RCS COMPUTATION OF AIRPLANE USING PARABOLIC EQUATION

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Abstract—The accurate analysis of scattering from objects with dimensions large compared to the wavelength using rigorous methods (finite element, FDTD, method of moments) with a personal computer is almost impractical. In asymptotic methods, physical optics (PO), geometrical theory of diffraction (GTD), the accurate modeling of the object's boundary is too cumbersome. The parabolic equation method gives accurate results in calculation of scattering from objects with dimensions ranging from one to tens of wavelengths. Solving parabolic equation with the marching method needs limited computer storage even for scattering calculations of large targets. In this paper, first the calculation procedure of radar cross section using parabolic equation in three dimension is studied and the necessary equations are derived. In order to show the validity of the parabolic equation, the RCS of a conducting sphere is calculated and the results are compared with analytic results. The airplane RCS has been computed by using a staircase model in the parabolic equation and the results are compared with physical optics results.

1. INTRODUCTION

Parabolic equation is an approximation of the wave equation which models energy propagating in a cone centered on a preferred direction, the paraxial direction. The parabolic equation was first introduced by Leontovich and Fock in order to study the diffraction of radiowaves around the earth [1]. By the advent of advanced computers closed

form solution of the parabolic equation was replaced by numerical solutions. Since then, the parabolic equation is being applied to radar, sonar, acoustic and wave propagation. The parabolic equation has been recently used in scattering calculations in acoustics [2] and electromagnetics [3].

2. THE PARABOLIC EQUATION FRAMEWORK

In this paper we concentrate on three dimensional analysis using parabolic equation. In all equations, the time dependence of the fields is assumed as $\exp(-j\omega t)$. For horizontal polarization, the electric field $\vec{E}$ only has non-zero component E_z, while for vertical polarization, the magnetic field $\vec{H}$ has the only one non-zero component H_z. The reduced function u is defined as

$$u(x, y, z) = \exp(-ikx)\psi(x, y, z) \tag{1}$$

In which $\psi(x, y, z)$ is the E_z component for horizontal polarization and H_z component for vertical polarization. The paraxial direction is assumed along the x axis. Assuming the refractive index of the medium, n, the field component ψ satisfies the following three dimensional wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + k^2 n^2 \psi = 0 \tag{2}$$

Using equations (1) and (2), the wave equation in terms of u is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + 2ik\frac{\partial u}{\partial x} + k^2(n^2 - 1)u = 0 \tag{3}$$

Considering $Q = \sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial y^2} + \frac{1}{k^2}\frac{\partial^2}{\partial z^2} + n^2}$, (3) is reduced to

$$\frac{\partial^2 u}{\partial x^2} + 2ik\frac{\partial u}{\partial x} + k^2(Q^2 - 1)u = 0 \tag{4}$$

Which can be written as

$$\left[\frac{\partial}{\partial x} + ik(1 + Q)\right]\left[\frac{\partial}{\partial x} + ik(1 - Q)\right]u = 0 \tag{5}$$

Decomposing equation (5), the following pair of equations is obtained

$$\frac{\partial u}{\partial x} = -ik(1 - Q)u \tag{6a}$$

$$\frac{\partial u}{\partial x} = -ik(1 + Q)u \tag{6b}$$

The solution to (6a) corresponds to the forward propagating waves while that of (6b) concerns backward propagating waves.

3. SCATTERED FIELDS CALCULATIONS

The simplest approximation of (6a) is obtained using the first order expansion of Taylor series. Using this approximation, the standard parabolic equation is obtained. We assume Q as

$$Q = \sqrt{Y + Z + 1} \tag{7}$$

In which

$$Y = \frac{1}{k^2} \frac{\partial^2}{\partial y^2}, \qquad Z = \frac{1}{k^2} \frac{\partial^2}{\partial z^2} + n^2 - 1$$

using the Feit and Fleck approximation to decouple Y and Z [4], we will have

$$\sqrt{Y + Z + 1} \sim \sqrt{Y + 1} + \sqrt{Z + 1} - 1 \tag{8}$$

Using the first order Taylor series of each radical of (8) and Substituting in (6a) yields

$$\frac{\partial u}{\partial x} = \frac{ik}{2}(Y + Z)u \tag{9}$$

With regard to the definition of Y and Z, equation (9) is reduced to the following form

$$\frac{\partial u}{\partial x} - \frac{i}{2k}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{ik}{2}(n^2 - 1)u = 0 \tag{10}$$

