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DIFFRACTION OF PLANE WAVE BY TWO PARALLEL SLITS IN AN INFINITELY LONG IMPEDANCE PLANE USING THE METHOD OF KOBAYASHI POTENTIAL

A. Imran and Q. A. Naqvi

Department of Electronics
Quaid-i-Azam University Islamabad
Islamabad, Pakistan

K. Hongo

EM Laboratory
Sakura, Japan

Abstract—We studied the diffraction of E-polarized electromagnetic plane wave by two parallel slits in an infinitely long impedance plane. Analysis is based on the concept of Kobayashi potential. Imposition of required boundary conditions leads to dual integral equations. The dual integral equations can be reduced to matrix equations with the infinite unknowns by using the properties of Weber-Schafheitlin's discontinuous integrals and Jacobi's polynomials. Matrix elements are given in terms of indefinite integrals which are difficult to evaluate analytically. The matrix elements are solved numerically. Diffracted far fields in the upper half space are studied.

1. INTRODUCTION

The problem of diffraction of electromagnetic waves by an infinite slit in a conducting screen has been studied extensively [1–4]. Morse and Rubenstein [1] treated the problem of diffraction of acoustic wave by using the method of separation of variables. Using the plane wave spectrum representation of electromagnetic fields, Clemmow [2] derived dual integral equations for the diffracted field by a slit. He discussed the approximate treatment of two complementary cases assuming that the slit width is much greater or much smaller than the wavelength. Hongo [5] studied diffraction from two parallel slits in a conducting

plane using Kobayashi potential method. In the present work, the Hongo's work [5] has been extended to slits in an impedance plane.

Kobayashi potential is an analytical technique for solving the mixed boundary value problems and was developed by Iwao Kobayashi in the beginning of 1930's. The method has been successfully used in solving various problems in electromagnetics and acoustic [9, 10]. The method uses the discontinuity properties of Weber-Schafheitlin's integrals and is closely related to the method of moments approach.

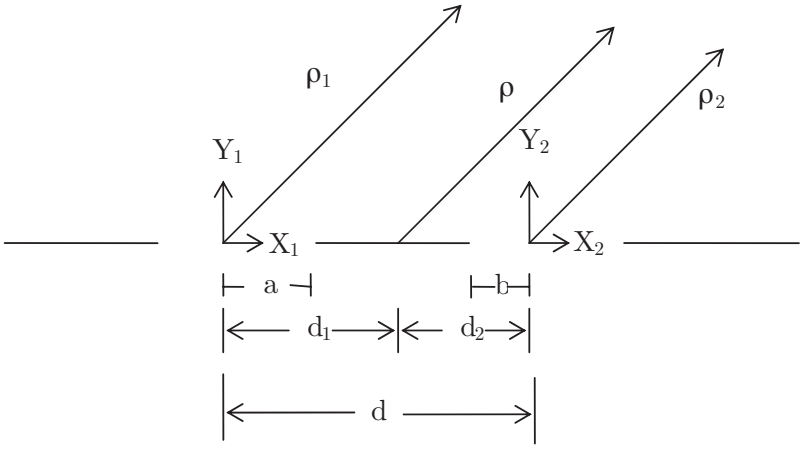

Figure 1. Two parallel infinitely long slits in an impedance plane.

2. FORMULATION

Consider two slits of width $2a$ and $2b$ in an impedance plane of negligible thickness with Z_+ and Z_- as impedances of the upper and lower faces respectively. The slit having width $2a$ is termed as slit 1 while slit 2 has the width of $2b$. The distance between the centers of the two slits is d. The geometry is shown in Figure 1. It may be noted that (x_1, y_1) are the local coordinates for slit 1 while (x_2, y_2) are the local coordinates for slit 2. E-polarized electromagnetic plane wave has been considered as an incident wave. That is

$$E_z^i = \exp[jk(x\cos\phi_0 + y\sin\phi_0)]$$

where ϕ_0 is the angle of incidence. The reflected field may be written as

$$E_z^r = -\frac{Z_0 - Z_+\sin\phi_0}{Z_0 + Z_+\sin\phi_0}\exp\Big[jk(x\cos\phi_0 - y\sin\phi_0)\Big]$$

The diffracted fields in the upper and lower half spaces may be written in terms of unknowns as

$$E_z^{d+} = \int_0^\infty \{g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\}\exp\left[-\sqrt{\xi^2-\kappa_a^2}y_a\right]d\xi + \int_0^\infty \{g_3(\xi)\cos(x_b\xi) + g_4(\xi)\sin(x_b\xi)\}\exp\left[-\sqrt{\xi^2-\kappa_b^2}y_b\right]d\xi \quad \text{for } y>0$$

$$E_z^{d-} = \int_0^\infty \{h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\}\exp\left[\sqrt{\xi^2-\kappa_a^2}y_a\right]d\xi + \int_0^\infty \{h_3(\xi)\cos(x_b\xi) + h_4(\xi)\sin(x_b\xi)\}\exp\left[\sqrt{\xi^2-\kappa_b^2}y_b\right]d\xi \quad \text{for } y<0$$

where $\kappa_a = ka$, $\kappa_b = kb$, $x_a = \frac{x_1}{a}$, $y_a = \frac{y_1}{a}$, $y_b = \frac{y_2}{b}$, $x_b = \frac{x_2}{b}$. Using the discontinuity properties of Weber-Schafheitlin's integrals the above coefficients for slit 1 may be written as [5]

$$g_1(\xi) = \frac{j\kappa_a}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta_+}\sum_{m=0}^\infty A_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$g_2(\xi) = \frac{j\kappa_a}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta_+}\sum_{m=0}^\infty B_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$h_1(\xi) = \frac{j\kappa_a}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta_-}\sum_{m=0}^\infty C_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$h_2(\xi) = \frac{j\kappa_a}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta_-}\sum_{m=0}^\infty D_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}}$$

Similarly coefficients corresponding to slit 2 may be written as

$$g_3(\xi) = \frac{j\kappa_b}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta_+}\sum_{m=0}^\infty E_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$g_4(\xi) = \frac{j\kappa_b}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta_+}\sum_{m=0}^\infty F_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$h_3(\xi) = \frac{j\kappa_b}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta_-}\sum_{m=0}^\infty G_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}}$$

$$h_4(\xi) = \frac{j\kappa_b}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta_-}\sum_{m=0}^\infty H_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}}$$

where ζ_+ and ζ_- are the normalized impedances of upper and lower faces of the impedance plane and A_m, B_m, C_m, D_m, E_m, F_m, G_m and H_m are the expansion coefficients to be determined.

