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A COMBINED BI-CGSTAB (l) AND WAVELET TRANSFORM METHOD FOR EM PROBLEMS USING METHOD OF MOMENTS

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Abstract—An efficient technique for the solution of large-scale electromagnetic radiation and scattering problems arising from the surface integral equations and the method of moments is developed. The conventional MoM basis and testing functions are used to discretize the integral equations resulting in a dense impedance matrix. A block-partitioned wavelet transform is then employed to sparsify the matrix. Full advantage is taken of the sparse nature of the mathematical model to solve the system of equations by means of the recently introduced Stabilized Bi-Conjugate Gradient method (Bi-CGSTAB (l)). Various problems are considered involving perfect electric conductor and dielectric material. Results are compared to the corresponding results obtained via the direct solution, or LU decomposition, of the original MoM dense matrix. Excellent results are obtained in a very efficient manner. By block partitioning the MoM impedance matrix as it is built and performing the wavelet transform on the matrix blocks, analysis of very large electromagnetic problems becomes possible in a very efficient and accurate manner.

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1. INTRODUCTION

Method of moments (MoM) is one of the most powerful numerical techniques that is employed to analyze a large spectrum of electromagnetic problems. Despite its effectiveness and popularity, MoM has always suffered from the fact that the discritized geometry under consideration results in a dense matrix that requires a prohibitively expensive memory and CPU time for large-scale scattering and radiation problems. Even when an iterative scheme is applied to solve the dense system, the computational cost is normally on an unacceptable order of $O(pN^2)$, where p is the number of iterations and N is the size of the system [1]. Methods to reduce the computational complexity associated with the MoM matrix include fast multipole method (FMM) [2], impedance matrix localization method [3], the adaptive integral method [4], and the wavelet bases method [5–11] and recently, the adaptive basis function [12]. The wavelet bases techniques may be applied either directly in which wavelets are used as basis and testing functions, or indirectly which sparsifies the impedance matrix via a wavelet matrix transform. In either case, the $O(pN^2)$ computational cost of an iterative scheme can be dramatically reduced and a sparse solver can be implemented to efficiently solve the sparsified system of equations.

The wavelet transform approach employs two major bases namely the Daubechies orthogonal wavelet (DOW) [13], and the nonorthogonal cardinal spline wavelet (NCSW) [14]. Despite the fact that NCSW generates a sparser impedance matrix, wavelet matrices have less nonzero elements in DOW, which results in DOW to be the more effective wavelet transform. Additionally, the NCSW transform makes the condition number of the impedance matrix very large whereas the

DOW transform does not affect the condition number. Consequently, the DOW transform proves to be the best choice for sparsifying the impedance matrix. In its conventional form, however, this transform requires the storage of the full MoM matrix, which is not practical for large EM radiation and scattering problems. However, if the block sub-matrices wavelet transform method [15, 16] is used, the MoM matrix can be partitioned and transformed to sparse sub-matrices, block by block eliminating the need to generate and store the full MoM matrix.

There has been an extensive interest in constructing an efficient iterative algorithm for the solution of the sparse system obtained through the application of the wavelet transform. Recently, G. Sleijpe and D. Fokkema have introduced a new Bi-conjugate Gradient Stabilized method (Bi-CGSTAB) that combines GMRES (l) and Bi-CG to profit from both techniques [17]. The new algorithm is designed to deal with linear equations involving unsymmetric matrices with complex spectrum. It is known that for a large class of equations including systems obtained by discretizing advection dominated PDE's, the Bi-CGSTAB method stagnates and fails to generate an accurate solution. This is shown to be due the fact that for this type of PDE's, the resulting matrix has almost pure imaginary eigenvalues. The CG-like algorithms involve constructing a so-called shadow residual vector of the form $\mathbf{r}_k = q_k(A)r_k$ where q_k is a polynomial of degree k. In many cases, however, the CGS choice of q_k results in the amplification of the residual vector r_k rather than reducing it [17]. Bi-CGSTAB (l) overcomes this problem by forming an l-degree minimal residual polynomials (MR-polynomials) p_m after each l-step. This new algorithm proves to be a robust technique when dealing with large and sparse system of equations.

