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PROPAGATION OF A LASER BEAM THROUGH A PLANE AND FREE TURBULENT HEATED AIR FLOW: DETERMINATION OF THE STOCHASTIC CHARACTERISTICS OF THE LASER BEAM RANDOM DIRECTION AND SOME EXPERIMENTAL RESULTS

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Abstract—The propagation of waves in a random medium is a very complex phenomena which presents numerous difficulties in its experimental approach, and in its theoretical analysis. In this work, the case of a laser beam direction during its random propagation through a hot free jet of air, is considered using geometrical optics. Some experiments are done in the jet and from the hypothesis of the Markovian process, the main stochastic characteristics of the laser beam direction are studied. In addition, the sensitivity of the probability density of the beam random direction with respect to the jet turbulent diffusion is determined.

6 Conclusion

References

1. INTRODUCTION

In several laser based systems such as laser radar, remote sensing, satellite communication, distance measuring, detection, various techniques for obtaining local or global information about turbulence are encountered. One usually uses the statistical properties of the laser beam random propagation through the corresponding turbulent medium. Sometimes, the inverse problem is also studied, that is, the possibility to obtain local or global stochastic properties of the laser beam random propagation using turbulence flow information. About these techniques, we can cite contributions by Wilson [1], Sirazetdinov et al. [2], Joia et al. [3], Gagnaire and Tailland [4], Benzirar et al. [5]. In [1], an experimental method for obtaining statistical properties of turbulent fluctuations in a circular jet is described. In [2], it is experimentally proved that the laser angular divergence increases with decreasing cross-angle. Results of different experiments showed that the average angle of the narrow one-micron beam disturbed by a jet aircraft engine is less than that of the ten-micron beam which is characterized by a large diffraction divergence, and that of the half-micron beam stronger subjected to disturbances. In [3], the propagation of a laser beam through a plane turbulent jet is considered; the intensity fluctuations arising in the beam are measured and their spectra and variance determined at various distances from the jet. Another problem has been treated in [4] where the temperature fluctuations created in a hot jet of air are measured from a laser beam intensity fluctuations. The propagation of a laser beam through a hot turbulent jet is studied again in [5]: Benzirar et al. have determined the value of the jet diffusion coefficient defined as a proportionality factor between the mean square of the deflection angle fluctuations and the length of the corresponding finite laser beam path. In the above works [3, 4] and [5], the statistical properties of the laser beam direction are not studied. The present paper is devoted to the determination of the characteristics of a laser beam random direction during its propagation, using geometrical optics and the hypothesis of the Markovian process. It is well known [6–8] and proved in [9] that the hypothesis of Markovian process which is used in the present paper leads to the Einstein-Fokker-Planck-Kolmogorov (EFPK) equation. For solving this equation the calculation procedure in our work uses the value of the diffusion coefficient obtained in [5]. Our calculus are essentially numerical and are done by using the experimental impact of the laser beam on the plane of the photoelectric cell. For theoretical analysis about interaction between waves and turbulent flows, Klyatskin and Tatarskii [6] have presented a consistent write up on the principal results and have described the conditions for the wave propagation process to be considered as a diffusion process.

In order to understand the present work, the paper consists of four sections. The first section is devoted to the description of the experimental setup and to the approximations that we have made for justifying the validity of the geometrical optics.

In the second section, we describe the modified experimental setup which allows the separation of the laser beam intensity fluctuations from the total fluctuations (intensity and direction), in order to prove that, in the jet considered, the intensity fluctuations may be neglected.

In the third section, we are concerned with the validity of the hypothesis of Markovian process for the laser beam random direction. It follows the description of the numerical calculation procedure for solving the EFPK equation. From that solution, the main stochastic characteristics of the laser beam direction are determined.

In the last section of the present paper, the methods of measurement are described and the results and various plots are presented and discussed.

2. EXPERIMENTAL CONDITIONS AND SOME REMARKS ABOUT THE PHYSICS OF THE PROBLEM

The experimental setup is shown schematically in Fig. 1. We are concerned with a light beam created from a 1 mW He-Ne laser and having initial diameter $a = 0.8\,\text{mm}$, wavelength $\lambda = 6328\,\text{A}$ and spectral width $\Delta\lambda = 6.4 \times 10^{-4}\,\text{A}$. This laser beam is placed in the (xx_1) plane and traverses the jet at a distance $x_1 = 200\,\text{mm}$ from the nozzle exhaust. The distance corresponding to the mean trajectory of the laser beam in the jet is taken along the Ox axis and is equal to : $X = 200\,\text{mm}$. We choose (x, x_1, x_2) as the three cartesian coordinates axis. Outside the jet, at a distance $d = 500\,\text{mm}$ from the jet border, is placed a photoelectric cell (PC1) on which the laser beam can produce an impact after having traversed the jet. The very important character of this cell is that the electric signals transmitted by the cell depend only on the two coordinates x_1 and x_2 of the beam impact position.

Before heating the jet, the ambient medium is at rest and the jet is at ambient temperature. Under these conditions, the beam trajectory is nearly rectilinear and produces on the cell, a constant impact which is taken as the origin O of the cell plane. When the jet is heated,

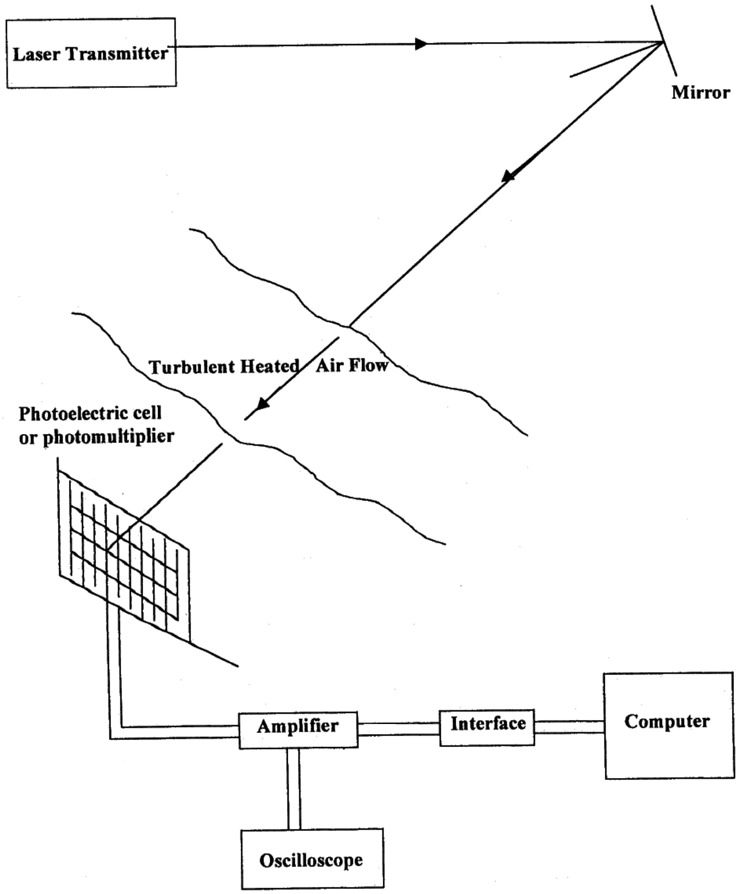


Figure 1. Experimental setup 1.

we can measure along the whole mean path of the laser beam, some parameters which allow us to assume that the jet is plane, that is, the mean temperature $\langle T \rangle$ by means of a thermocouple, the rms of the temperature fluctuations by using an anemometer with cold wire, and the mean velocity $\langle U \rangle$ by means of a Pitot's tube.