The boundary conditions associated with the geometry are

$$H_x^t\Big|_{y=0_+} = H_x^t\Big|_{y=0_-} \qquad \text{for} \qquad |x_a| \leq 1 \tag{1a}$$

$$H_x^t\Big|_{y=0_+} = H_x^t\Big|_{y=0_-} \qquad \text{for} \qquad |x_b| \leq 1 \tag{1b}$$

$$E_z^t\Big|_{y=0_+} = E_z^t\Big|_{y=0_-} \qquad \text{for} \qquad |x_a| \leq 1 \tag{1c}$$

$$E_z^t\Big|_{y=0_+} = E_z^t\Big|_{y=0_-} \qquad \text{for} \qquad |x_b| \leq 1 \tag{1d}$$

where superscript 't' stands for total. Boundary condition (1a) has been used. Relation $x_1 - d = x_2$ relates the two local coordinate systems Using the additional theorem for the trigonometric functions and then comparing even and odd functions, following is obtained

$$\begin{aligned}
&\frac{1}{a}\int_0^\infty \left[\sqrt{\xi^2-\kappa_a^2}\right]\{g_1(\xi)+h_1(\xi)\}\cos(x_a\xi)\,d\xi \\
&+\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right][\{g_3(\xi)+h_3(\xi)\}\cos(x_a\xi R)\cos(d_a\xi R) \\
&\qquad -\{g_4(\xi)+h_4(\xi)\}\cos(x_a\xi R)\sin(d_a\xi R)]d\xi \\
=\ &\frac{2j\kappa_a\sin\phi_0}{1+\zeta_+\sin\phi_0}\cos(\kappa_a x_a\cos\phi_0)
\end{aligned} \tag{2a}$$

and

$$\begin{aligned}
&\frac{1}{a}\int_0^\infty \left[\sqrt{\xi^2-\kappa_a^2}\right]\{g_2(\xi)+h_2(\xi)\}\sin(x_a\xi)\,d\xi \\
&+\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right][\{g_3(\xi)+h_3(\xi)\}\sin(x_a\xi R)\sin(d_a\xi R) \\
&\qquad +\{g_4(\xi)+h_4(\xi)\}\sin(x_a\xi R)\cos(d_a\xi R)]d\xi \\
=\ &-\frac{2\kappa_a\sin\phi_0}{1+\zeta_+\sin\phi_0}\sin(\kappa_a x_a\cos\phi_0)
\end{aligned} \tag{2b}$$

where $d_a = \frac{d}{a}$ and $R = \frac{a}{b}$

Expanding the trigonometric and Bessel functions in terms of orthogonal set of Jacobi's polynomials and using the expansion formula

$$x^{-m/2}J_m(\xi\sqrt{x}) = \sum_{m=0}^{m=\infty} 2(2n+m+1)\frac{\Gamma(n+m+1)}{\Gamma(n+1)\Gamma(m+1)}\frac{J_{(2n+m+1)}}{\xi}p_n^m(x)$$

where $p_n^m(x)$ is defined by

$$p_n^m = \frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+1)} x^{-m/2} \int_0^\infty J_m(\sqrt{x}\xi) J_{2n+m+1}(\xi) d\xi$$

following is obtained

$$\begin{aligned}
&\frac{1}{a}\int_0^\infty \left[\sqrt{\xi^2-\kappa_a^2}\right] \{g_1(\xi)+h_1(\xi)\} J_{2n+\frac{1}{2}}(\xi) \frac{1}{\sqrt{\xi}} d\xi \\
&+\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right] \left[\{g_3(\xi)+h_3(\xi)\} J_{2n+\frac{1}{2}}(\xi R)\cos(d_a\xi R)\right] \frac{1}{\sqrt{R\xi}} d\xi \\
&-\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right] \left[\{g_4(\xi)+h_4(\xi)\} J_{2n+\frac{1}{2}}(\xi R)\sin(d_a\xi R)\right] \frac{1}{\sqrt{R\xi}} d\xi \\
&= \frac{2j\kappa_a \sin\phi_0}{1+\zeta_+ \sin\phi_0} \frac{J_{2n+1/2}(\kappa_a \cos\phi_0)}{(\kappa_a \cos\phi_0)^{1/2}} \qquad (3a)
\end{aligned}$$

and result corresponding to equation (2b) is

$$\begin{aligned}
&\frac{1}{a}\int_0^\infty \left[\sqrt{\xi^2-\kappa_a^2}\right] \{g_2(\xi)+h_2(\xi)\} J_{2n+\frac{3}{2}}(\xi) \frac{1}{\sqrt{\xi}} d\xi \\
&+\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right] \left[\{g_3(\xi)+h_3(\xi)\} J_{2n+\frac{3}{2}}(\xi R)\sin(d_a\xi R)\right] \frac{1}{\sqrt{R\xi}} d\xi \\
&+\frac{1}{b}\int_0^\infty \left[\sqrt{\xi^2-\kappa_b^2}\right] \left[\{g_4(\xi)+h_4(\xi)\} J_{2n+\frac{3}{2}}(\xi R)\cos(d_a\xi R)\right] \frac{1}{\sqrt{R\xi}} d\xi \\
&= -\frac{2\kappa_a \sin\phi_0}{1+\zeta_+ \sin\phi_0} \frac{J_{2n+3/2}(\kappa_a \cos\phi_0)}{(\kappa_a \cos\phi_0)^{1/2}} \qquad (3b)
\end{aligned}$$

