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THE ITERATIVE MULTI-REGION ALGORITHM USING A HYBRID FINITE DIFFERENCE FREQUENCY DOMAIN AND METHOD OF MOMENTS TECHNIQUES

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Abstract—This paper presents a hybrid technique, which combines the desirable features of two different numerical methods, finite difference frequency domain (FDFD) and the method of moments (MoM), to analyze large-scale electromagnetic problems. This is done by dividing the computational domain into smaller sub-regions and solving each sub-region using the appropriate numerical method. Once each sub-region is analyzed, independently, an iterative approach takes place to combine the sub-region solutions to obtain a solution for the complete domain. As a result, a considerable reduction in the computation time and required computer memory is achieved.

1. INTRODUCTION

Numerical analyses of large-scale electromagnetic problems require long computational time and large computer memory. One of the goals of ongoing computational electromagnetics research is to develop time and memory efficient algorithms in order to solve real life problems. A class of time and memory efficient algorithms is developed through which, the computational domain is divided into smaller sub-regions and then the sub-regions solutions, after introducing the effect of interactions between these sub-regions, are used to provide the entire domain solution. A group of methods that decomposes the computational domain into sub-domains is known as the domain decomposition methods (DDM) [1–17]. These methods in general require common boundaries between sub-domains and boundary conditions are enforced on sub-domain interfaces. There are usually two approaches used with the applications of the coupling effects: the direct method imposes the continuity of the fields on the partition interfaces and generates a global coupling matrix [17], whereas the iterative method [1, 4] ensures the coupling between the adjacent elements by the transmission condition (TC) as described in [1]. It is possible to solve each sub-domain with the same method such as with finite element method (FEM) [4] or finite difference frequency domain method [8]. However, some DDM methods have the flexibility that in each sub-domain the most efficient method can be used independently to solve Maxwell's equations [7]. Therefore the complexity of the problem can be reduced, and a time and memory efficiency algorithm can be achieved. Another advantage of the DDM methods is that they are suitable to develop parallel processing techniques [14–16], and thus enable highly scalable algorithms.

In order to economically provide efficient solution to large-scale electromagnetic problems, especially those that involve open boundaries such as the scattering from multiple objects, decomposing the computation domain into separate sub-regions would be preferable. It is then necessary to develop accurate procedures to support the interaction between the unconnected sub-regions. Some hybrid-techniques based on combinations of method of moments (MoM), finite element (FE), finite difference time domain (FDTD), and physical optics (PO) have been used to solve a class of these problems, in which part of the problem is usually large compared to other parts [18–21].

This paper presents a hybrid technique, which combines the desirable features of two different numerical methods, finite difference frequency domain (FDFD) and the method of moments (MoM), to analyze large-scale electromagnetic problems. This is done by using them individually and then applying an iterative procedure between the two solutions, simulating number of sub-regions, to calculate the scattering from multiple objects similar to that described in [22]. This iterative procedure is referred to as iterative multi-region (IMR) technique, which requires the solution of fields in the sub-regions a number of times instead of one solution of the complete computational domain. This technique effectively reduces the size of the required memory, especially for practical and three-dimensional problems. Furthermore, the CPU time reduction can be achieved if the separation between the sub-regions is relatively large and/or coarser grids are used in some of the sub-regions, which may not be possible if only one domain is used for the solution of the entire problem. In this paper the presented technique is applied on two-dimensional scatterers, and the bistatic echo widths are calculated. This hybrid FDFD/MoM approach takes advantage of the capability of the FDFD to analyze inhomogeneous bodies with arbitrary material properties and that of the MoM to model large metallic structures with less computational memory requirements. Both numerical methods, provide a much stable solution relative to other available methods and a more convenient procedure for performing the interaction between the sub-regions based on well-known theorems. This procedure is the first step in the development of such a hybrid technique for the solution of three dimensional real world problems with available computer resources.

