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SCATTERING OF INHOMOGENEOUS TWO-DIMENSIONAL PERIODIC DIELECTRIC GRATINGS

M. Khalaj-Amirhosseini

College of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran

Abstract—A general method is proposed to frequency domain analysis of inhomogeneous two-dimensional periodic gratings. Each component of electromagnetic fields is expressed by several spatial harmonic plane waves. Then, two differential equations are obtained for the reflection and transmission matrices, using wave-splitting approach. Solving these equations gives us the co- and cross-polarized reflection and transmission coefficients. The method is studied using some examples.

1. INTRODUCTION

Laterally periodic planar layers (gratings) are used in many areas such as electromagnetics [1–3], integrated optics [4], electron beams [5], holography and so on. On the other hand, inhomogeneous planar layers are widely used in electromagnetics as optimum shields and filters and so on [6, 7]. Therefore, many efforts have been done to analyze gratings [1–3] and [8–10] or inhomogeneous planar layers [11, 12]. However, only some efforts have been done to analyze simultaneously inhomogeneous and periodic gratings, e.g., [13] for one-dimensional periodicity. The subject of this paper is finding the scattering from an inhomogeneous two-dimensional periodic grating, illuminated by a TM or TE polarized electromagnetic plane wave. Each component of electromagnetic fields is expressed by several spatial harmonic plane waves. Then, two differential equations are obtained for the reflection and transmission matrices, using wave-splitting approach. To solve these differential equations, the inhomogeneous gratings are subdivided to several thin homogeneous gratings (layers), at first. Then, total co- and cross-polarized reflection and transmission coefficients are obtained using

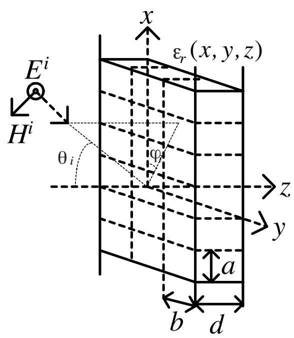

Figure 1. A typical grating illuminated by a plane wave.

finite difference method. The method is verified using analysis of some special types of gratings.

2. THE WAVES OUTSIDE THE GRATINGS

Figure 1 shows a typical inhomogeneous two-dimensional periodic dielectric grating with the thickness of d and periods of a and b. It is assumed that the incident plane wave propagates obliquely towards positive x, y and z direction with an angle of incidence φ_i and θ_i, electric filed strength of E_i and the angular frequency of ω. The incident wave consists of two different polarizations, TE and TM. Thus we can write like as following

$$\vec{E}_i = E_i\left(\alpha_{TE}\hat{a}_{TE} + \alpha_{TM}\hat{a}_{TM}\right)\exp\left(-j(k_{x0}x + k_{y0}y + k_{z0}z)\right) \tag{1}$$

in which

$$k_{x0} = k_0 \sin\theta_i \cos\varphi_i \tag{2}$$
$$k_{y0} = k_0 \sin\theta_i \sin\varphi_i \tag{3}$$
$$k_{z0} = k_0 \cos\theta_i \tag{4}$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the wave number in the free space. Also, α_{TE} and α_{TM} are the weighting coefficients of TE and TM polarizations, respectively, when $0 \leq \alpha_{TE}, \alpha_{TM} \leq 1$ and $\alpha_{TE}^2 + \alpha_{TM}^2 = 1$. Furthermore, $\hat{a}_{TE}$ and $\hat{a}_{TM}$ are the unit vectors related to TE and TM polarizations, respectively, given by

$$\hat{a}_{TE} = -\sin\varphi_i\hat{a}_x + \cos\varphi_i\hat{a}_y \tag{5}$$
$$\hat{a}_{TM} = \cos\theta_i\cos\varphi_i\hat{a}_x + \cos\theta_i\sin\varphi_i\hat{a}_y - \sin\theta_i\hat{a}_z \tag{6}$$