Let $\zeta_+ = \zeta_- = \zeta$ and putting the values of $g_1 \sim g_2$ and $h_1 \sim h_2$ we have

$$\begin{aligned}
&\sum_{m=0}^\infty \int_0^\infty \left[\frac{j\kappa_a \sqrt{\xi^2-\kappa_a^2}\frac{1}{a}}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta}\right] J_{2m+\frac{1}{2}}(\xi) J_{2n+\frac{1}{2}}(\xi) [A_m + C_m] \frac{1}{\xi} d\xi \\
&+\int_0^\infty \left[\frac{j\kappa_b \sqrt{\xi^2-\kappa_b^2}\frac{1}{b}}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta}\right] J_{2m+\frac{1}{2}}(\xi) J_{2n+\frac{1}{2}}(\xi R)\cos(d_a\xi R) \\
&\times [E_m + G_m] \frac{1}{\sqrt{R\xi}} d\xi \\
&-\int_0^\infty \left[\frac{j\kappa_b \sqrt{\xi^2-\kappa_b^2}\frac{1}{b}}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta}\right] J_{2m+\frac{3}{2}}(\xi) J_{2n+\frac{1}{2}}(\xi R)\sin(d_a\xi R)
\end{aligned}$$

$$\times [F_m + H_m] \frac{1}{\sqrt{R}\xi} d\xi$$
$$= \frac{2j\kappa_a \sin\phi_0}{1+\zeta\sin\phi_0} \frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}} \tag{4a}$$

and

$$\sum_{m=0}^{\infty} \int_0^{\infty} \left[\frac{\frac{1}{a} j\kappa_a \sqrt{\xi^2-\kappa_a^2}}{j\kappa_a + \sqrt{\xi^2-\kappa_a^2}\zeta} \right] J_{2m+\frac{3}{2}}(\xi) J_{2n+\frac{3}{2}}(\xi) [B_m + D_m] \frac{1}{\xi} d\xi$$
$$+ \int_0^{\infty} \left[\frac{\frac{1}{b} j\kappa_b \sqrt{\xi^2-\kappa_b^2}}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta} \right] J_{2m+\frac{1}{2}}(\xi) J_{2n+\frac{3}{2}}(\xi R) \sin(d_a \xi R)$$
$$\times [E_m + G_m] \frac{1}{\sqrt{R}\xi} d\xi$$
$$+ \int_0^{\infty} \left[\frac{\frac{1}{b} j\kappa_b \sqrt{\xi^2-\kappa_b^2}}{j\kappa_b + \sqrt{\xi^2-\kappa_b^2}\zeta} \right] J_{2m+\frac{3}{2}}(\xi) J_{2n+\frac{3}{2}}(\xi R) \cos(d_a \xi R)$$
$$\times [F_m + H_m] \frac{1}{\sqrt{R}\xi} d\xi$$
$$= -\frac{2\kappa_a \sin\phi_0}{1+\zeta\sin\phi_0} \frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}} \tag{4b}$$

Above two expressions may be written as

$$\sum_{m=0}^{\infty} \Big\{ K_a(2m+1/2, 2n+1/2; \kappa_a) A_m + K_a(2m+1/2, 2n+1/2; \kappa_a) C_m$$
$$+ K_{ca}(2m+1/2, 2n+1/2; d_a) E_m + K_{ca}(2m+1/2, 2n+1/2; d_a) G_m$$
$$- K_{sa}(2m+3/2, 2n+1/2; d_a) F_m - K_{sa}(2m+3/2, 2n+1/2; d_a) H_m \Big\}$$
$$= \frac{2j\kappa_a \sin\phi_0}{1+\zeta\sin\phi_0} \frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}} \tag{5a}$$
$$\sum_{m=0}^{\infty} \Big\{ K_a(2m+3/2, 2n+3/2; \kappa_a) B_m + K_a(2m+3/2, 2n+3/2; \kappa_a) D_m$$
$$+ K_{sa}(2m+1/2, 2n+3/2; d_a) E_m + K_{Sa}(2m+1/2, 2n+3/2; d_a) G_m$$
$$+ K_{ca}(2m+3/2, 2n+3/2; d_a) F_m + K_{Ca}(2m+3/2, 2n+3/2; d_a) H_m \Big\}$$
$$= -\frac{2\kappa_a \sin\phi_0}{1+\zeta\sin\phi_0} \frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}} \tag{5b}$$

where

$$K_a(m,n,\kappa_a)=\frac{1}{a}\int_0^\infty\left[\frac{j\kappa_a\sqrt{\xi^2-\kappa_a^2}}{j\kappa_a+\sqrt{\xi^2-\kappa_a^2}\zeta}\right]J_m(\xi)J_n(R\xi)\frac{1}{\xi}d\xi$$

$$K_{ca}(m,n,d_a)=\frac{1}{b}\int_0^\infty\left[\frac{j\kappa_b\sqrt{\xi^2-\kappa_b^2}}{j\kappa_b+\sqrt{\xi^2-\kappa_b^2}\zeta}\right]J_m(\xi)J_n(R\xi)\cos(Rd_a\xi)\frac{1}{\sqrt{R}\xi}d\xi$$

$$K_{sa}(m,n,d_a)=\frac{1}{b}\int_0^\infty\left[\frac{j\kappa_b\sqrt{\xi^2-\kappa_b^2}}{j\kappa_b+\sqrt{\xi^2-\kappa_b^2}\zeta}\right]J_m(\xi)J_n(R\xi)\sin(Rd_a\xi)\frac{1}{\sqrt{R}\xi}d\xi$$

