Progress In Electromagnetics Research, PIER 56, 1–18, 2006

CHARACTERISTICS OF A MICROSTRIP-FED MINIATURE PRINTED HILBERT SLOT ANTENNA

A. T. M. Sayem and M. Ali

Department of Electrical Engineering
University of South Carolina
Swearingen Building, Columbia, SC 29208, USA

Abstract—The resonant frequencies and input impedance of a microstrip-fed third order Hilbert slot antenna are studied as function of antenna and substrate parameters. A distinct dual-band characteristic is observed. Findings from HFSS simulation are validated by prototype fabrication and experimental measurement. The two resonant frequencies maintain a ratio of approximately 1.4 for narrow slots. A design parameter, L_{ax} is identified which needs to be about $0.80\lambda_g$ at the lowest operating frequency. The Hilbert slot provides a unidirectional pattern when operated against a metallic ground plane while maintaining dual-band characteristics.

1. INTRODUCTION

Study of small or miniaturized antennas and their radiation characteristics date back several decades [1–6]. Lately researchers have focused on investigating small self-resonant antennas, such as helical, meander, zigzag, and fractal type structures to achieve significant size reduction [7–14]. Among those considerable interest has been shown in self-similar space filling fractal geometries. These include studies on the Sierpinski gasket antenna, fractal dipoles and loops, and the Hilbert type antenna.

An overview of the fractal antenna concept and its application in arrays was given in [15]. In [16] a comparison between a Hilbert wire antenna and other self-resonant wire antennas was presented in terms of lowering the antenna resonant frequency. Similar yet more elaborate studies were presented in [17] that addressed properties, such as radiation resistance, quality factor (Q), and efficiency of Hilbert, modified Minkowski fractal, Koch fractal, and other small space filling antennas. Recently efforts have been placed to develop novel antennas using the fractal concept [18–21].

Hilbert type antennas have drawn specific interest among researchers to achieve miniaturization [22–28]. In [22] the resonant properties of a wire Hilbert antenna in free space were investigated. A vertical Hilbert monopole antenna was discussed in [24]. Chen et al. [25] proposed a printed Hilbert antenna that worked in the lower frequency band (UHF). In [26] Zhu et al. have used the Method of Moments (MoM) to conduct a parametric study of bandwidth, cross-polarization level and feed location of a matched Hilbert antenna. It was shown that input impedance of the order of 50Ω may be achieved by feeding the antenna closer to the end rather than feeding it at the centre. Recently a dual band miniature Hilbert PIFA was proposed for wireless mobile phone applications [28].

Majority of the work on Hilbert antennas reported to date were based on the dipole or monopole platform. In this paper we introduce a miniaturized Hilbert slot antenna fed by a microstrip transmission line. Considering a third order Hilbert curve [26] the characteristics of the slot antenna are studied using HFSS [29]. The antenna resonant frequencies, impedance, and patterns are studied as function of its geometrical parameters, substrate thickness, dielectric constant, transmission line parameter, and the presence and absence of a ground plane. It is observed that the third order Hilbert slot has an inherent dual-band return loss characteristic. When placed against a metallic ground plane the antenna retains its dual-band performance with very small antenna to reflector spacing. The Hilbert slot antenna in its first resonant frequency is 50% shorter than its printed dipole counterpart.

2. ANTENNA CONFIGURATION

The geometry of the proposed Hilbert slot antenna is shown in Fig. 1. The slot was constructed based on a third order Hilbert curve. The antenna was printed and fed by a 50Ω microstrip transmission line on a dielectric substrate with thickness, h and dielectric constant, ε_r. The Hilbert slot was constructed on the ground plane side of a substrate measuring 100 mm by 100 mm. The parameters of the antenna are L, w, w_L, and x where the latter two represent the width of the feed transmission line and the length of the microstrip T-junction. Study of the antenna was conducted using Ansoft HFSS 9.0 results of which were then confirmed by experimental measurements.

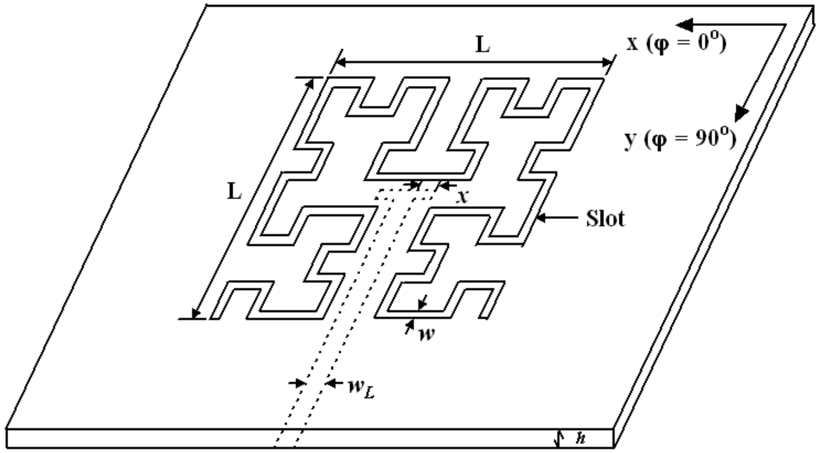

Figure 1. Geometry of the proposed Hilbert slot antenna. Parameters: substrate thickness = h, dielectric constant = ε_r, antenna length = L, slot width = w, spread of T-junction = x, transmission line width = w_L.

3. COMPUTATIONAL RESULTS

As a starting point an antenna model with the following parameters was constructed in HFSS: $L = 31.7$ mm, $w = 2$ mm, $w_L = 2.5$ mm, $h = 0.813$ mm and $\varepsilon_r = 2.2$ (Duroid 5880). First the effect of the length of the microstrip T-junction, x on the antenna impedance and $|S_{11}|$ characteristics were studied.