Regard to the periodicity of the geometry shown in Fig. 1, the electric and magnetic fields are pseudo-periodic functions in x and y with a period of a and b. One can use the following Fourier series expansion for an arbitrary three-dimensional function $F(x, y, z)$, which is periodic with respect to x and y with a period of a and b, respectively.

$$F(x,y,z) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} (F)_{m,n}\Big|_{z=z} \exp\left(-j(U_m x + V_n y)\right) \tag{7}$$

$$(F)_{m,n}\Big|_{z=z} = \frac{1}{ab}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} F(x,y,z)\exp\left(j(U_m x + V_n y)\right) dxdy \tag{8}$$

in which

$$U_m = \frac{2\pi m}{a} \tag{9}$$

$$V_n = \frac{2\pi n}{b} \tag{10}$$

In fact, $(F)_{m,n} = (F)_{m,n}(z)$ denotes the m, n-th Fourier coefficients of $F(x, y, z)$.

The electric and magnetic fields reflected or transmitted from gratings, can be represented as $\vec{F}_r = F_{rx}\hat{a}_x + F_{ry}\hat{a}_y + F_{rz}\hat{a}_z$ and $\vec{F}_t = F_{tx}\hat{a}_x + F_{ty}\hat{a}_y + F_{tz}\hat{a}_z$, respectively, where F represents E or H $(F = E, H)$. Each component of these fields are expressed by infinite spatial harmonic plane waves, given by

$$F_{rw}(x,y,z) = \left[\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(F_{rw})_{m,n}\exp\left(-j(U_m x + V_n y) + \gamma_{m,n} z\right)\right] \exp\left(-j(k_{x0}x + k_{y0}y)\right) \tag{11}$$

$$F_{tw}(x,y,z) = \left[\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(F_{tw})_{m,n}\exp\left(-j(U_m x + V_n y) - \gamma_{m,n}(z-d)\right)\right] \exp\left(-j(k_{x0}x + k_{y0}y)\right) \tag{12}$$

where w represents x, y or $z(w = x, y, z)$ and also

$$\gamma_{m,n} = \begin{cases} \sqrt{(k_x)_m^2 + (k_y)_n^2 - k_0^2} = \alpha_{m,n}; \\ \qquad \text{when} \;\; k_0 < \sqrt{(k_x)_m^2 + (k_y)_n^2} \\ j\sqrt{k_0^2 - ((k_x)_m^2 + (k_y)_n^2)} = j(k_z)_{m,n}; \\ \qquad \text{when} \;\; k_0 > \sqrt{(k_x)_m^2 + (k_y)_n^2} \end{cases} \tag{13}$$

in which

$$(k_x)_m = k_{x0} + U_m \tag{14}$$
$$(k_y)_n = k_{y0} + V_n \tag{15}$$

are the transverse wave numbers. It should be noticed that in a numerical computation, it is better to truncate the Fourier series expansion of electromagnetic field components by setting

$$(F)_{m,n} = 0 \ \text{ if } \ |m| \geq M \ \text{ or } \ |n| \geq N, \tag{16}$$

where M and N are two positive integers. We use such a truncation $(-M \leq m \leq M, -N \leq n \leq N\ )$ in the following sections.

3. THE WAVES INSIDE THE GRATINGS

In this section, the frequency domain equations of inhomogeneous gratings are reviewed. From the Faraday and Ampere Laws, the following six equations are obtained.

$$\partial_z E_x = -j\omega\mu_0 H_y + \partial_x E_z \tag{17}$$
$$\partial_z E_y = j\omega\mu_0 H_x + \partial_y E_z \tag{18}$$
$$\partial_z H_x = j\omega\varepsilon_0\varepsilon_r(x,y,z) E_y + \partial_x H_z \tag{19}$$
$$\partial_z H_y = -j\omega\varepsilon_0\varepsilon_r(x,y,z) E_x + \partial_y H_z \tag{20}$$
$$E_z = \frac{1}{j\omega\varepsilon_0}\varepsilon_r^{-1}(x,y,z)(\partial_x H_y - \partial_y H_x) \tag{21}$$
$$H_z = \frac{-1}{j\omega\mu_0}(\partial_x E_y - \partial_y E_x) \tag{22}$$

