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T-MATRIX ANALYSIS OF ELECTROMAGNETIC WAVE DIFFRACTION FROM A DIELECTRIC COATED FOURIER GRATING

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Abstract—This paper describes exactly a new formulation of the T-matrix method with R-matrix expression for the electromagnetic wave diffraction efficiency from dielectric coated metallic Fourier grating. We found that the parameters of numerical calculation are widely applied by using R-matrix expression in dielectric coating media whose thickness or depth groove on the Fourier grating is large. The absorption phenomena of diffraction efficiency in particular incident angle are observed in the two cases. One of the factor is a guided mode in the dielectric coated layer. Other factor is resonance absorption that occurs by plasmon anomalies on the substrate for the TM polarization.

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1. INTRODUCTION

In order to prevent the oxidizing by air, the Fourier grating is composed of the coated thin dielectric layer on the metallic grating surface.

There are a slight influence to the diffraction characteristics, that is, it is thin coated layer in comparison with wavelength for protection membrane, but the thick coated dielectric layer is used not prevention of oxidizing in recently.

When a numerical calculation is executed in thin coated dielectric layer, the problem by using a T-matrix expression in the conventional method with extended boundary condition has not yet been considered. However, the coated dielectric layer is not always thinly in comparison with wavelength, the thick coated layer is chosen for the propagation of a guided mode wave or the control of a diffraction efficiency.

A divergence of the T-matrix expression by evanescent mode of the diffraction must be considered in the case of these problems. The resonance absorption by surface plasmon anomalies exists that the energy of the incident wave in diffraction efficiency by a Fourier grating with dielectric coating medium has been absorbed [3]. This resonance absorption depends on oscillation of the surface wave and the guided wave toward the period of the grating that has already been studied in details for the past decay [1–3].

On the other hand, the absorption of incident light wave is confirmed, because the energy is transmitted toward the period of the grating with the surface wave in a coating dielectric medium. We must study interesting phenomena of the diffraction characteristics of these surface waves in the Fourier grating with the dielectric coating media because the resonance absorption anomalies by coupling diffracted evanescent wave of the -1th mode with surface plasmon wave occur [3].

For the sinusoidal metallic grating with dielectric coating media, the rigorous solution of differential method [4], the mode matching method [3] have already been done, however, the sufficient analysis for the Fourier grating with dielectric coating media has not yet carried out by T-matrix method with R-matrix expression.

In this paper, we examine the mechanism of the incident light wave that is absorbed in the grating. The rigorous formulation of the diffraction problem by the Fourier grating with large thickness of the dielectric coating is described by using the T-matrix analysis [6] with the R-matrix expression [2]. If thickness of the coated dielectric layer is large, the absorption by the guided mode along the direction of Fourier grating is observed.

And it find that the absorption anomalies by surface plasmon occur the particular incident angle of the TM polarization.

Their results are discussed in details, that is not only the diffraction efficiencies versus a thickness of the coated dielectric layer, a groove depth of the grating and the incident angle, the electromagnetic fields are illustrated in maps on the amplitude of field distribution and Poynting vector.

2. GEOMETRY OF THE PROBLEM AND THE DIELECTRIC COATED FOURIER GRATING

Let us consider the electromagnetic wave diffraction from a dielectric coated Fourier grating illuminated by a plane wave. The dielectric coated Fourier grating structure and the geometry of the problem are shown in Fig. 1. We assume a two-dimensional problem where the surface vary periodically in the x direction and does not vary in the y direction, the plane of incidence is the x-z plane for the incident angle θ_{inc}.

The incident region and substrate of the grating are filled up by material of homogeneous isotropic medium (permittivity ε_0 and ε_2, the permeability μ_0 and μ_2), respectively, and also dielectric coating (uniform medium for the permittivity ε_1 and the permeability μ_1) is considered.

When $S_\iota(\iota = 0, 1)$ is defined, the surface boundary profiles $f_\iota(x)$

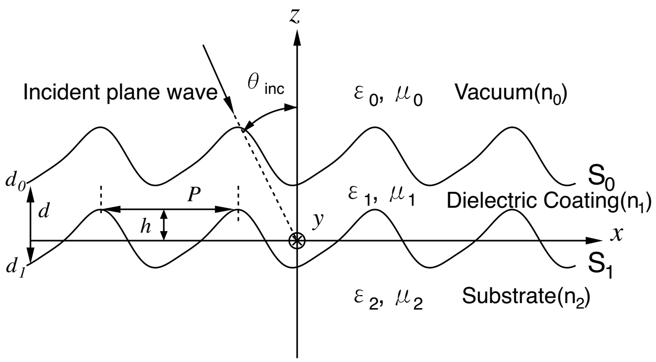


Figure 1. Geometory of the problem and the Fourier grating with dielectric overcoating in which the dielectric coating layer $[SiO(\sqrt{\varepsilon_1}(= n_1 = 1.54, \lambda = 650\,\text{nm})]$, substrate of Fourier grating $[Au(\sqrt{\varepsilon_2}(= n_2 = 0.142 - j3.374, \lambda = 650\,\text{nm})]$ are used.

yield

$$S_\iota : \; f_\iota(x) + d_\iota \quad (\iota = 0, 1 : \text{numbering of the boundary surface}) \tag{1}$$

where $d_0(\iota = 0)$ means the thickness d of the dielectric coating, and $d_1(\iota = 1)$ is zero width in this problem.