2. FDFD FORMULATION

Starting from Maxwell's equations for the total electric and magnetic fields

$$\nabla \times \overline{E}^{tot} = -j\omega\mu\overline{H}^{tot}, \qquad \nabla \times \overline{H}^{tot} = j\omega\varepsilon\overline{E}^{tot} \tag{1}$$

and then by separating the total field components into incident and scattered field components, we obtain

$$\nabla \times (\overline{E}^i + \overline{E}^s) = -j\omega\mu(\overline{H}^i + \overline{H}^s), \quad \nabla \times (\overline{H}^i + \overline{H}^s) = +j\omega\varepsilon(\overline{E}^i + \overline{E}^s). \tag{2}$$

The superscripts i and s are used to denote the incident and scattered fields, where the incident field is the field that would exist in the computational domain with no scatterers. This formulation is independent on the type of incident field. If the computational domain is free space then the incident field satisfies Maxwell's equations, such that

$$\nabla \times \overline{E}^i = -j\omega\mu_0\overline{H}^i, \qquad \nabla \times \overline{H}^i = +j\omega\varepsilon_0\overline{E}^i. \tag{3}$$

Substitution of equations (3) into (2) yields

$$\overline{H}^s + \frac{1}{j\omega\mu}\nabla \times \overline{E}^s = \left(\frac{\mu_0}{\mu} - 1\right)\overline{H}^i \tag{4}$$

$$\overline{E}^s - \frac{1}{j\omega\varepsilon}\nabla \times \overline{H}^s = \left(\frac{\varepsilon_0}{\varepsilon} - 1\right)\overline{E}^i. \tag{5}$$

In this paper, the FDFD formulation for the two-dimensional TM_z case is briefly presented, where the details of such formulation can be found in [23]. For an incident TM_z plane wave the field components can be given as

$$\begin{aligned}
E_z^i(x,y) &= E_0 e^{jk_0(x\cos\phi^i + y\sin\phi^i)} \\
H_x^i(x,y) &= -\sin\phi^i\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0 e^{jk_0(x\cos\phi^i + y\sin\phi^i)} \\
H_y^i(x,y) &= \cos\phi^i\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0 e^{jk_0(x\cos\phi^i + y\sin\phi^i)}
\end{aligned} \tag{6}$$

where E_0 is the magnitude of the incident electric field, k_0 is the wave number, ε_0, and μ_0 are the permittivity and the permeability of free space. The incident angle with respect to the x-axis of the global coordinates system is ϕ^i. Having defined the incident fields, equation (5) can be written for the z-component of the scattered electric field in the form

$$E_z^s - \frac{1}{j\omega\varepsilon_{zx}}\frac{\partial H_y^s}{\partial x} + \frac{1}{j\omega\varepsilon_{zy}}\frac{\partial H_x^s}{\partial y} = \left(\frac{\varepsilon_0}{\varepsilon_{zi}} - 1\right) E_z^i. \tag{7}$$

Using equation (4) and assuming that the electric field components $E_y^s = 0$ and $E_x^s = 0$ (for the TM_z case); the magnetic field components H_x and H_y can be expressed in terms of the z-component of the scattered electric field as

$$\begin{aligned} H_x^s &= -\frac{1}{j\omega\mu_{xy}}\frac{\partial E_z^s}{\partial y} + \left(\frac{\mu_0}{\mu_{xi}} - 1\right) H_x^i \\ H_y^s &= +\frac{1}{j\omega\mu_{yx}}\frac{\partial E_z^s}{\partial x} + \left(\frac{\mu_0}{\mu_{yi}} - 1\right) H_y^i \end{aligned}. \tag{8}$$