This equation is the standard parabolic equation. Equation (10) is the narrow angle approximation of parabolic equation in three dimension and calculates the total fields in forward direction. Scattered field and RCS of object can be calculated by using equation (10). Integration domain is considered as a box which embraces the object. Integration domain must be truncated in the transverse plane. In order to do this we used perfectly matched layer (PML) in transverse plane. At first PML has been introduced as an absorbing boundary condition to solve Maxwell's equations. PML has been used as an absorbing boundary condition to solve parabolic equation by Collino [5]. Important advantage of PML is its efficiency for all incident angles
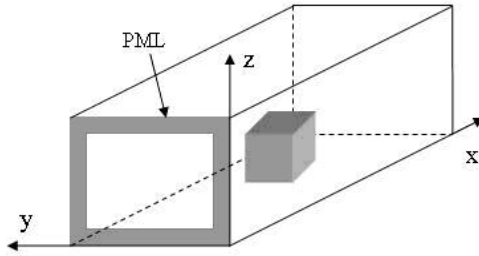
**Figure 1.** Integration domain with PML absorbing boundary condition.

by using it in a few grid points of integration domain. Integration domain with PML absorbing boundary condition is shown in Figure 1.

We discretise equation (10) on rectangular grid by using finite difference method. In order to discretise parabolic equation the Crank-Nicolson scheme is usually utilized. In this paper we use another scheme which shows better stability compared to the Crank-Nicolson scheme [6]. We define region $(m\Delta x, y, z)$ as range m. This scheme, despite Crank-Nicolson's scheme, in which second order derivatives with respect to y and z are calculated by averaging between range m and range $m - 1$, calculates second order derivatives just in range m. This work decreases the accuracy of discretising scheme with respect to the Crank-Nicolson and requires smaller Δx. The boundary of the object must be modeled accurately in scattering problems, therefore a smaller Δx is needed. Discretising equation (10) for free space yields

$$\frac{u_{i,j}^m - u_{i,j}^{m-1}}{\Delta x}$$
$$= \frac{i}{2k} \left(\frac{u_{i-1,j}^m - 2u_{i,j}^m + u_{i+1,j}^m}{\Delta y^2} + \frac{u_{i,j-1}^m - 2u_{i,j}^m + u_{i,j+1}^m}{\Delta z^2} \right) \quad (11)$$

By using equation (11), we can calculate fields in range m versus range $m - 1$. Positions of grid points, while can be determined regarding to equation (11), are shown in Figure 2. In two dimensional analysis by parabolic equation, we have to invert a triangular matrix to obtain u at range x_m. In three dimensional case matrix coefficient is a very large sparse matrix and we can not solve resulting equations with direct inversion. In this paper we used conjugate gradient method to calculate u at range x_m [7].

In order to calculate fields in all points of integration domain, first, the fields should be determined at range x_0. The incident field is
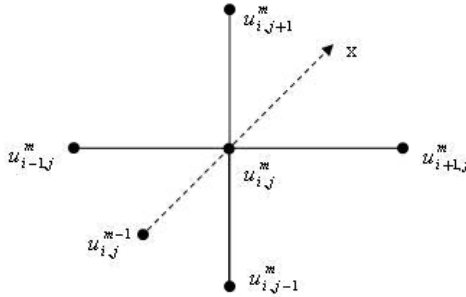
Figure 2. Positions of grid points regarding to equation (11).

assumed as a plane wave with unit amplitude as

$$u(x, y, z) = \exp\left(ik(x(\cos\theta - 1) + y\sin\theta\cos\varphi + z\sin\theta\sin\varphi)\right) \quad (12)$$

In which θ, φ are the angles of incident plane wave with x and y axis respectively.