About the physics of the problem, it is useful to specify that all measurements were done when the jet turbulence is fully developed and does not depend on time, in order to obtain stable results. Furthermore, we assume that the pressure at ant point of the jet remains nearly constant and is equal to the air pressure P_o, that is, the pressure gradient for two neighboring points inside the jet can be neglected. This hypothesis holds if the variations of pressure in the heated jet spread at very high speed. Consequently, the thermic turbulence can be considered as the unique cause of the refractive index fluctuations.

The inner scale ℓ and the outer scale L of the jet turbulence being measured in [5] under the same conditions ($\ell = 1\,\text{mm}$, $L = 10\,\text{mm}$), we can say that the classical conditions $X \gg L$, and $\lambda \ll \ell$ are then satisfied in the jet considered. It is well known that under those two conditions, the Maxwell equations give the stochastic Helmholtz equation for the wave electric field, which then leads to the equation of the geometrical optics approximation. It is obvious that the above conclusion is valid if diffraction may be neglected, that is if the size of the first Fresnel zone $\sqrt{\lambda X}$ is less than the inner scale of the turbulence, and if the intensity fluctuations of the laser beam remain very weak. Knowing that $\sqrt{\lambda X} = 0.356\,\text{mm}$ and $\ell = 1\,\text{mm}$, we can say that the condition $\sqrt{\lambda X} < \ell$ is satisfied. It remains to prove that the laser beam intensity fluctuations can be neglected in the jet considered. That is the purpose of the section below.

3. THE EXPERIMENTAL SETUP FOR THE SEPARATION OF THE INTENSITY FLUCTUATIONS FROM THE TOTAL FLUCTUATIONS

With the aim to estimate the intensity fluctuations, the previous experimental system can be modified and the new system is shown in Fig. 5. The laser beam already used in Section 2 is expanded in diameter by a first afocal system, after passing through the heated jet. It then falls onto two holes P_1 and P_2 (equal diameter $= 2a_o$, $P_1P_2 = d_o$) situated on a vertical screen, symmetrically with respect to the initial direction of the laser beam. By means of a second nearly afocal system, the interference pattern can be obtained on a plane (x_1, x_2) where the aperture of a photomultiplier is placed. Only three fringes are sufficient and permit to know all geometrical properties of the interference pattern. If X_o denotes the first zero of the Bessel's function J_1 and x_o the first zero of the function $x_2 = \cos^2 x_1$, it is well-known that the interference pattern contains only three fringes if the condition $X_o/x_o = 3$ is satisfied. Knowing that the lenses L_1, L_2, and the objectives l_1, l_2 have focal distances $F_1 = 100\,\text{mm}$, $F_2 = 100\,\text{mm}$, $f_1 = 17\,\text{mm}$ and $f_2 = 4.7\,\text{mm}$ respectively, the above condition leads to $a_0/d \approx 0.814$. In order to obtain large and well-illuminated fringes, we have taken: $2a_o = 0.4\,\text{mm}$ and $2d_o = 0.5\,\text{mm}$.

4. NUMERICAL CALCULATIONS PROCEDURE

Before describing the calculation procedure, we have to give the reason which allows us to use the hypothesis of Markovian process for the laser beam random direction. By way of proof, we use the works done by Benzirar et al. [5]. We can say that the above authors have determined by using the same hypothesis, under the same experimental conditions, the diffusion coefficient D defined as:

$$D = -\left(\frac{\langle\mu^2\rangle}{2}\right)\int_0^{\infty}\Delta R_\mu(0,0,r)dr \tag{1}$$

where μ denotes the refractive index fluctuations, R_μ the correlation coefficient of μ and Δ the Laplacian operator. After having compared the value that they obtained with the value measured by means of the hot wire technique by Gagnaire in [4], they showed that the model based on the hypothesis of Markovian process may be applied for the laser beam random direction in the jet considered.

Let us define the unit vector $\boldsymbol{U}$ characterizing the laser beam direction, by writing the cartesian components (U_x, U_y, U_z) of $\boldsymbol{U}$ as follows:

$$U_x = \sin\theta\cos\phi \tag{2a}$$
$$U_y = \sin\theta\sin\phi \tag{2b}$$
$$U_z = \cos\theta \tag{2c}$$

where ϕ is the polar angle and θ the azimuthal angle. Using the definition of Markovian process and taking into account the fact that for any small difference length of a given path, the laser beam direction may undergo only small variations, it is proved in [9] that the probability $P(\sigma,\theta,\phi)$ for the laser beam to have the direction $\boldsymbol{U}$, after having traversed the path of length σ, is governed by the second Kolmogorov's equation also named the EFPK equation:

$$\frac{\partial P}{\partial\sigma} = \frac{D}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial P}{\partial\theta}\right) + \frac{D}{\sin^2\theta}.\frac{\partial^2 P}{\partial\phi^2} \tag{3a}$$

where D is the jet diffusion coefficient previously defined.

Since the small difference between the small finite path $\Delta\sigma$ and the corresponding distance Δx along the x axis can be neglected, the Equation (3a) is replaced by:

$$\frac{\partial P}{\partial x} = \frac{D}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial P}{\partial\theta}\right) + \frac{D}{\sin^2\theta}.\frac{\partial^2 P}{\partial\phi^2} \tag{3b}$$

The laser beam initial direction before entering in the jet being the O_x axis, the initial condition associated to the Equation (3a) is:

$$P(\theta, \phi, x = 0) = \delta\left(\theta - \frac{\pi}{2}\right)\delta(\theta) \tag{3c}$$

where δ denotes the Dirac's distribution. Differentiating (3c) with respect to the parameter D and assuming $\frac{\partial^2 P}{\partial a \partial D} = \frac{\partial^2 P}{\partial D \partial a}$ for $a = \phi$, θ, we obtain:

$$\frac{\partial Q}{\partial x} = \frac{D}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial Q}{\partial\theta}\right) + \frac{D}{\sin^2\theta}.\frac{\partial^2 P}{\partial\phi^2} + \frac{1}{D}\frac{\partial P}{\partial x} \tag{4a}$$

where $Q = \frac{\partial P}{\partial D}$ represents the sensitivity of the probability of the direction of rays to the parameter D or the strength of the turbulent diffusion. The initial condition associated to the Equation (4a) and deduced from (3c) is:

$$Q(\phi, \theta, x = 0) = 0 \tag{4b}$$

The coordinates (x_1, x_2) of the laser beam impact on the photoelectric cell and the variable x are discretized according to the following relations:

$$\begin{aligned} x_1(L) &= x_{10} + L.c, \quad && L = 0, 1, 2, \ldots, L\max \quad (5a)\\ x_2(M) &= x_{20} + M.c, \quad && M = 0, 1, 2, \ldots, M\max \quad (5b)\\ x(0) &= 0, && \quad (5c)\\ x(N+1) - x(N) &= \Delta x, && N = 0, 1, 2, \ldots, N\max \quad (5d) \end{aligned}$$

