Progress In Electromagnetics Research, PIER 50, 163–186, 2005

ELECTROMAGNETIC FIELD FOR A HORIZONTAL ELECTRIC DIPOLE BURIED INSIDE A DIELECTRIC LAYER COATED HIGH LOSSY HALF SPACE

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Abstract—In this paper, analytical formulas have been derived for the electromagnetic field generated by a horizontal electric dipole inside high lossy half-space coated with a dielectric layer. This problem is corresponding to the electromagnetic field generated by a horizontal antenna in a submarine under an ice layer, or the measurement of the conductivity of the oceanic lithosphere with a horizontal antenna as the source, and a layer of sediment on the sea floor. These formulas obtained for the electromagnetic field can be employed to calculated the total field including the lateral-wave term and the trapped-surface-wave term. Because the wave number of the trapped-surface-wave term is different from that of the lateral-wave term, the interference appears in the total field. Additionally, this paper has presented the approximative formulas for a thin dielectric layer, which can be used for the communication in low frequencies region.

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1. INTRODUCTION

The electromagnetic wave propagation along the surface of a dielectric-layer coated high lossy half space is an interesting research topic for various practical applications. For example, a dielectric layer is frequently applied to superstrate to cover conducting strip in the integrated microstrip circuits and printed antennas in the frequency range of 1 GHz through 50 GHz, asphalt or cement to coat the ground for VHF communications in the frequency range of 100 MHz through 1000 MHz, and a thick ice layer to coat sea water for communications between two submarines in the frequency range of 30 kHz through 3 MHz at high-latitude regions.

From 1991 to 1994, King and Sandler [1–3] presented a set of analytical formulas about electromagnetic fields generated by a vertical or horizontal electric dipole over a conductor coated with a dielectric layer. In 1998, Wait [4] commented the 1994 King's paper and claimed that the paper had overlooked a trapped-surface-wave term, which varies as $\rho^{-1/2}$ in the far region without detail description. The debates between King and Wait rekindled the interest in the study on the electromagnetic field generated by a vertical electric dipole placed on a planar conductor covered with a dielectric layer coating [6, 8]. In previous works by the authors, it has been found that the electromagnetic field on the surface of the dielectric layer does include both the trapped-surface-wave term and the lateral-wave term, and the trapped-surface-wave varies as $\rho^{-1/2}$ in the horizontal direction and attenuates exponentially in the z direction. Later, further studies have been investigated on the electromagnetic field generated by a horizontal dipole over a dielectric-coated lossy material [9].

In 1986, J. M. Dunn [12] had already discussed the electromagnetic field generated by a horizontal dipole buried inside a high lossy material coated with a dielectric layer, and the problem had been embodied in King's monograph [13].

In this paper, the author analyzes the problem same as Dunn's paper, and finds Dunn had overlooked the trapped surface wave. The author has obtained the completed expressions of the electromagnetic field generated by an unit horizontal electric dipole buried inside a high lossy half-space coated with a dielectric layer. These expressions contain both the trapped surface wave and the lateral wave. And these formulas and computations can be applied to the communications in lower frequencies region.

2. THE INTEGRATED FORMULAS OF THE ELECTROMAGNETIC FIELD

The relevant geometry is illustrated in Fig. 1, where an unit horizontal electric dipole in the x-direction is located at $(0, 0, -d)$. Use is made of the time dependence $e^{-i\omega t}$, and comparing Fig. 1 in this paper with the corresponding one in [1], exchanging k_0 and k_2 and taking

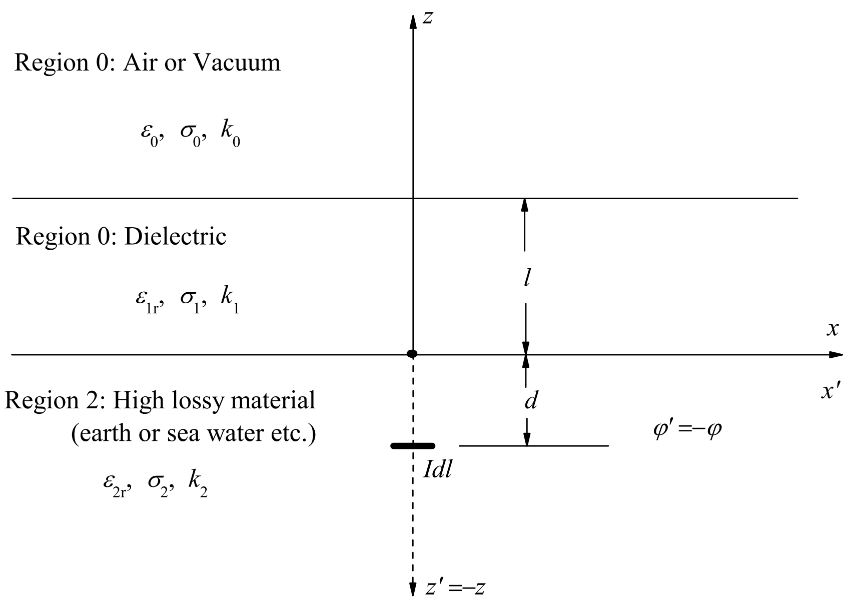

Figure 1. The poles and branches of the integrand in the λ complex plane.

$z' = -z$, $\varphi' = -\varphi$, the integrated formulas of the electromagnetic field are obtained readily.

$$\begin{aligned}
E_{2\rho}(\rho,\varphi',z') &= -\frac{\omega\mu_0}{4\pi k_2^2}\cdot\cos\varphi'[F_{\rho_0}(\rho,z'-d')-F_{\rho_0}(\rho,z'+d')\\
&\quad +F_{\rho_1}(\rho,z'+d')] \qquad (1)\\
E_{2\varphi'}(\rho,\varphi',z') &= \frac{\omega\mu_0}{4\pi k_2^2}\sin\varphi'[F_{\varphi_0'}(\rho,z'-d')-F_{\varphi_0'}(\rho,z'+d')\\
&\quad +F_{\varphi_1'}(\rho,z'+d')] \qquad (2)\\
E_{2z'}(\rho,\varphi',z') &= \frac{i\omega\mu_0}{4\pi k_2^2}\cos\varphi'[F_{z_0'}(\rho,z'-d')-F_{z_0'}(\rho,z'+d')\\
&\quad +F_{z_1'}(\rho,z'+d')] \qquad (3)\\
B_{2\rho}(\rho,\varphi',z') &= -\frac{\mu_0}{4\pi}\sin\varphi'[G_{\rho_0}(\rho,z'-d')-G_{\rho_0}(\rho,z'+d')\\
&\quad +G_{\rho_1}(\rho,z'+d')] \qquad (4)\\
B_{2\varphi'}(\rho,\varphi',z') &= -\frac{\mu_0}{4\pi}\cos\varphi'[G_{\varphi_0'}(\rho,z'-d')-G_{\varphi_0'}(\rho,z'+d')\\
&\quad +G_{\varphi_1'}(\rho,z'+d')] \qquad (5)\\
B_{2z'}(\rho,\varphi',z') &= \frac{i\mu_0}{4\pi}\sin\varphi'[G_{z_0'}(\rho,z'-d')-G_{z_0'}(\rho,z'+d')\\
&\quad +G_{z_1'}(\rho,z'+d')] \qquad (6)
\end{aligned}$$

The first and the second terms in (1) through (6) can be expressed as follows;

$$F_{\rho_0}(\rho,z'\pm d') = -e^{ik_2r}\left[\frac{2k_2}{r^2}+\frac{2i}{r^3}+\left(\frac{z'\pm d'}{r}\right)^2\left(\frac{ik_2^2}{r}-\frac{3k_2}{r^2}-\frac{3i}{r^3}\right)\right] \quad (7)$$

$$F_{\varphi_0'}(\rho,z'\pm d') = -e^{ik_2r}\left(\frac{ik_2^2}{r}-\frac{k_2}{r^2}-\frac{i}{r^3}\right) \quad (8)$$

$$F_{z_0'}(\rho,z'\pm d') = -e^{ik_2r}\left(\frac{\rho}{r}\right)\left(\frac{z'\pm d'}{r}\right)\left(\frac{k_2^2}{r}+\frac{3ik_2}{r^2}-\frac{3}{r^3}\right) \quad (9)$$

$$\begin{matrix} G_{\rho_0}(\rho,z'\pm d') \\ G_{\varphi_0'}(\rho,z'\pm d') \end{matrix} = -e^{ik_2r}\left(\frac{z'\pm d'}{r}\right)\left(\frac{ik_2}{r}-\frac{1}{r^2}\right) \quad (10)$$

$$G_{z_0'}(\rho,z'\pm d') = -e^{ik_2r}\left(\frac{\rho}{r}\right)\left(\frac{k_2}{r}+\frac{i}{r^2}\right) \quad (11)$$

In the above formulas, when using "$-$", $r = r_1 = \sqrt{\rho^2+(z'-d')^2}$, and when using "$+$", $r = r_2 = \sqrt{\rho^2+(z'+d')^2}$. Here, r_1 is the path length of the direct wave, r_2 is the path length of the reflected wave. At large distances between the source and the observation point, because of the high attenuation, both the direct and the reflected waves in region 2 are negligible.