4. COMPUTATION OF RADAR CROSS SECTION

After the calculation of fields over the entire computational domain, we can compute the fields within any arbitrary domain x as a function of the fields in the domain x_0 in free space as follows [8]

$$u(x, y, z) =$$
$$-\frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(x_0, y', z') \left[ik\frac{(x - x_0)}{d(y', z')} - \frac{1}{d(y', z')}\right] \cdot \frac{e^{ikd(y',z')}}{d(y', z')} dy' dz' \quad (13)$$
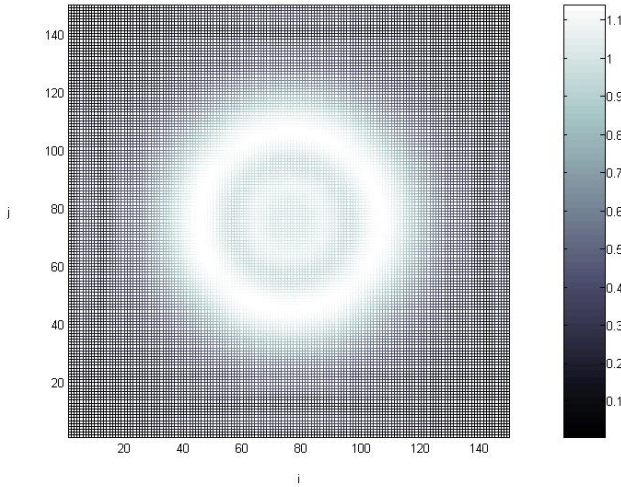
In which

$$d(y', z') = \sqrt{(x_0 - x')^2 + (y - y')^2 + (z - z')^2}$$

The radar cross section is defined as

$$\sigma(\theta, \varphi) = \lim_{r \to \infty} 4\pi r^2 \left|\frac{u^s(x, y, z)}{u^i(x, y, z)}\right|^2 \quad (14)$$

In which we have

$$x = r\cos\theta, \quad y = r\sin\theta\cos\varphi, \quad z = r\sin\theta\sin\varphi$$

Figure 3. Amplitude of the scattered field $u_s(m, i, j)$ from the conducting sphere in $x = 35\,\text{m}$ in $300\,\text{MHz}$.

Tending (x, y, z) to infinity along a given direction in (13), and assuming a unit amplitude for the incident wave, (14) yields [8]:

$$
\sigma(\theta, \psi) = \frac{k^2 \cos^2 \theta}{\pi} \left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u_s(x_0, y', z') e^{-ik \sin \theta (y' \cos \varphi + z' \sin \varphi)} dy' dz' \right|^2
$$

(15)

In which $u_s(x, y, z)$ is the scattered field.