It is proved in the following section that the quantities x_{10}, x_{20}, c, Lmax and Mmax will be dictated by the experimental conditions. The value of Nmax depends on Δx and the choice of a constant step Δx along the x direction is dictated by the flow nature (plane jet) The discretisation step Δx must be chosen such that $\Delta x \leq l_0$ where l_0 is the integral scale characterizing the dimension of the turbulent structures in which the propagation of light can be considered rectilinear ($l_0 \approx 8$ mm in [5]).

Assuming that the propagation of the laser ray is rectilinear from the jet border to the cell plane, it can be shown that the discrete values for the angles θ and ϕ are theoretically related to the coordinates (x_1, x_2) of the laser ray impact on a photoelectric cell placed outside the jet, according to the following relations :

$$\begin{aligned} \phi(L) &= \tan^{-1}[x_1(L)/(x\max + d)] && (6a)\\ \theta(M) &= \pi + \tan^{-1}[(x\max + d)/x_2(M)], \quad \text{if } x_2(M) < 0 && (6b)\\ \theta(M) &= \tan^{-1}[(x\max + d)/x_2(M)], \quad \text{if } x_2(M) > 0. && (6c)\\ \text{for } L &= 0, 1, 2, \ldots, L\max \text{ and } M = 0, 1, 2, \ldots, M\max. \end{aligned}$$

In relations (6), $x\max = X$ denotes the whole path distance of the laser beam propagation and d represents the distance between the jet border and the photoelectric cell.

For solving the equations (3b) and (4b) with the initial condition (3c) and (4b) respectively, the numerical scheme used is based on the following approximations :

$$P(\phi,\theta,x) \approx P_{L,M}^{N} \tag{7a}$$

$$\frac{\partial P}{\partial\theta}(\phi,\theta,x) \approx -A(M)P_{L,M-1}^{N} + B(M)P_{L,M}^{N} + C(M)P_{L,M+1}^{N} \tag{7b}$$

$$\frac{\partial^2 P}{\partial\theta^2}(\phi,\theta,x) \approx 2[I(M)P_{L,M-1}^{N} - E(M)P_{L,M}^{N} + F(M)P_{L,M+1}^{N}] \tag{7c}$$

$$\frac{\partial^2 P}{\partial\phi^2}(\phi,\theta,x) \approx 2[X(L)P_{L-1,M}^{N} - Y(L)P_{L,M}^{N} + Z(L)P_{L+1,M}^{N}] \tag{7d}$$

$$Q(\phi,\theta,x) \approx Q_{L,M}^{N} \tag{7e}$$

$$\frac{\partial Q}{\partial\theta}(\phi,\theta,x) \approx -A(M)Q_{L,M-1}^{N} + B(M)Q_{L,M}^{N} + C(M)Q_{L,M+1}^{N} \tag{7f}$$

$$\frac{\partial^2 Q}{\partial\theta^2}(\phi,\theta,x) \approx 2[I(M)Q_{L,M-1}^{N} - E(M)Q_{L,M}^{N} + F(M)Q_{L,M+1}^{N}] \tag{7g}$$

$$\frac{\partial^2 Q}{\partial\phi^2}(\phi,\theta,x) \approx 2[X(L)Q_{L-1,M}^{N} - Y(L)Q_{L,M}^{N} + Z(L)Q_{L+1,M}^{N}] \tag{7h}$$

$$\left(\frac{\partial P}{\partial x}\right)_{L,M}^{N+1/2} \approx \frac{2}{\Delta x}(P_{L,M}^{N+1/2} - P_{L,M}^{N}) \tag{7i}$$

with:

$$A(M) = \frac{h(M)}{h(M-1)[h(M-1)+h(M)]}, \tag{8a}$$

$$B(M) = \frac{[h(M)-h(M-1)]}{h(M-1)h(M)} \tag{8b}$$

$$C(M) = \frac{h(M-1)}{h(M)[h(M-1)+h(M)]}, \tag{8c}$$

$$D(M) = \frac{1}{h(M-1)[h(M-1)+h(M)]} \tag{8d}$$

$$E(M) = \frac{1}{h(M-1)h(M)}, \tag{8e}$$

$$F(M) = \frac{1}{h(M)[h(M-1)+h(M)]} \tag{8f}$$

$$X(L) = \frac{1}{g(L-1)[g(L-1)+g(L)]}, \tag{8g}$$

$$Y(L) = \frac{1}{g(L-1)g(L)} \tag{8h}$$

$$Z(L) = \frac{1}{g(L)[g(L-1)+g(L)]} \tag{8i}$$

and:

$$g(L) = \phi(L+1) - \phi(L), \text{ with } L = 0, 1, 2, \ldots, L\max -1 \tag{9a}$$

$$h(M) = \theta(M+1) - \theta(M) \text{ with } M = 0, 1, 2, \ldots, M\max -1 \tag{9b}$$

An explicit discretization scheme with alternating directions is used, according to the following formulas:

$$P_{L,M}^{N+1/2} = R(M)P_{L,M-1}^{N} + S(M)P_{L,M}^{N} + T(M)P_{L,M+1}^{N} \tag{10a}$$

$$P_{L,M}^{N+1} = U(L)P_{L-1,M}^{N+1/2} + V(L)P_{L,M}^{N+1/2} + W(L)P_{L+1,M}^{N+1/2} \tag{10b}$$

$$Q_{L,M}^{N+1/2} = R(M)Q_{L,M-1}^{N} + S(M)Q_{L,M}^{N} + T(M)Q_{L,M+1}^{N} + \frac{1}{D}\left(\frac{\partial P}{\partial x}\right)_{L,M}^{N+1/2} \tag{10c}$$

$$Q_{L,M}^{N+1} = U(L)P_{L-1,M}^{N+1/2} + V(L)P_{L,M}^{N+1/2} + W(L)P_{L+1,M}^{N+1/2} + \frac{1}{D}\left(\frac{\partial P}{\partial x}\right)_{L,M}^{N+1} \tag{10d}$$

with:

$$R(M) = D(\Delta x)[2D(M) - A(M).\cot(\theta(M))] \tag{11a}$$

$$S(M) = 1 - D(\Delta x)[2E(M) - B(M).\cot(\theta(M))] \tag{11b}$$

$$T(M) = D(\Delta x)[2F(M) + C(M).\cot(\theta(M))] \tag{11c}$$

$$\text{for } M = 0, 1, 2, \ldots, M\max -1,$$

and:

