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METAMATERIALS AND DEPOLARIZATION FACTORS

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Abstract—Depolarization factors of scatterers within anisotropic media are functions of not only the shape of the inclusion but also of the degree of anisotropy of the environment. In this contribution the depolarization factors are studied for anisotropic metamaterials. In such case, qualitatively new phenomena appear because the effective axial ratio of the scatterers, which determines the depolarization factors, may become complex. The negative real part of the depolatization factors is interpreted as "repolarization." The effect of the various parameters on the depolarization factors and effective dielectric parameters are analyzed and discussed for both three- and two-dimensional mixtures, with emphasis on the dissipative character of the homogenized metamaterials.

1. INTRODUCTION

It is perhaps not an exaggeration to note that the electromagnetics of wave–material interaction is experiencing a renaissance period. It is true that from the times of Faraday and Maxwell, matter has always been essential in the way electric and magnetic energy is concentrated and guided, and also it is fair to observe the surge of bianisotropics research in electromagnetics of the 1960's and 70's. However, the present time of entering into the new century is a period when we witness so much new research and development in our field that some may say that there is a paradigm shift in electromagnetics. Possibilities that new nanotechnologies offer are perhaps one of the driving forces in this process. Certainly one can argue and discuss how profound the changes in the long run really are but nevertheless these observations can serve as a motivation to study metamaterials from the homogenization and mixing point of view, as will be done in the present paper.

1.1. Metamaterials and Homogenization

The special property of metamaterials, their negative parameter values both for permittivity and permeability, does not cause in principle any particular troubles for someone interested in and working on homogenization of mixtures.† Very special effects are certainly to be expected when waves reflect and refract from layers of such media [6], but in the modeling of these materials, classical mixing formulas can be applied to negative parameter values to a surprisingly great extent.

† Here the term "metamaterial" is used to refer to media which are characterized by the simultaneously negative values for ϵ and μ and which have the potential to support backward waves. "Left-handed medium" is a more problematic term because of its association to handedness of the material structure (chirality) [1]. Rather, "metamaterial" is here synonymously used with "Veselago medium." The article by Veselago analyzing such media [2] appeared in 1967, and thoughts on such possibilities have been put forth in the Soviet Union already by Mandelstam in 1945 [3]. One could trace the origins of backward-wave media even one hundred years back (1904) to the writings of Lamb and Schuster, as has been observed by Holloway *et al.* [4]. However, concerning names and terminology, there is considerable confusion around as can be expected when an emerging field is opening up; see [5] for discussion on attempts to define metamaterials.

The assumption of isotropy helps considerably when homogenization theories are generalized to metamaterials. It is very common in today's metamaterial studies to assume isotropy in the medium response. Indeed, sometimes the material parameters are even assumed to be simply opposite to free space, *i.e.*, the relative permittivity and permeability values are taken to be $\epsilon = \mu = -1$.

However, recently interest is being focused on anisotropic metamaterials, too. After all, the very fabrication process of man-made metamaterials is based on layered element lattices, from which orthorhombic order or anisotropies of even less symmetry can emerge. And particularly interesting from the point of view of the present article are so-called *indefinite* media [7, 8]. These media are not only both anisotropic and metamaterials but the permittivity (and permeability) components in the eigenaxis directions are such that one or two of them are negative but the remaining two or one are positive.

Such anisotropy in metamaterial response has quite profound repercussions when heterogeneous mixing analysis is performed. The homogenization of "ordinary" anisotropic materials (with positive material parameter eigenvalues) requires special care since the averaging of the dipole moments has to be treated in a particular manner in non-isotropic environment. The shape of the inclusions that affects the polarization amplitude is not the only factor which determines the dipole moment to be averaged. Also the degree of anisotropy of the environment is essential.

Depolarization factors are very important parameters in the final homogenization process. Now, for the case of inclusions in anisotropic host medium, the depolarization factors can be calculated by combining the effects of both the shape and the external anisotropy through a single integral.