In equations (7) and (8) the permittivity and permeability parameters are indexed in such a way that these equations will be used for the perfectly matched layer (PML), that will be used to truncate the computational domain, and the non-PML regions as well in the FDFD solution region. Equations (7) and (8) can thus be reduced to one equation for TM_z case, which can be written in terms of the z-component of the scattered electric field as

$$\begin{aligned} &E_z^s - \frac{1}{j\omega\varepsilon_{zx}}\frac{\partial}{\partial x}\left[\frac{1}{j\omega\mu_{yx}}\frac{\partial E_z^s}{\partial x}\right] - \frac{1}{j\omega\varepsilon_{zy}}\frac{\partial}{\partial y}\left[\frac{1}{j\omega\mu_{xy}}\frac{\partial E_z^s}{\partial y}\right] \\ &= \left(\frac{\varepsilon_0}{\varepsilon_{zi}} - 1\right) E_z^i + \frac{1}{j\omega\varepsilon_{zx}}\frac{\partial}{\partial x}\left[\left(\frac{\mu_0}{\mu_{yi}} - 1\right) H_y^i\right] - \frac{1}{j\omega\varepsilon_{zy}}\frac{\partial}{\partial y}\left[\left(\frac{\mu_0}{\mu_{xi}} - 1\right) H_x^i\right]. \end{aligned} \tag{9}$$

The central finite difference approximation is applied for the derivatives in equation (9), leading to the following general form

$$a_{(i,j)}E_{(i-1,j)} + b_{(i,j)}E_{(i,j-1)} + c_{(i,j)}E_{(i,j)} + d_{(i,j)}E_{(i,j+1)} + e_{(i,j)}E_{(i+1,j)} = f_{(i,j)} \tag{10}$$

where the subscript "z" and the superscript "s" are omitted for simplifying the presentation of the equation, and the coefficients

a, b, c, d, and e are defined for the (i, j) cell as

$$a = \frac{1}{(\Delta x)^2\omega^2\varepsilon_{zx(i,j)}\mu_{yx\left(i-\frac{1}{2},j\right)}} \qquad b = \frac{1}{(\Delta y)^2\omega^2\varepsilon_{zy(i,j)}\mu_{xy\left(i,j-\frac{1}{2}\right)}}$$
$$d = \frac{1}{(\Delta y)^2\omega^2\varepsilon_{zy(i,j)}\mu_{xy\left(i,j+\frac{1}{2}\right)}} \qquad e = \frac{1}{(\Delta x)^2\omega^2\varepsilon_{zx(i,j)}\mu_{yx\left(i+\frac{1}{2},j\right)}}$$
$$c = 1 - a - b - d - e$$

while the permeabilities around cell (i, j) are given by

$$\mu_{yx\left(i-\frac{1}{2},j\right)} = \frac{1}{2}\left(\mu_{yx(i-1,j)} + \mu_{yx(i,j)}\right)$$
$$\mu_{yx\left(i+\frac{1}{2},j\right)} = \frac{1}{2}\left(\mu_{yx(i+1,j)} + \mu_{yx(i,j)}\right)$$
$$\mu_{xy\left(i,j-\frac{1}{2}\right)} = \frac{1}{2}\left(\mu_{xy(i,j-1)} + \mu_{xy(i,j)}\right)$$
$$\mu_{xy\left(i,j+\frac{1}{2}\right)} = \frac{1}{2}\left(\mu_{xy(i,j+1)} + \mu_{xy(i,j)}\right).$$

A linear set of equations can be constructed using equation (10). These equations can be arranged in a matrix form as $[A][E] = [Y]$, where $[A]$ is a highly sparse coefficients matrix of order N, where N is the number of nodes in the computational domain, $[E]$ is the vector of unknowns, in which each element represents one of the E_z scattered electric field components in the computation grid, and $[Y]$ is the excitation vector representing the right hand side of equation (10) and is a function of the incident field components, E_z^i, H_x^i, and H_y^i. The solution of this matrix equation for the vector $[E]$ yields the E_z^s field components at the cells of the computational domain.