$$U(L) = \frac{2D(\Delta x)X(L)}{\sin^2\theta(M)} \tag{12a}$$

$$V(L) = 1 - \frac{2D(\Delta x)Y(L)}{\sin^2\theta(M)} \tag{12b}$$

$$W(L) = \frac{2D(\Delta x)Z(L)}{\sin^2\theta(M)} \tag{12c}$$

$$\text{for } L = 0, 1, 2, \ldots, L\max -1$$

Assuming Dirichlet boundary conditions, we have for the numerical scheme, the following relations:

$$P^N_{0,M} = P^N_{L\max,M} = 0. \tag{13a}$$

$$Q^N_{0,M} = Q^N_{L\max,M} = 0 \tag{13b}$$

$$\text{for } M = 0, 1, 2, \ldots, M\max$$

and,

$$P^{N+1/2}_{L,0} = P^{N+1/2}_{L,M\max} = 0. \tag{14a}$$

$$Q^{N+1/2}_{L,0} = Q^{N+1/2}_{L,M\max} = 0. \tag{14b}$$

$$\text{for } L = 0, 1, 2, \ldots, L\max$$

5. MEASUREMENTS AND RESULTS

5.1. Experimental Measurements in the Jet

The thermocouple shown in Fig. 2 measures the mean temperature $\langle T\rangle$. For measuring the mean velocity $\langle U\rangle$, we use a moving graduated drum whose rotation around a vertical axis causes the vertical displacement of a cursor situated on a graduated ruler, such that 1 tower of the drum corresponds to 1 mm on the ruler. The drum helps to bring back a meniscus in order to measure the difference in level h between its position which depends on $\langle U\rangle$, and the initial level. That meniscus is illuminated and its displacement can be exactly detected by using a sight objective. By means of a Pitot's tube, the mean velocity can be measured because $\langle U\rangle$ is related to h by the relation $\langle U\rangle = [2\rho^{-1}(P_d - P_0)]^{1/2} = 4h^{1/2}$ where P_d is the dynamic pressure, P_0 is the static pressure equal to the atmospheric pressure and ρ is the air specific mass. In Fig. 3, are presented the variations of $\langle T\rangle$ and $\langle U\rangle$. It is shown that the values of $\langle T\rangle = 50°C$ and $\langle U\rangle = 8\,\text{m/sec}$ remain constant along the whole mean path of the laser beam except in a short area at both borders of the stream.

The rms of the temperature fluctuations $\langle t^2\rangle^{1/2}$ are measured by means of the cold-wire anemometer technique [10] using a wire whose diameter and length are respectively equal to 10^{-6} mm and 0.4 mm, with a current intensity $I_o = 0.16\,\text{mA}$. Taking into account the fact that the thermic turbulence is considered as the unique cause of the refractive index fluctuations, the rms of the refractive index fluctuations $\langle\mu^2\rangle^{1/2}$ are evaluated by using the Dale-Gladstone law. After some calculations, we obtain the relation:

$$\langle\mu^2\rangle^{1/2} = \left(\frac{aP_0}{\langle T\rangle^2}\right)\langle t^2\rangle^{1/2}$$

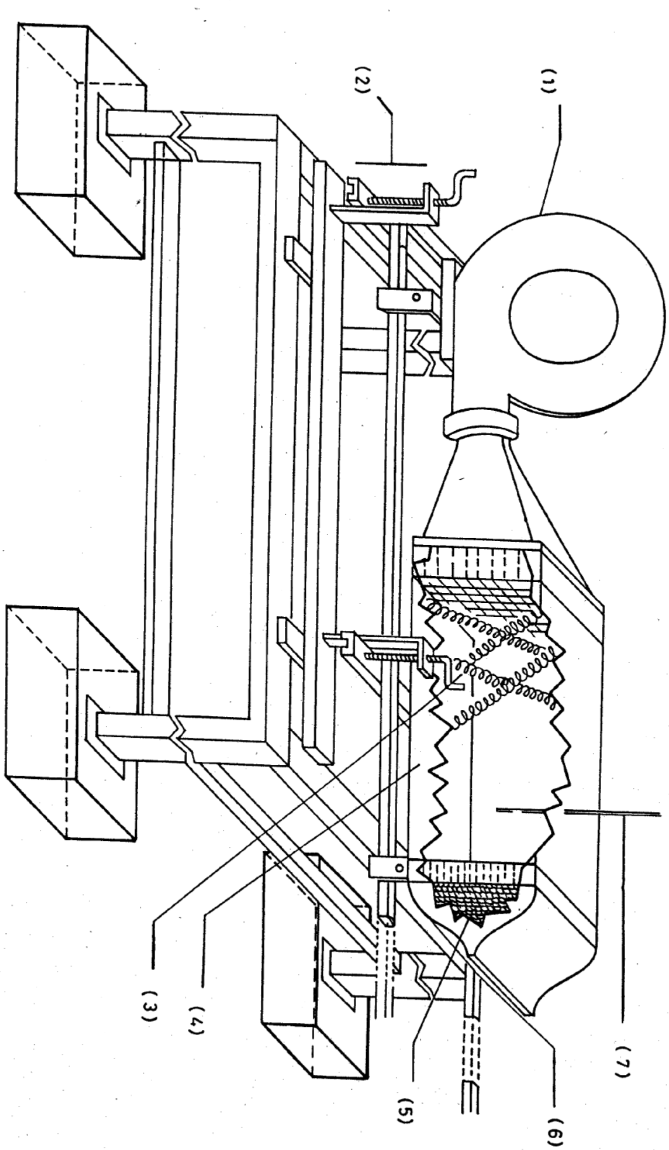


Figure 2. Wind Tunnel: (1) Ventilating fan - (2) Vertical displacement - (3) Heating resistances - (4) Box for flow homogeneity - (5) Filter against turbulence - (6) Nozzle - (7) Thermocouple.

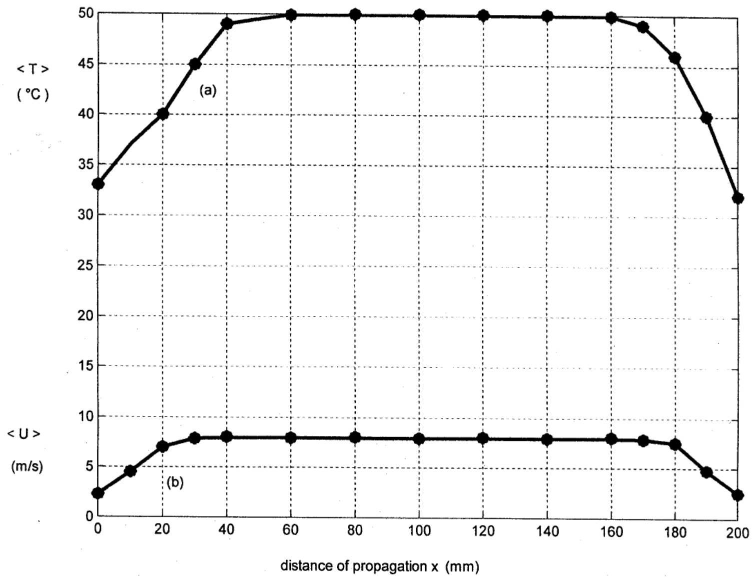


Figure 3. Statistical properties of the turbulent jet as function of the propagation distance x: (a) Mean temperature. (b) Mean speed.

where $a = 79 \times 10^{-6}\,\mathrm{K(mb)}^{-1} = \alpha/R$, α being the Dale-Gladstone constant and R a constant of the perfect gas.