The electric and magnetic fields inside the inhomogeneous gratings, i.e., $\vec{F} = F_x\hat{a}_x + F_y\hat{a}_y + F_z\hat{a}_z$ where ($F = E, H$ and $w = x, y, z$), can be written as follows

$$F_w(x,y,z) = \left[\sum_{m=-\infty}^{m=\infty}\sum_{n=-\infty}^{n=\infty}(F_w)_{m,n}\exp\left(-j(U_m x + V_n y)\right)\right] \exp\left(-j(k_{x0}x + k_{y0}y)\right) \tag{23}$$

Using the Fourier series expansion of the field components and that of the permittivity functions in (17)–(22), the following matrix equations are obtained for the Fourier coefficients of the fields.

$$\frac{d}{dz}\begin{bmatrix}\mathbf{e}\\ \mathbf{h}\end{bmatrix} = \mathbf{W}(z)\begin{bmatrix}\mathbf{e}\\ \mathbf{h}\end{bmatrix} \tag{24}$$

where $\mathbf{e} = [\mathbf{e}_x \quad \mathbf{e}_y]^T$ and $\mathbf{h} = [\mathbf{h}_x \quad \mathbf{h}_y]^T$, are the electric and magnetic fields vectors, respectively, in which

$$\begin{aligned}\mathbf{f}_w(z) &= [(F_w)_{-M,-N}\ (F_w)_{-M,-N+1} \cdots (F_w)_{-M,N} \cdots (F_w)_{0,0} \\ &\quad \cdots (F_w)_{M,-N}\ (F_w)_{M,-N+1} \cdots (F_w)_{M,N}]^T \end{aligned} \tag{25}$$

($\mathbf{f} = \mathbf{e}, \mathbf{h}$ and $w = x, y$) represents the Fourier coefficients of the electric and magnetic field components. Also, $\mathbf{W}(z)$ is a matrix as follows

$$\mathbf{W}(z) = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{W}_1(z) & \mathbf{W}_2(z) \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_3(z) & \mathbf{W}_4(z) \\ \mathbf{W}_5 & \mathbf{W}_6(z) & \mathbf{0} & \mathbf{0} \\ \mathbf{W}_7(z) & \mathbf{W}_8 & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{26}$$

in which eight $(2M+1)\times(2N+1)$ by $(2M+1)\times(2N+1)$ sub-matrices have been defined as the following

$$\mathbf{W}_1(z) = \frac{-j}{\omega\varepsilon_0}\mathbf{K}_x\mathbf{Q}(z)\mathbf{K}_y \tag{27}$$

$$\mathbf{W}_2(z) = -j\omega\mu_0\mathbf{I} + \frac{j}{\omega\varepsilon_0}\mathbf{K}_x\mathbf{Q}(z)\mathbf{K}_x \tag{28}$$

$$\mathbf{W}_3(z) = j\omega\mu_0\mathbf{I} - \frac{j}{\omega\varepsilon_0}\mathbf{K}_y\mathbf{Q}(z)\mathbf{K}_y \tag{29}$$

$$\mathbf{W}_4(z) = \frac{j}{\omega\varepsilon_0}\mathbf{K}_y\mathbf{Q}(z)\mathbf{K}_x \tag{30}$$

$$\mathbf{W}_5 = \frac{j}{\omega\mu_0}\mathbf{K}_x\mathbf{K}_y \tag{31}$$

$$\mathbf{W}_6(z) = j\omega\varepsilon_0\mathbf{P}(z) - \frac{j}{\omega\mu_0}\mathbf{K}_x\mathbf{K}_x \tag{32}$$