Here, we first use the conventional expansion and testing functions to discretize the integral equations involved in the solution of various electromagnetic problems. To prevent storing and transforming the whole impedance matrix, which makes the analysis of large-scale problems impossible, a simple block-partitioning scheme is employed to store and transform smaller blocks of the full MoM matrix. Upon performing the discrete wavelet transform (DWT), a generally high level of sparsification is achieved, which enables us to implement the Bi-CGSTAB (l) algorithm to solve the sparse system very efficiently. In order to accelerate the convergence rate, we perform an incomplete LU factorization (ILU) of the transformed impedance matrix where the factorization is computationally inexpensive. It is also shown that based on the complexity of the problems, an appropriate level of threshold can be set to capture enough details needed from the original system in order to obtain an accurate result when the wavelet

transform is employed. Several problems are considered to demonstrate the efficiency and applicability of the present method. Results are compared with the corresponding results obtained from the direct solution of the dense MoM system and excellent agreement is observed in all examples.

2. FORMULATION

2.1. Discrete Wavelet Transform

When a commonly used local expansion and testing functions are used, the resulting MoM system is expressed as

$$ZJ = E, \tag{1}$$

where Z, J, and E denote the method of moment matrix, the induced current vector, and the excitation vector, respectively. Note that Z is normally a complex, non-Hermitian, dense matrix of size $N \times N$. If a wavelet matrix W is introduced, the matrix Equation (1) is then transformed into

$$Z'J' = E' \tag{2}$$

where

$$Z' = WZW^T, \qquad J' = (W^T)^{-1}J, \qquad E' = WE. \tag{3}$$

Equation (2) then can be solved for J' and the direct solution is then obtained as

$$J = W^T J' \tag{4}$$

Matrix W has to be chosen in such a way that Z' contains many very small elements which can be set to zero due to a pre-selected threshold without adversely affecting the accuracy of the solution approximation. This can be achieved by constructing W from the Daubechies orthogonal wavelet (DOW) [13]. The transformation starts by defining the scaling and wavelet functions ϕ and ψ that satisfy the following two-scale relations

$$\begin{aligned} \phi(x) &= \sqrt{2}\sum_n h_n \phi(2x-n), \\ \psi(x) &= \sqrt{2}\sum_n g_n \phi(2x-n), \end{aligned} \tag{5}$$

where h_n and g_n denote decomposition filter coefficients. The Daubechies wavelets with m vanishing moments have filters that are each of length $2m + 2$, which are related as

$$g_k = (-1)^k h_{2m+1-k}, \qquad k = 0, \cdots, 2m+1 \tag{6}$$

The filter coefficients for $m = 1$ are given as

$$\begin{aligned} [h_0, h_1, h_2, h_3] &= [0.4830, 0.8365, 0.2241, -0.1294] \\ [g_0, g_1, g_2, g_3] &= [-0.1294, -0.2241, 0.8365, -0.4830] \end{aligned} \tag{7}$$

The l-level wavelet transform W can then be written as the product of one-level wavelet transform U_k

$$\begin{aligned} W &= U_{n-l+1} \cdots U_{n-1} U_n, \qquad \text{and} \\ U_{n-j} &= \begin{bmatrix} \begin{bmatrix} H_{n-j} \\ G_{n-j} \end{bmatrix} & \\ & I_{N-N/2j} \end{bmatrix} \end{aligned} \tag{8}$$

where I_k denotes the identity matrix and H_{n-j} and G_{n-j} are low and high pass matrices of size $N/2^{j+1} \times N/2^j$ defined by their filter coefficients h_k and g_k. Using the above wavelet transform technique, the sparse system of Equations (2) is obtained. This technique however, requires the storage and consequently transformation of the full MoM matrix, which is not feasible. In what follows, we describe a simple yet effective block- partitioning scheme that overcomes this problem.

2.2. Block-Partitioning Scheme

Conventionally, wavelet transform sparsifies the MoM matrix by operating on the full size of the matrix. Yu and Kishk used the wavelet transform to sparsify the dense MoM matrix around the boundaries of the current components to avoid the discontinuities between the current components [15, 16]. That made the wavelet transform matrix to be partitioned to diagonally blocked sub-matrices. We found that this form of the wavelet transform matrix does not need the full dense MoM matrix. Therefore, we introduce the idea of block partitioning the original MoM matrix during its generation by dividing and storing the matrix in as many blocks around the current components or any expected discontinuities as deem necessary. We then sparsify each block and its corresponding excitation vector and finally combine only the non-zero elements to solve for the unknown vector. To illustrate the procedure, assume that the diagonally blocked wavelet transform