Applying the remaining boundary conditions and proceeding as above we finally get the following set of expressions

$$\begin{aligned}&\sum_{m=0}^{\infty}\Big\{K_b(2m+1/2,2n+1/2;\kappa_b)E_m+K_b(2m+1/2,2n+1/2;\kappa_b)G_m\\&+K_{cb}(2m+1/2,2n+1/2;d_b)A_m+K_{cb}(2m+1/2,2n+1/2;d_b)C_m\\&+K_{sb}(2m+3/2,2n+1/2;d_b)B_m+K_{sb}(2m+3/2,2n+1/2;d_b)D_m\Big\}\\=&\ \frac{2j\kappa_b\sin\phi_0}{1+\zeta\sin\phi_0}\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\end{aligned}\tag{5c}$$

$$\begin{aligned}&\sum_{m=0}^{\infty}\Big\{K_b(2m+3/2,2n+3/2;\kappa_b)F_m+K_b(2m+3/2,2n+3/2;\kappa_b)H_m\\&-K_{sb}(2m+1/2,2n+3/2;d_b)A_m-K_{sb}(2m+1/2,2n+3/2;d_b)C_m\\&+K_{cb}(2m+3/2,2n+3/2;d_b)D_m+K_{cb}(2m+3/2,2n+3/2;d_a)B_m\Big\}\\=&\ -\frac{2\kappa_b\sin\phi_0}{1+\zeta\sin\phi_0}\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\end{aligned}\tag{5d}$$

$$\begin{aligned}&\sum_{m=0}^{\infty}\Big\{G_a(2m+1/2,2n+1/2;\kappa_a)A_m-G_a(2m+1/2,2n+1/2;\kappa_a)C_m\\&+G_{ca}(2m+1/2,2n+1/2;d_a)E_m-G_{ca}(2m+1/2,2n+1/2;d_a)G_m\\&-G_{sa}(2m+3/2,2n+1/2;d_a)F_m+G_{sa}(2m+3/2,2n+1/2;d_a)H_m\Big\}\\=&\ -\frac{2\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\end{aligned}\tag{5e}$$

$$\sum_{m=0}^{\infty}\Big\{G_a(2m+3/2,2n+3/2;\kappa_a)B_m - G_a(2m+3/2,2n+3/2;\kappa_a)D_m$$
$$+G_{sa}(2m+1/2,2n+3/2;d_a)E_m - G_{sa}(2m+1/2,2n+3/2;d_a)G_m$$
$$+G_{ca}(2m+3/2,2n+3/2;d_a)F_m - G_{ca}(2m+3/2,2n+3/2;d_a)H_m\Big\}$$
$$= -\frac{2j\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}} \tag{5f}$$

$$\sum_{m=0}^{\infty}\Big\{G_b(2m+1/2,2n+1/2;\kappa_b)E_m - G_b(2m+1/2,2n+1/2;\kappa_b)G_m$$
$$-G_{cb}(2m+1/2,2n+1/2;d_b)C_m + G_{cb}(2m+1/2,2n+1/2;d_b)A_m$$
$$-G_{sb}(2m+3/2,2n+1/2;d_b)D_m + G_{sb}(2m+3/2,2n+1/2;d_a)B_m\Big\}$$
$$= -\frac{2\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}} \tag{5g}$$

$$\sum_{m=0}^{\infty}\Big\{G_b(2m+3/2,2n+3/2;\kappa_b)F_m - G_b(2m+3/2,2n+3/2;\kappa_b)H_m$$
$$+G_{sb}(2m+1/2,2n+3/2;d_b)C_m - G_{sb}(2m+1/2,2n+3/2;d_b)A_m$$
$$-G_{cb}(2m+3/2,2n+3/2;d_b)D_m + G_{cb}(2m+3/2,2n+3/2;d_b)B_m\Big\}$$
$$= -\frac{2j\zeta\sin\phi_0}{1+\sin\phi_0}\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}} \tag{5h}$$

where

$$K_b(m,n,\kappa_b) = \int_0^{\infty}\left[\frac{\frac{1}{b}j\kappa_b\sqrt{\xi^2-\kappa_b^2}}{j\kappa_b+\sqrt{\xi^2-\kappa_b^2}}\right]J_m(\xi)J_n(q\xi)\frac{1}{\xi}d\xi$$
$$K_{cb}(m,n,d_b) = \int_0^{\infty}\left[\frac{\frac{1}{a}j\kappa_a\sqrt{\xi^2-\kappa_a^2}}{j\kappa_a+\sqrt{\xi^2-\kappa_a^2}}\right]J_m(\xi)J_n(q\xi)\cos(qd_b\xi)\frac{1}{\sqrt{q}\xi}d\xi$$
$$K_{sb}(m,n,d_b) = \int_0^{\infty}\left[\frac{\frac{1}{a}j\kappa_a\sqrt{\xi^2-\kappa_a^2}}{j\kappa_a+\sqrt{\xi^2-\kappa_a^2}}\right]J_m(\xi)J_n(q\xi)\sin(qd_b\xi)\frac{1}{\sqrt{q}\xi}d\xi$$
$$G_a(m,n,\kappa_a) = \int_0^{\infty}\left[\frac{j\kappa_a}{j\kappa_a+\sqrt{\xi^2-\kappa_a^2}\zeta}\right]J_m(\xi)J_n(R\xi)\frac{1}{\xi}d\xi$$
$$G_{ca}(m,n,d_a) = \int_0^{\infty}\left[\frac{j\kappa_b}{j\kappa_b+\sqrt{\xi^2-\kappa_b^2}\zeta}\right]J_m(\xi)J_n(R\xi)\cos(Rd_a\xi)\frac{1}{\sqrt{R}\xi}d\xi$$

$$G_{sa}(m,n,d_a)=\int_0^{\infty}\left[\frac{j\kappa_b}{j\kappa_b+\sqrt{\xi^2-\kappa_b^2}\zeta}\right]J_m(\xi)J_n(R\xi)\sin(Rd_a\xi)\frac{1}{\sqrt{R\xi}}d\xi$$

and $d_b=\frac{d}{b},\ q=\frac{b}{a}$.