$$\mathbf{W}_7(z) = -j\omega\varepsilon_0\mathbf{P}(z) + \frac{j}{\omega\mu_0}\mathbf{K}_y\mathbf{K}_y \tag{33}$$

$$\mathbf{W}_8 = -\frac{j}{\omega\mu_0}\mathbf{K}_y\mathbf{K}_x \tag{34}$$

In (27)–(34), $\mathbf{I}$ is an identity matrix and also

$$\mathbf{P}(m,n,z) = (\varepsilon_r)_{m-m',n-n'}(z) \tag{35}$$

and

$$\mathbf{Q}(m,n,z) = (\varepsilon_r^{-1})_{m-m',n-n'}(z) \tag{36}$$

($m, m' = -M, \dots, M$ and $n, n' = -N, \dots, N$) are the convolution matrices associated with $\varepsilon_r(z)$ and $\varepsilon_r^{-1}(z)$, respectively. Also,

$$\begin{aligned} \mathbf{K}_x &= \operatorname{diag}\left([(k_x)_{-M} \cdots (k_x)_{-M} \cdots\cdots (k_x)_0 \cdots (k_x)_0 \right. \\ &\quad \left. \cdots\cdots (k_x)_M \cdots (k_x)_M]\right) \qquad (37) \\ \mathbf{K}_y &= \operatorname{diag}\left([(k_y)_{-N} \cdots (k_y)_0 \cdots (k_y)_N \cdots\cdots (k_y)_{-N} \right. \\ &\quad \left. \cdots (k_y)_0 \cdots (k_y)_N]\right) \qquad (38) \end{aligned}$$

are diagonal matrices containing the transverse wave numbers. One sees that (24) is a problem with two-point boundary conditions for gratings and hence is difficult to solve.

4. ANALYSIS OF GRATINGS USING WAVE-SPLITTING APPROACH

In this section, the scattering from inhomogeneous two-dimensional periodic gratings is determined. For this purpose, the vacuum wave-splitting approach [13] is used. This approach changes the problem with two-point boundary conditions to a problem with one-point boundary conditions. In this approach, the transverse electric and magnetic fields are mapped to forward and backward transverse fields in outside the gratings (vacuum region). One can see that this mapping may be done as follows

$$\begin{bmatrix} \mathbf{e}^+ \\ \mathbf{e}^- \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix} \qquad (39)$$

where $\mathbf{e}^+ = [\mathbf{e}_{TM}^+ \;\; \mathbf{e}_{TE}^+]^T$ and $\mathbf{e}^- = [\mathbf{e}_{TM}^- \;\; \mathbf{e}_{TE}^-]^T$ are the forward and backward transverse fields vectors, respectively, in which both TM and TE modes are existed. Also, $\mathbf{A}$ is a matrix as follows

$$\mathbf{A} = \frac{1}{2} \begin{bmatrix} \mathbf{C} & \mathbf{S} & -\mathbf{Z}_{TM}\mathbf{S} & \mathbf{Z}_{TM}\mathbf{C} \\ -\mathbf{S} & \mathbf{C} & -\mathbf{Z}_{TE}\mathbf{C} & -\mathbf{Z}_{TE}\mathbf{S} \\ \mathbf{C} & \mathbf{S} & \mathbf{Z}_{TM}\mathbf{S} & -\mathbf{Z}_{TM}\mathbf{C} \\ -\mathbf{S} & \mathbf{C} & \mathbf{Z}_{TE}\mathbf{C} & \mathbf{Z}_{TE}\mathbf{S} \end{bmatrix} \qquad (40)$$