3.1. Microstrip T-Junction

Computed $|S_{11}|$ (dB) data for the proposed Hilbert slot antenna with the microstrip T-junction extension, x as the parameter are shown in Fig. 2. A distinct dual-frequency characteristic is observed. The first and second resonant frequencies are around 2.0 and 2.9 GHz, respectively. The effect of changing the length of the microstrip T-junction [see Fig. 1] is negligible on the antenna resonant frequencies. The ratio of the two resonant frequencies (high/low) is approximately 1.4. The microstrip T-junction has significant effect on the overall impedance matching of the antenna as observed from Figs. 2 and 3. For smaller values of x the high-band matching is superior to the low-band. As x increases the low-band impedance matching starts to improve and for $x = 1$ mm both bands are fairly well matched. It is observed that

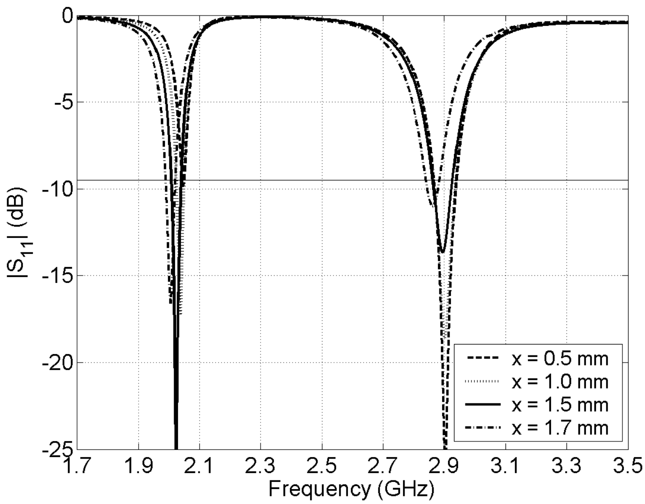

Figure 2. Computed $|S_{11}|$ versus frequency for the proposed Hilbert slot antenna with the length of the microstrip T-junction, x as the parameter. Antenna is printed on 0.813 mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). Other parameters: $L = 31.7$ mm and $w = 2$ mm, $w_L = 2.5$ mm.

for $x = 1.5$ mm the bandwidth in the low and high bands are 1.7% and 2.2% within -9.5 dB $|S_{11}|$. Further increase in x deteriorates the high-band performance.

The nature of the antenna matching with the feed transmission line can be explained from the Smith chart plots of Fig. 3. As indicated there are two types of loops: (1) Loops A — representing the low-frequency resonance and (2) Loops B — representing the high-frequency resonance. When the length of the microstrip T-junction, x is small (0.5 mm) the high-frequency loop is fairly close to the center of the Smith chart indicating a good match. The low-frequency loop for the same case moves away from the center of the chart indicating a poor match. When x is increased the low-frequency loops move toward their left and the high-frequency loops move toward their right. The relative movement of the low-frequency loops is greater than their high-frequency band counterparts. This helps achieve a good match for the low-frequency band without deteriorating the high-frequency band. The change in the lengths of the microstrip T-junction does not introduce any significant rotation for the impedance loops. They move left or right very much on a straight line.

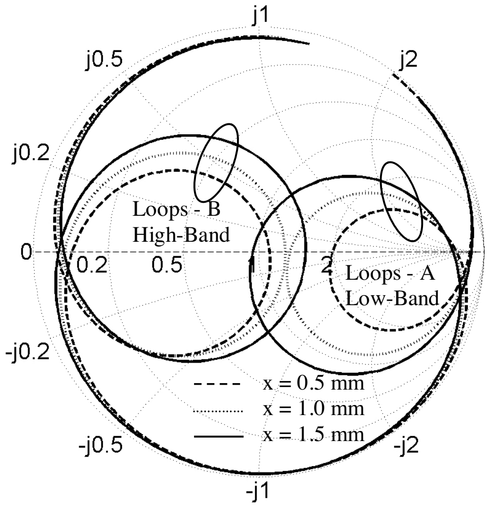

Figure 3. Computed input impedance versus frequency for the proposed Hilbert slot antenna with the length of the microstrip T-junction, x as the parameter. Antenna is printed on 0.813 mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). Other parameters: $L = 31.7$ mm and $w = 2$ mm, $w_L = 2.5$ mm.

3.2. Substrate Thickness (h)

Fig. 4 depicts the influence of the substrate thickness, h on the $|S_{11}|$. For each change in substrate thickness the width of the microstrip transmission line was adjusted accordingly to achieve 50Ω. As expected the dual-resonance characteristic is unaffected by the change in substrate thickness. With increasing thickness the antenna resonant frequencies at both bands shift lower (approximately 100 MHz when the substrate thickness varies from 0.4 mm to 1.0 mm). This is also expected since with increasing substrate thickness, the effective dielectric constant increases slightly which translates into a reduction in the resonant frequency. The ratio between the high and low resonant frequencies remains unchanged to 1.4.

The input impedances of the proposed Hilbert antenna for the same cases are shown in Fig. 5. It is clear that changing the substrate thickness also has significant effect on the input impedance of the antenna, with thickness increasing the change for loops B are more pronounced than loops A. This is the exact opposite of Fig. 3 where loops A were changing more drastically than B when x was adjusted. Therefore, for instance if a substrate thickness of 0.4 mm is desirable x

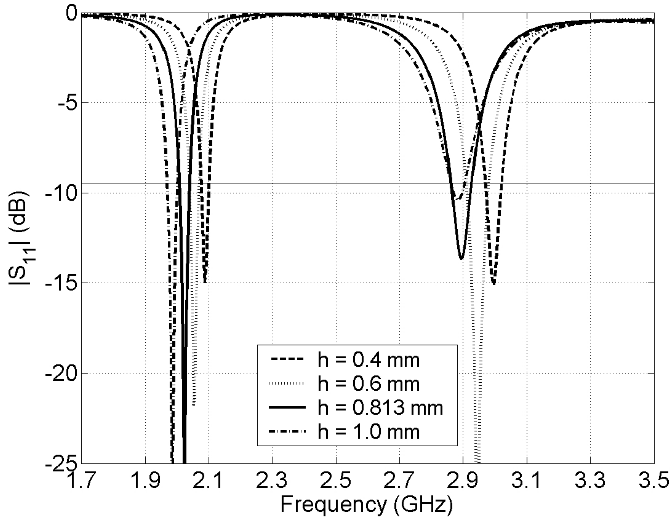

Figure 4. Computed $|S_{11}|$ versus frequency for the proposed Hilbert slot antenna with substrate thickness, h as the parameter. Antenna is printed on Duroid 5880 substrate ($\varepsilon_r = 2.2$). T-junction spread, $x = 1.5\,\text{mm}$. Other parameters: $L = 31.75\,\text{mm}$ and $w = 2\,\text{mm}$.