While two surfaces have same smooth profile, the Fourier grating structure is expressed by

$$f_\iota(x) = -h\{\cos(Kx) + \gamma \cos(2Kx + \delta)\} \tag{2}$$

where $K = 2\pi/P$, P is the period of Fourier grating, h is the amplitude of fundamental sinusoidal wave, $h\gamma$ and δ are the amplitude and phase of second hormonic wave, respectively.

3. FORMULATION BY USING EXTINCTION THEOREM

Here, we obtain the two-dimensional wave equation when the Maxwell's equations are derived in the rectangular coordinates for the each region as $\partial/\partial y \equiv 0$, therefore, electric and magnetic field do not vary in the y direction, and time dependence $\exp(j\omega t)$ is suppressed throughout in this paper.

$$\frac{\partial^2 \psi_i}{\partial x^2} + \frac{\partial^2 \psi_i}{\partial z^2} + k_i^2 \psi_i = 0 \tag{3}$$

$$\psi_i = \begin{cases} E_{iy} & : \text{ TE wave} \\ H_{iy} & : \text{ TM wave} \end{cases} \quad (i = 0, 1, 2 : \text{numbering of the media})$$

where, $k_i(=\omega\sqrt{\varepsilon_i\mu_i})$ denotes propagation constant in the ith medium, ω is the angular frequency. We yield wave number k_{xm} of the diffraction grating satisfying condition for mth mode number in the x direction as

$$k_{xm} = k_0 \sin\theta_{inc} + m\frac{2\pi}{P} \tag{4}$$

where θ_{inc} is the incident angle, above wave number k_{xm} can be also replaced by $k_{xm} = k_i\alpha_{im}$. Also, we define wave number k_{izm} satisfying the radiation condition in the z direction, it is replaced by $k_{izm} = k_i\beta_{im}$, that is

$$\beta_{im} = \begin{cases} \sqrt{1-\alpha_{im}^2} & 1 \geq \alpha_{im}^2 \\ -j\sqrt{\alpha_{im}^2 - 1} & 1 \leq \alpha_{im}^2 \end{cases} \qquad (i = 0, 1, 2) \tag{5}$$

The incident wave $\psi^{inc}(\bar{r})$, fields $\psi_0(\bar{r})$ in the incident medium, fields $\psi_2(\bar{r})$ in the substrate, and fields $\psi_1(\bar{r})$ in the coating medium are defined, respectively. In the each region, the electric and magnetic fields satisfy the integral representations by applying extinction theorem as the following:

- incident medium

$$\psi^{inc}(\bar{r}) - \int_P d\bar{\sigma}_0' \cdot \left[G_0(\bar{r},\bar{r}')\nabla'\psi_0(\bar{r}) - \psi_0(\bar{r}')\nabla' G_0(\bar{r},\bar{r}')\right]_{z'=f_0(x')+d} = \begin{cases} \psi_0(\bar{r}) & z > S_0 \\ 0 & z < S_0 \end{cases} \tag{6}$$

- dielectric coating medium

$$\int_P d\bar{\sigma}_1' \cdot \left[G_1(\bar{r},\bar{r}')\nabla'\psi_1(\bar{r}) - \psi_1(\bar{r}')\nabla' G_1(\bar{r},\bar{r}')\right]_{z'=f_1(x')} - \int_P d\bar{\sigma}_0' \cdot \left[G_1(\bar{r},\bar{r}')\nabla'\psi_1(\bar{r}) - \psi_1(\bar{r}')\nabla' G_1(\bar{r},\bar{r}')\right]_{z'=f_0(x')+d} = \begin{cases} 0 & z > S_0 \\ \psi_1(\bar{r}) & \\ 0 & z < S_1 \end{cases} \tag{7}$$

- substrate medium

$$\int_P d\bar{\sigma}_1' \cdot \left[G_2(\bar{r},\bar{r}')\nabla'\psi_2(\bar{r}) - \psi_2(\bar{r}')\nabla' G_2(\bar{r},\bar{r}')\right]_{z'=f_1(x')} = \begin{cases} 0 & z > S_1 \\ \psi_2(\bar{r}) & z < S_1 \end{cases} \tag{8}$$

where integral contour P means one period along the both surfaces, G_i $(i = 0, 1, 2)$ denotes the two-dimensional Green function $G_i(\bar{r},\bar{r}')$

satisfying periodic property that is represented as

$$G_i(\bar{r},\bar{r}') = -\frac{j}{2k_iP}\sum_{m=-\infty}^{\infty}\frac{1}{\beta_{im}}\exp\left[-j\{k_{xm}(x-x')+k_{izm}|z-z'|\}\right] \quad (i=0,1,2) \tag{9}$$

where, $\bar{r}$, $\bar{r}'$ are the position vector of the observation and the secondary source on the surface, respectively. And, $d\bar{\sigma}'_\iota$ is the normal direction vector defined by

$$d\bar{\sigma}'_\iota = dx'\left[\hat{z}-\frac{df_\iota(x')}{dx'}\hat{x}\right] \tag{10}$$

$\hat{x}$ and $\hat{z}$ are the unit vectors x and z, respectively.