The third terms in (1) and (2) can be expressed as

$$F_{\rho_1}(\rho, z'+d') = F_{\rho_2}(\rho, z'+d') + F_{\rho_3}(\rho, z'+d') \tag{12}$$

$$F_{\varphi'_1}(\rho, z'+d') = F_{\varphi'_2}(\rho, z'+d') + F_{\varphi'_3}(\rho, z'+d') \tag{13}$$

where

$$\begin{matrix} F_{\rho_2}(\rho, z'+d') \\ F_{\varphi'_2}(\rho, z'+d') \end{matrix} = \frac{1}{2}\int_0^\infty \gamma_2(Q_3+1)[J_0(\lambda\rho) \mp J_2(\lambda\rho)] e^{i\gamma_2(z'+d')} \cdot \lambda d\lambda \tag{14}$$

$$\begin{matrix} F_{\rho_3}(\rho, z'+d') \\ F_{\varphi'_3}(\rho, z'+d') \end{matrix} = -\frac{k_2^2}{2}\int_0^\infty \gamma_2^{-1}(P_3-1)[J_0(\lambda\rho) \pm J_2(\lambda\rho)] e^{i\gamma_2(z'+d')} \cdot \lambda d\lambda \tag{15}$$

The third term in (3) can be expressed as

$$F_{z'_1}(\rho, z'+d') = \int_0^\infty (Q_3+1) J_1(\lambda\rho) e^{i\gamma_2(z'+d')} \cdot \lambda^2 d\lambda \tag{16}$$

Similarly,

$$G_{\rho_1}(\rho, z'+d') = G_{\rho_2}(\rho, z'+d') + G_{\rho_3}(\rho, z'+d') \tag{17}$$

$$G_{\varphi'_1}(\rho, z'+d') = G_{\varphi'_2}(\rho, z'+d') + G_{\varphi'_3}(\rho, z'+d') \tag{18}$$

where

$$\begin{matrix} G_{\rho_2}(\rho, z'+d') \\ G_{\varphi'_2}(\rho, z'+d') \end{matrix} = \frac{1}{2}\int_0^\infty (Q_3+1)[J_0(\lambda\rho) \pm J_2(\lambda\rho)] e^{i\gamma_2(z'+d')} \cdot \lambda d\lambda \tag{19}$$

$$\begin{matrix} G_{\rho_3}(\rho, z'+d') \\ G_{\varphi'_3}(\rho, z'+d') \end{matrix} = -\frac{1}{2}\int_0^\infty (P_3-1)[J_0(\lambda\rho) \mp J_2(\lambda\rho)] e^{i\gamma_2(z'+d')} \cdot \lambda d\lambda \tag{20}$$

$$G_{z'_1}(\rho, z'+d') = -\int_0^\infty (P_3-1)\gamma_2^{-1} J_1(\lambda\rho) e^{i\gamma_2(z'+d')} \cdot \lambda^2 d\lambda \tag{21}$$

where Q_3 and P_3 are the reflected coefficients of the electric-type wave and the magnetic-type wave, respectively. They are

$$\frac{\gamma_2}{2}(Q_3+1) = \frac{\frac{k_2^2\gamma_2}{k_1^2}\left(\frac{k_1^2\gamma_0}{k_0^2} - i\gamma_1\tan\gamma_1 l\right)}{\gamma_2 + \frac{k_2^2\gamma_0}{k_0^2} - i\left(\frac{k_2^2\gamma_1}{k_1^2} + \frac{k_1^2\gamma_0\gamma_2}{k_0^2\gamma_1}\right)\tan\gamma_1 l} \tag{22}$$

$$\frac{k_2^2}{2\gamma_2}(P_3-1) = -\frac{k_2^2\left(\frac{\gamma_1}{\gamma_0} - i\tan\gamma_1 l\right)}{\gamma_1 + \frac{\gamma_1\gamma_2}{\gamma_0} - i\left(\gamma_2 + \frac{\gamma_1^2}{\gamma_0}\right)\tan\gamma_1 l} \tag{23}$$

where

$$\gamma_m = \sqrt{k_m^2 - \lambda^2}, \quad m = 0, 1, 2 \tag{24}$$

In the next step, it is necessary to evaluate the above Sommerfeld integrals.

3. EVALUATION FOR THE ELECTRIC-TYPE TERMS OF THE ELECTROMAGNETIC FIELD

The terms in (14), (16) and (19) including the factor (Q_3+1), which depends on the coefficient of the electric-type wave, are defined by the electric-type terms. Similarly, the terms in (15), (20) and (21) including the factor (P_3-1), which depends on the coefficient of the magnetic-type wave, are defined by the magnetic-type terms. Substituting (22) into (14), the following expressions are obtained readily.

$$\begin{array}{c} F_{\rho 2}(\rho, z'+d') \\ F_{\varphi 2}(\rho, z'+d') \end{array} = \int_0^\infty \frac{\left(\frac{\gamma_0}{k_0^2} - i\frac{\gamma_1}{k_1^2}\tan\gamma_1 l\right)\gamma_2}{\frac{\gamma_2}{k_2^2} + \frac{\gamma_0}{k_0^2} - i\left(\frac{\gamma_1}{k_1^2} + \frac{k_1^2\gamma_0\gamma_2}{k_0^2k_2^2\gamma_1}\right)\tan\gamma_1 l} \cdot e^{i\gamma_2(z'+d')}\left[J_0(\lambda\rho) \mp J_2(\lambda\rho)\right]\lambda d\lambda \tag{25}$$

Taking into account the following relations

$$J_n(\lambda\rho) = \frac{1}{2}\left[H_n^{(1)}(\lambda\rho) + H_n^{(2)}(\lambda\rho)\right] \tag{26}$$

$$H_n^{(1)}(-\lambda\rho) = (-1)^{n+1}\mathrm{H}_n^{(2)}(\lambda\rho) \tag{27}$$

(25) can be re-written as

$$\begin{aligned} \begin{matrix} F_{\rho 2}(\rho, z'+d') \\ F_{\varphi_2'}(\rho, z'+d') \end{matrix} &= \frac{1}{2}\int_{-\infty}^{\infty} \frac{\left(\frac{\gamma_0}{k_0^2} - i\frac{\gamma_1}{k_1^2}\tan\gamma_1 l\right)\gamma_2}{q(\lambda)} \\ &\quad \cdot e^{i\gamma_2(z'+d')}\left[H_0^{(1)}(\lambda\rho) \mp H_2^{(1)}(\lambda\rho)\right]\lambda d\lambda \end{aligned} \tag{28}$$

where

$$q(\lambda) = \frac{\gamma_2}{k_2^2} + \frac{\gamma_0}{k_0^2} - i\left(\frac{\gamma_1}{k_1^2} + \frac{k_1^2\gamma_0\gamma_2}{k_0^2 k_2^2\gamma_1}\right)\tan\gamma_1 l \tag{29}$$

In order to evaluate the Sommerfeld integrals, the poles of the integrand should be considered. The integrand of (28) has the poles λ_j to satisfy the equation $q(\lambda) = 0$. When k_2 is much larger than k_1 and k_0, using the limit of $k_2 \to \infty$, we can convert the equation $q(\lambda) = 0$ to

$$q^*(\lambda) = \frac{\gamma_0}{k_0^2} - i\frac{\gamma_1}{k_1^2}\tan\gamma_1 l = 0 \tag{30}$$

Equation (30) has been discussed in [6], when the thickness l of the dielectric layer satisfy $(n-1)\pi < \sqrt{k_1^2 - k_0^2}l < n\pi$, equation (30) has n roots on the up-half space of the plane of λ. If $k_2 \neq \infty$, and is much larger than k_1 and k_0, Following the manner addressed in [6, 8], under the same condition, (29) still has n roots and may be solved by Newton method numerically.