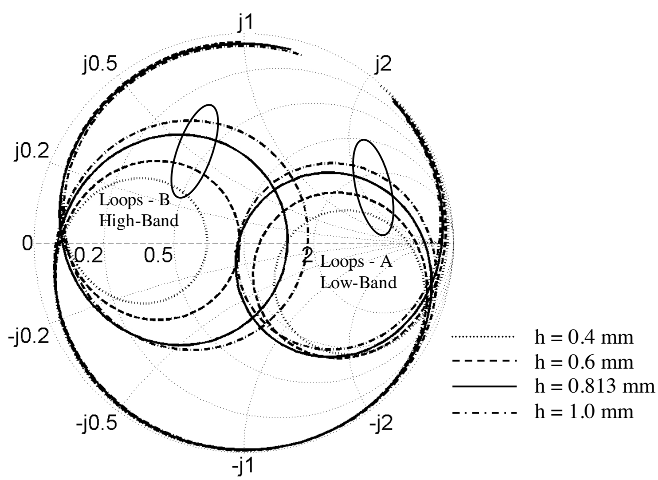

Figure 5. Computed input impedance versus frequency for the proposed Hilbert slot antenna with h as the parameter. Antenna is printed on Duroid 5880 substrate ($\varepsilon_r = 2.2$). Other antenna parameters: $L = 31.75\,\text{mm}$ and $w = 2\,\text{mm}$, $x = 1.5\,\text{mm}$.

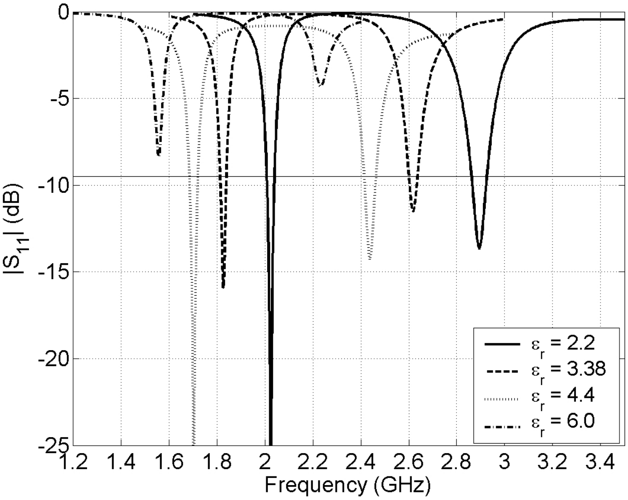

Figure 6. Computed $|S_{11}|$ versus frequency for the proposed Hilbert slot antenna with ε_r as the parameter. Antenna is printed on 0.813 mm thick substrate. Other antenna parameters: $L = 31.75$ mm and $w = 2$ mm and $x = 1.5$ mm.

needs to be increased so that loops A and B move in their respective directions and result in optimum matching. Similarly, when $h = 1.0$ mm we may need to decrease x slightly to achieve a better high band matching.

3.3. Dielectric Constant (ε_r)

Fig. 6 shows the $|S_{11}|$ data with varying dielectric constant. The low and high resonant frequencies shift lower as ε_r increases. For dielectric constants up to 4.4 the $|S_{11}|$ characteristics are good. With further increase in ε_r to 6.0 the $|S_{11}|$ data degrade which can be improved by proper choice of thickness and T-junction spread, x.

3.4. Antenna Parameter Adjustment

Simulations were conducted to observe the antenna resonant characteristics as function of antenna length, L and slot width, w. Duroid 5880 substrate ($\varepsilon_r = 2.2$) with thickness, $h = 0.813$ mm was considered. The length of the microstrip T-junction extension, x was fixed at 1.5 mm. Antenna length was varied as 20, 31.7, and 44 mm. The $|S_{11}|$ characteristics for this study are shown plotted in Fig. 7.

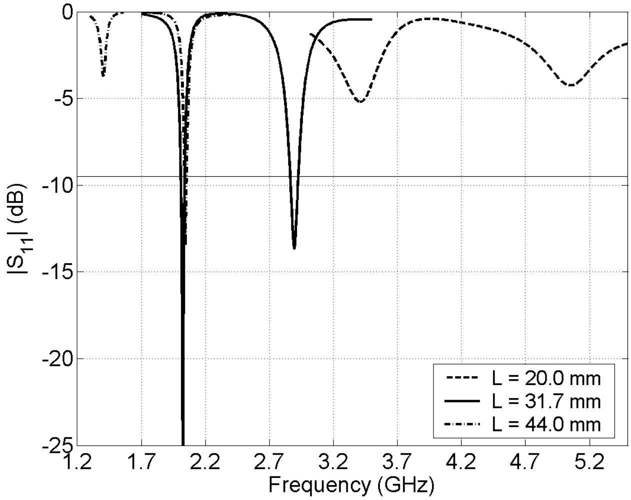

Figure 7. Computed $|S_{11}|$ versus frequency for the proposed Hilbert slot antenna with antenna length, L as the parameter. Antenna is printed on 0.813 mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). T-junction spread, $x = 1.5\,\text{mm}$. Other antenna parameter: $w = 2\,\text{mm}$.