In Fig. 4, are plotted the variations of $\langle t^2\rangle^{1/2}$ and $\langle \mu^2\rangle^{1/2}$. It appears that $\langle t^2\rangle^{1/2} = 2.75°C$ and $\langle \mu^2\rangle^{1/2} = 2.116 \times 10^{-4}$. These values remain constant along the whole mean path of the laser beam except in a short area at both borders of the jet.

The above measurements $\langle T\rangle$, $\langle U\rangle$, $\langle t^2\rangle^{1/2}$ and $\langle \mu^2\rangle^{1/2}$ done along the whole mean path of the laser beam are essential and serve to justify that the jet considered is plane.

5.2. Measurement of the Intensity Fluctuations

In the absence of the jet, the interference pattern is obtained from the experimental set up described in Section 3; the width of the fringes is equal to 9 mm. This value must be compared to the theoretical width equal to 8.85 mm.

For measuring the intensity fluctuations, we use a phototransmitting cell (PC2) which is fed by an electrical tension equal to 12 V. That cell is simultaneously connected to an oscilloscope and to

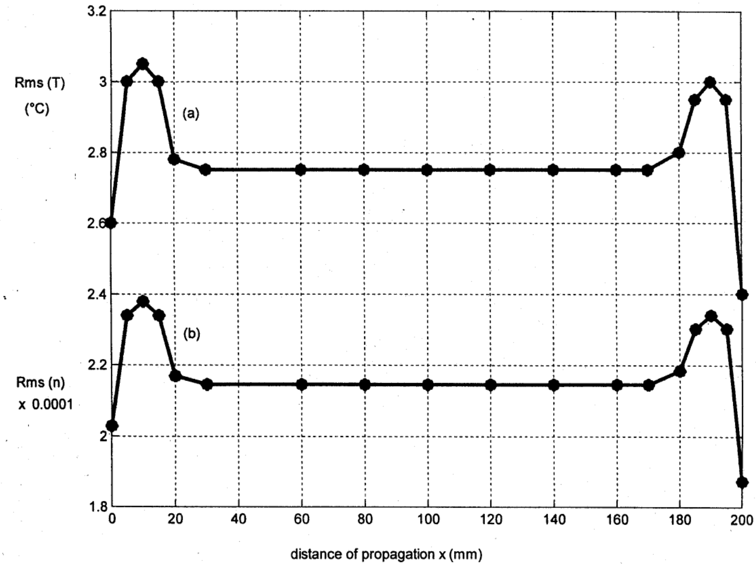


Figure 4. Statistical properties of the turbulent jet as function of the propagation distance x. (a) rms of temperature fluctuations $\sqrt{\langle t^2 \rangle}$. (b) rms of refractive index fluctuations $10^4\sqrt{\langle \mu^2 \rangle}$.

a rms-meter. The important character of the cell PC2 is the fact that the transmitted electrical tension is proportional to the incident radiation intensity and does not depends on the direction of the incident radiation if the impact zone is situated inside the sensitive zone of the cell. For separating the intensity fluctuations from the total fluctuations (intensity fluctuations + direction fluctuations), the measurements are done as follows. For a given point M situated in the interference pattern, we choose as a reference the mean value of the light intensity $\langle I \rangle$ corresponding to M. This value is measured by means of an oscilloscope in the absence of the jet. After that, the laser traverses the heated jet and reaches the photomultiplier connected to the rms-meter and placed in the interference pattern at the point M. Under these conditions, that photomultiplier measures the quantity $\sqrt{\langle I_t^2 \rangle}/\langle I \rangle$ where $\sqrt{\langle I_t^2 \rangle}$ represents the rms of the total fluctuations.

In the absence of the optical system, various polarizers are then placed in front of the laser-transmitter in order to adjust the mean value of the light intensity. When the value of $\langle I \rangle$ corresponding to the point M is obtained, the laser beam after having traversed the heated jet, is received by the cell PC2 connected to the rms-meter. The quantity $\sqrt{\langle i^2 \rangle}/\langle I \rangle$ is then measured, $\sqrt{\langle i^2 \rangle}$ being the rms of the intensity fluctuations.

With the 15 mW He-Ne laser beam ($\lambda = 6328$ A, $\Delta\lambda = 6.4 \times 10^{-4}$ A, initial diameter = 0.8 mm), several comparisons between the quantities $\sqrt{\langle i^2 \rangle}/\langle I \rangle$ and $\sqrt{\langle I_t^2 \rangle}/\langle I \rangle$ have been done for several points of the interference pattern, that is, for several mean values of the light intensity. For the values of $\langle I \rangle$ corresponding to the following values of the electrical tension: 0.40 Volt, 0.65 Volt, 0.85 Volt, 1.10 Volt, 1.35 Volt and 1.65 Volt, the quantity $\sqrt{\langle i^2 \rangle}/\langle I \rangle$ is respectively equal to 10%, 10.5%, 11%, 10.4%, 11.5% and 10% whereas the quantity $\sqrt{\langle I_t^2 \rangle}/\langle I \rangle$ is respectively equal to 85%, 76%, 64%, 58%, 45%, and 35%. It appears that the intensity fluctuations are all the more strong as the power of the laser beam is high. In the particular case of the laser beam used in Section 2, the light power is equal to 1 mW and corresponds to 0.18 Volt, and the intensity fluctuations are equal to 4.2% of the total fluctuations. Therefore, we can conclude that the laser beam intensity fluctuations may be neglected during our experiments if we use a laser transmitter whose power is equal to 1 mW.

5.3. Determination of the Main Stochastic Characteristics of the Laser Beam Random Direction

When the jet is heated, the experiment shows that the luminous trace produced by the laser beam on the photoelectric cell (PC1) can be contained inside a reduced square (S) whose size is $s = 20$ mm and whose center is not the initial impact O. The parameters x_{10} and x_{20} used in the numerical calculation procedure are then determined, that is,: $x_{10} = -15 + 7c$ (mm) and $x_{20} = -15 + 12c$ (mm). The experimental luminous trace produced by the laser ray is shown in Fig. 6a and must be compared to the luminous trace predicted by the Markovian model drawn in Fig. 6b.