matrix, the full dense MoM matrix, and the excitation vector are given as

$$W = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & W_L \end{bmatrix},$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1K} \\ Z_{21} & Z_{22} & \cdots & Z_{2K} \\ \vdots & \vdots & \cdots & \vdots \\ Z_{K1} & Z_{K2} & \cdots & Z_{KK} \end{bmatrix}, \quad \& \quad E = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_K \end{bmatrix} \tag{9}$$

where W_i is a multi-level transform matrix, which is constructed from one-level transform matrix P_n^i [19]:

$$W_i = P_{n-1+1}^i \cdots P_{n-1}^i P_n^i \tag{10}$$

$$P_{n-j}^i = \begin{bmatrix} \begin{bmatrix} H_{n-j}^i \\ G_{n-j}^i \end{bmatrix} & [0] \\ [0] & I_{N_i - N_i/2^j} \end{bmatrix} \tag{11}$$

W_i expresses the decomposition equations. W_i can be expressed as adaptive wavelet packet transform matrix, but the decomposition tree of wavelet transform approaches the predefined decomposition tree [18]. $W_i (i = 1, 2, \cdots, L)$ are orthogonal and could be the same as one another for the matrices of the same size. Z_{ij} is a sub-matrix of order L_j, and E_i is a sub-vector of length L_i. The transformed matrix Z', the transformed vector E', and the unknown vector J' become

$$Z' = \begin{bmatrix} W_1^T Z_{11} W_1 & W_1^T Z_{12} W_2 & \cdots & W_1^T Z_{1K} W_K \\ W_2^T Z_{21} W_1 & W_2^T Z_{22} W_2 & \cdots & W_2^T Z_{2K} W_K \\ \vdots & \vdots & \cdots & \vdots \\ W_K^T Z_{K1} W_1 & W_K^T Z_{K2} W_2 & \cdots & W_K^T Z_{KK} W_K \end{bmatrix},$$

$$E' = \begin{bmatrix} W_1^T E_1 \\ W_2^T E_2 \\ \vdots \\ W_K^T E_K \end{bmatrix} \quad \& \quad J' = \begin{bmatrix} W_1^T J_1 \\ W_2^T J_2 \\ \vdots \\ W_K^T J_K \end{bmatrix}, \tag{12}$$

where J_i is a sub-vector of length L_i. Notice that each block of the transformed matrix Z' can be transformed independently, which makes it possible to generate the matrix block by block and perform the transformation and storage in the appropriate position in the sparse matrix. After solving the sparse system, the original unknown current coefficient can be obtained as $J = W^T J'$.

Note that unknown vector J is obtained from J' through inverse wavelet transform after J' is obtained through the iterative solution. We also note that if the blocks are partitioned into the same size, W_i and W_j become the same identical transform matrices thus reducing the computational effort even further by producing only a single transform matrix W. The next natural step is to implement an efficient iterative scheme that can accurately approximate the desired solution with minimum computational complexity.

2.3. The Bi-CGSTAB (l) Algorithm

There are wide range of iterative solvers for the solution of large, complex, sparse system of linear equations of the form $Ax = b$ among which the Conjugate Gradient (CG), the Bi-Conjugate Gradient (Bi-CG), and the Quasi Minimal Residual (QMR) are the well known methods. There are instances for which these techniques yield a slow convergence rate and they might even diverge and fail to provide approximate solutions. In the CG-like algorithms, for example, a shadow residual vector $\mathbf{r}_k = q_k(A)r_k$, where q_k is a polynomial of degree k, is generated to minimize the residual vector r_k. However, for some practical problems, the choice of q_k used in CGS results in increasing the residual effect and hence divergence is encountered. To overcome this problem, van der Vorst [19] introduced a new algorithm called Bi-CGSTAB that takes for the q_k the product of the 1-step minimal residual (MR) polynomials. This degree 1 of polynomials, takes the form $1 - w_k t$ for some optimum w_k. For many problems, this choice of q_k results in a faster and smoother convergence when compared to CGS and Bi-CG methods. However, there are instances for which w_k becomes nearly zero, which causes stagnation or even breakdown [17]. Gutknecht introduced a variant algorithm called Bi-CGSTAB2 that attempts to avoid this stagnation by employing a second degree of MR polynomials [20]. Despite its attractive properties, this algorithm has the problem of producing degenerate MR polynomials that results in the next second-degree polynomials as well as the Bi-CG iteration coefficients to be polluted by large errors that could affect the performance of the algorithm severely and results in near breakdown. More recently, G. Sleijpe and D. Fokkema presented a generalized algorithm, Bi-CGSTAB (l) that overcomes

