Progress In Electromagnetics Research, PIER 53, 271–281, 2005

FREQUENCY-DOMAIN ANALYSES OF NONLINEARLY LOADED ANTENNA ARRAYS USING SIMULATED ANNEALING ALGORITHMS

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Abstract—In this paper, simulated annealing algorithms are applied to the analyses of nonlinearly loaded antenna arrays. The analysis is first transformed into an optimization problem and then be solved by simulated annealing algorithms. Numerical examples show that the results calculated by the proposed method are consistent with those of other published papers. Nearly global optimum solutions can be obtained since the simulated annealing algorithm is inherently a direct searching method. It should be noted that the array mutual coupling effects are included in the analyses of this paper.

1. INTRODUCTION

There have been many studies for the frequency-domain analyses of nonlinearly loaded antenna arrays [1–8]. In those studies, the problems are transformed into equivalent microwave circuits and then solved by different frequency-domain approaches. In this paper, a new frequency-domain approach for the analyses of nonlinearly loaded antenna arrays including mutual coupling effects is presented. In this new approach, the analysis of a nonlinear problem is first transformed into an optimization problem. This optimization problem is then solved by simulated annealing algorithms [9, 10].

The simulated annealing algorithm has widespread application in engineering optimization. It utilizes the behavior of systems with many degrees of freedom in thermal equilibrium at a finite temperature to find the minimum of a given function with many parameters. However, the application of the simulated annealing algorithm in this study is for analysis, but not for optimization. To our knowledge, this paper is the first study to apply this algorithm to such problems of

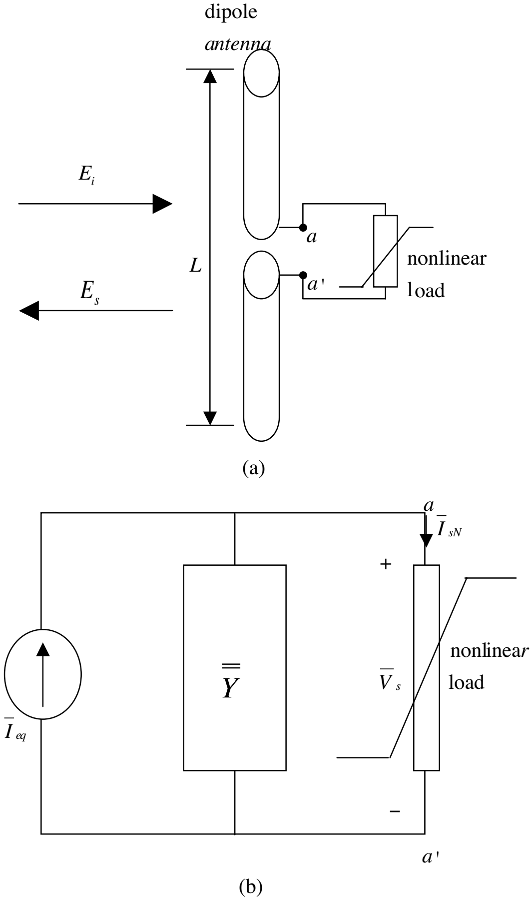

Figure 1. (a) Schematic diagram of a single nonlinearly loaded antenna, and (b) its equivalent microwave circuit.

nonlinearly loaded antenna arrays. Numerical examples show that the results calculated by the proposed methods are consistent with the results of other published papers. Nearly global optimum solutions can be obtained since the algorithm is a direct searching method of optimization and requires no gradient operations in the iteration procedures.

2. FORMULATIONS

For simplicity, a single nonlinearly loaded antenna element is considered first. As shown in Fig. 1(a), the system is illuminated by a plane wave E_i. According to [1, 2, 5, 6], the analysis becomes an equivalent microwave circuit problem for the solution of terminal voltage $\overline{V}_s$ at different harmonic frequencies, as shown in Fig. 1(b). In Fig. 1(b), the $\overline{I}_{eq}$ is the short-circuit current at the antenna terminal due to the incident wave and the $\overline{I}_{sN}$ is the current of the nonlinear load. Each variable in Fig. 1(b) is described in the frequency domain. For example,

$$\overline{V}_s = \begin{bmatrix} V_{s0} & V_{s_1} & V_{s_2} & \cdots & V_{s_{2P-1}} & V_{s_{2P}} \end{bmatrix}^T, \tag{1}$$

represents the Fourier series coefficients of the time-domain signal $v_s(t)$, i.e.,

$$v_s(t) = V_{s_0} + \sum_{p=1}^{P} \left\{ V_{s_{2p-1}} \cos \omega_p t + V_{s_{2p}} \sin \omega_p t \right\}. \tag{2}$$