As expected, with increase in antenna length, L the two resonant frequencies shift lower. The ratio between the high and low resonant frequencies remains fixed at approximately 1.4. No attempts were made to improve the tuning of the poorly matched antennas for the sake of brevity. However, it is clear from previous studies [Figs. 3 and 5] that antenna matching can be improved by adjusting the microstrip T-junction length and the substrate thickness.

The effect of the slot width on the input return loss of the Hilbert antenna is studied in Fig. 8. Slot width varies as 0.25, 0.50, 1.0, and 2.0 mm. Once again Duroid 5880 substrate ($\varepsilon_r = 2.2$) with thickness, $h = 0.813\,\text{mm}$ was considered. The length of the microstrip T-junction extension, x was fixed at 1.5 mm. Antenna length, L was 31.7 mm. From Fig. 8 it is apparent that similar to the first case studied in Fig. 3 all four slot widths show two distinct resonant loops representing the low and high frequency bands. Interestingly for a particular slot width the two impedance loops lie approximately on a straight line. Thus it is plausible that the poor impedance matches for the narrower slots can be remedied by adjusting the length of the microstrip T-junction, x. It is also observed that with increase in slot width the impedance loops go through a clockwise rotation resembling that of an addition of a shunt capacitor.

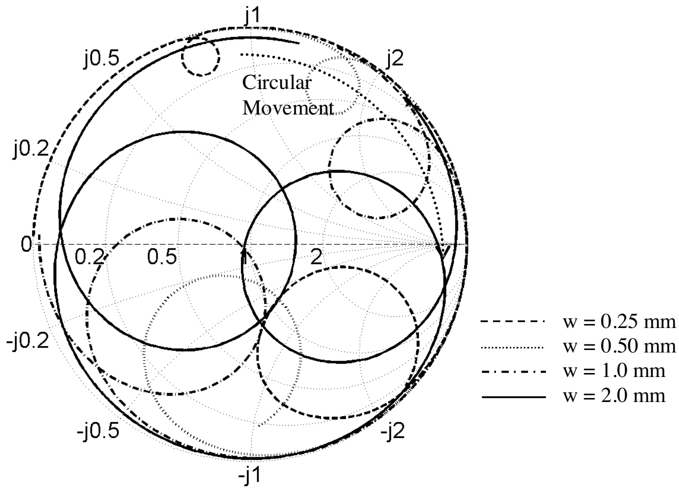

Figure 8. Computed input impedance versus frequency for the proposed Hilbert slot antenna with w as the parameter. Antenna is printed on 0.813 mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). Other antenna parameters: $L = 31.75$ mm and $x = 1.5$ mm.

To demonstrate the effectiveness of the microstrip T-junction in antenna impedance tuning the slot antenna with width 0.5 mm was considered. Corresponding impedance plots for $x = 1.5$, 3.0, and 4.5 mm are shown in Fig. 9. Clearly by increasing x the impedance loops move along a straight line and help achieve a better match.

4. EXPERIMENTAL RESULTS

To validate the performance of the proposed Hilbert slot antenna two antennas were fabricated and measured. First, an antenna with the parameters of $L = 31.7$ mm, $w = 2$ mm, $W_L = 2.5$ mm, $h = 0.813$ mm, and $x = 1.5$ mm was fabricated on Duroid 5880 ($\varepsilon_r = 2.2$). A comparison of the measured and simulated data for this antenna is shown in Fig. 10 along with an inset photograph of the actual antenna. The measured data is in good agreement with the simulated. The resonant frequencies are about the same and the bandwidths are also similar.

Computed and measured data for the Hilbert slot antenna on a 0.95 mm thick FR4 ($\varepsilon_r = 4.4$) substrate are shown in Fig. 11. These data are also in good agreement. In both cases, measured bandwidths at the high resonant frequency are wider than computed.

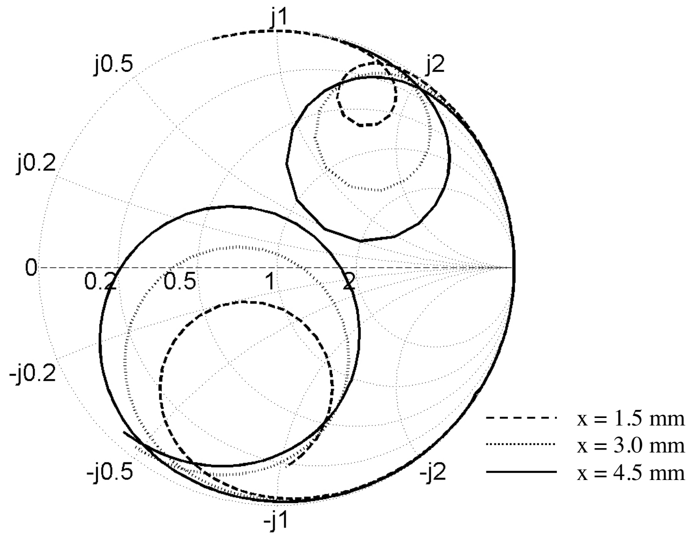

Figure 9. Computed input impedance versus frequency for the proposed Hilbert slot antenna with x as the parameter. Antenna is printed on 0.81 mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). Other antenna parameters: $w = 0.5$ mm, $L = 31.7$ mm.