3. APPROXIMATE SOLUTIONS OF EXPANSION CO-EFFICIENTS

The integrals K_{ca}, K_{cb}, K_{sb},K_{sa}, G_{ca}, G_{cb}, G_{sb} and G_{sa} may be simplified using saddle point method and the integrals K_a, G_a,K_b, G_b may be computed using the standard methods. Equations from (5a) to (5h) may be written in the matrix form as

$$\begin{aligned}&[K_a(2m+1/2,2n+1/2;\kappa_a)]\,[A_m+C_m]\\&+[K_{ca}(2m+1/2,2n+1/2;d_a)]\,[E_m+G_m]\\&-[K_{sa}(2m+3/2,2n+1/2;d_a)]\,[F_m+H_m]\\=&\ \frac{2j\kappa_a\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right]\end{aligned}\tag{6a}$$

$$\begin{aligned}&[K_a(2m+3/2,2n+3/2;\kappa_a)]\,[B_m+D_m]\\&+[K_{ca}(2m+3/2,2n+3/2;\kappa_a)]\,[F_m+H_m]\\&+[K_{sa}(2m+1/2,2n+3/2;d_a)]\,[E_m+G_m]\\=&\ -\frac{2\kappa_a\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right]\end{aligned}\tag{6b}$$

$$\begin{aligned}&[K_b(2m+1/2,2n+1/2;\kappa_b)]\,[E_m+G_m]\\&+[K_{cb}(2m+1/2,2n+1/2;d_b)]\,[A_m+C_m]\\&+[K_{sb}(2m+3/2,2n+1/2;d_b)]\,[B_m+D_m]\\=&\ \frac{2j\kappa_b\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right]\end{aligned}\tag{6c}$$

$$\begin{aligned}&[K_b(2m+3/2,2n+3/2;\kappa_b)]\,[F_m+H_m]\\&+[K_{cb}(2m+3/2,2n+3/2;d_b)]\,[B_m+D_m]\\&-[K_{sb}(2m+1/2,2n+3/2;d_b)]\,[A_m+C_m]\\=&\ -\frac{2\kappa_b\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right]\end{aligned}\tag{6d}$$

$$
\begin{aligned}
&[G_a(2m+1/2,2n+1/2;\kappa_a)]\,[A_m-C_m]\\
&+[G_{ca}(2m+1/2,2n+1/2;d_a)]\,[E_m-G_m]\\
&+[G_{sa}(2m+3/2,2n+1/2;d_a)]\,[-F_m+H_m]\\
&= -\frac{2\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right] \qquad (6e)
\end{aligned}
$$

$$
\begin{aligned}
&[G_a(2m+3/2,2n+3/2;\kappa_a)]\,[B_m-D_m]\\
&+[G_{ca}(2m+3/2,2n+3/2;\kappa_a)]\,[F_m-H_m]\\
&+[G_{sa}(2m+1/2,2n+3/2;d_a)]\,[E_m-G_m]\\
&= -\frac{2j\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right] \qquad (6f)
\end{aligned}
$$

$$
\begin{aligned}
&[G_b(2m+1/2,2n+1/2;\kappa_b)]\,[E_m-G_m]\\
&+[G_{cb}(2m+1/2,2n+1/2;d_b)]\,[A_m-C_m]\\
&+[G_{sb}(2m+3/2,2n+1/2;d_b)]\,[B_m-D_m]\\
&= -\frac{2\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right] \qquad (6g)
\end{aligned}
$$

$$
\begin{aligned}
&[G_b(2m+3/2,2n+3/2;\kappa_b)]\,[F_m-H_m]\\
&+[G_{cb}(2m+1/2,2n+3/2;d_b)]\,[B_m-D_m]\\
&-[G_{sb}(2m+1/2,2n+3/2;d_b)]\,[A_m-C_m]\\
&= -\frac{2j\zeta\sin\phi_0}{1+\zeta\sin\phi_0}\left[\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right] \qquad (6h)
\end{aligned}
$$

The above equations may be solved by using block Gauss-Seidel procedure. Since K_{ca}, K_{cb}, K_{sb}, K_{sa}, G_{ca}, G_{cb}, G_{sb} and G_{sa} are the coupling integrals so for large separation between the slits they all go to zero and we get zeroth order solutions as given below

$$
\begin{aligned}
[A_m+C_m]^0 &= \frac{2j\kappa_a\sin\phi_0}{1+\zeta\sin\phi_0}\,[K_a(2m+1/2,2n+1/2;\kappa_a)]^{-1}\\
&\quad\times\left[\frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right]\\
[B_m+D_m]^0 &= -\frac{2\kappa_a\sin\phi_0}{1+\zeta\sin\phi_0}\,[K_a(2m+3/2,2n+3/2;\kappa_a)]^{-1}\\
&\quad\times\left[\frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right]
\end{aligned}
$$