For the statistical procedure, the cell plane is cross ruled in 1600 squares of same size $c = 0.5$ mm used in Equation (5). The values of the integers $L\max$ and $M\max$ defined in (5) are then determined, that is, $L\max = M\max = 40$. The initial impact of the laser beam corresponds to the integers $L = L_0 = 23$ and $M = M_0 = 18$; these values are used in the discretized initial condition for numerical calculations of the laser impact position probabilities as follows: $P(x = 0, L, M) = \delta_{LL_0}\delta_{MM_0}$, δ being the Kronecker delta.

In the course of the experiment, 2048 ray impact points on the photoelectric cell (PC1) have been obtained and allow, by using

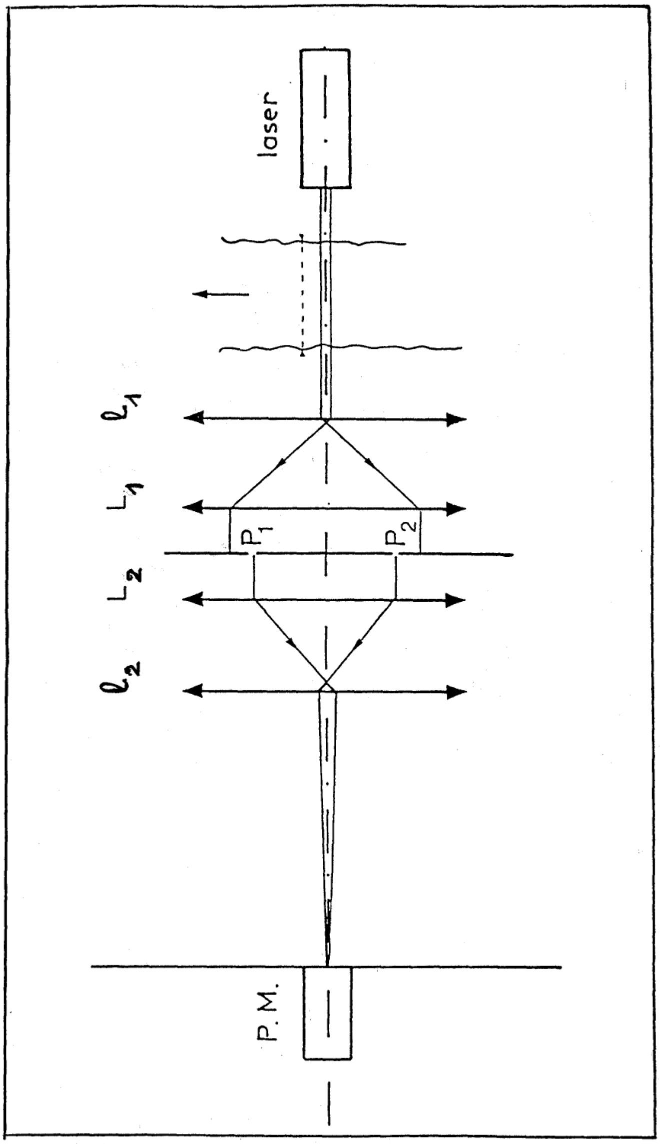
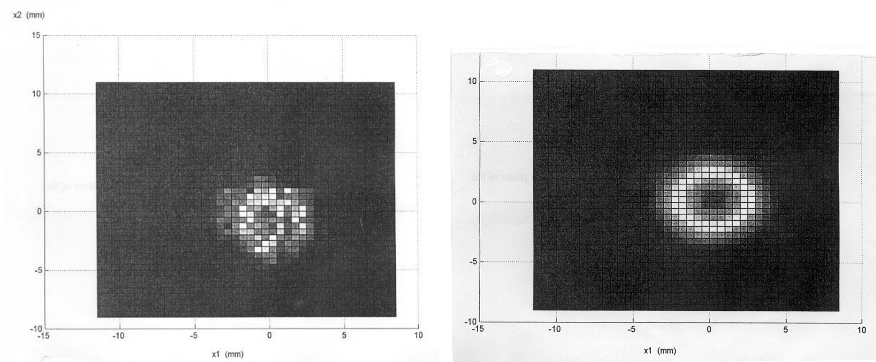
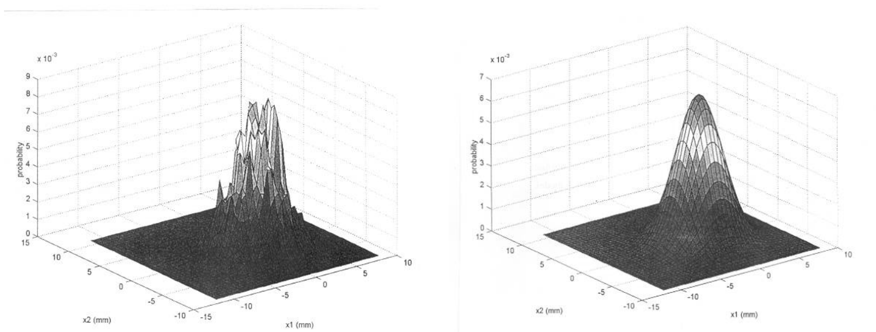


Figure 5. Experimental Setup 2 used for estimating intensity fluctuations.



(a) (b)

Figure 6. Luminous trace produced by the laser ray on the plane of the photoelectric cell: (a) Experimental results (b) Results obtained from the Markovian model.



(a) (b)

Figure 7. Probabilities of the laser ray impact position on the plane of the cell: (a) Experimental results, (b) Markovian model.

the experimental setup 1, the measurement of the laser impact position probabilities. In Fig. 7a are plotted the probability values experimentally obtained. Those values must be compared to those derived from the Markovian model and plotted in Fig. 7b in which $D = 2.98 \times 10^{-9}$ is the value obtained in [5]. After having determined the probability $P(x, \phi, \theta)$ for any distance x by means of the numerical procedure , some statistical properties for the laser beam direction can be deduced as function of the distance of propagation, that is, the maximum of the probability of the laser ray direction, the non-normalized marginal probability densities for x_1 and x_2, the mean value