3. TM$_z$ MOM FORMULATION FOR PEC CYLINDERS

The MoM technique is used to analyze the problem of a two-dimensional perfectly electric conductor (PEC) structures, by solving the electric field integral equation (EFIE). The EFIE for the unknown current density, J, induced on the surface of the analyzed structure, is then solved by applying the boundary condition on the total tangential electric field along a closed contour surrounding the object assuming that the object was located in free space. Thus the EFIE can be written as

$$-E_z^i(\overline{r}) = -j\omega\mu_0 \int_c J_z(\ell') \left[\frac{1}{4j} H_0^{(2)}\left(k_0|\overline{\rho} - \overline{\rho}'|\right)\right] \partial\ell' \underline{\hat{z}} \tag{11}$$

where $H_0^{(2)}$ is the Hankel function of the second kind and zeroth order, $\overline{\rho}$ and $\overline{\rho}'$ are the position vectors for the observation and source points, respectively. Expanding the unknown current component, $J_z(\ell')$, and using the point matching technique, the integral equation in (11) reduces to [24]

$$\sum_{n=1}^{N} I_n \Pi_n(\ell') \left\{ -\frac{\omega\mu_0}{4} \int_c J_z(\ell') H_0^{(2)}\left(k_0|\overline{\rho}_m - \overline{\rho}'|\right) \partial\ell' \right\} = -E_0 e^{jk_0(x_m \cos\phi^{inc}\hat{x} + y_m \sin\phi^{inc}\hat{y})}, \quad m = 1, 2, \ldots, N \tag{12}$$

where ρ_m is the position vector of the matching points and $\{\Pi_n(\ell'),\ n = 1, 2, \ldots, N\}$ is the set of pulse basis functions defined on the segments ΔC_n of the contour C by

$$\Pi_n(\rho) = \begin{cases} 1, & \rho \in \Delta C_n \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

Equation (12) may be written in a matrix form as

$$[Z_{mn}][I_n] = [V_m] \tag{14}$$

where

$$V_m = -E_z^i(\rho_m) \tag{15}$$

$$Z_{mn} = -\frac{\omega\mu_0}{4} \int_{\Delta C_n} H_0^{(2)}\left(k_0|\overline{\rho}_m - \overline{\rho}'|\right) \partial\ell'. \tag{16}$$

4. HYBRID FDFD — MOM TECHNIQUE

The iterative technique developed here is based on dividing the original electromagnetic scattering problem of a large domain into smaller problems in separated sub-regions and then an interaction between the small problems is to be taken into consideration. Dividing the original problem into smaller problems provides the benefit of minimizing the domain size and thus saving a huge memory in addition to the saving in the computational time, especially with large separation between some of the smaller domains. Therefore instead of dealing with one large domain one would be dealing with multiple smaller regions. In addition to this, one can also use the advantage of different numerical approaches to solve each region separately. In this paper for instance, a hybrid FDFD/MoM approach is presented, which uses the advantage of the capability of the FDFD to analyze inhomogeneous bodies with arbitrary material properties and that of the MoM to model large metallic structures with less computational memory requirements. Thus less computational time and memory consumption can be achieved.

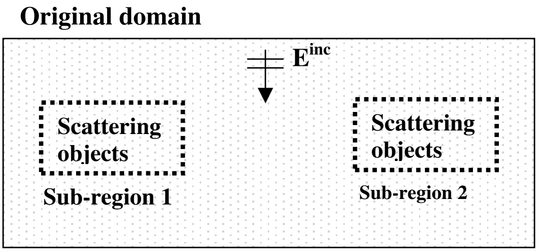

Figure 1. Original domain of a large scattering problem showing possible decomposition to two sub-regions.