In addition, the integrand has two branch points at $\lambda = k_0$ and $\lambda = k_2$, the branch cuts and the poles are taken on the λ complex plane as shown in Fig. 2.

On both sides of the branch cut Γ_0, the phase of Γ_0 will change to 180°, but Γ_2 remains the same value, and vice versa for the branch cut Γ_2. Under these conditions, (28) may be re-written as

$$\begin{aligned} \begin{matrix} F_{\rho 2}(\rho, z'+d') \\ F_{\varphi 2}(\rho, z'+d') \end{matrix} &= \pi i \sum_j \frac{\gamma_2^*\left(\frac{\gamma_0^*}{k_0^2} - \frac{i\gamma_1^*}{k_1^2}\tan\gamma_1^* l\right)}{q'(\lambda_j)} \\ &\quad \cdot e^{i\gamma_2^*(z'+d')}\lambda_j\left[H_0^{(1)}(\lambda_j\rho) \mp H_2^{(1)}(\lambda_j\rho)\right] \\ &\quad + \frac{1}{2}\int_{\Gamma_0+\Gamma_2} \frac{\gamma_2\left(\frac{\gamma_0}{k_0^2} - \frac{i\gamma_1}{k_1^2}\tan\gamma_1 l\right)}{q(\lambda)} \\ &\quad \cdot e^{i\gamma_2(z'+d')}\lambda\left[H_0^{(1)}(\lambda\rho) \mp H_2^{(1)}(\lambda\rho)\right] d\lambda \end{aligned} \tag{31}$$

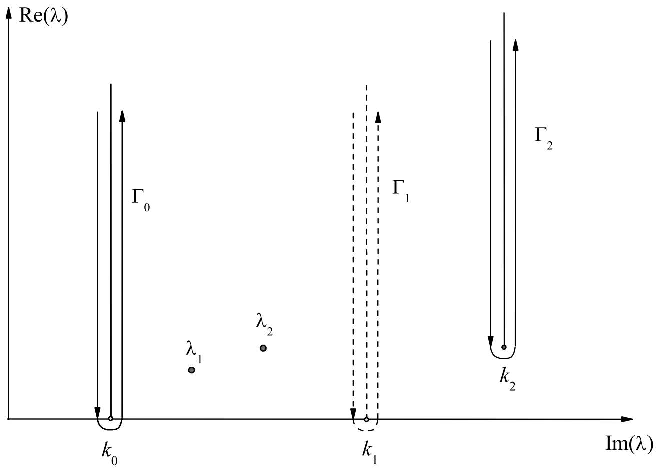

Figure 2. The poles and branches of the integrand in the λ complex plane.

where

$$\begin{aligned} q'(\lambda_j) &= -\frac{\lambda_j}{k_2^2\gamma_2^*} - \frac{\lambda_j}{k_0^2\gamma_0^*} - i\tan\gamma_1^* l \\ &\quad \cdot \left[-\frac{\lambda_j}{k_1^2\gamma_1^*}\frac{k_1^2}{k_0^2k_2^2}\left(-\frac{\lambda_j\gamma_2^*}{\gamma_1^*\gamma_0^*} - \frac{\lambda_j\gamma_0^*}{\gamma_1^*\gamma_2^*} + \frac{\gamma_0^*\gamma_2^*\lambda_j}{\gamma_1^{*3}}\right)\right] \\ &\quad + i\sec^2\gamma_1^* l \cdot \left(\frac{\gamma_1^*}{k_1^2} + \frac{k_1^2\gamma_0^*\gamma_2^*}{k_0^2k_2^2\gamma_1^*}\right)\frac{\lambda_j l}{\gamma_1^*} \end{aligned} \tag{32}$$

$$\gamma_m^* = \sqrt{k_m^2 - \lambda_j^2} \qquad m = 0, 1, 2 \tag{33}$$

The first term in (31) is the sum of residues at the poles, if the thickness l of the dielectric layer satisfy $(n-1)\pi < \sqrt{k_1^2 - k_0^2}l < n\pi$, the integrand should have n poles. On both sides of the branch cut Γ_0, let

$$\lambda = k_0(1 + i\tau^2) \tag{34}$$

thus,

$$H_n^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_0\rho}}\exp\left[i\left(k_0\rho - \frac{n\pi}{2} - \frac{\pi}{4}\right) - k_0\rho\tau^2\right] \tag{35}$$

If $k_0\rho \gg 1$, it may be seen that the dominant contribution of the integral along Γ_0 comes from the neighborhood of k_0, and following approximation may be taken

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0 e^{i\frac{3\pi}{4}} \sqrt{2} \cdot \tau \tag{36}$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \gamma_1' = \sqrt{k_1^2 - k_0^2} \tag{37}$$

$$\gamma_2 = \sqrt{k_2^2 - \lambda^2} \approx \gamma_2' = \sqrt{k_2^2 - k_0^2} \tag{38}$$

Substituting (36)–(38) into the integral (31) along the branch cut Γ_0, following expressions may be obtained

$$\begin{aligned}
&\frac{1}{2}\int_{\Gamma_0} \frac{\gamma_2\left(\frac{\gamma_0}{k_0^2} - i\frac{\gamma_1}{k_1^2}\tan\gamma_1 l\right)}{q(\lambda)} \cdot e^{i\gamma_2(z'+d')}\left[H_0^{(1)}(\lambda\rho) \mp H_{(2)}^{(1)}(\lambda\rho)\right] \cdot \lambda \cdot d\lambda \\
&\quad = 2k_0^2\sqrt{\frac{2}{\pi k_0 \rho}}\exp\left[ik_0\rho + i\gamma_2'(z'+d') + i\frac{\pi}{4}\right] \cdot \frac{\gamma_2'}{1 - i\frac{k_1^2 \gamma_2{}'}{k_2^2\gamma_1{}'}\tan\gamma_1{}' l} \\
&\quad \cdot(-A + e^{i\frac{\pi}{4}}\Delta)\left(\sqrt{\frac{\pi}{k_0\rho}} + \pi i\sqrt{2}\Delta e^{-iP^*}F(P^*)\right)\left\{\begin{array}{c} 1 \\ -\frac{i}{k_0\rho} \end{array}\right\}
\end{aligned} \tag{39}$$

where

$$\Delta = \frac{\frac{k_0\gamma_2'}{k_2^2} - i\frac{k_0\gamma_1'}{k_1^2}\tan\gamma_1' l}{\sqrt{2}\left(1 - i\frac{k_1^2\gamma_2'}{k_2^2\gamma_1'}\tan\gamma_1' l\right)} \tag{40}$$

$$A = e^{-i\frac{\pi}{4}}\frac{k_0\gamma_1'}{\sqrt{2}k_1^2}\tan\gamma_1' l \tag{41}$$

$$P^* = k_0\rho\Delta^2 \tag{42}$$

$$F(P^*) = \frac{1}{2}(1+i) + \int_0^{P^*}\frac{e^{it}}{\sqrt{2\pi t}}dt \tag{43}$$

$F(P^*)$ is Fresnel integral.