In (1) and (2), the P is an integer representing the number of harmonics and "T" denotes the transposition. The $\overline{\overline{Y}}$ in Fig. 1(b) is defined as

$$\overline{\overline{Y}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & G_1 & B_1 & 0 & \vdots & 0 \\ 0 & -B_1 & G_1 & \ddots & 0 & \vdots \\ \vdots & 0 & 0 & \ddots & \ddots & 0 \\ 0 & \vdots & \vdots & 0 & G_P & B_P \\ 0 & 0 & \cdots & 0 & -B_P & G_P \end{bmatrix} \tag{3}$$

where G_p and B_p denote the real part and imaginary part of the antenna input admittance at the p-th harmonic ($p = 1, \cdots, P$). It should be noted that $\overline{I}_{eq}$ and $\overline{\overline{Y}}$ can be obtained by moment methods [11].

The analysis then becomes the solution of terminal voltage $\overline{V}_s$ at different harmonic frequencies. Assume $\overline{v}_s(t)$ is a vector with its components sampled from $v_s(t)$ and the relation between the time-domain and frequency-domain variable is given as

$$\begin{cases} \overline{V}_s &= \overline{\overline{D}}\,\overline{v}_s(t) \\ \overline{v}_s(t) &= \overline{\overline{D}}^{-1}\overline{V}_s \end{cases}. \tag{4}$$

Applying KCL at terminal a-a' of Fig. 1(b), we have an error vector $\overline{\varepsilon}$ given by

$$\overline{\varepsilon} = \overline{\overline{Y}}\,\overline{V}_s - \overline{I}_{eq} + \overline{\overline{D}} f\left(\overline{\overline{D}}^{-1}\,\overline{V}_s\right) \to \overline{0}, \tag{5}$$

where $f(\cdot)$ is the i-v characteristics of the nonlinear device. The problem then becomes to find an optimum vector $\overline{V}_s$ that makes the error vector $\overline{\varepsilon}$ approach zero, i.e.,

$$\text{Minimize } g(\overline{V}_s) = \|\overline{\varepsilon}\|^2 = \left\|\overline{\overline{Y}}\,\overline{V}_s - \overline{I}_s + \overline{\overline{D}} f\left(\overline{\overline{D}}^{-1}\overline{V}_s\right)\right\|^2 \to 0. \tag{6}$$

In this study, simulated annealing algorithms [9, 10] are used to find an optimum $\overline{V}_s$ that satisfies (6). The iteration procedures of simulated annealing algorithms are described in detail in [9]. Initially, a random $\overline{V}_s$ is selected in the feasible region and its cost function $g(\overline{V}_s)$ is evaluated. The iteration is given as

$$(\overline{V}_s)_{new} = (\overline{V}_s)_{old} + r s_i \hat{e}_i, \text{ for } i = 0, 1, \cdots, 2P, \tag{7}$$

where r is a random number in the range $[-1, 1]$, $\hat{e}_i$ is the coordinate direction and s_i is its step size. If this point is outside the feasible region, the i-th component of $\overline{V}_s$ is adjusted to be a random point in the feasible region. A new cost function value $g[(\overline{V}_s)_{new}]$ is then evaluated. If $g[(\overline{V}_s)_{new}] \leq g[(\overline{V}_s)_{old}]$, the point is accepted and $(\overline{V}_s)_{old}$ is replaced by $(\overline{V}_s)_{new}$. If $g[(\overline{V}_s)_{new}] > g[(\overline{V}_s)_{old}]$, the acceptance or rejection about the point depends on a probability "*prob*" defined as

$$prob = e^{\frac{g\left[(\overline{V}_s)_{new}\right] - g\left[(\overline{V}_s)_{old}\right]}{Temp}}, \tag{8}$$

where $Temp$ is called "temperature" in the simulated annealing algorithm. A random number r in the range $[0, 1]$ is generated. If $r < prob$, the point of the new step is accepted. Otherwise it is rejected. This procedure is for the purpose of reducing the chance of getting stuck in local solutions and referred to as the Metropolis criterion [10].