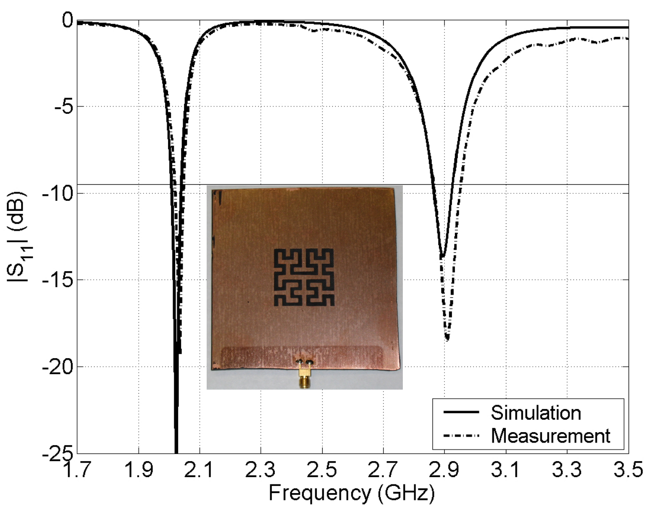

Figure 10. Comparison between computed and measured $|S_{11}|$ data for Hilbert slot antenna. Antenna is printed on $h = 0.813$ mm thick Duroid 5880 substrate ($\varepsilon_r = 2.2$). T-junction spread, $x = 1.5$ mm. Other antenna parameter: $L = 31.75$ mm, $w = 2$ mm.

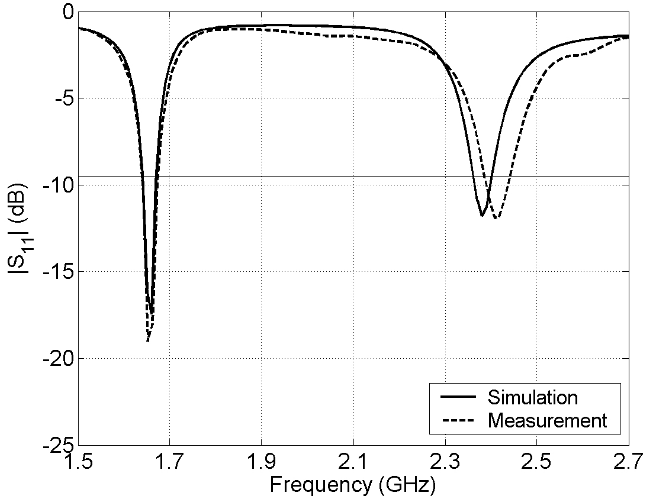

Figure 11. Comparison between computed and measured $|S_{11}|$ data for Hilbert slot antenna. Antenna is printed on $h = 0.95$ mm thick FR4 substrate ($\varepsilon_r = 4.4$). T-junction spread, $x = 1.0$ mm. Other antenna parameter: $L = 31.75$ mm, $w = 2$ mm, $w_L = 1.8$ mm.

5. RADIATION CHARACTERISTICS AND UNIDIRECTIONAL BEAM

Computed radiation patterns of the Hilbert slot antenna are shown in Fig. 12. Patterns were computed at 2.02 and 2.89 GHz for an antenna with the following parameters: $h = 0.813$ mm, $\varepsilon_r = 2.2$, $L = 31.75$ mm, $w = 2$ mm, and $x = 1.5$ mm. In the absence of a ground plane, the radiation patterns are bi-directional. At 2.02 GHz the peak gain is 3.3 dBi. In the $\phi = 0°$ plane E_ϕ is the dominant component and in the $\phi = 90°$ plane E_θ is the dominant component. Maximum cross-polarization level at 2.02 GHz is -8.5 dB which occurs at $\phi = 0°$. The cross-polarization at $\phi = 90°$ is much smaller (-30 dB). The nature of the cross-polarization is consistent with the radiation characteristics of a third-order wire Hilbert antenna [26]. At 2.89 GHz the peak gain is 4.2 dBi and in both planes E_θ is the dominant component. The beam in both planes is bifurcated which indicates a higher order mode of radiation.

Antenna characteristics were also studied by adding a metallic reflector plane of size 200 mm by 200 mm to achieve unidirectional patterns. The reflector plate was placed 3.0 mm below the microstrip transmission line. Rohacell foam ($\varepsilon_r = 1.04$) was used to support the

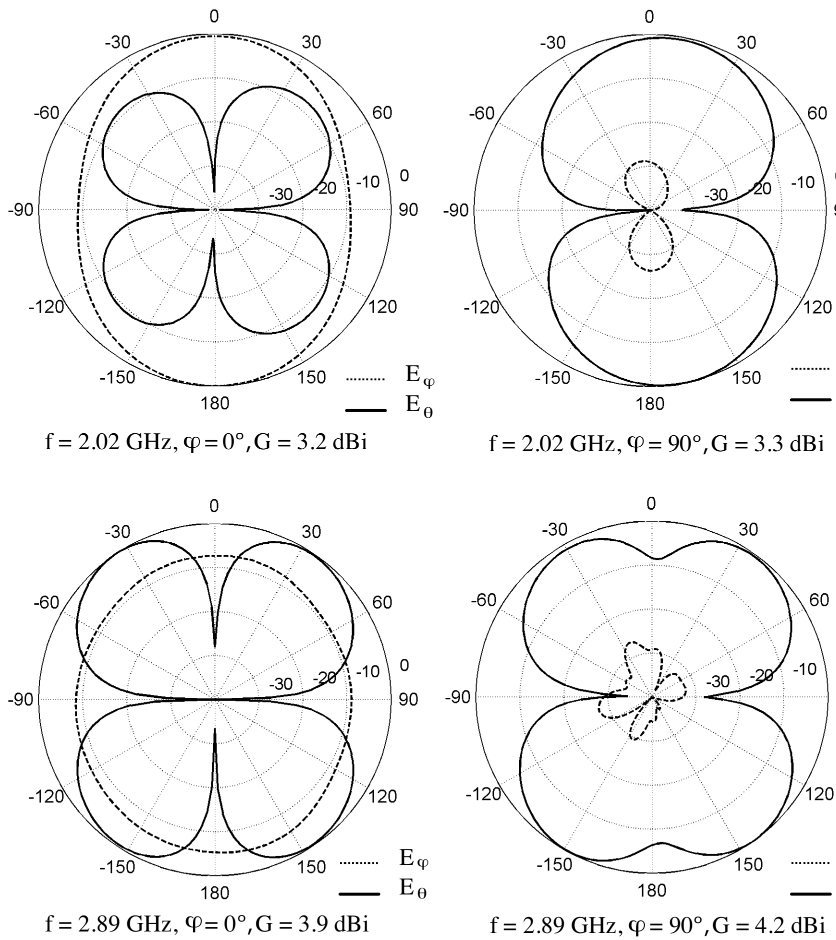

Figure 12. Computed radiation patterns for the proposed Hilbert slot antenna at both resonant frequencies. Antenna is printed on $h = 0.813\,\text{mm}$ thick Rogers 5880 substrate ($\varepsilon_r = 2.2$). Other antenna parameters: $L = 31.75\,\text{mm}$, $w = 2\,\text{mm}$, $x = 1.5\,\text{mm}$.