Since Region 2 is a high lossy medium, so the integral along the branch cut Γ_2 can be neglected. Therefore, (31) may be simplified as

$$\begin{array}{l} F_{\rho 2}(\rho, z'+d') \\ F_{\varphi_2'}(\rho, z'+d') \end{array} = \pi i \sum_j \frac{\gamma_2^*\left(\frac{\gamma_0^*}{k_0^2} - \frac{i\gamma_1^*}{k_1^2}\tan\gamma_1^* l\right)}{q'(\lambda_j)} e^{i\gamma_2^*(z'+d')}\lambda_j$$

$$\cdot\left[H_0^{(1)}(\lambda_j\rho)\mp H_2^{(1)}(\lambda_j\rho)\right]-\frac{2k_0^2(A-e^{i\frac{\pi}{4}}\Delta)\gamma_2'}{1-i\dfrac{k_1^2\gamma_2'}{k_2^2\gamma_1'}\tan\gamma_1' l}$$
$$\cdot\sqrt{\frac{2}{\pi k_0\rho}}\exp\left[ik_0\rho+i\gamma_2'(z'+d')+i\frac{\pi}{4}\right]\begin{Bmatrix}1\\-\frac{i}{k_0\rho}\end{Bmatrix}$$
$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+\sqrt{2}\cdot\pi\cdot i\cdot\Delta\cdot e^{-iP^*}F(P^*)\right]\tag{44}$$

Apparently, in (44), the first term is the trapped-surface-wave term, and the second one is the lateral-wave term. Similarly, $F_{z_1'}(\rho,z'+d')$ and $G_{\rho_2}(\rho,z'+d')$ may be re-written as

$$\begin{aligned}F_{z_1'}(\rho,z'+d') &= \int_0^\infty(Q_3+1)J_1(\lambda\rho)e^{i\gamma_2(z'+d')}\cdot\lambda^2\cdot d\lambda\\&= 2\pi i\sum_j\frac{\dfrac{\gamma_0^*}{k_0^2}-\dfrac{i\gamma_1^*}{k_1^2}\tan\gamma_1^* l}{q'(\lambda_j)}e^{i\gamma_2^*(z'+d')}\lambda_j^2H_1^{(1)}(\lambda_j\rho)\\&\quad+\frac{2k_0^3\left(A-e^{i\frac{\pi}{4}}\Delta\right)}{1-i\dfrac{k_1^2\gamma_2'}{k_2^2\gamma_1'}\tan\gamma_1' l}\cdot\exp\left[ik_0\rho+i\gamma_2'(z'+d')+i\frac{\pi}{4}\right]\\&\quad\cdot\sqrt{\frac{2}{\pi k_0\rho}}\left[\sqrt{\frac{\pi}{k_0\rho}}+\sqrt{2}i\pi\cdot\Delta\cdot e^{-iP^*}F(P^*)\right]\end{aligned}\tag{45}$$

$$\begin{aligned}\begin{matrix}G_{\rho_2}(\rho,z'+d')\\G_{\varphi_2'}(\rho,z'+d')\end{matrix} &= i\pi\sum_j\frac{\dfrac{\gamma_0^*}{k_0^2}-\dfrac{i\gamma_1^*}{k_1^2}\tan\gamma_1^* l}{q'(\lambda_j)}\left[H_0^{(1)}(\lambda_j\rho)\pm H_2^{(1)}(\lambda_j\rho)\right]\\&\quad\cdot e^{i\gamma_2^*(z'+d')}\lambda_j-\frac{2k_0^2(A-e^{i\frac{\pi}{4}}\Delta)}{1-i\dfrac{k_1^2\gamma_2'}{k_2^2\gamma_1'}\tan\gamma_1' l}\sqrt{\frac{2}{\pi k_0\rho}}\\&\quad\cdot\exp\left[ik_0\rho+i\gamma_2'(z'+d')+i\frac{\pi}{4}\right]\begin{Bmatrix}-\frac{i}{k_0\rho}\\1\end{Bmatrix}\\&\quad\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+\sqrt{2}i\pi\cdot\Delta\cdot e^{-iP^*}F(P^*)\right]\end{aligned}\tag{46}$$

4. EVALUATION OF THE TERMS FOR THE MAGNETIC-TYPE WAVE

Substituting (23) into (15), the following expressions may be obtained

$$\begin{matrix} F_{\rho 3}(\rho, z'+d') \\ F_{\varphi_3'}(\rho, z'+d') \end{matrix} = \int_0^\infty \frac{k_2^2\left(\frac{\gamma_1}{\gamma_0} - i\tan\gamma_1 l\right)}{\gamma_2 p(\lambda)} e^{i\gamma_2(z'+d')}[J_0(\lambda\rho) \pm J_2(\lambda\rho)]\lambda d\lambda \tag{47}$$

where

$$p(\lambda) = \frac{\gamma_1}{\gamma_0} + \frac{\gamma_1}{\gamma_2} - i\left(1 + \frac{\gamma_1^2}{\gamma_0\gamma_2}\right)\tan\gamma_1 l \tag{48}$$

The electromagnetic field due to a horizontal electric dipole over a lossless half-space coated with a dielectric layer have been addressed in [9]. When $k_2 \to \infty$, (48) has been simplified as (27) as shown in [9]. It has been pointed out that when the thickness l of the dielectric layer satisfies the condition $(n - \frac{1}{2})\pi \leq \sqrt{k_1^2 - k_0^2} l \leq (n + \frac{1}{2})\pi$, equation $p(\lambda) = 0$ has n roots. If $k_2 \neq \infty$ and k_2 is much larger than k_0 and k_1, equation $p(\lambda) = 0$ still has n roots, and the roots $\lambda_j (j = 1, 2.....n)$ can be solved numerically by Newton method.

Therefore, (47) can be re-written as

$$\begin{aligned} \begin{matrix} F_{\rho 3}(\rho, z'+d') \\ F_{\varphi_3'}(\rho, z'+d') \end{matrix} &= \pi i \sum_j \frac{k_2^2\left(\frac{\gamma_1^*}{\gamma_0^*} - i\tan\gamma_1^* l\right)}{\gamma_2^* p'(\lambda_j)} e^{i\gamma_2^*(z'+d')}\lambda_j \\ &\quad \cdot \left[H_0^{(1)}(\lambda_j\rho) \pm H_2^{(1)}(\lambda_j\rho)\right] \\ &\quad + \frac{1}{2}\int_{\Gamma_0} \frac{k_2^2\left(\frac{\gamma_1}{\gamma_0} - i\tan\gamma_1 l\right)}{\gamma_2 p(\lambda)} e^{i\gamma_2(z'+d')} \\ &\quad \cdot \left[H_0^{(1)}(\lambda\rho) \pm H_2^{(1)}(\lambda\rho)\right]\lambda d\lambda \end{aligned} \tag{49}$$

where

$$\begin{aligned} p'(\lambda) &= \lambda\gamma_1\left(\frac{1}{\gamma_0^3} + \frac{1}{\gamma_2^3}\right) - \frac{\lambda}{\gamma_1}\left(\frac{1}{\gamma_0} + \frac{1}{\gamma_2}\right) - i\frac{\lambda\tan\gamma_1 l}{\gamma_0\gamma_2}\left(\frac{\gamma_1^2}{\gamma_2^2} + \frac{\gamma_1^2}{\gamma_0^2} - 2\right) \\ &\quad + i\frac{\lambda l}{\gamma_1}\sec^2\gamma_1 l\left(1 + \frac{\gamma_1^2}{\gamma_0\gamma_2}\right) \end{aligned} \tag{50}$$

Since the dominant contribution of the integral along the branch cut Γ_0 comes from the neighborhood of k_0, the second term in (49) may be simplified as

$$\begin{aligned}
&\frac{1}{2}\int\limits_{\Gamma_0} \frac{k_2^2\left(\dfrac{\gamma_1}{\gamma_0} - i\tan\gamma_1 l\right)}{\gamma_2 p(\lambda)} e^{i\gamma_2(z'+d')}\left[H_0^{(1)}(\lambda\rho) \pm H_2^{(1)}(\lambda\rho)\right]\lambda d\lambda \\
&\quad = \frac{2k_0^2k_2^2\tan\gamma_1' l}{\gamma_2'\left(\dfrac{\gamma_1'}{\gamma_2'} - i\tan\gamma_1' l\right)}\cdot\sqrt{\frac{2}{\pi k_0\rho}}\exp\left[ik_0\rho + i\gamma_2'(z'+d') - i\frac{\pi}{4}\right] \\
&\quad \cdot\left\{\begin{matrix} -\frac{i}{k_0\rho} \\ 1 \end{matrix}\right\}\cdot\left(B + e^{i\frac{\pi}{4}}\cdot\Delta'\right)\cdot\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\Delta' e^{-iP_2}F(P_2)\right] \qquad (51)
\end{aligned}$$

where

$$\Delta' = \frac{\gamma_1' - i\dfrac{\gamma_1'^2}{\gamma_2'}\tan\gamma_1' l}{\sqrt{2}k_0\left(\dfrac{\gamma_1'}{\gamma_2'} - i\tan\gamma_1' l\right)} \qquad (52)$$

$$B = e^{-i\frac{\pi}{4}}\frac{\gamma_1'}{k_0\sqrt{2}\tan\gamma_1' l} \qquad (53)$$