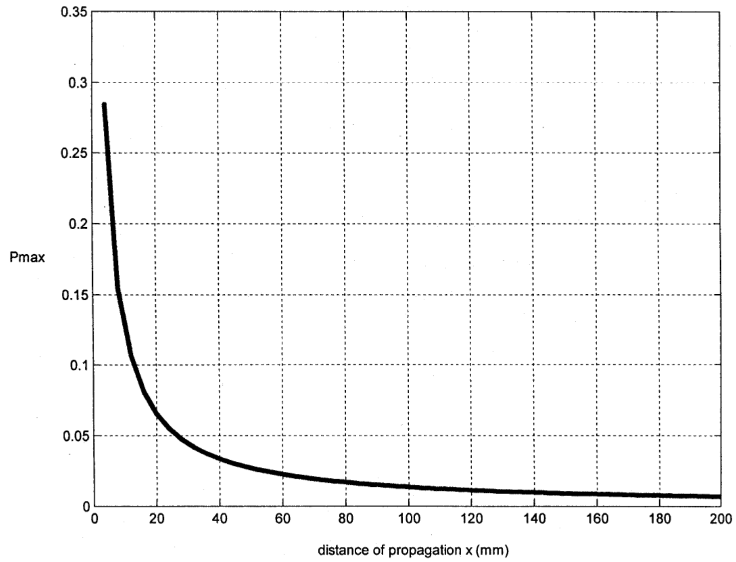


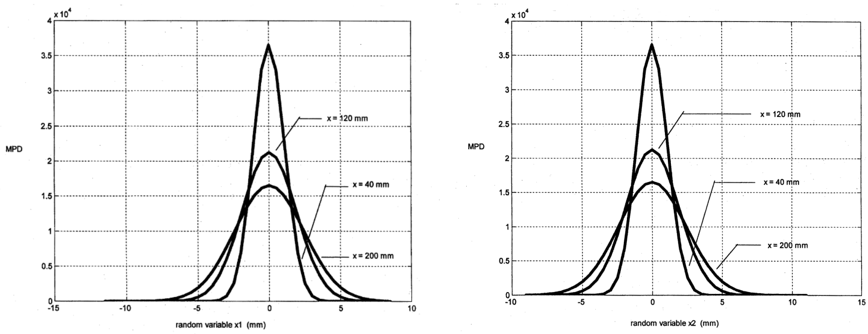
Figure 8. Maximum value of the probabilities of the laser ray direction as function of the propagation distance x.

of x_1 and x_2, the rms of x_1 and x_2, the asymmetric coefficients A_c for x_1 and x_2 and the corresponding flattening coefficients F_c. (For a given random variable a, A_c and F_c are defined as: $A_c = \langle a^3\rangle/\langle a^2\rangle^{3/2}$ and $F_c = \langle a^4\rangle/\langle a^2\rangle^2$).

It is shown in Fig. 8, that the maximum value of the probabilities of the laser beam direction decreases rapidly for $x \leq 20\,\text{mm}$, as the propagation distance x increases. For the values of x situated between $x = 20\,\text{mm}$ and $x = 200\,\text{mm}$, that maximum value continues to decrease, but it decreases slowly. From that, we can conclude that the jet diffusion effect for $x \leq 20\,\text{mm}$ is greater than the same diffusion effect corresponding to the values of x such that $20 \leq x \leq 200\,\text{mm}$.

That diffusion effect created by the thermic turbulence in the jet is particularly seen in the Figures 9a and 9b, where it is shown that the base of the probability density curve for three values of x ($x = 40\,\text{mm}$, $x = 120\,\text{mm}$, $x = 200\,\text{mm}$) increases as the propagation distance x increases, whereas the maximum value of the corresponding probabilities decreases.

At the jet border, the non-normalized marginal probability densities for the random variables x_1 or x_2 are equal to the corresponding Gaussian law. This important result is proved in Fig. 10



(a) (b)

Figure 9. Non-normalized marginal probability densities corresponding to three values of the distance of propagation $x = 40\,\mathrm{mm}$, $x = 120\,\mathrm{mm}$ and $x = 200\,\mathrm{mm}$: (a) For the random variable x_1. (b) For the random variable x_2.

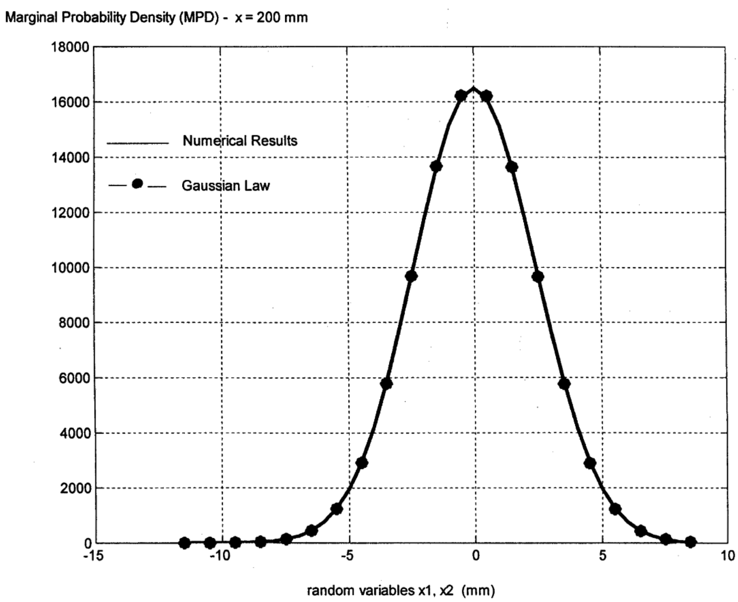
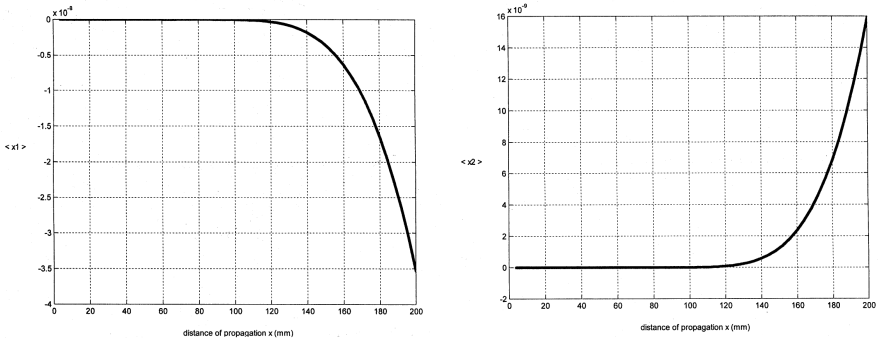


Figure 10. Comparison of the non-normalized marginal probability densities for x_1 at the jet border with the Gaussian law (Numerical results (—) and Gaussian law ($-*-$)).



(a) (b)

Figure 11. Mean values as function of the propagation distance: (a) For x_1. (b) For x_2.

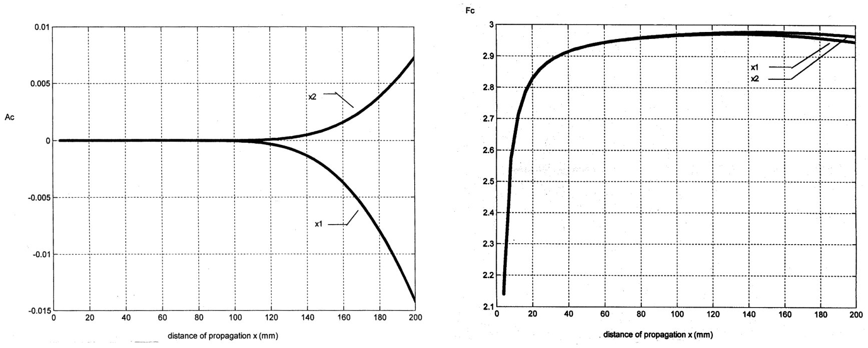
where comparisons between the corresponding numerical results and those derived from the Gaussian law, are carried out.