antenna and microstrip transmission line assembly above this reflector plate. Simulated and measured $|S_{11}|$ data for this antenna are shown in Fig. 13. The resonant frequencies of this antenna were somewhat lower (2.02 and 2.89 GHz) than they are with the reflector (2.2 and 3.0 GHz). An additional resonance at around 2.5 GHz is also visible. The simulated and measured $|S_{11}|$ are similar in general. At the high frequency the measured $|S_{11}|$ value at the resonant frequency is not

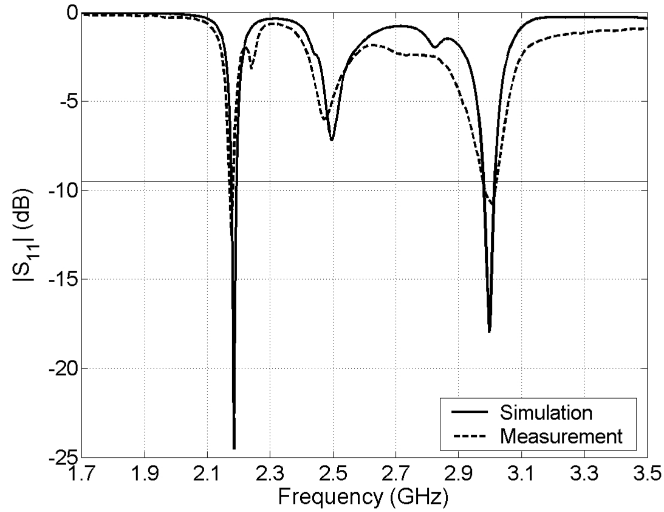

Figure 13. Computed and measured $|S_{11}|$ data for the Hilbert slot antenna on top of a metal reflector of size 200 mm by 200 mm at a distance of 3 mm from the microstrip line plane. Other parameters are: $h = 0.813\,\mathrm{mm}$, $\varepsilon_r = 2.2$, $L = 31.75\,\mathrm{mm}$, $w = 2\,\mathrm{mm}$, $x = 1.5\,\mathrm{mm}$.

as low as the simulated. This may be due to the differences between the simulation model and the actual measurement scenario. In the measurement scheme a coaxial line lies 3 mm above the metal sheet reflector in order to establish connection with the microstrip line.

Computed radiation patterns for the antenna with the metal reflector are shown in Fig. 14. Clearly the patterns are directional with the front to back ratios better than 20 dB. The cross-polarization at foresight at 2.2 GHz is extremely small.

6. DISCUSSION AND CONCLUSIONS

From the studies above we can conclude that the third-order Hilbert slot antenna has dual-resonance characteristics. Such an antenna is much smaller to a printed half-wave dipole or full-wave slot antenna (see Table 1). In addition, unlike a dipole it does not require a balun. The limitations observed are: (1) the ratio of the two resonant frequencies is approximately constant and (2) the antenna radiates both components of fields as expected from its multi-folded geometrical shape. The nearly constant ratio between the two resonant frequencies can be slightly changed by adopting different slot widths. The