$$
\begin{aligned}
[E_m + G_m]^0 &= \frac{2j\kappa_b \sin\phi_0}{1+\zeta\sin\phi_0}\left[K_b(2m+1/2, 2n+1/2;\kappa_b)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right] \\
[F_m + H_m]^0 &= -\frac{2\kappa_b \sin\phi_0}{1+\zeta\sin\phi_0}\left[K_b(2m+3/2, 2n+3/2;\kappa_b)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right] \\
[A_m - C_m]^0 &= -\frac{2\zeta \sin\phi_0}{1+\zeta\sin\phi_0}\left[G_a(2m+1/2, 2n+1/2;\kappa_a)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+1/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right] \\
[B_m - D_m]^0 &= -\frac{2j\zeta \sin\phi_0}{1+\zeta\sin\phi_0}\left[G_a(2m+3/2, 2n+3/2;\kappa_a)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+3/2}(\kappa_a\cos\phi_0)}{(\kappa_a\cos\phi_0)^{1/2}}\right] \\
[E_m - G_m]^0 &= -\frac{2\zeta \sin\phi_0}{1+\zeta\sin\phi_0}\left[G_b(2m+1/2, 2n+1/2;\kappa_b)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+1/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right] \\
[F_m - H_m]^0 &= -\frac{2j\zeta \sin\phi_0}{1+\zeta\sin\phi_0}\left[G_b(2m+3/2, 2n+3/2;\kappa_b)\right]^{-1} \\
&\quad \times \left[\frac{J_{2n+3/2}(\kappa_b\cos\phi_0)}{(\kappa_b\cos\phi_0)^{1/2}}\right]
\end{aligned}
$$

The first order solutions may be written as

$$
\begin{aligned}
[A_m + C_m]^1 &= [A_m + C_m]^0 - \left[K_a(2m+1/2, 2n+1/2;\kappa_a)\right]^{-1} \\
&\quad \times \left[K_{ca}(2m+1/2, 2n+1/2; d_a)\right][E_m + G_m]^0 \\
&\quad + \left[K_a(2m+1/2, 2n+1/2;\kappa_a)\right]^{-1} \\
&\quad \times \left[K_{sa}(2m+3/2, 2n+1/2; d_a)\right][F_m + H_m]^0 \\
&= [A_m + C_m]^0 + \left[\kappa_a\sqrt{\frac{\pi}{2}}\frac{\exp(-j[kd - \frac{\pi}{4}])}{(kd)3/2}\right] \\
&\quad \times \left[\left[K_a(2m+1/2, 2n+1/2;\kappa_a)\right]^{-1}\right]
\end{aligned}
$$

$$\times [J_{2n+1/2}(R\kappa_a)] \Big[-[J_{2m+1/2}(\kappa_a)]^T [E_m + G_m]^0 + j[J_{2m+3/2}(\kappa_a)]^T [F_m + H_m]^0 \Big] \quad (7a)$$

$$\begin{aligned}
[B_m + D_m]^1 &= [B_m + D_m]^0 - [K_a(2m+3/2, 2n+3/2; \kappa_a)]^{-1} \\
&\times [K_{ca}(2m+3/2, 2n+3/2; \kappa_a)] [F_m + H_m]^0 \\
&- [K_a(2m+3/2, 2n+3/2; \kappa_a)]^{-1} \\
&\times [K_{sa}(2m+1/2, 2n+3/2; d_a)] [E_m + G_m]^0 \\
&= [B_m + D_m]^0 + \left[\kappa_a \sqrt{\frac{\pi}{2}} \frac{\exp(-j[kd - \frac{\pi}{4}])}{(kd)3/2} \right] \\
&\times \Big[[K_a(2m+3/2, 2n+3/2; \kappa_a)]^{-1} \Big] \\
&\times [J_{2n+3/2}(R\kappa_a)] \Big[-[J_{2m+3/2}(\kappa_a)]^T \\
&\times [F_m + H_m]^0 - j[J_{2m+1/2}(\kappa_a)]^T [E_m + G_m]^0 \Big] \quad (7b)
\end{aligned}$$

$$\begin{aligned}
[E_m + G_m]^1 &= [E_m + G_m]^0 - [K_b(2m+1/2, 2n+1/2; \kappa_b)]^{-1} \\
&\times [K_{cb}(2m+1/2, 2n+1/2; d_b)] [A_m + C_m]^0 \\
&- [K_b(2m+1/2, 2n+1/2; \kappa_b)]^{-1} \\
&\times [K_{sb}(2m+3/2, 2n+1/2; d_b)] [B_m + D_m]^0 \\
&= [E_m + G_m]^0 + \left[\kappa_b \sqrt{\frac{\pi}{2}} \frac{\exp(-j[kd - \frac{\pi}{4}])}{(kd)3/2} \right] \\
&\times \Big[[K_b(2m+1/2, 2n+1/2; \kappa_b)]^{-1} \Big] \\
&\times [J_{2n+1/2}(q\kappa_b)] \Big[-[J_{2m+1/2}(\kappa_b)]^T [A_m + C_m]^0 \\
&- j[J_{2m+3/2}(\kappa_b)]^T [B_m + D_m]^0 \Big] \quad (7c)
\end{aligned}$$

$$\begin{aligned}
[F_m + H_m]^1 &= [F_m + H_m]^0 - [K(2m+3/2, 2n+3/2; \kappa_b)] \\
&\times [K_c(2m+3/2, 2n+3/2; d_b)] [B_m + D_m]^0 \\
&+ [K_b(2m+3/2, 2n+3/2; \kappa_b)] \\
&\times [K_s(2m+1/2, 2n+3/2; d_b)] [A_m + C_m]^0 \\
&= [F_m + H_m]^0 + \left[\kappa_b \sqrt{\frac{\pi}{2}} \frac{\exp(-j[kd - \frac{\pi}{4}])}{(kd)3/2} \right] \\
&\times \Big[[K_b(2m+3/2, 2n+3/2; \kappa_b)]^{-1} \Big]
\end{aligned}$$