$$P_2 = k_0\rho\Delta'^2 \qquad (54)$$

thus,

$$\begin{aligned}
\begin{matrix} F_{\rho_3}(\rho, z'+d') \\ F_{\varphi_3'}(\rho, z'+d') \end{matrix} &= \pi i\sum_j \frac{k_2^2\left(\dfrac{\gamma_1^*}{\gamma_0^*} - i\tan\gamma_1^* l\right)}{\gamma_2^* p'(\lambda_j)} e^{i\gamma_2^*(z'+d')}\lambda_j \\
&\quad\cdot\left[H_0^{(1)}(\lambda_j\rho) \pm H_2^{(1)}(\lambda_j\rho)\right] \\
&\quad + \frac{2k_0^2k_2^2\tan\gamma_1' l(B + e^{i\frac{\pi}{4}}\Delta')}{\gamma_2'\left(\dfrac{\gamma_1'}{\gamma_2'} - i\tan\gamma_1' l\right)} e^{ik_0\rho + i\gamma_2'(z'+d') - i\frac{\pi}{4}}\sqrt{\frac{2}{\pi k_0\rho}} \\
&\quad\cdot\left\{\begin{matrix} -\frac{i}{k_0\rho} \\ 1 \end{matrix}\right\}\left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\Delta' e^{-iP_2}F(P_2)\right] \qquad (55)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\begin{matrix} G_{\rho 3}(\rho, z'+d') \\ G_{\varphi_3'}(\rho, z'+d') \end{matrix} &= \pi i \sum_j \frac{\dfrac{\gamma_1^*}{\gamma_0^*} - i\tan\gamma_1^* l}{p'(\lambda_j)} e^{i\gamma_2^*(z'+d')}\lambda_j \\
&\quad \cdot \left[H_0^{(1)}(\lambda_j\rho) \mp H_2^{(1)}(\lambda_j\rho)\right] \\
&\quad + \frac{2k_0^2 \tan\gamma_1' l (B + e^{i\frac{\pi}{4}}\Delta')}{\left(\dfrac{\gamma_1'}{\gamma_2'} - i\tan\gamma_1' l\right)} \sqrt{\frac{2}{\pi k_0\rho}} e^{ik_0\rho + i\gamma_2'(z'+d') - i\frac{\pi}{4}} \\
&\quad \cdot \left\{ \begin{matrix} 1 \\ -\frac{i}{k_0\rho} \end{matrix} \right\} \left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\Delta' e^{-iP_2} F(P_2)\right] \qquad (56)
\end{aligned}$$

and

$$\begin{aligned}
G_{z_1'}(\rho, z'+d') &= \pi i \sum_j \frac{\dfrac{\gamma_1^*}{\gamma_0^*} - i\tan\gamma_1^* l}{\gamma_2^* p'(\lambda_j)} e^{i\gamma_2^*(z'+d')}\lambda_j^2 H_1^{(1)}(\lambda_j\rho) \\
&\quad - \frac{2k_0^3 \tan\gamma_1' l (B + e^{i\frac{\pi}{4}}\Delta')}{\gamma_2'\left(\dfrac{\gamma_1'}{\gamma_2'} - i\tan\gamma_1' l\right)} \cdot \sqrt{\frac{2}{\pi k_0\rho}} e^{ik_0\rho + i\gamma_2'(z'+d') - i\frac{\pi}{4}} \\
&\quad \cdot \left[\sqrt{\frac{\pi}{k_0\rho}} + i\sqrt{2}\pi\Delta' e^{-iP_2} F(P_2)\right] \qquad (57)
\end{aligned}$$

5. APPROXIMATION OF THE ELECTROMAGNETIC FIELD FOR THIN DIELECTRIC LAYER

Taking into account that two submarines are buried inside in the sea water covered with a layer of ice, and they communicate the message with the horizontal antenna each other. The relevant geometry of this problem is illustrated in Fig. 3. In this problem, the complex dielectric constants of there three kinds of medium satisfy the condition of $|k_0^2| \ll |k_1^2| \ll |k_2^2|$ and the thickness l of the dielectric layer satisfies the relation $|k_1^2 l^2| \ll 1$, the equation $q(\lambda) = 0$ has only one root λ_1, and equation $p(\lambda) = 0$ has no root. In this case, (40) and (41) may be simplified as

$$\Delta \approx \frac{\dfrac{k_0}{k_2} - ik_0 l}{\sqrt{2}} \ll 1 \qquad (58)$$

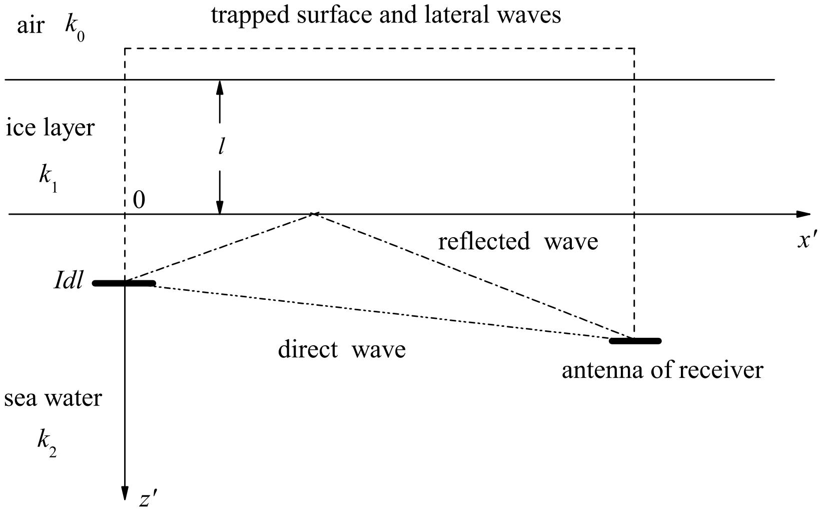

Figure 3. The geometry for two submarines are buried inside in the sea water covered with a layer of ice.

$$A \approx \frac{e^{-i\frac{\pi}{4}} k_0 l}{\sqrt{2}} \ll 1 \tag{59}$$

(52) and (53) may be approximated as

$$\Delta' \approx \frac{k_2}{\sqrt{2}k_0(1 - ik_2 l)} \gg 1 \tag{60}$$

$$B \approx \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}k_0 l} \gg 1 \tag{61}$$

So that the terms of the electric-type wave may be expressed as

$$\begin{aligned} \begin{matrix} F_{\rho_2}(\rho, z'+d') \\ F_{\varphi_2'}(\rho, z'+d') \end{matrix} &= \pi i \frac{\left(\frac{\gamma_0^*}{k_0^2} - i\frac{{\gamma_1^*}^2 l}{k_1^2}\right)\gamma_2^*}{q'(\lambda_1)} e^{i\gamma_2^*(z'+d')}\lambda_1 \\ &\quad \cdot \left[H_0^{(1)}(\lambda_1\rho) \mp H_2^{(1)}(\lambda_1\rho)\right] \\ &\quad + 2k_0^3\sqrt{\frac{1}{\pi k_0 \rho}}\, e^{ik_0\rho + ik_2(z'+d') + i\frac{\pi}{2}} \left\{ \begin{matrix} 1 \\ -\frac{i}{k_0\rho} \end{matrix} \right\} \\ &\quad \cdot \left[\sqrt{\frac{\pi}{k_0\rho}} + \pi i\left(\frac{k_0}{k_2} - ik_0 l\right) e^{-iP^*} F(P^*)\right] \end{aligned} \tag{62}$$

$$F_{z_1'}(\rho, z'+d') = 2\pi i \cdot \frac{\dfrac{\gamma_0^*}{k_0^2} - i\dfrac{\gamma_1^{*2} l}{k_1^2}}{q'(\lambda_1)} H_1^{(1)}(\lambda_1\rho)\lambda_1^2 e^{i\gamma_2^*(z'+d')} - 2\frac{k_0^4}{k_2}\sqrt{\frac{1}{\pi k_0 \rho}}\, e^{ik_0\rho + ik_2'(z'+d') + i\frac{\pi}{2}} \cdot \left[\sqrt{\frac{\pi}{k_0\rho}} + \pi i \left(\frac{k_0}{k_2} - ik_0 l\right) e^{-iP^*} F(P^*)\right] \quad (63)$$

$$\begin{matrix} G_{\rho_2}(\rho, z'+d') \\ G_{\varphi_2'}(\rho, z'+d') \end{matrix} = \pi i \frac{\dfrac{\gamma_0^*}{k_0^2} - i\dfrac{\gamma_1^* l}{k_1^2}}{q'(\lambda_1)}\, e^{i\gamma_2^*(z'+d')}\lambda_1 \cdot \left[H_0^{(1)}(\lambda_1\rho) \pm H_2^{(1)}(\lambda_1\rho)\right] + 2\frac{k_0^3}{k_2}\sqrt{\frac{1}{\pi k_0\rho}} e^{ik_0\rho + ik_2'(z'+d') + i\frac{\pi}{2}} \begin{Bmatrix} -\frac{i}{k_0\rho} \\ 1 \end{Bmatrix} \cdot \left[\sqrt{\frac{\pi}{k_0\rho}} + \pi i \left(\frac{k_0}{k_2} - ik_0 l\right) e^{-iP^*} F(P^*)\right] \quad (64)$$