in which

$$\begin{aligned} \mathbf{C} &= \operatorname{diag}\left[\cos(\varphi_{-M,-N}) \cdots \cos(\varphi_{M,N})\right] \qquad (41) \\ \mathbf{S} &= \operatorname{diag}\left[\sin(\varphi_{-M,-N}) \cdots \sin(\varphi_{M,N})\right] \qquad (42) \\ \mathbf{Z}_{TM} &= \eta_0 \operatorname{diag}\left[\cos(\theta_{-M,-N}) \cdots \cos(\theta_{M,N})\right] \qquad (43) \\ \mathbf{Z}_{TE} &= \eta_0 \operatorname{diag}\left[\cos^{-1}(\theta_{-M,-N}) \cdots \cos^{-1}(\theta_{M,N})\right] \qquad (44) \end{aligned}$$

are four diagonal matrices. In (41)–(44), the following functions have been used

$$\cos(\varphi_{m,n}) = \frac{(k_x)_m}{\sqrt{(k_x)_m^2 + (k_y)_n^2}} \tag{45}$$

$$\sin(\varphi_{m,n}) = \frac{(k_y)_n}{\sqrt{(k_x)_m^2 + (k_y)_n^2}} \tag{46}$$

$$\cos(\theta_{m,n}) = \begin{cases} \dfrac{\sqrt{k_0^2 - (k_x)_m^2 - (k_y)_n^2}}{k_0}; & \text{when } k_0 > \sqrt{(k_x)_m^2 + (k_y)_n^2} \\ -j\dfrac{\sqrt{(k_x)_m^2 + (k_y)_n^2 - k_0^2}}{k_0}; & \text{when } k_0 < \sqrt{(k_x)_m^2 + (k_y)_n^2} \end{cases} \tag{47}$$

Using (39) and (24), the differential equation for forward and backward waves is obtained as

$$\frac{d}{dz}\begin{bmatrix} \mathbf{e}^+ \\ \mathbf{e}^- \end{bmatrix} = \mathbf{B}(z)\begin{bmatrix} \mathbf{e} \\ \mathbf{h} \end{bmatrix} \tag{48}$$

where

$$\mathbf{B}(z) = \mathbf{A}\mathbf{W}(z)\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \tag{49}$$

Now, the transverse reflection and transmission coefficient matrices are defined, respectively as follows

$$\mathbf{e}^-(z) = \mathbf{\Gamma}_t(z)\mathbf{e}^+(z) \tag{50}$$

$$\mathbf{e}(d^+) = \mathbf{T}_t(z)\mathbf{e}^+(z) \tag{51}$$

Indeed, $\mathbf{\Gamma}_t(0)$ and $\mathbf{T}_t(0)$ are the reflection and transmission matrices, respectively for transverse electromagnetic fields on two main surfaces of grating. Differentiating (50)–(51) and using (48)–(49), yields the following differential equations.

$$\frac{d\mathbf{\Gamma}_t(z)}{dz} = \mathbf{B}_{21}(z)+\mathbf{B}_{22}(z)\mathbf{\Gamma}_t(z)-\mathbf{\Gamma}_t(z)\mathbf{B}_{11}(z)-\mathbf{\Gamma}_t(z)\mathbf{B}_{12}(z)\mathbf{\Gamma}_t(z) \tag{52}$$

$$\frac{d\mathbf{T}_t(z)}{dz} = -\mathbf{T}_t(z)\mathbf{B}_{11}(z) - \mathbf{T}_t(z)\mathbf{B}_{12}(z)\mathbf{\Gamma}_t(z) \tag{53}$$

The above differential equations can be solved numerically. First, the inhomogeneous grating is subdivided to K thin homogeneous gratings, whose thickness is very smaller than the wavelength. Then, the backward difference approximation is used to descritize (52)–(53). The following boundary conditions have to be used to solve the resulted difference equations step-by-step from $z = d$ to $z = 0$.

$$\mathbf{\Gamma}_t(d) = \begin{cases} \mathbf{0}; & \text{for open-end gratings} \\ -\mathbf{I}; & \text{for short-end gratings} \end{cases} \tag{54}$$

$$\mathbf{T}_t(d) = \begin{cases} \mathbf{I}; & \text{for open-end gratings} \\ \mathbf{0}; & \text{for short-end gratings} \end{cases} \tag{55}$$

In (54)–(55), we have considered two types of application for gratings calling them as open-end (as shown in Fig. 1) and short-end (coated by a perfect electric conductor) gratings. The short-end gratings can be utilized as the walls of anechoic chambers.