$$\times[J_{2n+3/2}(q\kappa_b)]\left[-[J_{2m+3/2}(\kappa_b)]^T[B_m+D_m]^0\right.$$
$$\left.+\,j[J_{2m+1/2}(\kappa_b)]^T[A_m+C_m]^0\right] \quad (7d)$$

$$\begin{aligned}[A_m-C_m]^1 &= [A_m-C_m]^0-[G_a(2m+1/2,2n+1/2;\kappa_a)]^{-1}\\ &\times[G_{ca}(2m+1/2,2n+1/2;d_a)][E_m-G_m]^0\\ &+[G_a(2m+1/2,2n+1/2;\kappa_a)]^{-1}\\ &\times[G_{sa}(2m+3/2,2n+1/2;d_a)][F_m-H_m]^0\\ &= [A_m-C_m]^0+\left[\kappa_a\zeta\sqrt{\frac{\pi}{2}}\frac{\exp(-j[kd-\frac{\pi}{4}])}{(kd)3/2}\right]\\ &\times\left[[G_a(2m+1/2,2n+1/2;\kappa_a)]^{-1}\right]\\ &\times[J_{2n+1/2}(R\kappa_a)]\left[[J_{2m+1/2}(\kappa_a)]^T[E_m-G_m]^0\right.\\ &\left.-\,j[J_{2m+3/2}(\kappa_a)]^T[F_m-H_m]^0\right] \quad (7e)\end{aligned}$$

$$\begin{aligned}[B_m-D_m]^1 &= [B_m-D_m]^0-[G_a(2m+3/2,2n+3/2;\kappa_a)]^{-1}\\ &\times[G_{ca}(2m+3/2,2n+3/2;d_a)][F_m-H_m]^0\\ &-[G_a(2m+3/2,2n+3/2;\kappa_a)]^{-1}\\ &\times[G_{sa}(2m+1/2,2n+3/2;d_a)][E_m-G_m]^0\\ &= [B_m-D_m]^0+\left[\kappa_a\zeta\sqrt{\frac{\pi}{2}}\frac{\exp(-j[kd-\frac{\pi}{4}])}{(kd)3/2}\right]\\ &\times\left[[G_a(2m+3/2,2n+3/2;\kappa_a)]^{-1}\right]\\ &\times[J_{2n+3/2}(R\kappa_a)]\left[[J_{2m+3/2}(\kappa_a)]^T[F_m-H_m]^0\right.\\ &\left.+\,j[J_{2m+1/2}(\kappa_a)]^T[E_m-G_m]^0\right] \quad (7f)\end{aligned}$$

$$\begin{aligned}[E_m-G_m]^1 &= [E_m-G_m]^0-[G_b(2m+1/2,2n+1/2;\kappa_b)]^{-1}\\ &\times[G_{cb}(2m+1/2,2n+1/2;d_b)][A_m-C_m]^0\\ &-[G_b(2m+1/2,2n+1/2;\kappa_b)]^{-1}\\ &\times[G_{sb}(2m+3/2,2n+1/2;d_b)][B_m-D_m]^0\\ &= [E_m-G_m]^0+\left[\kappa_b\zeta\sqrt{\frac{\pi}{2}}\frac{\exp(-j[kd-\frac{\pi}{4}])}{(kd)3/2}\right]\\ &\times\left[[G_b(2m+1/2,2n+1/2;\kappa_b)]^{-1}\right]\end{aligned}$$

$$\times[J_{2n+1/2}(q\kappa_b)]\left[[J_{2m+1/2}(\kappa_b)]^T[A_m-C_m]^0\right.$$
$$\left.+\,j[J_{2m+3/2}(\kappa_b)]^T[B_m-D_m]^0\right] \quad (7g)$$

$$\begin{aligned}[F_m-H_m]^1 &= [F_m-H_m]^0-[G_b(2m+3/2,2n+3/2;\kappa_b)]^{-1}\\ &\times[G_{cb}(2m+3/2,2n+3/2;d_b)]\,[B_m-D_m]^0\\ &+[G_b(2m+3/2,2n+3/2;\kappa_b)]^{-1}\\ &\times[G_{sb}(2m+1/2,2n+3/2;d_b)]\,[A_m-C_m]^0\\ &= [F_m-H_m]^0+\left[\kappa_b\zeta\sqrt{\frac{\pi}{2}}\frac{\exp(-j[kd-\frac{\pi}{4}])}{(kd)3/2}\right]\\ &\times\left[[G_b(2m+3/2,2n+3/2;\kappa_b)]^{-1}\right]\\ &\times[J_{2n+3/2}(q\kappa_b)]\left[[J_{2m+3/2}(\kappa_b)]^T[B_m-D_m]^0\right.\\ &\left.-\,j[J_{2m+1/2}(\kappa_b)]^T[A_m-C_m]^0\right] \quad (7h)\end{aligned}$$

where $[A]^T$ means the transpose of matrix A. The zeroth order co-efficients give the fields by slit 1 or slit 2 as if they are isolated and the first order co-efficients yield the field when the first order interaction between the slits are taken into account. Similarly we can introduce the higher order interaction terms by iteration. Since impedance face supports the surface waves so when the observation point is far from the surface, these waves can be neglected and diffracted field dominates. Diffracted far field in the upper half space can be evaluated by applying the saddle point method. The result is given

$$\begin{aligned}E_z^d &= \sqrt{\frac{\pi}{2}}\frac{\sin\phi}{1+\zeta\sin\phi}\frac{1}{\sqrt{k\rho}}\exp\left[-jk\rho+j\frac{\pi}{4}\right]\\ &\times\sum_{m=0}^{\infty}\left\{A_m\frac{J_{2m+1/2}(\kappa_a\cos\phi)}{\sqrt{\kappa_a\cos\phi}}+jB_m\frac{J_{2m+3/2}(\kappa_a\cos\phi)}{\sqrt{\kappa_a\cos\phi}}\right\}\\ &\times\exp[-jkd_1\cos\phi]\\ &+\left\{E_m\frac{J_{2m+1/2}(\kappa_b\cos\phi)}{\sqrt{\kappa_b\cos\phi}}+jF_m\frac{J_{2m+3/2}(\kappa_b\cos\phi)}{\sqrt{\kappa_b\cos\phi}}\right\}\\ &\times\exp[jkd_2\cos\phi] \quad (8)\end{aligned}$$

Similarly the corresponding expression for the lower half space may be derived.