these shortcomings of Bi-CGSTAB and Bi-CGSTAB2. This method proves to be the most reliable yet efficient technique to obtain an approximate solution to a large, complex, sparse system. For $l = 1$, this scheme becomes the Bi-CGSTAB algorithm. The polynomials q_k are selected to be the product of l-step MR-Polynomials of the following form when $k = ml + l$

$$q_k = q_{ml+l} = p_m p_{m-1} \cdots p_0, \tag{13}$$

where the p_i's are of degree l, $p_i(0) = 1$, and p_m minimizes $\|p_m(A)q_{k-1}(A)r_k\|_2$. An l-degree MR-polynomial p_m is formed after each l-step. In the intermediate steps $k = ml+i$, $i = 1, \cdots, l-1$ simple factors t^i are used to reconstruct the p_m from these powers. In this way, certain near-breakdowns can be avoided totally in these steps. It is also worthwhile to mention that in exact algorithm if Gutknech's Bi-CGSTAB2 algorithm does not break down, it produces the same result as Bi-CGSTAB (l) if $l = 2$. The complete Bi-CGSTAB (l) algorithm may be found in [17] for interested readers.

2.4. Preconditioning Process

Often times, the sparse matrix Z' resulting from the wavelet transformation of the original impedance matrix has a very large condition number of even up to tens of thousands and would certainly result in a very slow converging or diverging system. Therefore, the incorporation of a preconditioning scheme is highly advised. Since Z' is a sparse matrix, the sparsity property needs to be explored when the preconditioning is applied. Among different techniques available, one of the most efficient and computationally inexpensive preconditioners is the incomplete LU decomposition (ILU) [21]. If we denote the nonzero structure of Z' by $NZ(Z')$, i.e., the set of all pairs (i, j) such that $z'_{ij} \neq 0$, then ILU is nothing but an (i, j, k) version of Gaussian elimination, which is essentially restricted to the $NZ(Z')$ part of the matrix. With the ILU preconditioner, highly accurate results are obtained very efficiently for a wide variety of problems.

3. NUMERICAL RESULTS

In order to illustrate the accuracy and efficiency of the present technique, several numerical examples are discussed here. First, we consider the problem of evaluating the surface electric current distribution $\mathbf{J}_S$ induced on a perfect electric conductor, PEC, sphere with radius $a = 2.5\lambda$ due to a θ-polarized plane wave incident in the direction of $\theta = \pi$. The conventional MoM basis and testing functions

are used to discretize the Electric Field Integral Equation (EFIE) and a dense impedance matrix of the size 256×256 is obtained. The wavelet transform as discussed above is performed with a threshold of 10^{-4} resulting in a spars system with a sparsification rate of 73%. The Bi-CGSTAB (l) is then used to solve the transformed system with l ranging from 1 to 8. The number of iterations along with the 2-norm of the residual vector is listed in Table 1.

Table 1. Results of the iterative solution of EFIE equation with $N = 256$.

Order l	Number of Iterations	$\|r_k\|_2$
2	5	7.9E-09
3	4	4.8E-08
4	2	7.7E-08
5 and up	2	E-010

Two equivalent surface current components are obtained. The magnitudes of the current coefficients are computed and compared with the corresponding results obtained through direct LU decomposition of the original impedance matrix. Fig. 1 shows the magnitude of the current coefficients versus the basis function order, which are arranged to start from the direction of the plane wave incidence. The first half of the curve represents the t-directed current coefficients and the second half for the φ-directed current component. An excellent agreement between both results is observed.

Next, the same problem is treated using the Magnetic Field Integral Equation (MFIE) resulting in the same matrix size but different structure. The sparsified system has a 70% sparsification rate. Again, the iterative solver is employed to obtain an approximate solution to the current density and results are seen in Table 2 and Fig. 2 in which the comparison is made with the direct solution. As seen excellent agreement is observed. Although, the results in Fig. 1 and Fig. 2 are for the same problem, we notice that the solution obtained from the EFIE has some oscillation as compared to those obtained in Fig. 2 from MFIE.