Consider a problem of two scatterers apart from each other by a distance equivalent to the wavelength λ, as shown in Fig. 1. The first step is to divide the original computational domain problem into sub-regions, namely in this case region 1 and region 2. Assuming that the analysis for region 1 is performed using the MoM solution, while that of region 2 is performed using the FDFD method. Since in this paper the MoM is used to simulate conducting structures only, electric current is calculated due to an incident plane wave excitation based on equation (14). While the scattered electromagnetic fields due to an incident wave are calculated on region 2 (FDFD solution). Based on the equivalence principle, fictitious electric and magnetic currents are calculated over imaginary surfaces surrounding the scatterers of region 2. As for region 1, once the electric current is computed using the MoM solution, electromagnetic fields radiated by this current are calculated over an imaginary surface where new electric and magnetic currents are generated. Electromagnetic fields radiated by these currents are then calculated at the other sub-regions grid nodes, for region 1 (MoM solution), at the positions of the excitation vector. These fields are considered as the new excitation for that region and the cycle of calculation of scattered fields, fictitious currents and radiated fields are repeated as a new iteration. The iteration process between sub-regions continues until a convergence criterion is achieved. The sum of all calculated scattered fields through iterations gives the total scattered field, which is found to be equivalent to the scattered field calculated from the solution of the original problem with acceptable tolerance. This iterative procedure denoted as IMR technique is illustrated in Fig. 2.

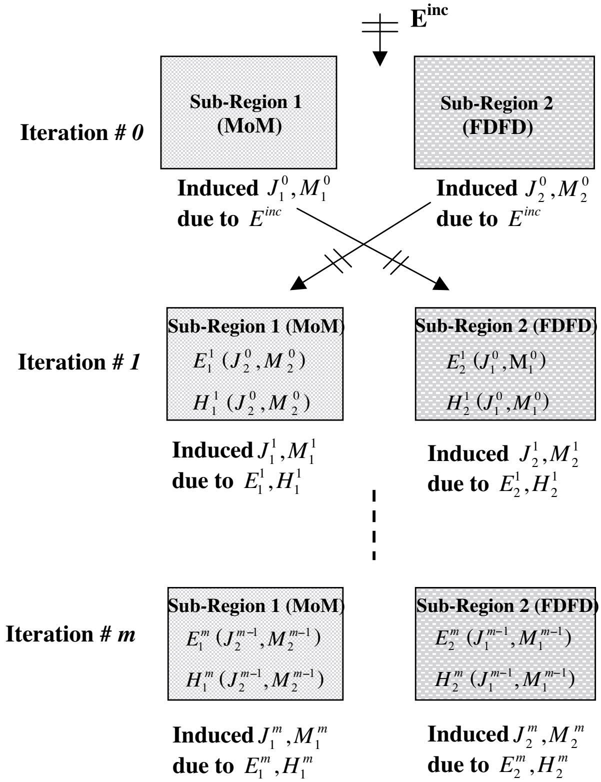

Figure 2. Scheme for the iterative procedure applied by converting the electric and magnetic currents to field components generated on the other region.

5. NUMERICAL RESULTS

In this section, numerical results are provided to prove the validity of this hybridization method (FDFD/MoM) using the IMR technique. Figure 3 shows a test configuration to prove such idea, where two cylinders placed along the x-axis are excited by a TM_z plane wave incident at $\phi^i = 180°$. A conducting cylinder of radius $0.5\,\lambda$ is placed on the left side of a dielectric cylinder of radius $1\,\lambda$, where the latest has relative permittivity (ε_r) equals to 2.2 and relative permeability (μ_r) equals to 1. The two cylinders are separated by $0.5\,\lambda$. The MoM solution is used to solve the scattered field from the conducting circular cylinder, while the FDFD is used to solve that of the dielectric cylinder. Once the currents are generated, the interaction between

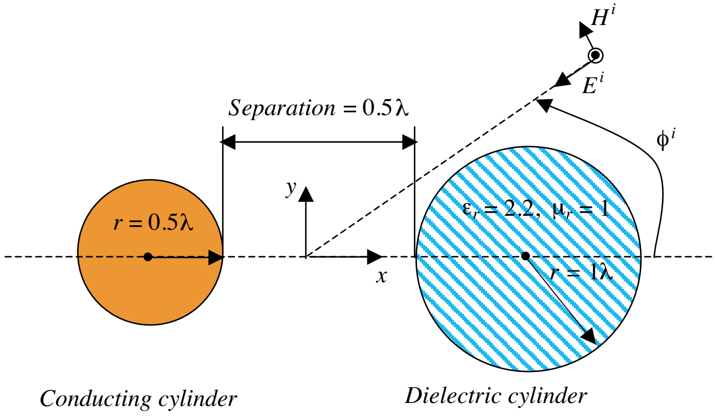

Figure 3. Geometry of one conducting and one dielectric cylinder.