Then, we apply the following boundary conditions on the surface

$$\psi_2(\bar{r}') = \psi_1(\bar{r}')|_{z'=f_1(x')} \tag{11a}$$

$$d\bar{\sigma}'_1\cdot\nabla'\psi_2(\bar{r}') = \nu_2 d\bar{\sigma}'_1\cdot\nabla'\psi_1(\bar{r}')|_{z'=f_1(x')} \tag{11b}$$

$$\psi_1(\bar{r}') = \psi_0(\bar{r}')|_{z'=f_0(x')+d} \tag{11c}$$

$$d\bar{\sigma}'_0\cdot\nabla'\psi_1(\bar{r}') = \nu_1 d\bar{\sigma}'_0\cdot\nabla'\psi_0(\bar{r}')|_{z'=f_0(x')+d} \tag{11d}$$

where

$$\nu_i = \begin{cases}\mu_i/\mu_{i-1} & : \text{ TE wave}\\ \varepsilon_i/\varepsilon_{i-1} & : \text{ TM wave}\end{cases} \qquad (i=1,2) \tag{12}$$

Furthermore, fields expression by using Fourier series expansion on surface S_ι $(\iota=0,1)$ of the grating is assumed

$$\psi_1(\bar{r}') = 2\sum_{n=-\infty}^{\infty}\alpha^s_{1n}\exp(-jk_{xn}x'),\ \ z'=f_1(x') \tag{13a}$$

$$d\bar{\sigma}'_1\cdot\nabla'\psi_1(\bar{r}') = -2jk_1dx'\sum_{n=-\infty}^{\infty}\beta^s_{1n}\exp(-jk_{xn}x'),\quad z'=f_1(x') \tag{13b}$$

$$\psi_0(\bar{r}') = 2\sum_{n=-\infty}^{\infty}\alpha^s_{0n}\exp(-jk_{xn}x'),\ \ z'=f_0(x')+d \tag{13c}$$

$$d\bar{\sigma}'_0\cdot\nabla'\psi_0(\bar{r}') = -2jk_0dx'\sum_{n=-\infty}^{\infty}\beta^s_{0n}\exp(-jk_{xn}x'),\ \ z'=f_0(x')+d \tag{13d}$$

where, α^s_{1n}, β^s_{1n}, α^s_{0n} and β^s_{0n} are the unknown coefficients to be determined from the boundary conditions.

3.1. Field Expansion in the Incident Medium

For the incident medium, we expand to get the modal plane wave in Eq. (6). The incident wave ψ^{inc} and reflected diffraction wave ψ_0^r having the unknown coefficients of reflected waves b_m and incident wave a_m are explicitly expanded as

$$\psi_0^r = \sum_{m=-\infty}^{\infty} b_m \frac{\exp\{-j(k_{xm}x + k_{0zm}z)\}}{\sqrt{\beta_{0m}}} \qquad z > f_0 + d \tag{14a}$$

$$\psi^{inc} = \sum_{m=-\infty}^{\infty} a_m \frac{\exp\{-j(k_{xm}x - k_{0zm}z)\}}{\sqrt{\beta_{0m}}} \qquad z < f_0 + d \tag{14b}$$

Because incident plane wave has a single mode in free space, it is determined on the surface $z = f_0(x') + d$

$$a_m = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \tag{15}$$

By applying the boundary conditions (11a) and (11b), and Fourier series expansions (13a) and (13b), normalizing by the phase factor $\Lambda_m^{\pm} = \exp\{\pm jk_{0zm}d\}$, the unknown coefficients b_m, a_m can be expressed in the matrix form as

$$\begin{bmatrix} b'_m \\ a'_m \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \begin{bmatrix} \beta_{0n}^s \\ \alpha_{0n}^s \end{bmatrix} \tag{16}$$

where $b'_m = \Lambda_m^+ b_m$, $a'_m = \Lambda_m^- a_m$ and

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \begin{bmatrix} -Q_D^+(k_0, f_0) & -Q_N^+(k_0, f_0) \\ Q_D^-(k_0, f_0) & Q_N^-(k_0, f_0) \end{bmatrix}. \tag{17}$$

where $Q_D^{\pm}$, $Q_N^{\pm}$ are the Dirichlet matrices and the Neumann matrices, respectively, as later expression.