5. RCS RESULTS

In this section airplane RCS results which are calculated by the parabolic equation is presented. In order to show the validity of the parabolic equation solution, the RCS of a conducting sphere with radius 10λ is calculated using narrow angle PE. The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter (corresponding to $300\,\text{MHz}$). The angle of incident wave is zero. Integration domain size is 35λ in x direction and 30λ in y and z directions. The grid spacing in the x, y and z directions are assumed $\lambda/2$, $\lambda/5$ and $\lambda/5$ respectively. Near filed results in planes $x = 35\,\text{m}$ and $y = 15\,\text{m}$ are shown in Figures 3 and 4 respectively. RCS of the conducting sphere is illustrated in Figure 5. Parabolic equation results and analytic results, where obtained from the extraction of Hankel
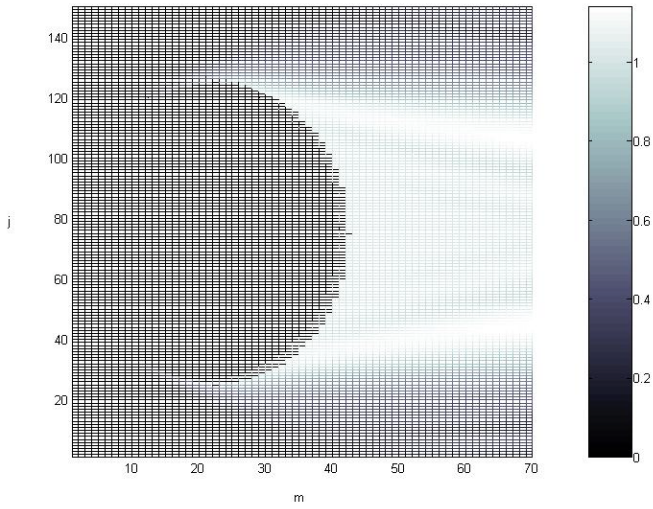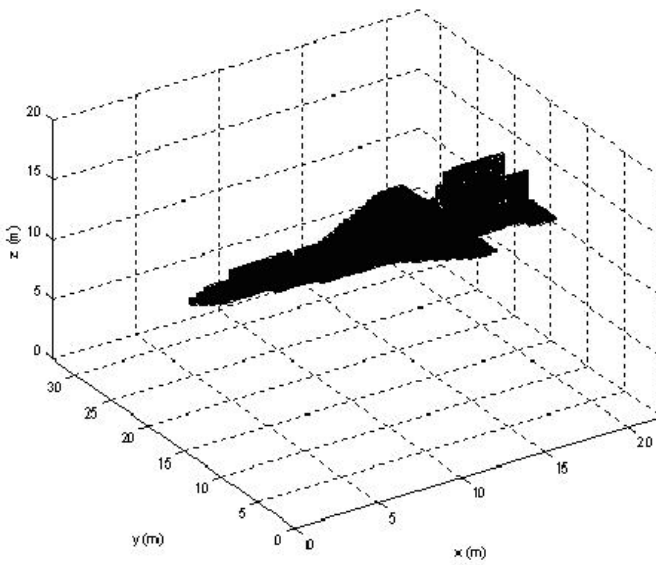
Figure 4. Amplitude of the scattered field $u_s(m, i, j)$ from the conducting sphere in $y = 15\,\mathrm{m}$ in $300\,\mathrm{MHz}$.
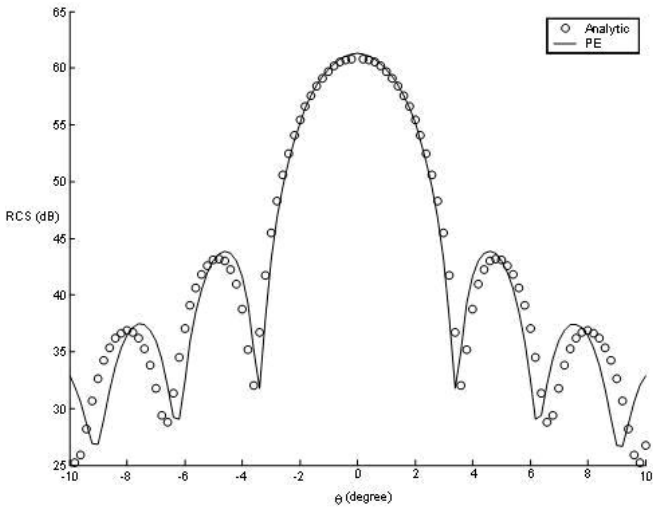
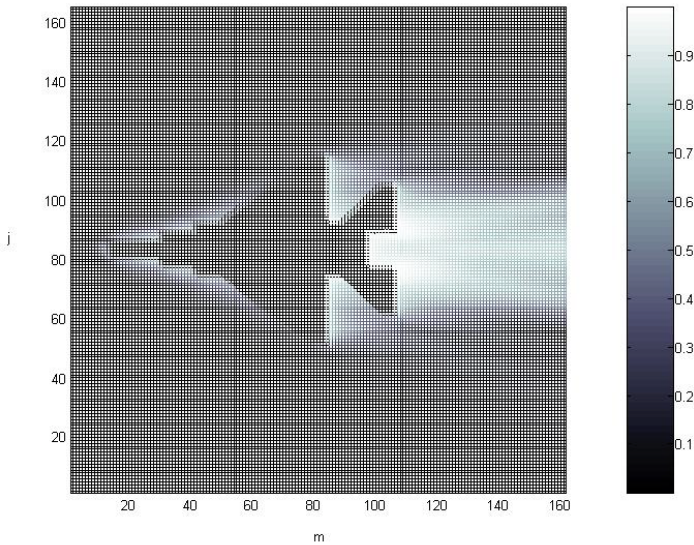**Figure 5.** RCS of the conducting sphere with radius $10\,\lambda$.
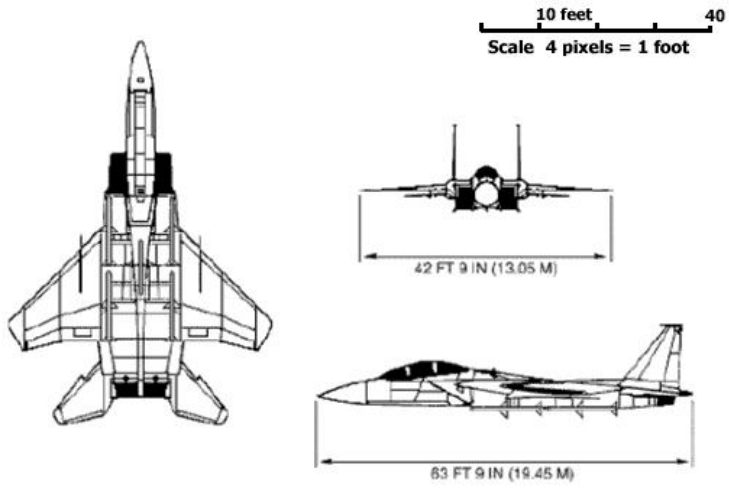
Figure 6. The geometry of the actual airplane and its dimensions.