At each temperature $Temp$ in (8), N_T loops with the same temperature are repeated. Each loop consists of N_C cycles. A cycle involves taking a random step in each of the $2P + 1$ directions in (7) successively. To speed the convergence, the step size s_i in (7) should be adaptive and proportional to the acceptance rate in Metropolis criterion [10]. The temperature is then iterated by a reducing factor $r_T(< 1)$, i.e., $Temp$ is replaced by $r_T \cdot Temp$ in the next iteration loop of temperature, until acceptable $g(\overline{V}_s) = \|\overline{\varepsilon}\|^2 \to 0$ is obtained. An optimum vector $\overline{V}_s$ can then be found to make the error vector $\overline{\varepsilon}$ in (5) or (6) approach zero.

The above analyses can be extended to problems of nonlinearly loaded antenna arrays including mutual coupling effects. For an N-element nonlinearly loaded antenna array illuminated by a plane wave E_i, as shown in Fig. 2(a), the equivalent circuit can be expressed as Fig. 2(b), where port k represents the k-th antenna terminal. It should be noted that the array mutual coupling effects are included in the linear network. The circuit equations are similar to those described above except the definitions in (1) and (3) are modified as

$$\begin{aligned}\overline{V}_s \;=\; & \left[V_{s,1_0} \quad V_{s,1_1} \quad V_{s,1_2} \quad \cdots \quad V_{s,1_{2P-1}} \quad V_{s,1_{2P}} \quad ; \cdots\cdots ; \right. \\ & \left. V_{s,N_0} \quad V_{s,N_1} \quad V_{s,N_2} \quad \cdots \quad V_{s,N_{2P-1}} \quad V_{s,N_{2P}}\right]^T, \end{aligned} \tag{9}$$

and

$$\overline{\overline{Y}} = \begin{bmatrix} \overline{\overline{Y}}_{11} & \cdots & \overline{\overline{Y}}_{1N} \\ \vdots & \ddots & \vdots \\ \overline{\overline{Y}}_{N1} & \cdots & \overline{\overline{Y}}_{NN} \end{bmatrix}, \tag{10}$$

where $\overline{\overline{Y}}_{ij}$ denotes the mutual admittances between the i-th and the j-th antenna elements at different harmonics. After the cost functions $\overline{\varepsilon}$ are determined, simulated annealing algorithms can be used to find the optimum $\overline{V}_s$ of (9) that satisfies (6).

3. NUMERICAL SIMULATION RESULTS

In this section, two numerical examples are given to illustrate the formulations described above. Without loss of generality, dipole antennas are considered for simplicity. Each dipole has length of 0.467λ and length-to-diameter ratio of 74.2. The incident wave is monochromic with frequency ω and assumed to have strength $E_i = 1.0\,\text{volt/m}$. The i-v characteristic $f(\cdot)$ of the nonlinear device in Fig. 1 and Fig. 2 is given as