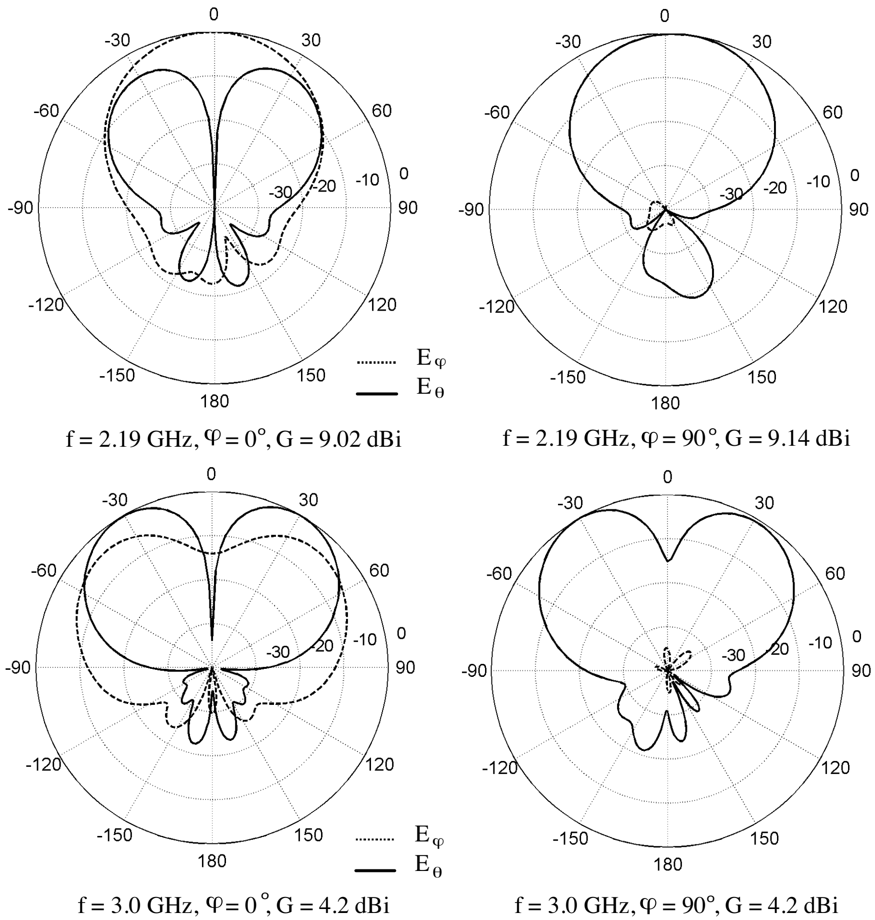

Figure 14. Computed radiation patterns for the proposed Hilbert slot antenna on top of a metal reflector of size 200 mm by 200 mm at a distance of 3 mm from the microstrip line plane. Antenna is printed on $h = 0.813$ mm thick Rogers 5880 substrate ($\varepsilon_r = 2.2$). Other antenna parameters: $L = 31.75$ mm, $w = 2$ mm, $x = 1.5$ mm.

polarization issue may not be a factor when applied to mobile wireless applications where these types of antennas with multiple components may be an advantage. When used as a directional antenna as suggested in the last example further research is needed in terms of examining the cross-polarization. Also $|S_{11}|$ performance needs to be improved.

The next question that must be answered is how about designing the antenna for a particular frequency band? In this regard we tabulated all our data in terms of a common design parameter called L_{ax}. This parameter is defined in Fig. 15. Based on the guided wavelength for a slot-line given in [30] we calculated the L_{ax} in λ.

Table 1. Computed Guided Wavelength (λ_g) and L_{ax}.

Case	Low Resonant Frequency (f_l) GHz	Guided Wavelength (λ_g) mm	(L_{ax}) in λ_g
$L=20.0$, $h=0.813$, $w=2.0$, $w_L=2.5$, $\varepsilon_r=2.2$	3.400	81.8	0.84
$L=31.7$, $h=0.813$, $w=2.0$, $w_L=2.5$, $\varepsilon_r=2.2$	2.025	139.0	0.82
$L=44.0$, $h=0.813$, $w=2.0$, $w_L=2.5$, $\varepsilon_r=2.2$	1.402	202.0	0.80
$\varepsilon_r=4.4$, $L=31.7$, $h=0.813$, $w=2.0$, $w_L=1.6$	1.701	149.0	0.76
$\varepsilon_r=6$, $L=31.7$, $h=0.813$, $w=2.0$, $w_L=1.2$	1.560	151.0	0.75
$h=0.4$, $L=31.7$, $w=2.0$, $w_L=1.2$, $\varepsilon_r=2.2$	2.088	140.2	0.81
$h=0.6$, $L=31.7$, $w=2.0$, $w_L=1.9$, $\varepsilon_r=2.2$	2.054	139.4	0.81
$h=0.813$, $L=31.7$, $w=2.0$, $w_L=2.5$, $\varepsilon_r=2.2$	2.025	139.0	0.82
$h=1.0$, $L=31.7$, $w=2.0$, $w_L=3.1$, $\varepsilon_r=2.2$	1.984	140.2	0.81
$h=1.2$, $L=31.7$, $w=2.0$, $w_L=3.7$, $\varepsilon_r=2.2$	1.958	140.6	0.81

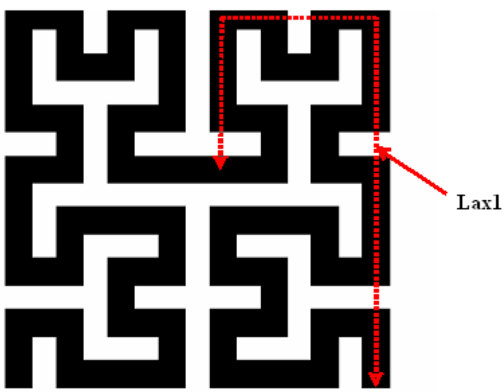

Figure 15. Axial Length of the Proposed Hilbert slot antenna. ($L_{ax} = 2 * L_{ax1}$).

The equations used for this purpose are:

For $2.22 \leq \varepsilon_r \leq 3.8$

$$\lambda_g/\lambda_0 = 1.045 - 0.365 \ln \varepsilon_r + \frac{6.3(\boldsymbol{w}/\boldsymbol{h})\varepsilon_r^{0.945}}{(238.64 + 100\boldsymbol{w}/\boldsymbol{h})} - \left[0.148 - \frac{8.81(\varepsilon_r + 0.95)}{100\varepsilon_r}\right] \ln(\boldsymbol{h}/\lambda_0) \quad (1)$$

For $3.8 \leq \varepsilon_r \leq 9.8$

$$\lambda_g/\lambda_0 = 0.9217 - 0.277 \ln \varepsilon_r + 0.0322(\boldsymbol{w}/\boldsymbol{h}) \left[\frac{\varepsilon_r}{(\boldsymbol{w}/\boldsymbol{h} + 0.435)}\right]^{1/2} - 0.01 \ln(\boldsymbol{h}/\lambda_0) \left[4.6 - \frac{3.65}{\varepsilon_r^2 \sqrt{\boldsymbol{w}/\lambda_0}(9.06 - 100\boldsymbol{w}/\lambda_0)}\right] \quad (2)$$

It is clear that for all the cases studied a third order Hilbert slot antenna can be designed with an approximate axial length, $L_{ax} = 0.80\lambda_g$ where λ_g is the guided wavelength given in [30].

While actual practical applications of such antennas must satisfy the specific bandwidth, gain, and pattern characteristics, these miniature printed slot antennas with dual-band characteristics clearly show potentials for future mobile wireless applications.

ACKNOWLEDGMENT

This work was supported in part by the National Science Foundation (NSF) Career Award ECS-0237783.