Since $P_2 = k_0\rho\Delta'^2 \gg 1$, the function $\mathrm{erfc}(\sqrt{x})$ may be approximated as

$$\mathrm{erfc}(\sqrt{x}) = \frac{1}{\pi} e^{-x} \cdot \sum_{k=0}^{3} \frac{(-1)^k \Gamma\left(k + \frac{1}{2}\right)}{x^{k+1/2}} + R_n \quad (65)$$

The lateral wave term of (55) may be simplified as

$$\left[\sqrt{\frac{\pi}{k_0\rho}} + \pi i\sqrt{2} \cdot \Delta' \cdot e^{-iP^2} F(P_2)\right] \approx \sqrt{\frac{\pi}{k_0\rho}} \left[\frac{-ik_0^2}{k_2^2}\frac{(1-ik_2 l)^2}{k_0\rho} + \frac{3k_0^4}{k_2^4}\frac{(1-ik_2 l)^4}{(k_0\rho)^2} + \cdots\right] \quad (66)$$

thus,

$$\begin{matrix} F_{\rho_3}(\rho, z'+d') \\ F_{\varphi_3'}(\rho, z'+d') \end{matrix} = \frac{2k_0^4}{k_1}\frac{1}{k_0\rho} e^{ik_0\rho + ik_2(z'+d') - i\frac{\pi}{2}} \cdot \begin{Bmatrix} -\frac{i}{k_0\rho} \\ 1 \end{Bmatrix} \cdot \left[\frac{1}{k_0\rho} + \frac{3i(1-ik_2 l)^2}{k_2^2\rho^2}\right] \quad (67)$$

$$\begin{matrix} G_{\rho_3}(\rho, z'+d') \\ G_{\varphi_3'}(\rho, z'+d') \end{matrix} = \frac{2k_0^4}{k_1 k_2}\frac{1}{k_0\rho} e^{ik_0\rho + ik_2(z'+d') - i\frac{\pi}{2}} \cdot \begin{Bmatrix} 1 \\ -\frac{i}{k_0\rho} \end{Bmatrix}$$

$$\cdot\left[\frac{1}{k_0\rho}+\frac{3i(1-ik_2l)^2}{k_2^2\rho^2}\right] \tag{68}$$

$$G_{z_1'}(\rho,z'+d') = -\frac{2k_0^5}{k_1k_2^2}\frac{1}{k_0\rho}e^{ik_0\rho+ik_2(z'+d')-i\frac{\pi}{2}} \cdot\left[\frac{1}{k_0\rho}+\frac{3i(1-ik_2l)^2}{k_2^2\rho^2}\right] \tag{69}$$

Substituting (62)–(64) and (67)–(69) into (1)–(6), neglecting the direct wave and the reflected wave in the conductor, the final expressions of the electromagnetic field for the thin dielectric layer are

$$\begin{aligned}
E_{2\rho}(\rho,\varphi',z') &= -\frac{\omega\mu_0}{4\pi k_2^2}\cos\varphi'\left\{\pi i\frac{\left(\frac{\gamma_0^*}{k_0^2}-i\frac{\gamma_1^{*2}l}{k_1^2}\right)\gamma_2^*}{q'(\lambda_1)}e^{i\gamma_2^*(z'+d')}\cdot\lambda_1\right.\\
&\quad\cdot\left[H_0^{(1)}(\lambda_1\rho)-H_2^{(1)}(\lambda_1\rho)\right]\\
&\quad+\ 2k_0^3\cdot e^{ik_0\rho+ik_2(z'+d')+i\frac{\pi}{2}}\left[\frac{1}{k_0\rho}+\frac{i}{k_1k_0^2\rho^3}\right.\\
&\quad\left.\left.+i\sqrt{\frac{\pi}{k_0\rho}}\cdot\left(\frac{k_0}{k_2}-ik_0l\right)\cdot e^{-iP^*}F(P^*)\right]\right\}
\end{aligned} \tag{70}$$

$$\begin{aligned}
E_{2\varphi'}(\rho,\varphi',z') &= \frac{\omega\mu_0}{4\pi k_2^2}\sin\varphi'\left\{\pi i\frac{\left(\frac{\gamma_0^*}{k_0^2}-\mathrm{i}\frac{\gamma_1^{*2}l}{k_1^2}\right)\gamma_2^*}{q'(\lambda_1)}e^{i\gamma_2^*(z'+d')}\cdot\lambda_1\right.\\
&\quad\cdot\left[H_0^{(1)}(\lambda_1\rho)+H_2^{(1)}(\lambda_1\rho)\right]\\
&\quad+\ 2k_0^3e^{ik_0\rho+ik_2(z'+d')}\cdot\frac{1}{k_0\rho}\left[\frac{1}{k_0\rho}-\frac{i}{k_1\rho}\right.\\
&\quad\left.\left.+i\sqrt{\frac{\pi}{k_0\rho}}\left(\frac{k_0}{k_2}-ik_0l\right)\cdot e^{-iP^*}F(P^*)\right]\right\}
\end{aligned} \tag{71}$$

$$\begin{aligned}
E_{2z'}(\rho,\varphi',z') &= \frac{i\omega\mu_0}{4\pi k_2^2}\cos\varphi'\left\{2\pi i\frac{\frac{\gamma_0^*}{k_0^2}-i\frac{\gamma_1^{*2}l}{k_1^2}}{q'(\lambda_1)}\cdot e^{i\gamma_2^*(z'+d')}H_1^{(1)}(\lambda_1\rho)\lambda_1^2\right.\\
&\quad-\ \frac{2k_0^4}{k_2}e^{ik_0\rho+ik_2(z'+d')+i\frac{\pi}{2}}\cdot\left[\frac{1}{k_0\rho}+i\sqrt{\frac{\pi}{k_0\rho}}\left(\frac{k_0}{k_2}-ik_0l\right)\right.
\end{aligned}$$

$$\cdot e^{-iP^*}F(P^*)\Big]\Bigg\} \tag{72}$$

$$\begin{aligned} B_{2\rho}(\rho,\varphi',z') &= -\frac{\mu_0}{4\pi}\sin\varphi'\Bigg\{\pi i\frac{\dfrac{\gamma_0^*}{k_0^2}-i\dfrac{\gamma_1^{*2}l}{k_1^2}}{q'(\lambda_1)}e^{i\gamma_2^*(z'+d')}\lambda_1 \\ &\quad\cdot\Big[H_0^{(1)}(\lambda_1\rho)+H_2^{(1)}(\lambda_1\rho)\Big] \\ &\quad+2\frac{k_0^3}{k_2}e^{ik_0\rho+ik_2(z'+d')}e^{-i\frac{\pi}{2}}\frac{1}{k_0\rho}\Bigg[\frac{i}{k_0\rho}+\frac{1}{k_1\rho} \\ &\quad-\sqrt{\frac{\pi}{k_0\rho}}\left(\frac{k_0}{k_2}-ik_0l\right)e^{-iP^*}F(P^*)\Bigg]\Bigg\} \end{aligned} \tag{73}$$

$$\begin{aligned} B_{2\varphi'}(\rho,\varphi',z') &= -\frac{\mu_0}{4\pi}\cos\varphi'\Bigg\{\pi i\frac{\dfrac{\gamma_0^*}{k_0^2}-i\dfrac{\gamma_1^{*2}l}{k_1^2}}{q'(\lambda_1)}e^{i\gamma_2^*(z'+d')}\cdot\lambda_1 \\ &\quad\cdot\Big[H_0^{(1)}(\lambda_1\rho)-H_2^{(1)}(\lambda_1\rho)\Big] \\ &\quad+2\frac{k_0^3}{k_2}e^{ik_0\rho+ik_2(z'+d')+i\frac{\pi}{2}}\Bigg[\frac{1}{k_0\rho}+\frac{i}{k_0^2k_1\rho^3} \\ &\quad+i\sqrt{\frac{\pi}{k_0\rho}}\left(\frac{k_0}{k_2}-ik_0l\right)e^{-iP^*}F(P^*)\Bigg]\Bigg\} \end{aligned} \tag{74}$$

$$B_{2z'}(\rho,\varphi',z') = \frac{i\mu_0}{4\pi}\sin\varphi'\frac{2k_0^3}{k_1k_2^2\rho^2}e^{ik_0\rho+ik_2(z'+d')}e^{-i\frac{\pi}{2}} \tag{75}$$