After determining $\mathbf{\Gamma}_t(0)$ and $\mathbf{T}_t(0)$, the complete (not transverse) reflection and transmission coefficient matrices are obtained as follows

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_{TM,TM} & \mathbf{\Gamma}_{TM,TE} \\ \mathbf{\Gamma}_{TE,TM} & \mathbf{\Gamma}_{TE,TE} \end{bmatrix} = \begin{bmatrix} \mathbf{D}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{\Gamma}_t(0) \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{56}$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_{TM,TM} & \mathbf{T}_{TM,TE} \\ \mathbf{T}_{TE,TM} & \mathbf{T}_{TE,TE} \end{bmatrix} = \begin{bmatrix} \mathbf{D}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{T}_t(0) \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{57}$$

where

$$\mathbf{D} = \mathrm{diag}\left[\cos(\theta_{-M,-N}) \cdots \cos(\theta_{M,N})\right] \tag{58}$$

Of course, only two columns of matrices $\mathbf{\Gamma}$ and $\mathbf{T}$ corresponding to scattering due to fundamental spatial harmonic incidence, i.e., $m = n = 0$, are our interesting.

5. EXAMPLES AND RESULTS

In this section, two special types of inhomogeneous planar layers are analyzed using the presented method.

Type 1: (Homogeneous Planar Layer)

Consider a homogeneous planar layer with the following parameter

$$\varepsilon_r(x, y, z) = \varepsilon_{r0} \tag{59}$$

The Fourier coefficients of the permittivity functions of this type of planar layer will be as follows

$$(\varepsilon_r^{\pm 1})_{m,n} = \varepsilon_{r0}^{\pm 1} \delta(m)\delta(n) \tag{60}$$

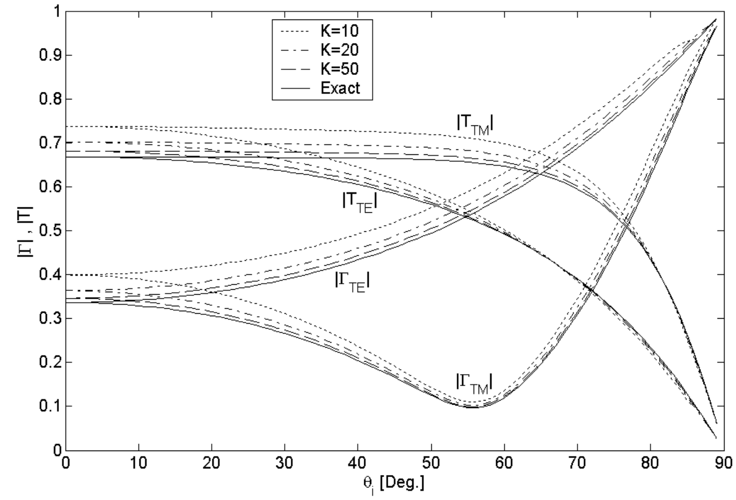

Figure 2. The amplitude of the reflection and transmission coefficients of open-end homogeneous planar layer versus θ_i.

It is simple to show that the solution is independent to the parameters a, b, M and N for this type of planar layers.