$$i = f(v) = \frac{1}{75}v + 4v^3. \tag{11}$$

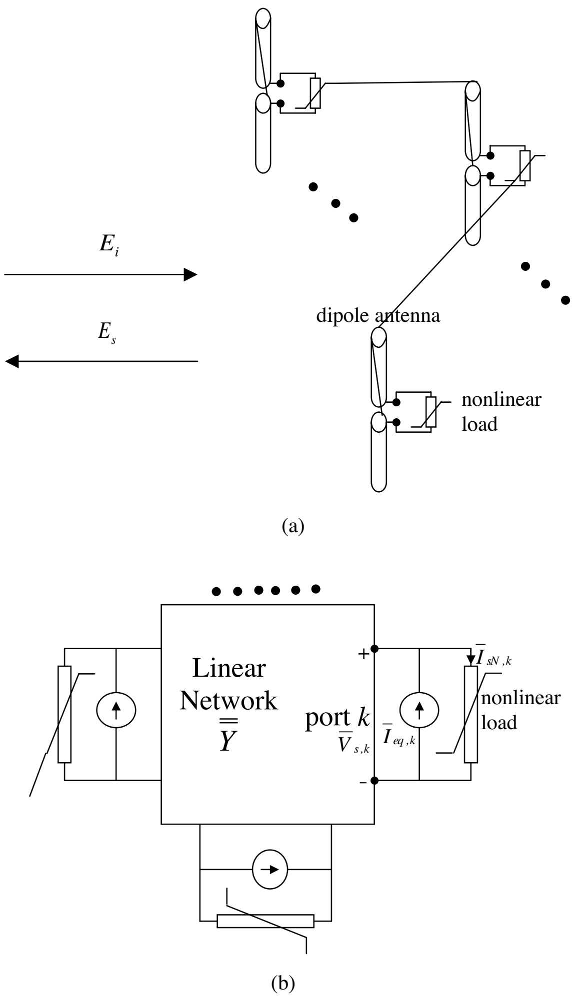

Figure 2. (a) Schematic diagram of a nonlinearly loaded antenna array, and (b) its equivalent microwave circuit.

In the simulated annealing algorithm, the temperature starts at $Temp = 50$ and the parameters of iteration procedures are chosen as $N_T = 5$, $N_C = 20$ and $r_T = 0.5$.

In the first example, a single nonlinearly loaded dipole is considered. Following the procedures described above, the final voltages of terminal a-a' at different mixing frequencies are shown in Fig. 3. For comparison, the results in Fig. 3(a) of [6] are converted

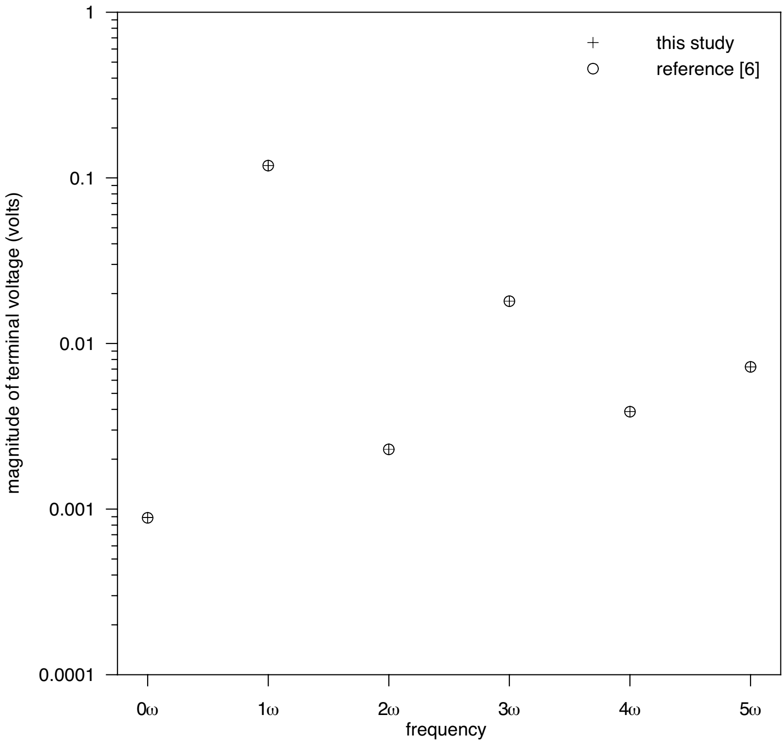

Figure 3. Spectrum of voltages at the dipole-terminal of a single nonlinearly loaded dipole by using the approaches of this study and reference [6].

into antenna terminal voltages and are also plotted in Fig. 3. It shows that they are in good agreement.

In the second example, two parallel dipole antennas with each antenna loaded with a nonlinear lumped load are considered. Each array element is the same as that of the first example. The spacing between the two dipoles is 0.75λ. Due to the symmetry, the resulting terminal voltages of the two dipoles are the same. Following the procedures given above, the final terminal voltages of each dipole at various mixing frequencies are given in Fig. 4. For comparison, the results in Fig. 4(a) of [6] are also plotted in Fig. 4. It shows that they are in good agreement. It should be noted that the mutual coupling effects within the array have been considered in this example.