**Figure 7.** Airplane staircase model (dimensions are in meters).

Figure 8. Amplitude of the scattered field $u_s(m, i, j)$ from the airplane in $z = 11.3\,\mathrm{m}$ for $300\,\mathrm{MHz}$.

and Legendre functions [9], are shown with solid line and circle marker respectively in Figure 5. As it can be seen, there is a good agreement between the analytical results and the parabolic equation results up to angles about 10 degrees.

In order to calculate the scattered fields from the airplane, its staircase model is utilized. Dimensions of airplane and its staircase model are shown in Figures 6 and 7 respectively. Integration domain size in order to calculate scattered fields from the airplane has been considered as 40λ, 30λ and 30λ in the x, y and z directions respectively. The grid spacings in the x, y and z directions are assumed $\lambda/4$, $\lambda/5$ and $\lambda/5$ respectively.

The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter. The angle of incident wave is zero. Near field results in planes $z = 11.3\,\mathrm{m}$ and $x = 25\,\mathrm{m}$ are shown in Figures 8 and 9 respectively. RCS results of airplane calculated with narrow angle PE in $\varphi = 0°$ and $\varphi = 90°$ planes are presented in Figures 10 and 11 respectively. Solid lines represent parabolic equation results and diamond markers represent physical optics results.
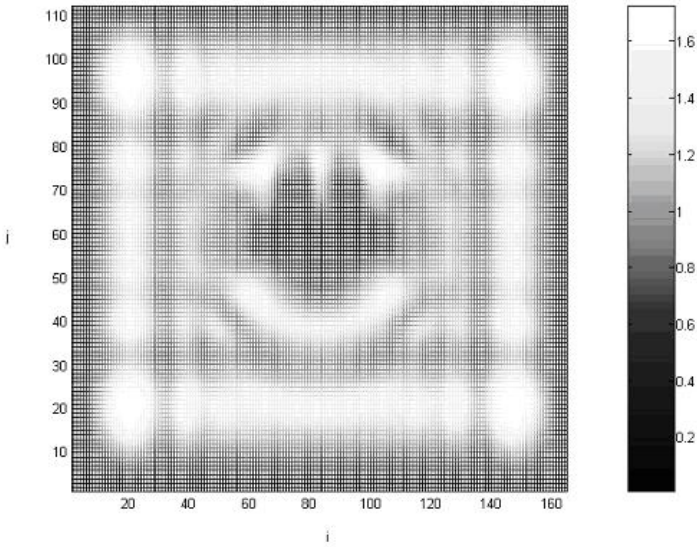
Figure 9. Amplitude of the scattered field $u_s(m, i, j)$ from the airplane in $x = 25 \, \text{m}$ for $300 \, \text{MHz}$.
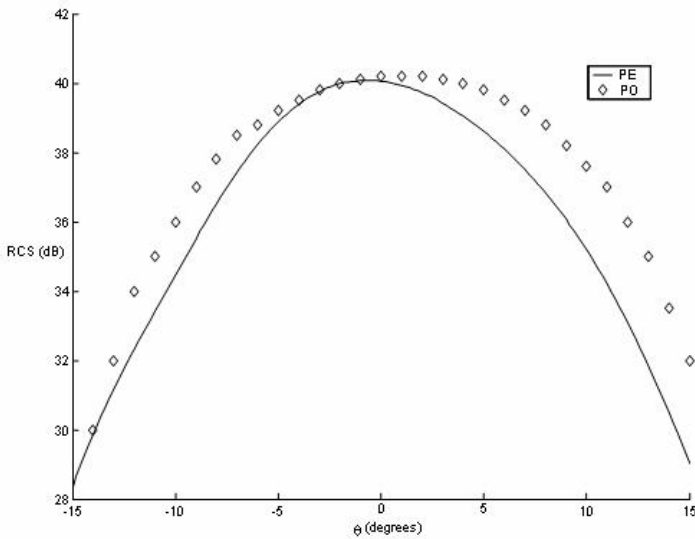
Figure 10. RCS of airplane in $\varphi = 0°$ plane in $300 \, \text{MHz}$.
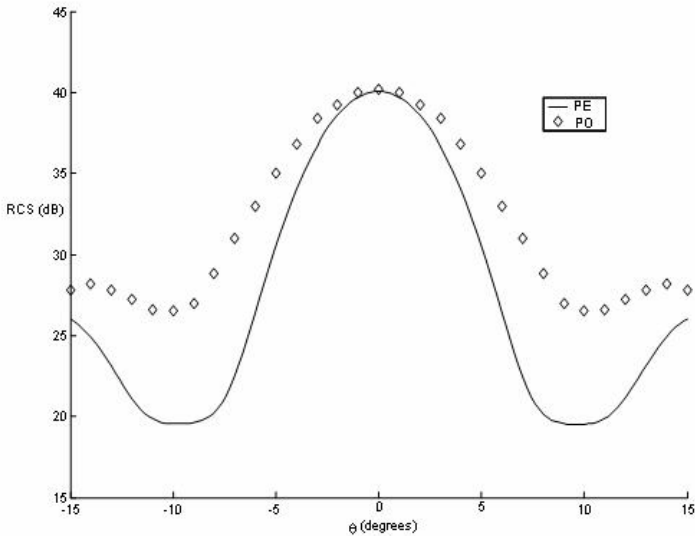
Figure 11. RCS of airplane in $\varphi = 90°$ plane in 300 MHz.

6. CONCLUSION

Parabolic equation method provides advantages in calculating scattered fields from objects with dimensions large compared to the wavelength. In this paper RCS of an airplane was calculated by three dimensional parabolic equation method. In order to show the validity of the parabolic equation, at first RCS of a conducting sphere with radius 10λ was calculated and the results were compared with analytic results. As it can be seen, there is a good agreement between the analytical results and the parabolic equation results up to angles about 10 degrees. RCS and scattered fields from the airplane has been presented by using its staircase model in 300 MHz and the results were compared with physical optic results. Good agreement between two methods can be seen especially in $\varphi = 0°$ plane.

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