the two cylinders takes place by applying the IMR technique in the same manner described in Section 4. One of the advantages of using the MoM technique is to reduce the computational time required to solve for conducting structures, thus the consumed time by the hybrid, IMR – FDFD/MoM, solution after some iteration is then going to be less than that used for the same number of iterations by the IMR – FDFD/FDFD solution. Figure 4 shows the far field calculations for the problem defined in Fig. 3, where a comparison between three different approaches is presented: the full domain solution based on FDFD, the IMR – FDFD/FDFD solution, and the hybrid IMR – FDFD/MoM solution. It can be clearly seen from the figure, the strong match between the FDFD/FDFD and the hybrid FDFD/MoM solutions after 4 iterations, where both solutions approach the full domain solution using the FDFD technique. Table 1 shows a comparison between the three approaches, regarding the total computational domain size, which is involved in constructing the matrix solution, as well as the total computational time for each problem. Table 1 indicates that the total computational size using the IMR – FDFD/FDFD technique, for the problem defined in Fig. 3, is 30% less than solving the classical FDFD solution applied to the whole problem; this is due to the flexibility in discretizing each domain separately and thus no obligation on using a smaller discretization when defining a simple structure. As for the hybrid IMR – FDFD/MoM technique, the required computational size is 60% less than that required to solve for the whole problem, which proves the efficiency of using the hybrid FDFD/MoM solution together with the IMR technique, regarding the computational memory consumption. Concerning the computational

Table 1. Comparison between the full domain solution, the IMR – FDFD/FDFD, and the IMR – FDFD/MoM techniques, regarding both the computational domain size and the computational time for the problem illustrated in Fig. 3.

	Full domain (FDFD)	IMR – FDFD/FDFD	IMR – FDFD/MoM
Total Domain size (cells)	9,163	6,498	3,694
Computational time	36 sec.	71 sec.	49 sec.

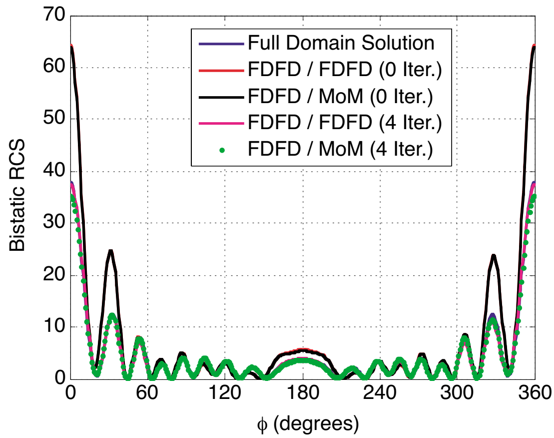

Figure 4. Bistatic echo width a conducting and dielectric cylinder excited by a TMz plane wave incident at $\phi^i = 180°$.

time, the hybrid IMR – FDFD/MoM converged to the full domain solution after 4 iterations in 49 seconds, while the IMR – FDFD/FDFD converged after the same number of iterations in 71 seconds, both compared to the full domain solution that took 36 seconds on a Pentium 4 machine with a processor of 3.2 GHz and 2 GB RAM.