REFERENCES

1. Chu, L. J., "Physical limitations on omni-directional antennas," *J. Appl. Phys.*, Vol. 19, 1163–1175, Dec. 1948.
2. Fenwick, R., "A new class of electrically small antennas," *IEEE Trans. Antennas and Propagat.*, Vol. 13, Issue 3, 379–383, May 1965.
3. Wheeler, H. A., "Small antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-23, 462–469, July 1975.
4. Hansen, R. C., "Fundamental limitations of small antennas," *Proc. IEEE*, Vol. 69, 170–182, Feb. 1981.
5. McLean, J. S., "A reexamination of the fundamental limits on the radiation Q of electrically small antennas," *IEEE Trans. Antennas Propagat.*, Vol. 44, No. 5, 672–676, May 1996.

6. Thiele, G. A., P. L. Detweiler, and R. P. Penno, "On the lower bound of the radiation Q for electrically small antennas," *IEEE Trans. Antennas Propagat.*, Vol. 51, No. 6, 1263–1269, June 2003.
7. Nakano, H., H. Tagami, A. Yoshizawa, and J. Yamauchi, "Shortening ratios of modified dipoles," *IEEE Trans. Antennas Propagat.*, Vol. AP-32, No. 4, 385–387, Apr. 1984.
8. Rashed, J. and C. T. Tai, "A new class of resonant antennas," *IEEE Trans. Antennas Propagat.*, Vol. 39, No. 9, 1428–1430, Sept. 1991.
9. Ali, M., S. S. Stuchly, and K. Caputa, "Characteristics of bent wire antennas," *Journal of Electromagnetic Waves and Applications*, Vol. 9, No. 9, 1149–1162, 1995.
10. Ali, M., S. S. Stuchly, and K. Caputa, "A wide-band dual meander-sleeve antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 10, No. 9, 1223–1236, 1996.
11. Baliarda, C. P., J. Romeu, and A. Cardama, "The Koch monopole: A small fractal antenna," *IEEE Trans. Antennas and Propagat.*, Vol. 48, No. 11, 1773–1781, November 2000.
12. Ali, M., M. Okoniewski, and S. S. Stuchly, "Study of a printed meander antenna using the FDTD method," *Microwave Opt. Technol. Lett.*, Vol. 37, 440–444, June 2003.
13. Ali, M., G. J. Hayes, H.-S. Hwang, and R. A. Sadler, "Design of a multi-band internal antenna for third generation mobile phone handsets," *IEEE Trans. Antennas and Propagat.*, Vol. 51, No. 7, 1452–1461, July 2003.
14. Petko, J. S. and D. H. Werner, "Miniature reconfigurable three-dimensional fractal tree antennas," *IEEE Trans. Antennas and Propagat.*, Vol. 52, No. 8, 1945–1956, August 2004.
15. Gianvittorio, J. P. and Y. Rahmat-Samii, "Fractal antennas: A novel antenna miniaturization technique, and applications," *IEEE Ant. and Propag. Magazine*, Vol. 44, No. 1, 20–36, February 2002.
16. Best, S. R. and J. D. Morrow, "The effectiveness of space-filling fractal geometry in lowering resonant frequency," *IEEE Antennas Wireless Propagat. Lett.*, Vol. 1, Issue 5, 112–115, 2002.
17. Best, S. R., "A comparison of the resonant properties of small space-filling fractal antennas," *IEEE Antennas Wireless Propagat. Lett.*, Vol. 2, Issue 13, 197–200, 2003.
18. Gelman, Y., E. B. Ari, and R. Shavit, "Multi-band bowtie antenna based on fractal geometry," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 4, 3441–3444, Monterey, CA, June 2004.

19. Dehkhoda, P. and A. Tavakoli, "Circularly polarized microstrip fractal antennas," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 4, 3453–3456, Montery, CA, June 2004.
20. Lule, E., T. Babij, and TimeDerivative, Inc., "Koch island fractal ultra wideband dipole antenna," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 3, 2516–2519, Monterey, CA, June 2004.
21. Li, X., X. X. Yin, T. J. Cui, and W. Hong, "A new version of printed Sierpinski multiband fractal antenna," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 4, 3457–3460, Monterey, CA, June 2004.
22. Vinoy, K. J., K. A. Jose, V. K. Varadan, and V. V. Varadan, "Hilbert curve fractal antenna: A small resonant antenna for VHF/UHF applications," *Microwave Opt. Technol. Lett.*, Vol. 29, No. 4, 215–219, May 2001.
23. Romeu, J. and S. Blanch, "A three dimensional Hilbert antenna," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 4, 550–553, San Antonio, TX, June 2002.
24. Anguera, J., C. Puente, and J. Soler, "Miniature monopole antenna based on the fractal Hilbert curve," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 4, 546–549, San Antonio, TX, June 2002.
25. Chen, X., S. S. Naeini, and Y. Liu, "A down sized printed Hilbert antenna for UHF band," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 2, 581–584, Columbus, OH, June 2003.
26. Zhu, J., A. Hoorfar, and N. Engheta, "Bandwidth, cross-polarization, and feed-point characteristics of matched Hilbert antennas," *IEEE Antennas Wireless Propagat. Lett.*, Vol. 2, Issue 1, 2–5, 2003.
27. Sayem, A. T. M., M. Ali, and H. S. Hwang, "Miniaturized dual-band Hilbert slot antenna for wireless application," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 3, 3119–3122, Monterey, CA, June 2004.
28. Azad, M. Z. and M. Ali, "A compact Hilbert planar inverted-F antenna (PIFA) for dual-band mobile phone applications," *Proc. IEEE Antennas and Propagation Society Int. Symp.*, Vol. 3, 3127–3130, Monterey, CA, June 2004.
29. *Ansoft HFSS*, version 9.0, Ansoft Corporation, Pittsburgh, PA.
30. Janaswamy, R. and D. H. Schaubert, "Characteristic impedance of a wide slotline on low-permittivity substrates," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-34, 900–902, Aug. 1986.