3.2. Fields Expansion in the Substrate

The fields expression in the substrate medium as described in Eq. (8) can be similarly expressed by using a modal expansion of the plane wave in Eq. (14a). Since the substrate is infinite extended medium in the $-z$ direction, unknown coefficient B_m satisfying radiation condition is zero and unknown coefficients A_m meaning the transmitted wave ψ_2^t is represented as

$$0 = \sum_{m=-\infty}^{\infty} B_m \frac{\exp\{-j(k_{xm}x + k_{2zm}z)\}}{\sqrt{\beta_{2m}}} \qquad z > f_1 \tag{18a}$$

$$\psi_2^t = \sum_{m=-\infty}^{\infty} A_m \frac{\exp\{-j(k_{xm}x - k_{2zm}z)\}}{\sqrt{\beta_{2m}}} \quad z < f_1 \tag{18b}$$

By applying the boundary conditions (11c) and (11d), and Fourier series expansions (13c) and (13d), we also obtain the matrix form for unknown coefficients B_m, A_m

$$\begin{bmatrix} B_m \\ A_m \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \beta_{1n}^s \\ \alpha_{1n}^s \end{bmatrix} \tag{19}$$

where

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -\nu_2 \frac{k_1}{k_2} Q_D^+(k_2, f_1) & -Q_N^+(k_2, f_1) \\ \nu_2 \frac{k_1}{k_2} Q_D^-(k_2, f_1) & Q_N^-(k_2, f_1) \end{bmatrix} \tag{20}$$

where $Q_D^{\pm}$, $Q_N^{\pm}$ are the Dirichlet matrices and the Neumann matrices, respectively, as later expression.

3.3. Fields Expansion in the Coating Medium

The fields expansion in the dielectric coating medium will be described by using a modal expansion of the plane wave in the previous section. We use the R-matrix expression with exchanged submatrix in the extinction theorem. Thus, we avoid numerical singularity by the T-matrix expression so that the relations between unknown coefficients α_{1n}^s, β_{1n}^s on S_1 and unknown coefficients α_{0n}^s, β_{0n}^s on S_0 are represented by transition matrix in the coating medium. In this case, let us note $\bar{r}'$ in Eq. (7), in which the prime means the position vector of the secondary source on the surface. The first team in Eq. (7) means the position vector on the substrate surface defined by $f_1(x')$, the second team in Eq. (7) denotes the position on the dielectric coating medium surface by $f_0(x') + d$.

Next, we consider fields expansion in the dielectric coated medium. When the field $\psi_1(\bar{r})$ is expanded with a property for the periodic Green function, it is as following:

$$\begin{aligned} \psi_1(\bar{r}) &= \sum_{m=-\infty}^{\infty} b_{1m} \frac{\exp\{-j(k_{xm}x + k_{1zm}z)\}}{\sqrt{\beta_{1m}}} \\ &- \sum_{m=-\infty}^{\infty} a_{1m} \frac{\exp\{-j(k_{xm}x - k_{1zm}z)\}}{\sqrt{\beta_{1m}}} \end{aligned} \tag{21}$$

where b_{1m}, a_{1m} mean the unknown coefficients of down-going and up-going waves in the coating medium, respectively. By applying the boundary conditions (11a)–(11d) and Fourier series expansions (13a)–(13d), we have the matrix form for relation of the unknown coefficients.

$$[b_{1m}] = \left[Q_N^+(k_1, f_1)\right] \alpha_{1n}^s + \left[Q_D^+(k_1, f_1)\right] \beta_{1n}^s \tag{22a}$$

$$[a_{1m}] = \zeta_1^- \left[Q_N^-(k_1, f_0)\right] \alpha_{0n}^s + \nu_1 \frac{k_0}{k_1} \left[Q_D^-(k_1, f_0)\right] \beta_{0n}^s \tag{22b}$$

where α_{1n}^s, β_{1n}^s are the unknown expansion coefficients in Eqs. (13a), (13b) and α_{0n}^s, β_{0n}^s are the unknown expansion coefficients in Eqs. (13c), (13d).

The representation of the unknown coefficients B_m, A_m at the substrate medium in the previous Subsection 3.2 is obtained. These coefficients B_m, A_m can be solved for the expansion coefficients β_{1n}^s, α_{1n}^s. Similarly, the unknown coefficients b'_m, a'_m at the incident medium in subsection 3.1 are represented. These coefficients b'_m, a'_m can be also solved for the expansion coefficients β_{0n}^s, α_{0n}^s.

Substituting these coefficients B_m, A_m, b'_m and a'_m into Eqs. (22a), (22b), solving for A_m, b'_m by using R-matrix expression, therefore, the unknown coefficients A_m, b'_m are determined and the field distribution in the dielectric coating medium is found.