Now, consider a homogeneous planar layer with parameters of $\varepsilon_{r0} = 2 - j1$ and $d = 5\,\text{cm}$. Figure 2, compares the amplitude of the reflection and transmission coefficients of open-end layer versus θ_i for $f = 1.0\,\text{GHz}$ and $\varphi_i = 0°$, obtained from the exact solution and from the presented method with $K = 10$, 20 and 50. One sees a good agreement between the exact solutions and the solutions obtained from the proposed method. It is seen and also evident that, as the number of thin layers, K, increases the accuracy of the obtained solutions increases. The better accuracy for larger angles of incidence, may be due to larger wavelength along the thickness of the layer for these angles ($\lambda_z = \lambda/\cos\theta_i$, in which λ is the wavelength in the layer).

Type 2: (Wedge Grating)

Consider a dielectric wedge grating with the following parameter for $|x| < a/2$ and $|y| < b/2$

$$\varepsilon_r(x, y, z) = \begin{cases} \varepsilon_{r0} & |x| < \dfrac{a}{2d}z \text{ and } |y| < \dfrac{b}{2d}z \\ 1 & \text{otherwise} \end{cases} \tag{61}$$

The Fourier coefficients of the permittivity functions of this type of

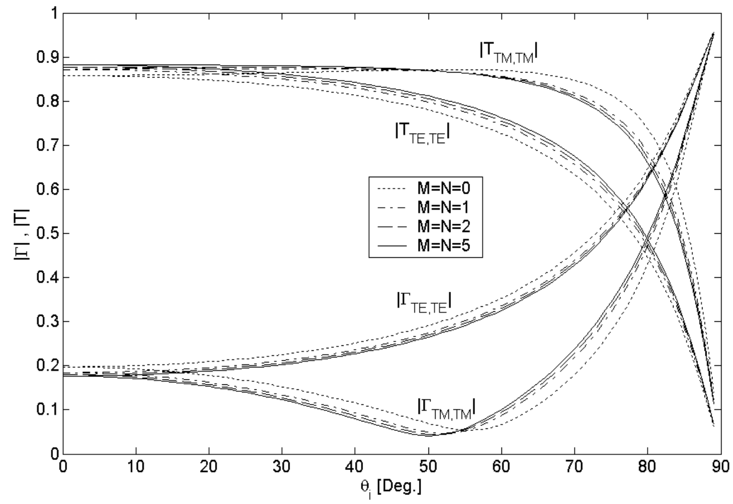

Figure 3. The amplitude of the co-polarized reflection and transmission coefficients of open-end wedge grating versus θ_i.

planar layer will be as follows

$$(\varepsilon_r^{\pm 1})_{m,n} = \begin{cases} \dfrac{\varepsilon_{r0}^{\pm 1} - 1}{\pi^2 mn} \sin(m\pi z/d)\sin(n\pi z/d); & m,\ n \neq 0 \\ \dfrac{\varepsilon_{r0}^{\pm 1} - 1}{\pi n}(z/d)\sin(n\pi z/d); & m = 0,\ n \neq 0 \\ \dfrac{\varepsilon_{r0}^{\pm 1} - 1}{\pi m}(z/d)\sin(m\pi z/d); & m \neq 0,\ n = 0 \\ 1 + \left(\varepsilon_{r0}^{\pm 1} - 1\right)(z/d)^2; & m = n = 0 \end{cases} \tag{62}$$

Now, assume that $\varepsilon_{r0} = 2 - j1$, $a = b = d = 5\,\mathrm{cm}$ and $f = 1.0\,\mathrm{GHz}$. With these assumptions, only the fundamental spatial harmonic wave, i.e., $m = n = 0$, is not evanescent. Figure 3 shows the amplitude of the co-polarized reflection and transmission coefficients of open-end grating for fundamental spatial harmonic versus θ_i, assuming $\varphi_i = 0^\circ$, $K = 50$ and $M = N = 0, 1, 2$ and 5. Figure 4 shows the amplitude of the cross-polarized reflection and transmission coefficients of open-end grating for fundamental spatial harmonic, also. Moreover, Figure 5 plots the amplitude of the co-polarized reflection coefficient of short-end grating for fundamental spatial harmonic versus θ_i. One sees in Figs. 3 and 5 reasonable curves and finds a good convergence with respect to increasing M and N, especially for larger angles of

