phi' = z/\bar{\rho} + O(\beta^2). \quad (45)$$

Let us introduce the angle $\bar{\phi}$ measured between the point (x, z), the edge of the screen, and the semi-axis x, for which †

$$\cos\bar{\phi} = (x - vt)/\bar{\rho}, \quad \text{and} \quad \sin\bar{\phi} = z/\bar{\rho}. \tag{46}$$

Then by (44) and (45)

$$\cos(\phi' \pm \theta') = \cos(\bar{\phi} \pm \theta) \pm \beta \sin\theta \sin(\bar{\phi} \pm \theta) + O(\beta^2), \tag{47}$$

and since

$$\cos(w + \epsilon) = \cos w - \epsilon \sin w + O(\epsilon^2), \quad \text{as} \quad \epsilon \to 0,$$

we have

$$\cos(\phi' \pm \theta') = \cos(\bar{\phi} \pm \theta \mp \beta \sin\theta) + O(\beta^2). \tag{48}$$

Hence

$$\phi' \pm \theta' \approx \bar{\phi} \pm \theta \mp \beta \sin\theta. \tag{49}$$

The formulas (43) through (49) give simple approximate transformations of the variables from the primed to the laboratory frame.

By using these formulas in (38) and (40) we arrive at the following approximate representations for the transverse electric and magnetic induction fields in E and H case

$$\begin{aligned}\begin{Bmatrix}\mathbf{E}_\perp \\ c\mathbf{B}_\perp\end{Bmatrix} = -\hat{\mathbf{y}}\Bigg\{ c\frac{H(ct-\bar{\rho})}{\sqrt{2}\pi}&\left[\sqrt{\frac{\bar{\rho}}{ct-\bar{\rho}}} + \beta\left(\frac{x\bar{\rho}}{2(ct-\bar{\rho})^{3/2}} + \frac{x-vt}{\sqrt{\bar{\rho}}\sqrt{ct-\bar{\rho}}} + \sqrt{\frac{ct-\bar{\rho}}{\bar{\rho}}}\right)\right] \\ &\times\left(\frac{\cos[(\bar{\phi}-\theta+\beta\sin\theta)/2]}{ct-\beta x+\bar{\rho}\cos(\bar{\phi}-\theta+\beta\sin\theta)} \pm \frac{\cos[(\bar{\phi}+\theta-\beta\sin\theta)/2]}{ct-\beta x+\bar{\rho}\cos(\bar{\phi}+\theta-\beta\sin\theta)}\right) \\ &+(1-\beta\cos\theta)\big[H(\theta-\bar{\phi}-\beta\sin\theta-\pi)\delta[ct-\beta x+\bar{\rho}\cos(\bar{\phi}-\theta+\beta\sin\theta)] \\ &\pm H(\bar{\phi}+\theta-\beta\sin\theta-\pi)\delta[ct-\beta x+\bar{\rho}\cos(\bar{\phi}+\theta-\beta\sin\theta)]\big]\Bigg\},\end{aligned} \tag{50}$$

respectively.

The remaining field components can be found by using the same substitutions in (33), (35), (39) and (41).

† $\bar{\rho}$, $\bar{\phi}$ and z form a cylindrical coordinate system in the laboratory frame S with its origin at the half-plane edge.

6. CONCLUSIONS

In this paper 2D problem of EM plane pulse diffraction by a moving half-plane has been considered. It was assumed that the half-plane is perfectly conducting and the surrounding medium is non-dispersive. The total fieeld has been constructed and its simplification was found for the case where the velocity of the screen is significantly smaller than the velocity of light. The results of this work can be exploited in solving a similar problem, where the source pulse has an envelope different from the Dirac function.

ACKNOWLEDGMENT

This work was partially supported by Polish Ministry of Education and Science with funds granted for research in 2005-2008 under grant no. 0 T00A 014 29.

REFERENCES

1. Lee, S. W. and R. Mittra, "Scattering of electromagnetic waves by a moving cylinder in free space," *Can. J. Phys.*, Vol. 45, No. 9, 2999–3007, 1967.
2. Tsandoulas, G. N., "Electromagnetic diffraction by a moving wedge," *Radio Sci.*, Vol. 3, No. 9, 887–893, 1968.
3. Ciarkowski, A. and B. Atamaniuk, "Electromagnetic diffraction by a moving half-plane," *J. Tech. Phys.*, Vol. 46, No. 4, 203–212, 2005.
4. De Cupis, P. G. Gerosa, and G. Schettini, "Electromagnetic scattering by an object in relativistic translational motion", *J. Electromagn. Waves Appl.*, Vol. 14, No. 8, 1037–1062, 2000.
5. De Cupis, P., G. Gerosa, and G. Schettini, "Gaussian beam diffraction by uniformly moving targets," *Atti della Fondazione Gorgio Ronchi*, Vol. LVI, Nos. 4–5, 799–811, 2001.
6. De Cupis, P., P. Burghignoli, G. Gerosa, and M. Marziale, "Electromagnetic wave scattering by a perfectly conducting wedge in uniform translational motion," *J. Electromagn. Waves Appl.*, Vol. 16, No. 8, 345–364, 2002.
7. Felsen, L. B. (ed.), *Transient Electromagnetic Fields*, Chap. 1, Springer-Verlag, 1976.
8. Ciarkowski, A., "On Sommerfeld precursor in Lorentz medium," *J. Tech. Phys.*, Vol. 43, No. 2, 187–203, 2002.

9. Ciarkowski, A., "Propagation of the main signal in a dispersive Lorentz medium," *Archives of Acoustics*, Vol. 27, No. 4, 339–349, 2002.
10. Ciarkowski, A., "Dependence of the Brillouin precursor form on the initial signal rise-time," *J. Tech. Phys.*, Vol. 44, No. 2, 181–192, 2003.
11. Born, M. and E. Wolf, *Principles of Optics*, Cambridge, 1964.
12. Noble, B., *Methods based on the Wiener-Hopf Technique*, Pergamon, 1958.
13. Ciarkowski, A., J. Boersma, and R. Mittra, "Plane wave diffraction by a wedge — A spectral domain approach," *IEEE Trans. Antennas Propagat.*, Vol. AP-32, No. 1, 20–29, 1984.
14. Jones, D. S., *The Theory of Electromagnetism*, Pergamon Press, 1964.