In this section, the R-matrix expression between α_{0n}^s, α_{1n}^s and β_{0n}^s, β_{1n}^s will be described. By using the periodic property of Green function in Eq. (9), applying the boundary conditions (11a) and Fourier series expansions (13a), we have the expression of the R-matrix form

$$\begin{bmatrix} \alpha_{0n}^s \\ \alpha_{1n}^s \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} \beta_{0n}^s \\ \beta_{1n}^s \end{bmatrix} \tag{23}$$

where

$$\begin{aligned}\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} &= \begin{bmatrix} -Q_N^+(k_1, f_0) & \zeta_1^- Q_N^+(k_1, f_1) \\ -Q_N^-(k_i, f_0) & \zeta_1^+ Q_N^-(k_i, f_1) \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} \nu_1 \frac{k_0}{k_1} Q_D^+(k_1, f_0) & -\zeta_1^- Q_D^+(k_1, f_1) \\ \nu_1 \frac{k_0}{k_1} Q_D^-(k_1, f_0) & -\zeta_1^+ Q_D^-(k_1, f_1) \end{bmatrix},\end{aligned} \tag{24}$$

$\zeta_1^\pm$ are the diagonal matrix elements which mean variation elements for the thickness d of the dielectric coating medium. Thus elements in Eq. (24) are expressed by phase factor

$$\zeta_1^\pm = \exp(\pm j k_{1zm} d) \tag{25}$$

Eq. (23) is expressed by the R-matrix form with the submatrix r_{11}, r_{12}, r_{21}, r_{22} in Eq. (24) which are based on Dirichlet $Q_D^\pm$ and Neumann $Q_N^\pm$

elements of the coating layers on the surface of two boundaries S_0 and S_1.

4. T-MATRIX FORMULATION

In order to obtain T-matrix formulation by using R-matrix expression in Eq. (23), expansion coefficients β^s_{0n} and β^s_{1n}, then, α^s_{0n} and α^s_{1n} are extinguished by substituting Eq. (23) into Eqs. (16) and (19), the coefficients β^s_{0n} and β^s_{1n} can be eliminated in the form of the recombined matrix for the immediately expression. The reflected coefficient b'_m in Eq. (16) and transmitted coefficient A_m in Eq. (19) are finally represented for the incident wave coefficients a'_m in Eq.(16) and $B_m \equiv 0$ satisfying radiation condition in the substrate. The total T-matrix formulation $[T_{mn}]$ with R-matrix expression is finally obtained, that is

$$\begin{bmatrix} b'_m \\ A_m \end{bmatrix} = [T_{mn}] \begin{bmatrix} a'_m \\ B_m \end{bmatrix} \tag{26}$$

where

$$[T_{mn}] = \begin{bmatrix} X_{11}r_{11} + X_{12} & X_{11}r_{12} \\ Y_{21}r_{21} & Y_{21}r_{22} + Y_{22} \end{bmatrix} \begin{bmatrix} X_{21}r_{11} + X_{22} & X_{21}r_{12} \\ Y_{11}r_{21} & Y_{11}r_{22} + Y_{12} \end{bmatrix}^{-1} \tag{27}$$

In Eqs. (24) and (27), the elements of the Dirichlet matrices $Q^{\pm}_D$ and Neumann matrices $Q^{\pm}_N$ are defined in the rigorous form:

$$\begin{aligned} Q^{\pm}_D(k_i, f_\iota) &= \frac{-1}{P\sqrt{\beta_{im}}} \int_{-P/2}^{P/2} dx' \exp\{\pm j k_{izm} f_\iota(x')\} \\ &\quad \times \exp\{-jx'(k_{xn} - k_{xm})\} \\ &\quad \iota = 1(i = 1, 2), \quad 0(i = 0, 1) \end{aligned} \tag{28a}$$

$$\begin{aligned} Q^{\pm}_N(k_i, f_\iota) &= \left[\frac{1 - \alpha_{in}\alpha_{im}}{\pm\beta_{im}}\right] Q^{\pm}_D(k_i, f_\iota) \\ &\quad \iota = 1(i = 1, 2), \quad 0(i = 0, 1) \end{aligned} \tag{28b}$$

where $f_\iota(x')$ is the Fourier grating profile defined by Eq. (2). The matrix elements Eqs. (28a), (28b) for the Fourier grating can be analytically expressed by the Bessel functions

$$\begin{aligned} Q^{\pm}_D(k_i, f_\iota) &= \frac{-1}{\sqrt{k_{izm}}} \sum_{l=-\infty}^{\infty} \exp\left\{\mp jl\left(\frac{\pi}{2} + \delta\right)\right\} \\ &\quad (\mp j)^{|n-m\pm 2l|} J_{|n-m\pm 2l|}(k_{izm}h) J_l(k_{izm}h\gamma) \\ &\quad \iota = 1(i = 1, 2), \quad 0(i = 0, 1) \end{aligned} \tag{29}$$

and $Q_N^{\pm}$ are determined form $Q_D^{\pm}$ that the relationship is already given by Eq. (28b).