6. CALCULATIONS AND DISCUSSIONS

Taking into account that a horizontal antenna of a submarine is buried inside in the sea water covered with a layer of ice, with the operating frequency f=1MHz, the conductivity and the relative dielectric constant of ice being approximately $\sigma_1 = 1\times10^{-5}$ S/m and $\varepsilon_{1r} = 3.2$, respectively, and the conductivity and the relative dielectric constant of sea water being $\sigma_2 \approx 4$ S/m, $\varepsilon_{2r} = 80$, respectively, the wave number of the electric-type trapped surface wave λ_1 varies as a function of the thickness l of ice layer. The real part and the imaginary part of λ_1/k_0 are shown in Figs. 4 and 5, respectively. In contrast, when Region 2 is a perfect conductor, the corresponding real part and the imaginary part of λ_1/k_0 are also plotted in the same figures. From the Figs. 4 and 5, it is seen that the real part for perfect conductor is closed

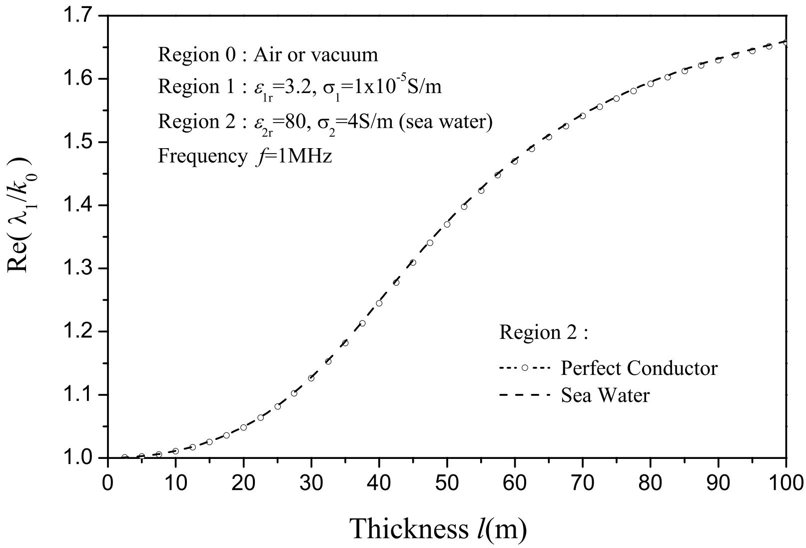

Figure 4. The real parts of λ_1/k_0 for electric-type trapped surface wave vary with thickness l of ice layer.

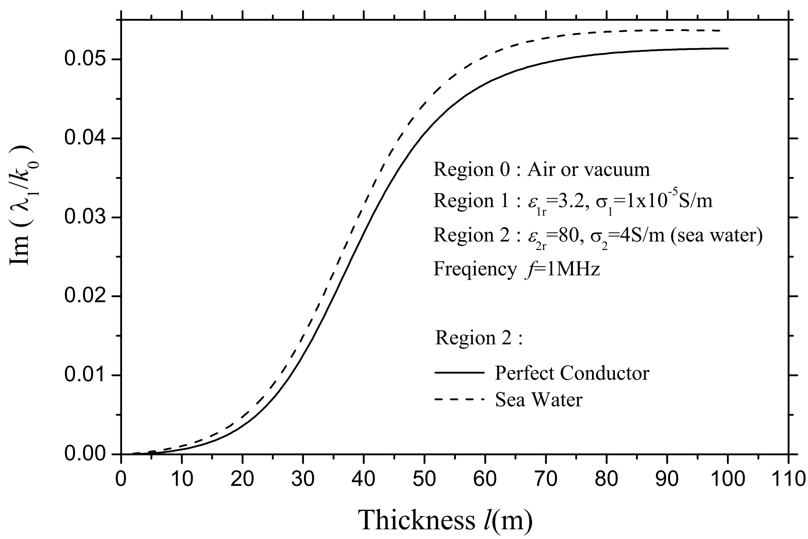

Figure 5. The imaginary parts of λ_1/k_0 for electric-type trapped surface wave vary with thickness l of ice layer.

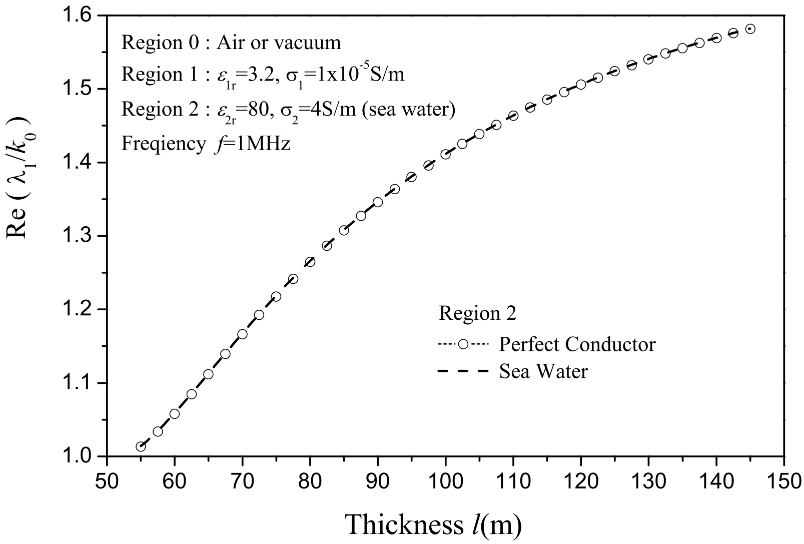

Figure 6. The real parts of λ_1/k_0 for magnetic-type trapped surface wave vary with thickness l of ice layer.

to that for the high lossy material, but the imaginary parts of λ_1/k_0 for both substrates have a little difference. The trapped surface wave for Region 2 being high lossy material attenuates faster than that for Region 2 being perfect conductor. Similarly, the real and imaginary parts of the wave number for the magnetic-type trapped surface wave as a function of the thickness l are shown in Figs. 6 and 7, respectively.

If the transmitting and receiving antennas are located at $d' = z' = 0.5\,\mathrm{m}$ underneath the ice layer, Figs. 8 and 9 show the amplitude of the component $E_{2\rho}$ at $\varphi' = 0°$ as a function of the propagation distances with the thicknesses of the ice layer $l = 2\,\mathrm{m}$ and $8\,\mathrm{m}$, respectively. In Figs. 10 and 11, the amplitude of $E_{2\varphi}$ in the direction $\varphi' = 90°$ as a function of propagation distance are shown with l=2m and 8m, respectively. From Figs. 9 and 11, it is seen that the interference effect will appear in the total field.

From Fig. 9 to Fig. 11, the curves of lateral wave is the same as Dunn's paper, and Dunn overlooked the trapped surface wave.