1.2. Scope and Structure of the Study

In the following, the behavior of these depolarization factors will be studied for the case of anisotropic metamaterials, and especially for indefinite media. They become complex for certain material parameter values. The results will be analyzed. Furthermore, predictions are calculated for the effective permittivity components using different parameter values for the mixture parameters. The mixing rule which is used in the analysis is a generalization of the classical Maxwell Garnett mixing formula [9], but it is most probable that the obtained results would be qualitatively similar for other, more sophisticated mixing rules. Both three-dimensional and two-dimensional mixtures are studied. For the three-dimensional case, the anisotropy is assumed uniaxial (meaning that the effective permittivity dyadic has only two components). The results are discussed and interpreted from the physical point of view, with a special care on losses and dissipation.

In mixing and homogenization studies in general, and those involving metamaterials in particular, one of the difficulties is the large number of parameters that can be varied in the problem. Even if we neglect the explicit bookkeeping on the degrees of freedom that are caused by the randomness that is often present in mixtures (although not in regular, periodic structures), there are still very many factors that determine the complexity of the mixture. In order to be able to extract dependencies of the effective properties on primary parameters, one needs to make simplifications and only allow variations with the most essential parameters. This is important especially in the present case when qualitatively new effects emerge from the interaction of metamaterial character of inclusions and anisotropic mixing.

The main simplifying assumption in the analysis to follow is—in addition to the fact that the analysis is based on the basic Maxwell Garnett mixing principle—that the effect of the magnetic permeability is bypassed. In other words, the permeability of all the mixing components and also the composite are assumed that of free space. It is true that the important property that is driving metamaterials applications is the negative values for *both* permittivity *and* permeability and hence it might seem strange not to account for the magnetic mixing. However, here we must remember the power of duality [10]: the magnetic and electric problems have the same mathematical form, and therefore the results for dielectric mixing can be directly exploited in the magnetic regime. Even if the origin of the magnetic polarizabilities in the physical and engineering point of view are very different from those of electric inclusions and responses, the advantage that we receive from the magnetic simplification is sufficient to motivate the assumption made in the analysis.

In the analysis to follow, the temporal dependence of $\exp(-\mathrm{i}\omega t)$ is assumed for the time-harmonic fields. This means that for passive (dissipative) materials, the imaginary part of the permittivity is *positive* [11]. Furthermore, all the permittivities are understood in units of free-space permittivity ϵ_0 ($\epsilon_0 \approx 8.854 \cdot 10^{-12}$ As/Vm). In other words, the subindex "r" is omitted from the ϵ-symbol.

2. DEPOLARIZATION FACTORS OF ELLIPSOIDS IN ANISOTROPIC MATERIAL

The essence of the analysis to follow can be best introduced using Figure 1 which illustrates two-phase mixtures. Spherical (in general, ellipsoidal) inclusions are randomly scattered in anisotropic