The next problem uses the MFIE to analyze the induced electric current density on a PEC sphere but with a larger number of unknowns which results in a dense matrix of size 1024×1024. The MoM matrix is wavelet-transformed resulting in a 92% sparsification rate. As expected, a much larger rate is obtained as the size of the problem space

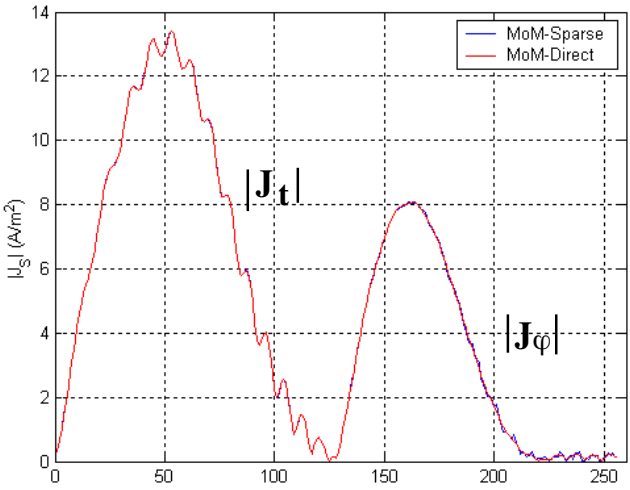

Figure 1. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a PEC sphere with $a = 2.5\lambda$ using EFIE formulation.

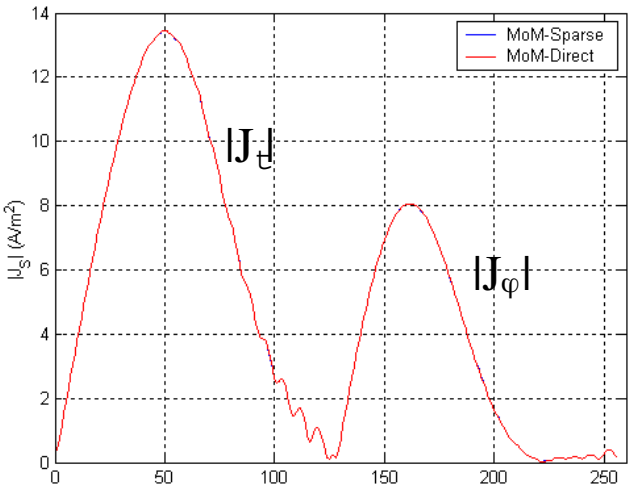

Figure 2. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a PEC sphere with $a = 2.5\lambda$ using MFIE formulation.

Table 2. Results of the iterative solution of MFIE equation with $N = 256$.

Order l	Number of Iterations	$\|r_k\|_2$
2	4	6.7E-010
3	3	3.9E-011
4	2	2.7E-07
5 and up	2	E-012

Table 3. Results of the iterative solution of MFIE equation with $N = 1024$.

Order l	Number of Iterations	$\|r_k\|_2$
4	6	1.7E-07
3	4	9.0E-07
2	3	3.5E-07
5 and up	3	E-08

is increased, which illustrates the effectiveness of the sparse system analysis. Results of the sparse solver and direct method are seen in Table 3 and Fig. 3. Excellent agreement with a dramatic saving in both CPU and memory is achieved.

To illustrate the effectiveness of this procedure when dealing with complex problems, we solve for the induced current density on a truncated dielectric cone loaded with conducting strips as shown in Fig. 4 with $\varepsilon_r = 4.0$, $a_1 = 0.11\lambda$, $a_2 = 0.71771\lambda$, $h = 0.812\lambda$, the conducting strips with period $p = \lambda/20$, strip width $w = 0.035\lambda$, and $w/p = 0.7$. With the conventional discretization of the integral equation, a matrix size of 512×512 is obtained. After the sparsification process, an 18% sparsification rate is obtained. It is clear that this rate is much lower than the previous problems. This can be clearly explained by the fact that for a complex geometry, more non-zero elements are needed to approximate the solution more accurately. However, as the problem size becomes larger the wavelet transformation naturally becomes more effective by producing more zero elements. Results of the sparse system analysis and the direct solution are given in Table 4 and Fig. 5. Excellent agreement is observed. The sparse solver is capable of capturing even the small