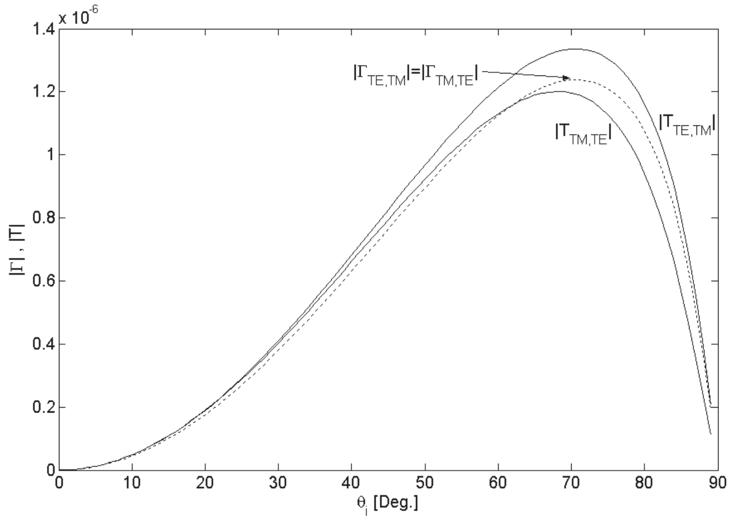

Figure 4. The amplitude of the cross-polarized reflection and transmission coefficients of open-end wedge grating versus θ_i.

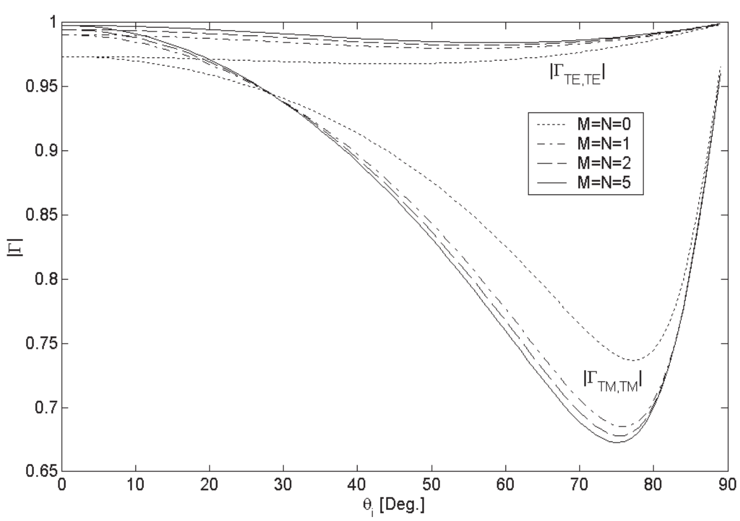

Figure 5. The amplitude of the co-polarized reflection coefficient of short-end wedge grating versus θ_i.

incidence (as before with respect to increasing K). So, it may be concluded that as the thickness or the periods of gratings (with respect to the wavelength) increase, the necessary parameters K or M and N increase, respectively.

6. CONCLUSION

A general and efficient method is proposed to frequency domain analysis of inhomogeneous two-dimensional periodic gratings. Each component of electromagnetic fields is expressed by several spatial harmonic plane waves. Then, two differential equations are obtained for the reflection and transmission matrices, using wave-splitting approach. To solve these differential equations, the inhomogeneous gratings are subdivided to several thin homogeneous gratings (layers), at first. Then total co- and cross- polarized reflection and transmission coefficients are obtained using finite difference method. It concluded that as the thickness or the periods of gratings (with respect to the wavelength) increase, the necessary number of thin layers and spatial harmonics increase, respectively.

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