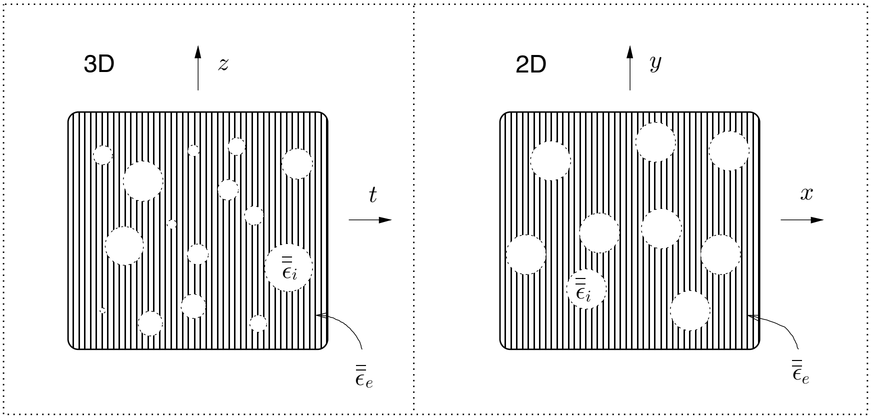

Figure 1. The mixtures under analysis: uniaxial three-dimensional case (with optical axis aligning with z) and anisotropic two-dimensional case. The permittivity dyadic of the anisotropic background is $\overline{\overline{\epsilon}}_e$ with the special axis along z (3D) and y (2D). The permittivity dyadic of inclusions is $\overline{\overline{\epsilon}}_i$. (The inclusions are taken to be isotropic (ϵ_i) in the calculations to follow.) In the figure, the inclusions are assumed equisized (although it is not necessary for homogenization). Hence in the 2D mixture the circles have the same dimension, but the cut of the 3D figure on an arbitrary level displays varying cross sections through spheres.

background. The figure shows two cases: three-dimensional volume and two-dimensional surface (where spheres are of course circles). In the case that the inclusions are considerable smaller in dimension than the wavelength of the operating electromagnetic field, there is sense in looking for an effective permittivity tensor for the macroscopic mixture. The homogenized continuum permittivity is a function of the component permittivities and their fractional volumes.

Indeed, the effective permittivity dyadic for a mixture where ellipsoidal inclusions with anisotropic permittivity dyadic $\overline{\overline{\epsilon}}_i$ are (all aligned) in anisotropic background $\overline{\overline{\epsilon}}_e$ can be calculated. The relative permittivity, according to the Maxwell Garnett mixing formula, reads

$$\overline{\overline{\epsilon}}_{\mathrm{eff}} = \overline{\overline{\epsilon}}_e + f\,(\overline{\overline{\epsilon}}_i - \overline{\overline{\epsilon}}_e) \cdot \left[\overline{\overline{\epsilon}}_e + (1-f)\overline{\overline{\mathrm{L}}} \cdot (\overline{\overline{\epsilon}}_i - \overline{\overline{\epsilon}}_e)\right]^{-1} \cdot \overline{\overline{\epsilon}}_e \qquad (1)$$

where the modified depolarization dyadic is [12, Section 5.5.2]

$$\overline{\overline{\mathrm{L}}} = \frac{\det\overline{\overline{\mathrm{A}}}}{2} \int_0^\infty ds\, \overline{\overline{\epsilon}}_e \cdot \frac{\left(\overline{\overline{\mathrm{A}}}^2 + s\overline{\overline{\epsilon}}_e\right)^{-1}}{\sqrt{\det\left(\overline{\overline{\mathrm{A}}}^2 + s\overline{\overline{\epsilon}}_e\right)}} \tag{2}$$

Here, the symmetric and positive-definite dyadic

$$\overline{\overline{\mathrm{A}}} = \sum_{i=x,y,z} a_i \mathbf{v}_i \mathbf{v}_i \tag{3}$$

with

$$\det\overline{\overline{\mathrm{A}}} = a_x a_y a_z \tag{4}$$

defines the ellipsoid as $\mathbf{r} \cdot \overline{\overline{\mathrm{A}}}^{-2} \cdot \mathbf{r} \leq 1$. The semiaxes of the ellipsoid are a_x, a_y, a_z. Note that the permittivities are *relative*, in other words dimensionless scalars or dyadics.

Before moving into the homogenization, let us take a look at the behavior of the depolarization dyadic components in case of uniaxial environments.