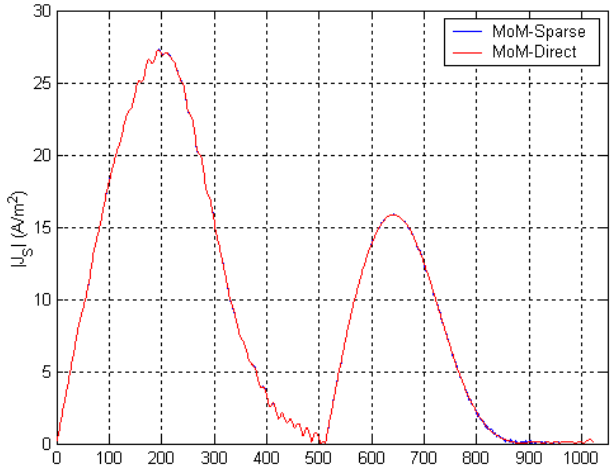

Figure 3. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a PEC sphere with $a = 2.5\lambda$ using MFIE formulation and a discretization size of half of that used for Fig. 2 results.

Table 4. Results of the iterative solution of EFIE equation with $N = 512$ for the complex geometry.

Order l	Number of Iterations	$\|r_k\|_2$
2	24	1.0E-07
3	23	6.0E-07
4	15	2.4E-08
5	9	6.8E-07
6	8	8.3E-09
7 & 8	8	E-09

details related to the current distribution. Note also the effect of increasing the order of the Bi-CGSTAB (l) order, l.

In order to examine the efficiency and adequacy of our method for larger problems, we consider the scattering from a PEC sphere with radius $a = 10\lambda$. EFIE is discretized using Galerkin method and a dense system of dimensions 2048×2048 is obtained. The wavelet transform is employed resulting in a sparse system of only 137,951 unknowns corresponding to a sparsification rate of 96.7%. The electric current

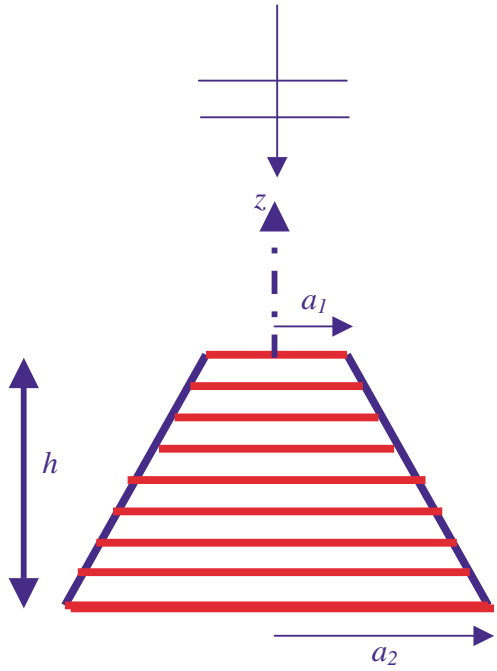

Figure 4. A Cross section of an axis-symmetric truncated dielectric cone loaded with conducting strips.

distribution is evaluated by both the sparse method and the direct LU decomposition technique and results are depicted in Fig. 6 where nearly identical current values are obtained. We note that the sparse solver uses the Bi-CGSTAB (2) which converges in 4 iterations.

We next consider a dielectric sphere with radius $a = 5\lambda$. A dense matrix system of dimensions 2048×2048 is obtained. The induced current coefficients are calculated by the Bi-CGSTAB (2) and the direct method and results are shown in Fig. 7 where again excellent agreement is observed. The sparse solver converges in 7 iterations with a sparsification rate of 92.6%.

In the following, we consider a dual reflector antenna. The main reflector is a parabolic reflector of 3 m diameter and focal to diameter ratio of 0.41. The subreflector is shaped reflector of diameter 0.386 m very close to the waveguide aperture as shown in Fig. 8. The frequency of operation is 5.75 GHz. The resulting MoM matrix is 2048×2048. The wavelet transform results in a 89% sparsification rate. The Bi-CGSTAB (2) converges in 7 iterations and the results are compared with the results obtained by the direct method and as it can be seen

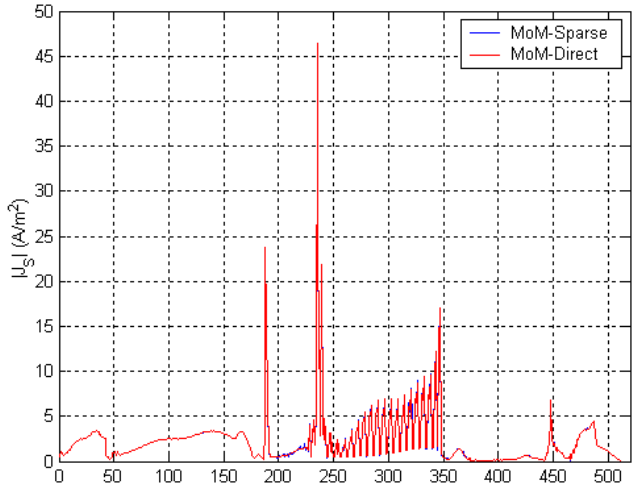

Figure 5. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a truncated dielectric cone loaded with conducting sphere.