This is the Fourier grating Eq. (2) whose profile is expressed by summation of two sinusoidal functions, which are rigorously formulated in this paper. Also, the one of the sinusoidal grating is obtained by setting $\gamma = 0$ in this Section [6], that is, analysis of the electromagnetic diffraction from a dielectric coated sinusoidal grating has already been studied [7].

5. DIFFRACTION EFFICIENCIES AND POYNTING VECTOR

The diffraction power of the reflected, transmitted waves and the incident power are obtained by calculating from time average of the Poynting vector in the each mode. By using the normalized power having the incident wave, we define diffraction efficiencies of the reflected ρ_m^r and the transmitted ρ_m^t waves for the mode number m

$$\rho_m^r = |b_m|^2 \tag{30}$$

$$\rho_m^t = \frac{k_2}{k_0}\frac{1}{\nu_2\nu_1}|A_m|^2 \tag{31}$$

where b_m is the expansion coefficient of reflected wave in Eq. (14a), A_m is the expansion coefficient of transmitted wave in Eq. (18b). Therefore, the total reflected ρ_{total}^r and transmitted ρ_{total}^t diffraction efficiencies are also defined by summing possible modes for the propagating

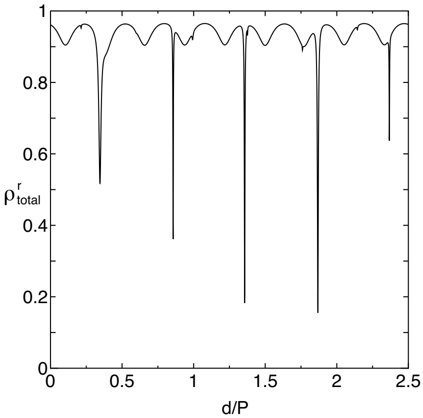
$$\rho_{total}^r = \sum_m \rho_m^r \qquad (m: \quad Re(\beta_{0zm}) > 0) \tag{32}$$

$$\rho_{total}^t = \sum_m \rho_m^t \qquad (m: \quad Re(\beta_{2m}) - Im(\beta_{2zm}) > 0) \tag{33}$$

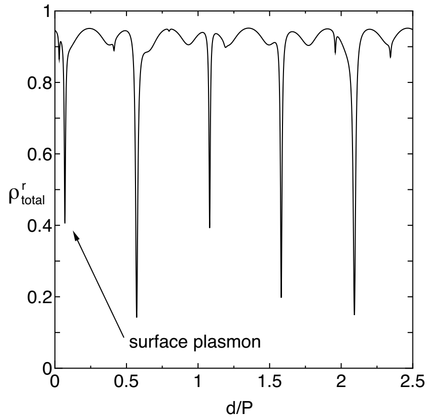
If the medium of all regions is the perfect dielectrics without loss, the energy conversion does supply a good numerical consistency check. The percentage power error ε_{err} is, then, defined by

$$\varepsilon_{err} = |1 - (\rho_{total}^r + \rho_{total}^t)| \times 100 \quad [\%] \tag{34}$$

In the perfect dielectrics of all region, thus, the energy conversion is calculated from time average of the Poynting vector. This percentage power error ≤ 1 [%] is obtained, however, the numerical data figures are omitted here in the paper.



(a) TE wave ($\theta_{inc} = 30°$)



(b) TM wave ($\theta_{inc} = 30°$)

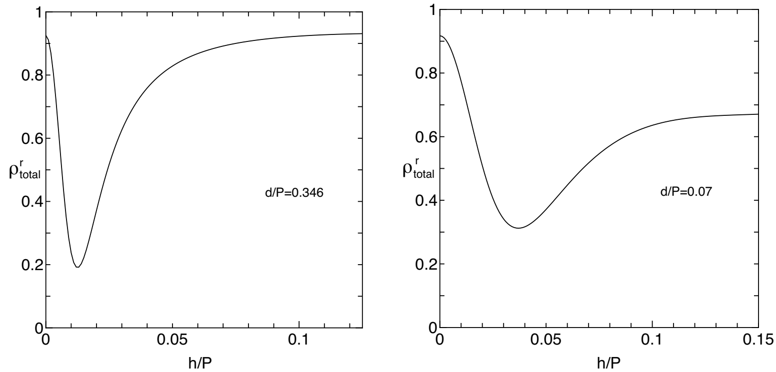
Figure 2. The total reflected diffraction efficiency ρ^r_{total} versus thickness d/P for incident angle $\theta_{inc} = 30°$.

6. NUMERICAL EXAMPLES AND DISCUSSION

In this section, all the parameters, $P = 800\,\text{nm}$, $h = 20\,\text{nm}$, $\gamma = 0.2$ and $\delta = \pi/2$, $\theta_{inc} = 30°$ and wavelength $\lambda = 650\,\text{nm}$, Au substrate ($n_2 = 0.142 - j3.374$), S_iO dielectric coating ($n_1 = 1.54$) are used.