2.1. Three-Dimensional Case

Assume that the inclusion is a sphere and of isotropic material ($\overline{\overline{\epsilon}}_i = \epsilon_i \overline{\overline{\mathrm{I}}}$). Furthermore, be the uniaxial environment permittivity as

$$\overline{\overline{\epsilon}}_e = \epsilon_t \left(\overline{\overline{\mathrm{I}}}_t + c\, \mathbf{u}_z \mathbf{u}_z\right) = \epsilon_t \overline{\overline{\mathrm{I}}}_t + \epsilon_z\, \mathbf{u}_z \mathbf{u}_z \tag{5}$$

where the degree of uniaxiality is c:

$$\epsilon_z = c\epsilon_t \tag{6}$$

and the transversal-to-z unit dyadic is $\overline{\overline{\mathrm{I}}}_t = \overline{\overline{\mathrm{I}}} - \mathbf{u}_z \mathbf{u}_z$.

Then, using (1), we can write for the two components of the homogenized medium:

$$\epsilon_{\text{eff},z} = \epsilon_z + f \frac{\epsilon_z(\epsilon_i - \epsilon_z)}{\epsilon_z + (1-f)L_z(\epsilon_i - \epsilon_z)} \tag{7}$$

$$\epsilon_{\text{eff},t} = \epsilon_t + f \frac{\epsilon_t(\epsilon_i - \epsilon_t)}{\epsilon_t + (1-f)L_t(\epsilon_i - \epsilon_t)} \tag{8}$$

Note that the depolarization factors here depend on c, and therefore the effective permittivity components are coupled.

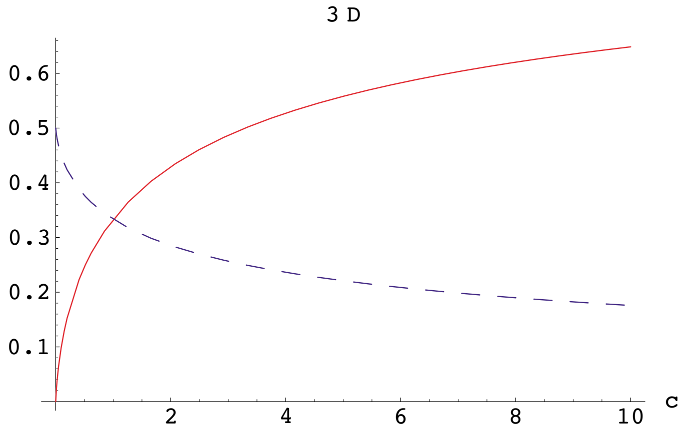

Figure 2. The depolarization factors for the three-dimensional case. Solid line—L_z, dashed line—L_t. The parameter c is the ratio ϵ_z/ϵ_t.

These are the well-known depolarization factors for spheroids [14].[‡] The axial ratio corresponds to the square root of the uniaxiality parameter c. For biaxial ellipsoids, the depolarization factors can be written in terms of elliptic integrals [15, 16].

Evaluation of the integral (1) for the uniaxial case gives us

$$L_z = \frac{c}{c-1}\left(1 - \frac{\arctan\sqrt{c-1}}{\sqrt{c-1}}\right) \tag{9}$$

$$L_t = \frac{1}{2(1-c)}\left(1 - c\frac{\arctan\sqrt{c-1}}{\sqrt{c-1}}\right) \tag{10}$$

Obviously, $L_z + 2L_t = 1$.

In Figure 2, the depolarization factors (9)–(10) are shown as functions of c. Note that even if the real geometric form of the inclusion is a sphere, still the dipole moment induced in it is dependent on the direction of the incident field, because different amounts of depolarization are created for orthogonal field excitations, as shown by the two curves in the figure.

If the anisotropy vanishes ($c = 1$), the depolarization factor components become equal, $L_z = L_t = 1/3$, as expected.

[‡] Ellipsoids of revolution: prolate and oblate spheroids depending whether the axis of the ellipsoid in the revolution direction is longer or shorter than the other two. Here, we have to deal with depolarization factors of oblate spheroids if $c > 1$ (positive uniaxiality), and prolate spheroids if $c < 1$ (negative uniaxiality) [13].