The explanations for the nonlinear phenomena of Fig. 3 and Fig. 4 are given in detail in [6]. Due to the linear term and the cubic term of

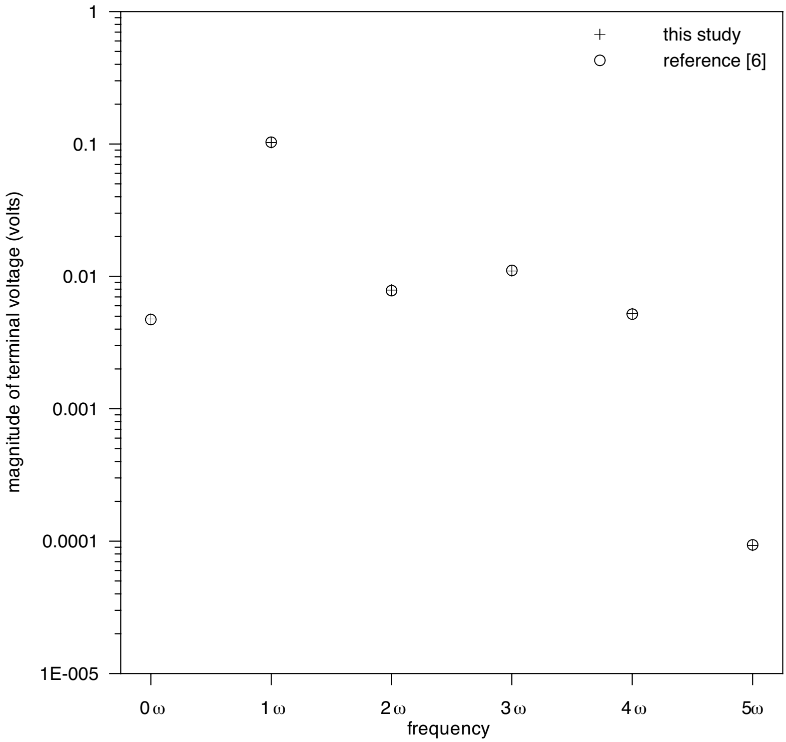

Figure 4. Spectrum of voltages at each dipole-terminal of a two-element nonlinearly loaded dipole array by using the approaches of this study and reference [6].

$f(\cdot)$ in (11), it is found that the maximum voltage component occurs at fundamental frequency ω and the voltage component at mixing frequency 3ω is slightly greater than those at mixing frequencies 2ω, 4ω, 5ω, and DC (0ω). The radiation and scattering characteristics of the above examples are given in [2, 5, 6] in detail. However, this paper has the same results using different approaches.

To realize the converging speed of the analyses described above, the square error $\|\overline{\varepsilon}\|^2$, i.e., the cost function $g(\overline{V}_s)$, for the first numerical example in each iteration loop of temperature in the simulated annealing algorithm is shown in Fig. 5. In this example, the threshold for stopping the iteration is set to be 1.E-12. It shows that iteration loops of temperature larger than 50 can give good results in this problem. Note that the convergence problem is not important in this paper since we have known that the optimum value of the cost