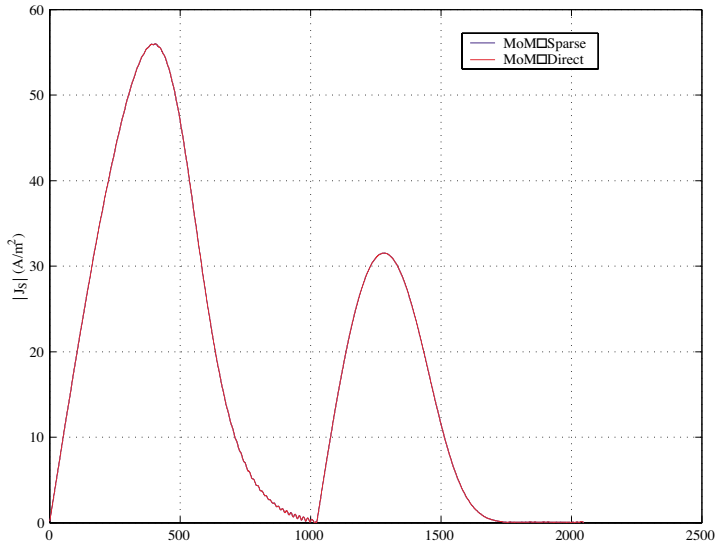

Figure 6. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a PEC sphere with $a = 10\lambda$.

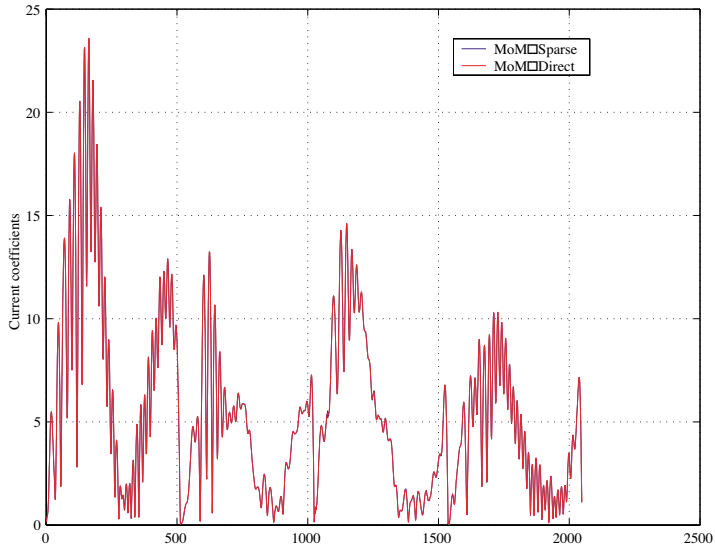

Figure 7. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a dielectric sphere with $a = 2.5\lambda$.

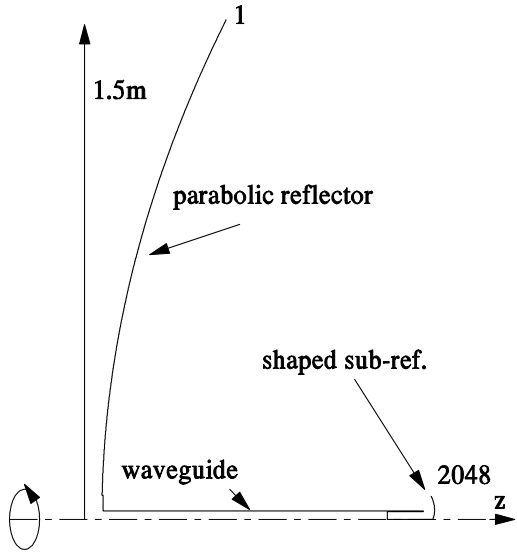

Figure 8. Generating curves for the parabolic reflector antenna with sub-reflector.