2.2. Two-Dimensional Case

In an analogous manner, the depolarization factors for two-dimensional mixtures can be studied. The situation in Figure 1 (right-hand side) means that the inclusions are aligned cylinders, axes along z, and the field is only allowed to be polarized in the plane perpendicular to z. In this case, the external anisotropy reads:

$$\bar{\bar{\epsilon}}_e = \epsilon_x \left(\mathbf{u}_x\mathbf{u}_x + c\,\mathbf{u}_y\mathbf{u}_y\right) = \epsilon_x\mathbf{u}_x\mathbf{u}_x + \epsilon_y\mathbf{u}_y\mathbf{u}_y \tag{11}$$

where

$$\epsilon_y = c\epsilon_x \tag{12}$$

Then the depolarization factors, for a circular inclusion, can be evaluated from (2)

$$L_y = \frac{\sqrt{c}}{1+\sqrt{c}} \tag{13}$$

$$L_x = \frac{1}{1+\sqrt{c}} \tag{14}$$

Obviously, $L_x + L_y = 1$.§ Of course, for degenerated anisotropy, the two depolarization factors become equal, $L_x = L_y = 1/2$.

Consequently, the effective permittivity components for a 2D mixture where circular holes with isotropic permittivity ϵ_i occupy a volume fraction f in the anisotropic background that obeys permittivity (11), can be written as

$$\epsilon_{\text{eff},y} = \epsilon_y + f\frac{\epsilon_y(\epsilon_i - \epsilon_y)}{\epsilon_y + (1-f)L_y(\epsilon_i - \epsilon_y)} \tag{15}$$

$$\epsilon_{\text{eff},x} = \epsilon_x + f\frac{\epsilon_x(\epsilon_i - \epsilon_x)}{\epsilon_x + (1-f)L_x(\epsilon_i - \epsilon_x)} \tag{16}$$

where, again, the coupling comes through the depolarization factors, L_x and L_y, which are functions of c, and therefore on ϵ_y/ϵ_x-ratio. This can be observed in the illustrations to follow.

3. DEPOLARIZATION FACTORS FOR INDEFINITE MEDIA

There was not too much reason for fascination in the curves of Figure 2 (for the 3D case, and for 2D, Figure 3). One might say that they

§ In general, the three depolarization factors sum up to one, meaning that in the two-dimensional case one can interpret the result $L_x + L_y = 1$ in a way $L_z = 0$. This is certainly the case because there is no variation in the geometry along z-axis, whence all boundaries are tangential to z, and no depolarization can be born for fields with $\mathbf{u}_z \cdot \mathbf{E} \neq 0$.

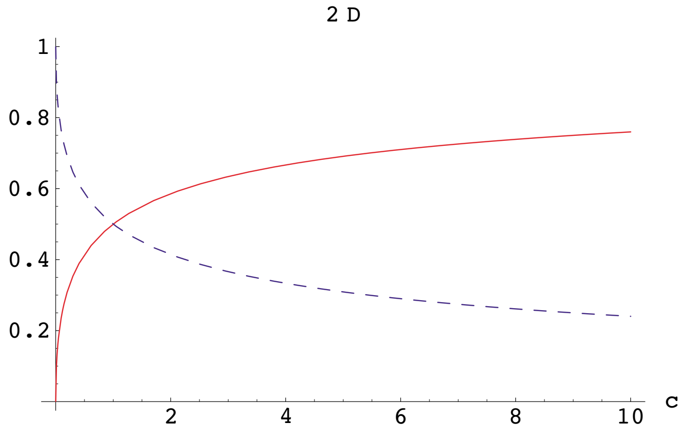

Figure 3. The depolarization factors for the two-dimensional case. Solid line—L_y, dashed line—L_x. The parameter c is the ratio ϵ_y/ϵ_x.

only reproduce the curves of depolarization factors of ellipsoid in isotropic environment, although the matter–geometry interaction has changed the horizontal axis of the depolarization diagrams: it is not the ellipticity of the ellipsoid but rather the anisotropy of the host which is the factor that determines depolarization.