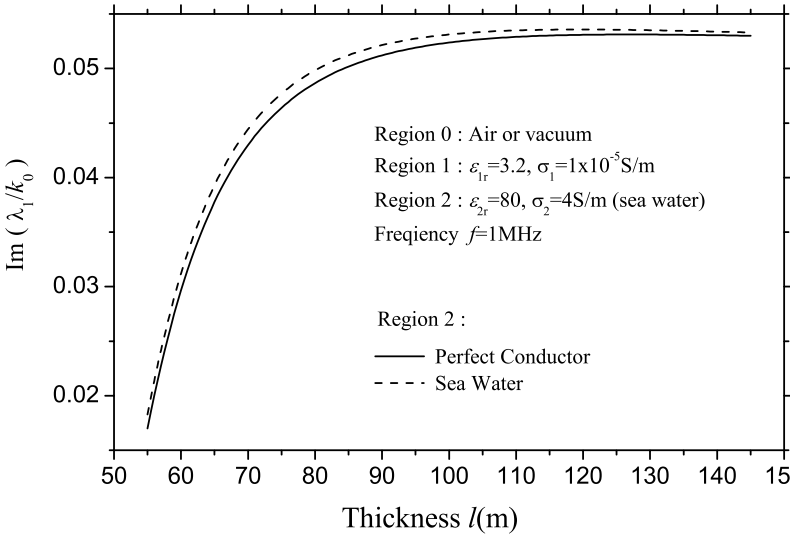

Figure 7. The imaginary parts of λ_1/k_0 for magnetic-type trapped surface wave vary with thickness l of ice layer.

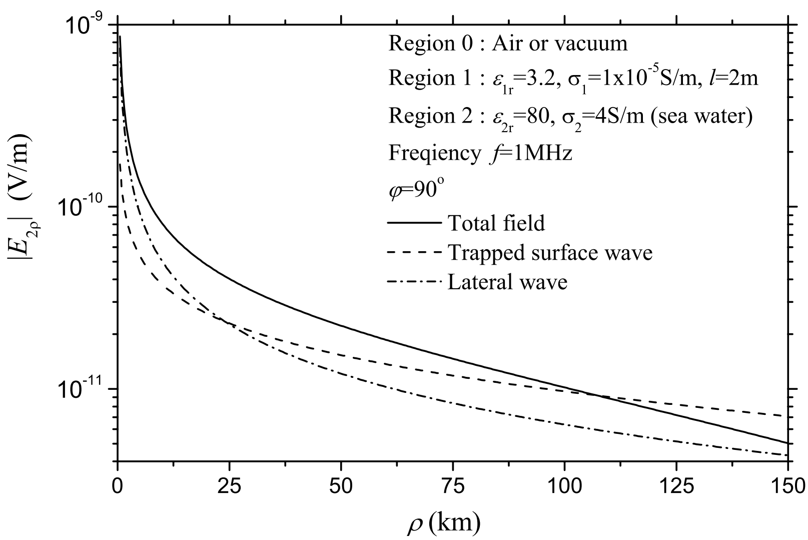

Figure 8. The components of $|E_{2\rho}|$ vary with ρ for $l = 2\,\mathrm{m}$ in direction $\varphi' = 0°$.

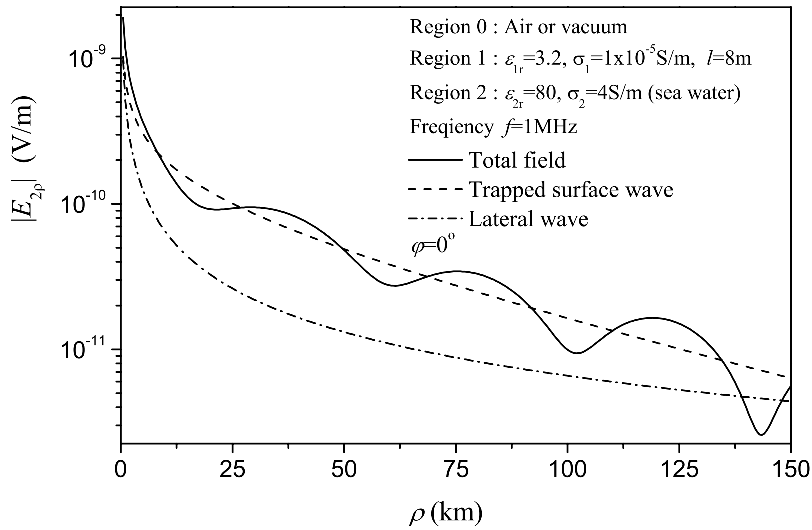

Figure 9. The components of $|E_{2\rho}|$ vary with ρ for $l = 8\,\mathrm{m}$ in direction $\varphi' = 0°$.

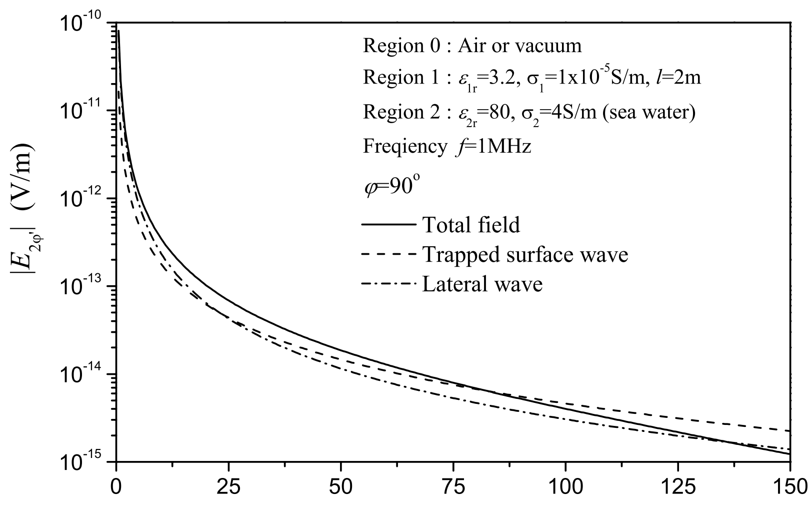

Figure 10. The components of $|E_{2\varphi}|$ vary with ρ for $l = 2\,\mathrm{m}$ in direction $\varphi' = 90°$.

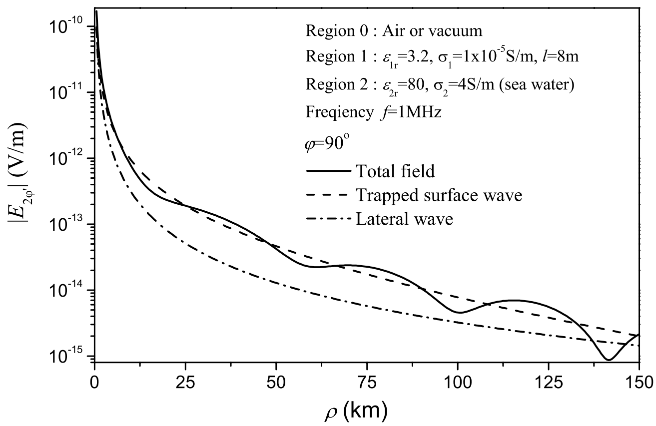

Figure 11. The components of $|E_{2\varphi}|$ vary with ρ for $l = 8\,\mathrm{m}$ in direction $\varphi' = 90°$.

7. CONCLUSIONS

From the above derivations and computations, it is concluded as following: (i) the horizontal antenna underneath the ice layer can efficiently generate both the trapped surface wave and the lateral wave, and the amplitudes of both the waves are at about the same level and can not be neglected; (ii) the wave number of the trapped surface wave is dependent on the thickness l of the dielectric layer as well as the electric parameters of the medium. If the dielectric is lossless, the trapped surface wave will attenuate in the rate of $\rho^{-1/2}$ in the far-zone region, and the trapped surface wave is dominant term; (iii) if the thickness of the dielectric layer is relatively large, the wave number of the trapped surface wave λ_i will be significantly different from the wave number of the lateral wave k_0.

When the thickness l of the dielectric layer satisfied $\sqrt{k_1^2 - k_0^2} \cdot l < \frac{\pi}{2}$, the electric-type trapped surface wave can be excited efficiently and the magnetic-type trapped surface wave cannot be excited. When the thickness l of the dielectric layer satisfied $\sqrt{k_1^2 - k_0^2} \cdot l > \frac{\pi}{2}$, both the electric-type and magnetic-type trapped surface waves can be excited efficiently. Under the condition of thick dielectric cases, the total field will be more complex. When l satisfies $n\pi < \sqrt{k_1^2 - k_0^2} \cdot l < (n+1)\pi$, the electric-type trapped surface wave has $n+1$ modes, and if l is in the condition of $n\pi - \frac{\pi}{2} < \sqrt{k_1^2 - k_0^2} \cdot l < n\pi + \frac{\pi}{2}$, the surface there are n modes of the magnetic-type trapped surface wave.

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