In Fig. 11a and Fig. 11b, the mean values $\langle x_1 \rangle$ and $\langle x_2 \rangle$ of the random variables x_1 and x_2 are plotted, as function of the propagation distance x. It can be concluded, from these curves that the mean values $\langle x_1 \rangle$ and $\langle x_2 \rangle$ are simultaneously equal to zero for $0 < x \leq 110\,\text{mm}$, that is, for these values, x_1 and x_2 may be considered as centered random variables. When the laser beam undergoes random perturbation during a propagation distance x such that $110\,\text{mm} \leq x \leq 200\,\text{mm}$, the mean value $\langle x_1 \rangle$ remains negative and decreases rapidly as the propagation distance increases, whereas the mean value $\langle x_2 \rangle$ remains positive and increases slowly. These laws concerning the mean values of x_1 and x_2 are the same as the corresponding curves describing the asymmetry coefficients A_c plotted in Fig. 12a.

In Fig. 12b, it is shown that, as the propagation distance increases, the values of the two flattening coefficients F_c for the random variables x_1 and x_2 respectively, remain positive and vary between 2.14 and 2.96, and that, the two curves showing those variations are nearly identical.

We can see in Fig. 13 that, the root mean square (rms) for x_1 and x_2 vary according to the same law, and that the corresponding values are contained between $0.332 \times 10^{-5}\,\text{m}^{-1}$ and $2.375 \times 10^{-5}\,\text{m}^{-1}$.

In Fig. 14, the sensitivity of the probability density of the laser beam directions to the strength of the turbulent diffusion of the jet, is plotted for the values of the propagation distance: $x = 40\,\text{mm}$ and $200\,\text{mm}$. From these curves, it can be concluded that the maximum sensitivity occurs for specified directions symmetrically situated with respect to the initial direction of the laser ray. For those directions,



(a) (b)

Figure 12. (a) Asymmetric coefficients A_c as function of the propagation distance. (b) Flattening coefficients F_c as function of the propagation distance.

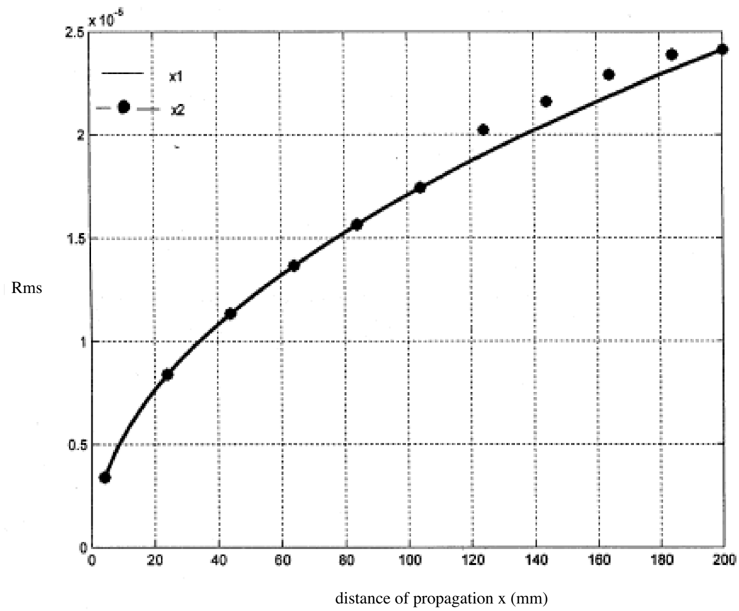
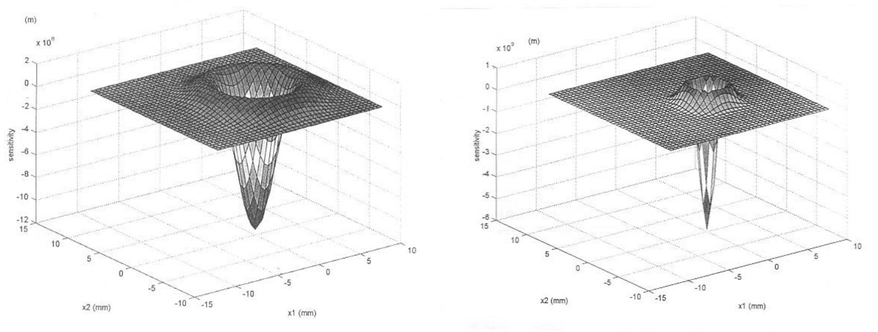


Figure 13. Root mean square of x_1 and x_2 as function of the propagation distance.



(a) (b)

Figure 14. Sensitivity of the probability density of the laser ray to the strength of the jet turbulent diffusion: (a) For $x = 40\,\text{mm}$, (b) For $x = 200\,\text{mm}$.

the probability density increases very much as the turbulent diffusion increases. Consequently, the corresponding maximum of sensitivity dP/dD remains positive. The above effect is caused by the fact that, for absolute values, there exists a highest maximum which occurs when the probability density of the laser ray direction reaches its maximum. For that value, the probability density decreases very much as the turbulent diffusion increases and consequently, the sensitivity dP/dD remains the negative lowest minimum. For the laser ray directions corresponding to the boundaries of the luminous trace produced by the ray on the plane of the photoelectric cell, the turbulent diffusion has no effect and consequently, the quantity dP/dD vanishes for any propagation distance.

6. CONCLUSION

In this paper, the main stochastic characteristics of the random propagation of a laser beam through a heated turbulent jet of air, are determined, using geometrical optics and Markovian process hypothesis, applied along the whole path of the laser beam. The approximation of geometrical optics is proved by the experimental conditions characterizing our experiments and the validity of the Markovian model is justified using the previous work done in [5].

By means of a numerical calculation procedure which uses our experimental results, this paper shows the possibility of obtaining information about random wave propagation from the diffusion coefficient of a turbulent hot jet of air, that is, from the spatial correlation of the refractive index fluctuations in the turbulent jet, without introducing any instrument into the flow.

It is useful to indicate that Chernov [8] has analytically solved the EFPK equation in a general case in which the diffusion coefficient remains constant and does not depends on spatial coordinates. In the present work, we did not use that general solution because we need to create a numerical calculation procedure which is able to take into account our specific experimental conditions related to the jet considered. About that, we can mention that our method of calculation has shown in Fig. 11 the difference in behavior of the mean values of random variables x_1 and x_2, whereas the two random variables are governed by the same marginal probability laws. It has been possible to put in evidence this interesting result because our experiments proved that the intervals of variations of x_1 and x_2 are different in the jet considered.

The ultimate aim is to use the present calculation procedure in order to elaborate from it, an efficient calculation procedure for subsequent works in which the laser beam will be set normal to the jet exhaust, that is, the case in which the diffusion coefficient depends on spatial coordinates. Consequently, we need to be sure that the present calculation procedure, made for constant diffusion coefficient, is valid. About that, Chernov's analytical solution can then be used to prove the validity of our calculation procedure as it is shown in Fig. 10.

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