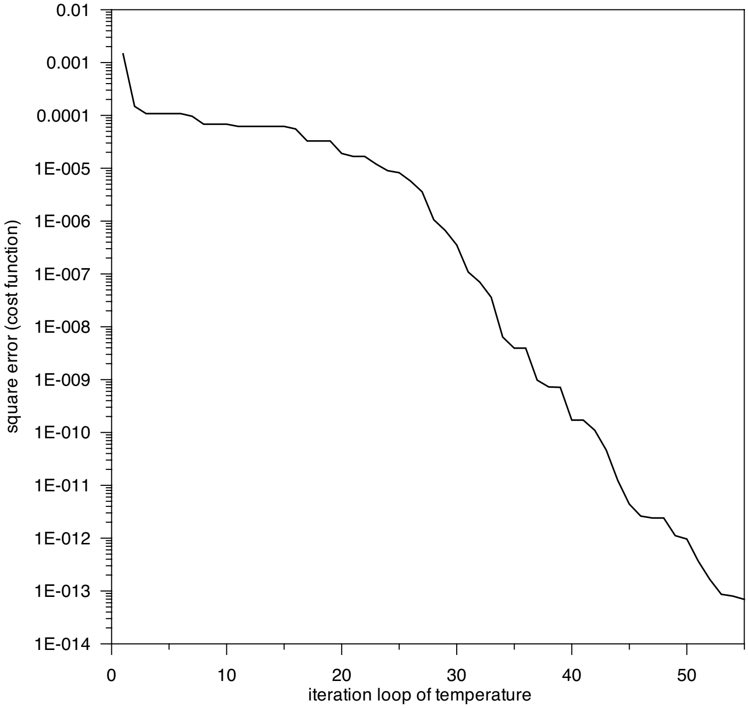

Figure 5. The square error $\|\overline{\varepsilon}\|^2$, i.e., the cost function $g(\overline{V}_s)$, (in Log scale) in each iteration loop of temperature in the simulated annealing algorithm for the first numerical example.

function is zero in our problems.

The above numerical example is computed using Fortran-90 codes on a PC with Intel Pentium 2.8 GHz CPU. The computation time of iteration procedure for one example is about 0.42 second.

4. CONCLUSION

In this paper, simulated annealing algorithms are applied to the frequency-domain analyses of nonlinearly loaded antenna arrays. The array mutual coupling effects are included in the proposed analyses. Numerical examples show that the results calculated by the proposed methods are consistent with the results of [6]. Nearly global optimum solution can be obtained due to the use of Metropolis criterion [10] in the iteration procedures. Similar to the genetic algorithms, neither a suitable guess of an initial solution nor the gradient operations are required in the simulated annealing algorithms. However, the simulated annealing algorithms are in general more efficient then the genetic algorithms in application of this problem because the genetic algorithms require the transformations between the binary and the decimal systems.

ACKNOWLEDGMENT

The author would like to express his sincere gratitude to Professor Tsung-Nan Lin, Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, for his helpful discussion about the simulated algorithm. The work in this paper was supported by the National Science Council, Taiwan, Republic of China, under Grant NSC 93-2611-E-006-027.

REFERENCES

1. Sarkar, T. K. and D. D. Weiner, "Scattering analysis of nonlinearly loaded antennas," *IEEE Trans. Antennas Propagat.,* Vol. AP-24, No. 3, 125–131, Mar. 1976.
2. Huang, C. C. and T. H. Chu, "Analysis of wire scatterers with nonlinear or time-harmonic loads in the frequency domain," *IEEE Trans. Antennas Propagat.,* Vol. AP-41, No. 1, 25–30, Jan. 1993.
3. Lee, K. C., "Analysis of large nonlinearly loaded antenna arrays under multi-tones excitation," *Microwave and Optical Technology Letters,* Vol. 25, No. 5, 319–323, June 2000.
4. Lee, K. C., "An efficient algorithm for the steady-state analysis of a nonlinearly loaded antenna array," *Journal of Electromagnetic Waves and Applications,* Vol. 14, No. 10, 1373–1382, October 2000.
5. Lee, K. C., "Two efficient algorithms for the analyses of a nonlinearly loaded antenna and antenna array in the frequency domain," *IEEE Trans. Electromagn. Compat.,* Vol. EMC-42, No. 4, 339–346, November 2000.
6. Lee, K. C., "Genetic algorithms based analyses of nonlinearly loaded antenna arrays including mutual coupling effects," *IEEE Trans. Antennas Propagat.,* Vol. AP-51, No. 4, 776–781, April 2003.
7. Lee, K. C. and T. H. Chu, "Analysis of injection-locked antenna array including mutual coupling effects," *IEEE Trans. Antennas Propagat.,* Vol. AP-52, No. 11, 2885–2890, November 2004.

8. Lee, K. C. and T. N. Lin, "Application of neural networks to analyses of nonlinearly loaded antenna arrays including mutual coupling effects," *IEEE Trans. Antennas Propagat.,* Vol. AP-53, 2005.
9. Kirkpatrick, S., C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science* Vol. 220, No. 4598, 671–680, May 1983.
10. Belegundu, A. D. and T. R. Chandrupatla, *Optimization Concepts and Applications in Engineering,* Chap. 7, Prentice-Hall, New Jersey, 1999.
11. Harrington, R. F., *Field Computation by Moment Methods,* Macmillan, New York, 1968.

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