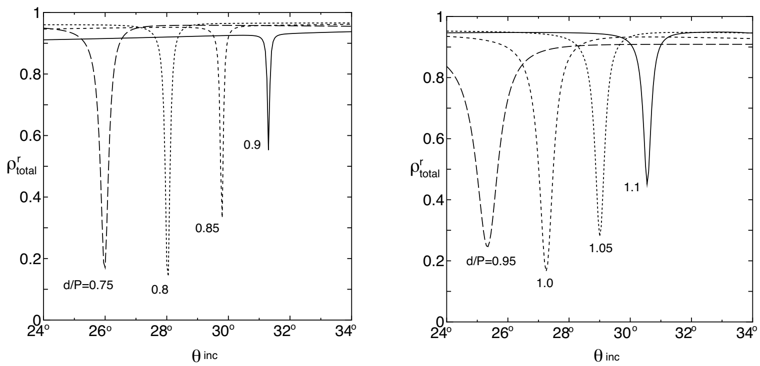
Fig. 2b shows the Fourier grating with the coating dielectric medium when the grating is illuminated by TM plane wave with incident angle $\theta_{inc} = 30°$ from incident medium (air, $n_0 = 1.0$). We find that the resonance absorption (surface plasmon) in the thin dielectric coating layer arises in the total reflected diffraction efficiency. The total reflected diffraction efficiency $\rho^r_{total}(= \sum |b_m|^2)$ is illustrated versus thickness d of the coating layer with period P, such as, d/P. Fig. 2a is TE polarization, we can not find that the resonance absorption in the thin dielectric coating layer arises in the diffraction efficiency as TM polarization. Also, the numerical results(ρ^r_0) by T-matrix method with R-matrix expression in the paper are confirmed in good agreement with the results by S-matrix method [4] in the special case of a sinusoidal profile ($\gamma = 0$) with incident angle ($\theta_{inc} = 42°$), however, the figures are omitted here.

In Fig. 3b, the reflected diffraction efficiency ρ^r_{total} ($d/P = 0.07$) versus groove depth h of the grating is illustrated. Fig. 3a is also TE polarization for $d/P = 0.346$. We find that the minimum value of diffraction efficiency ρ^r_{total} by shallow groove depth in this thickness of coating layer exists at $h/P = 0.0125$.



(a) TE wave ($d/P = 0.346$) (b) TM wave ($d/p = 0.07$)

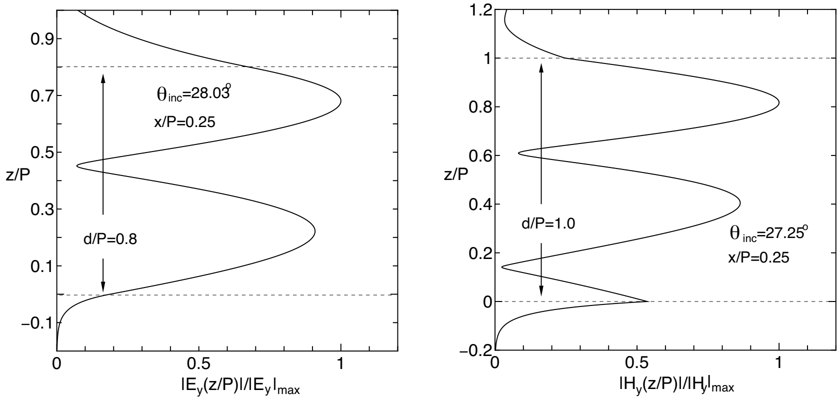
Figure 3. The total reflected diffraction efficiency ρ^r_{total} versus groove depth h/P.



(a) TE wave ($\theta_{inc} = 28.03°$) (b) TM wave ($\theta_{inc} = 27.25°$)

Figure 4. The total reflected diffraction efficiency ρ^r_{total} versus incident angle θ_{inc}.

When the TE plane wave is illuminated on incident angle $\theta_{inc} = 24° \sim 34°$, the reflected diffraction efficiency ρ^r_{total} versus θ_{inc} as shown in Fig. 4a is appeared an aspect varying of the absorption angle whose parameter d/P is the thickness of the a dielectric coating layer. We found that the maximum absorptions by guided wave in numerical value of ρ^r_{total} exist at the thickness $d/P = 0.8(\theta_{inc} = 28.03°)$ in the TE polarization. The TM polarization is also illustrated in Fig. 4b, for which the maximum absorption arises at the parameter $d/P = 1.0(\theta_{inc} = 27.25°)$.



(a) TE wave ($\theta_{inc} = 28.03°$) (b) TM wave ($\theta_{inc} = 27.25°$)

Figure 5. The amplitude of field distributions along z/P for $x/P = 0.25$.