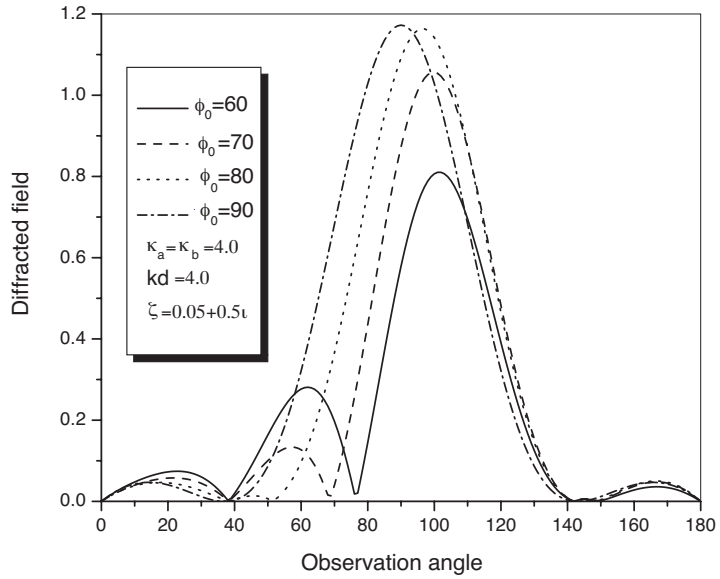

Figure 2. Variations of diffracted field with angle of incidance.

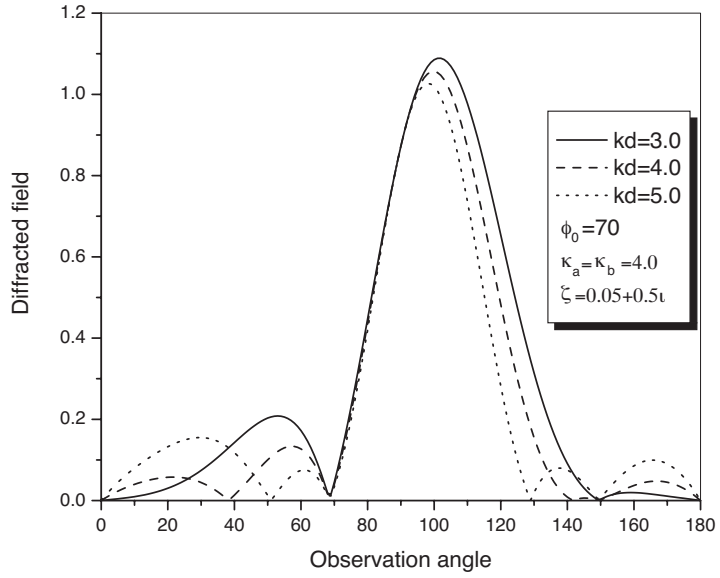

Figure 3. Variations of diffracted field with spacing between the slits.

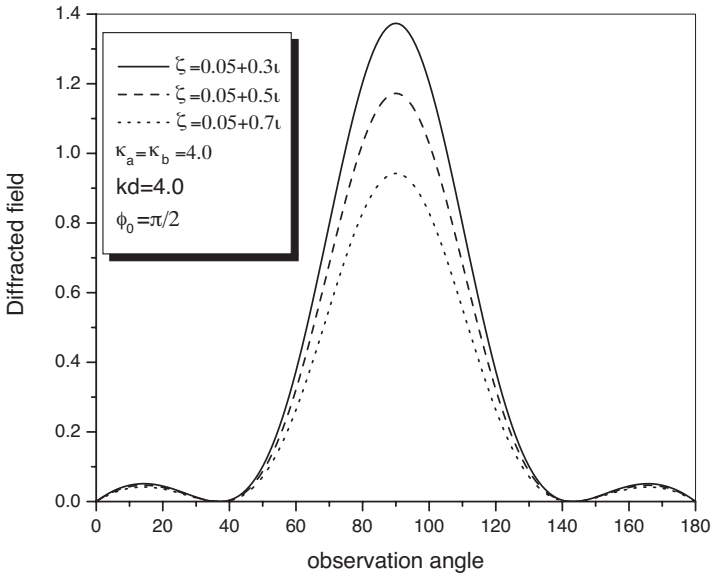

Figure 4. Variations of diffracted field with the impedance of plane.

4. NUMERICAL RESULTS AND DISCUSSION

In order to obtain the diffracted fields using equation (8) we must determine the expansion coefficients A_m, B_m, E_m and F_m by solving the simultaneous equations (7a)–(7h) for infinite number of unknowns. The matrix elements are computed numerically. The diffracted fields are determined for the upper half space for different values of angle of incidence ϕ_0, separation between the slits kd and impedance of plane containing slits. The plots are shown in Figure 2 to Figure 4. It is noted that as we increase the angle of incidence, the amplitude of the main lobe of the diffracted field also increases. It is also observed that the decrease of spacing between the slits moves the main lobe of the diffracted field towards the higher observation angles. Figure 4 shows the effects of variation of impedance of the plane on the diffracted field pattern. It is observed that the amplitude of the field decreases as we increase the value of impedance ζ.

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