Figure 5 shows another configuration that consists of two different scatterers, a conducting square of side 0.5 λ, and a dielectric ellipsoid of relative permittivity equals to 2.2 and relative permeability equals to 1. The radius of the ellipsoid in the x-direction is equal to 0.25 λ and in the y-direction 0.5 λ, the separation distance between the two scatterers is 0.25 λ. The proposed structure in Fig. 5 is excited by a TM_z plane wave incident at $\phi^i = 90°$. Figure 6 shows a comparison between the bistatic echo widths of the three approaches, as presented previously

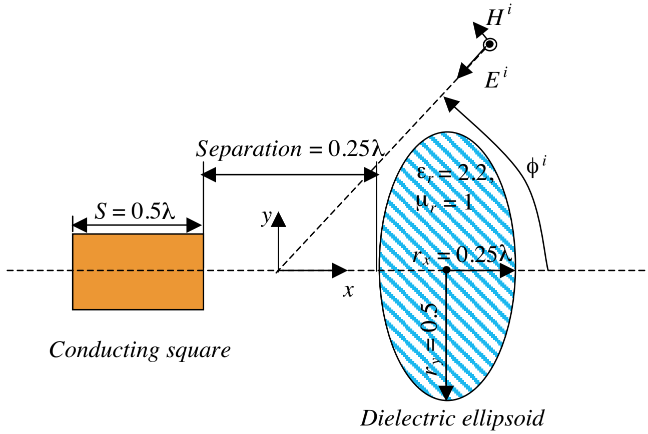

Figure 5. Geometry of a conducting square and a dielectric ellipsoid.

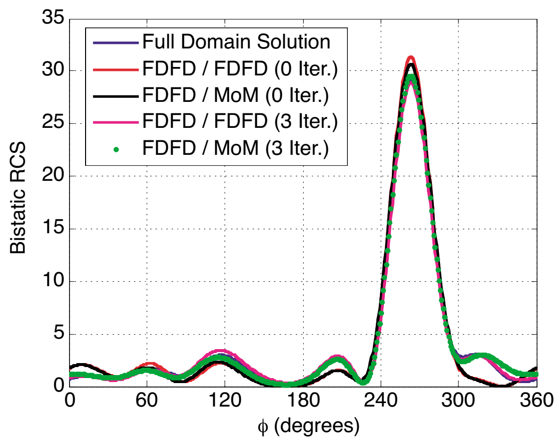

Figure 6. Bistatic echo width a conducting square and dielectric ellipsoid excited by a TMz plane wave incident at $\phi^i = 90°$.

in Fig. 4. Again, in the hybrid IMR– FDFD/MoM solution, the MoM solution is used to solve the scattered field from the conducting square, while the FDFD is used to solve that of the dielectric ellipsoid. Still a strong match can be noticed between the FDFD/FDFD and the hybrid FDFD/MoM solutions at 0 iteration and after 3 iterations, where both solutions approach the full domain solution. Table 2 shows the total computational size using the IMR – FDFD/FDFD technique, for the problem defined in Fig. 5, to be 53% less than solving the classical

Table 2. Comparison between the full domain solution, the IMR – FDFD/FDFD, and the IMR – FDFD/MoM techniques, regarding both the computational domain size and the computational time for the problem illustrated in Fig. 5.

	Full domain (FDFD)	IMR – FDFD/FDFD	IMR – FDFD/MoM
Total Domain size (cells)	15,776	7,392	4,416
Computational time	39 sec.	21 sec.	17 sec.

FDFD solution applied to the whole problem. While for the hybrid IMR – FDFD/MoM technique, the required computational size was 70% less than that required to solve for the whole problem. As for the computational time, the full domain solution required 39 seconds to finalize the simulation, while the IMR – FDFD/FDFD took 21 seconds to converge to the full domain solution after 3 iterations, and for the same number of iterations, the IMR – FDFD/MoM took 17 seconds.

6. CONCLUSION

In this paper an iterative multi-region technique is proposed to solve large-scale electromagnetic problems by dividing a problem into separate sub-regions and then using a hybrid MoM and FDFD solutions. This procedure starts by dividing the original computational domain into separate sub-regions where the solution is easily performed using either the MoM or the FDFD techniques in each sub-region followed by an iterative interaction process between the sub-regions. The new approach proposed here is found to be efficient in producing accurate results for the original problem with over 60% saving in the computer memory usage and with no significant change in the computational time.

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