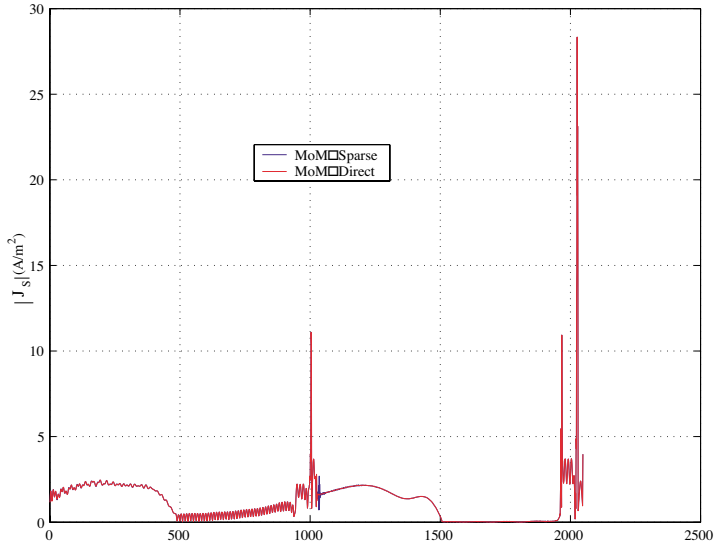

Figure 9. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique for a dual reflector antenna.

Table 5. Results of the iterative solution of MFIE equation with $N = 1024$ with the block-partitioning technique.

Order l	Number of Iterations	$\|r_k\|_2$
2	11	5.4E-07
3	6	3.4E-07
4	5	1.2E-07
5	5	6.8E-10
6	4	7.6E-07
7 & 8	3	E-09

in Fig. 9, identical results are obtained.

The final example serves the purpose of illustrating the effectiveness of block-partitioning technique described in Section 2.2. Again, MFIE is used to formulate the induced current density over a PEC sphere where now instead of generating a 1024×1024 matrix, four 512×512 sub-matrixes are generated separately and the wavelet

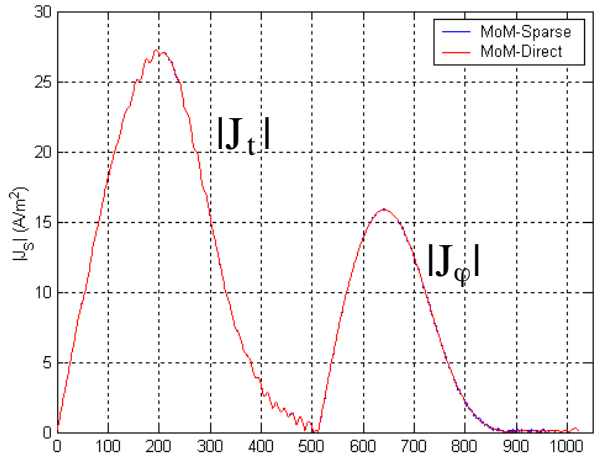

Figure 10. A comparison of the magnitude of the electric current coefficients evaluated by the direct LU decomposition and by the sparse technique.

transform in applied to each individual sub-matrix. After sparsification is performed, the whole system of transformed sub-matrices along with their corresponding excitation vector blocks are combined to solve for the unknown vector. Clearly, we can now avoid storing the whole MoM matrix at once hence saving in storage requirements of the MoM technique. A 91.75% sparsification rate is obtained showing a slight decrease in the population of non-zero entries as compared to example 3 where the whole MoM matrix was sparsified. Note that we chose four blocks of equal dimensions for simplicity purposes. Again, the current distribution is calculated using both the direct and the block-partitioning sparse method and results are depicted in Table 5 and Fig. 10. Excellent agreement is observed while a huge saving in CPU time and memory requirements is achieved. Note that convergence is obtained in only 7 iterations.

4. CONCLUSIONS

We presented a very efficient method for the solution of discretized integral equations obtained when the Method of Moments is employed to solve electromagnetic problems. Conventional expansion and testing functions were used to discretize the problem geometry. A Daubechies orthogonal wavelet transform was used to convert the dense impedance matrix to a sparse matrix. Full advantage was then taken from the

sparse behavior of the system by implementing a preconditioned Bi-CGSTAB (l) scheme that efficiently yields accurate solutions to the problems. With the addition of block transformation method, large-scale problems are effectively analyzed with minimal computation effort and storage requirements. Our proposed technique is capable of capturing very fine details associated with the electric current distribution as compared to the direct solution.

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