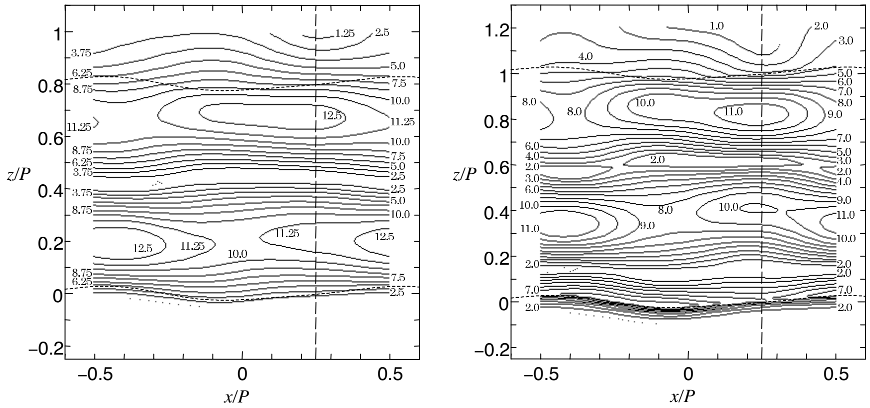
Fig. 5a shows the one dimensional electric field distribution $|E_y(z/P)|$ by normalizing maximum value $|E_y|_{max}$ along z axis for the fixed position $x/P = 0.25$, and parameters $d/P = 0.8$, $\theta_{inc} = 28.03°$, and the absorption results from the guided wave only in dielectric coating media.

Fig. 5b also shows the magnetic field distribution $|H_y(z/P)|$ by normalizing maximum value $|H_y|_{max}$ along z axis for the fixed position $x/P = 0.25$, and parameters $d/P = 1.0$, $\theta_{inc} = 27.25°$. We found that the absorption results from both the surface plasmon in grating and the guided wave in dielectric coating media.

Figs. 6 and 7 show two-dimensional equi-field distributions and Poynting vector, whose fixed line on $x/P = 0.25$ in Fig.5 which were marked by broken line as same parameters, respectively.

Fig. 6a is the mapped equi-electric field amplitude of two-dimensional distributions for TE polarization and $d/P = 0.8$, $\theta_{inc} = 28.03°$. The Fig. 6b shows also equi-magnetic field amplitude for TM polarization and $d/P = 1.0$, $\theta_{inc} = 27.25°$ similarly. We found that the large amplitudes (TM) of the field arise on surface of the grating.

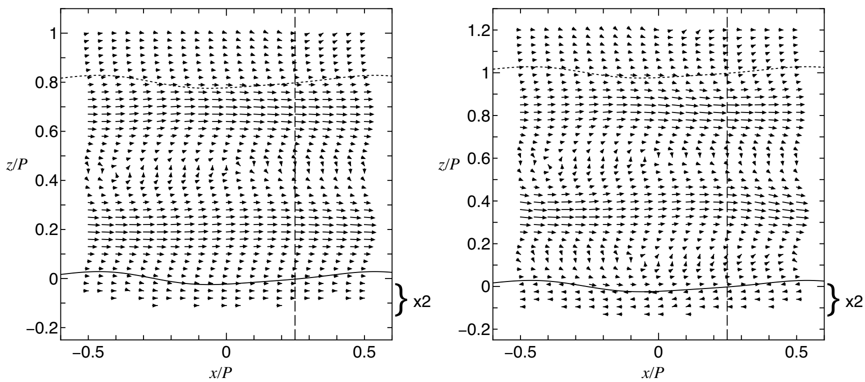
As shown in Fig. 7a, the two-dimensional map of the Poynting vector is illustrated in TE polarization. We found that the guided wave only in dielectric coating media is observed. Fig. 7b shows the case of TM polarization. In this TM polarization, it has been point out that surface plasmon anomalies occur in the Fourier grating with complex permittivity, we found that the resonance absorption arise from both the surface plasmon and the guided wave in dielectric coating media.



(a) TE wave ($\theta_{inc} = 28.03°$)

(b) TM wave ($\theta_{inc} = 27.25°$)

Figure 6. The two-dimensional equi-field distribution in the space (x, z).



(a) TE wave ($\theta_{inc} = 28.03°$)

(b) TM wave ($\theta_{inc} = 27.15°$)

Figure 7. A map of the Poynting vector in the space (x, z).

7. CONCLUSION

This numerical results were discussed in details, those were the diffraction efficiencies versus d/P, h/P and the incident angle. And, the electromagnetic fields were illustrated in maps on the amplitude of field distribution and Poynting vector.

In those numerical examples, it depends on existence of guided mode that the absorption phenomena occur in a certain particular incident angle. We shown that the absorption arises from the resonance absorption by surface plasmon anomalies on the TM polarization wave incidence.

This formulation can be applied to numerical calculation of the Fourier grating that the coated dielectric layer is thick, and that groove depth of the grating is comparatively deep although the coating layer is thin.

For future works, the analysis of the multilayer-coated Fourier grating will be treated by using the T-matrix method with R-matrix expression in the paper.

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