However, since the focus in this study is the possibility of having negative parameters for some of the permittivity components, let us generalize the depolarization factors into the field of indefinite media. In other words, into cases where the c-parameter is negative, or even complex. Results of these calculations for the three-dimensional mixture are shown in Figure 4.

3.1. Observations Regarding the Depolarization Factors

Interesting things certainly start to happen as c becomes negative.

(i) Looking first at the two upper subplots in Figure 4, where c is real, we can see that the depolarization factors are strongly varying functions as c crosses zero. Especially the absolute value of L_z has a strong discontinuity in its derivative.

(ii) For negative values of c, the depolarization factors are complex. Both L_z and L_t have simultaneously imaginary parts due to the requirement $L_z + 2L_t = 1$.

(iii) For c values negative but close to zero, the real part of L_z is small and the imaginary part grows quickly, being larger than the real

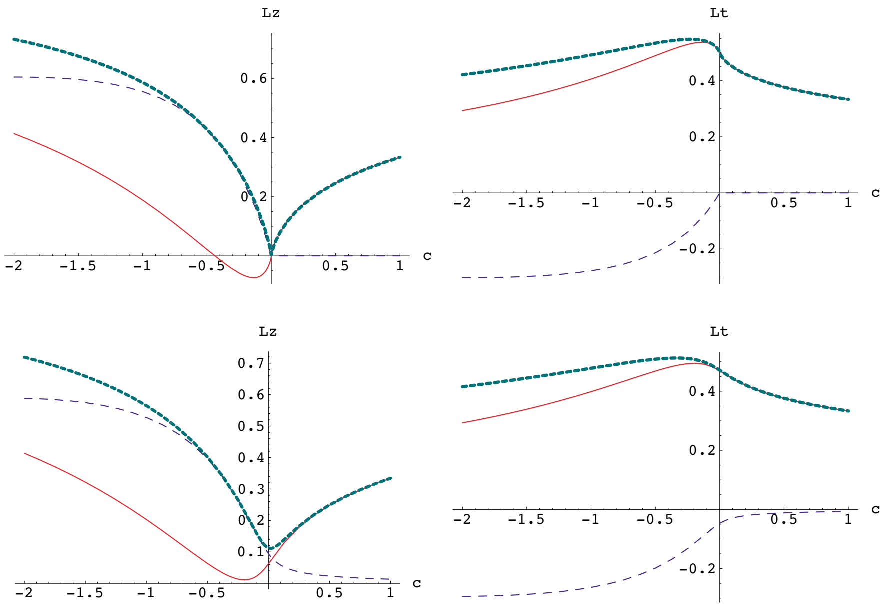

Figure 4. The depolarization factors for the three-dimensional case. Upper row: lossless. Lower row: with an imaginary part of $0.1 * \mathrm{i}$ added to c. (Solid line–real part, Dashed line–imaginary part, thick line–absolute value.)

part. On the other hand, for L_t the situation is the opposite: large real part and small imaginary part.

(iv) When c moves on to the left on the negative axis, L_z approaches fairly quickly the value 1, and also $L_t \to 0$, and their imaginary parts vanish. In fact, the approach to these limiting values happens more rapidly than on the positive axis of c. (Of course, also on the positive side, $\lim L_z = 1$ and $\lim L_t = 0$ as $c \to +\infty$.)

(v) A very interesting observation is the fact that the real part of L_z can be *negative*. This happens for values in the range $-0.439 < c < 0$. We could make the interpretation that for such values there is *repolarization* instead of ordinary depolarization. It may happen, though, that the simultaneous large imaginary part in L_z may mask any potentially useful effects of the repolarization that one might envisage.

(vi) When we allow a small imaginary part in c (the two lower figures), we can see that the curves remain qualitatively the same as in the lossless case but become "softer" functions of the real part of c.

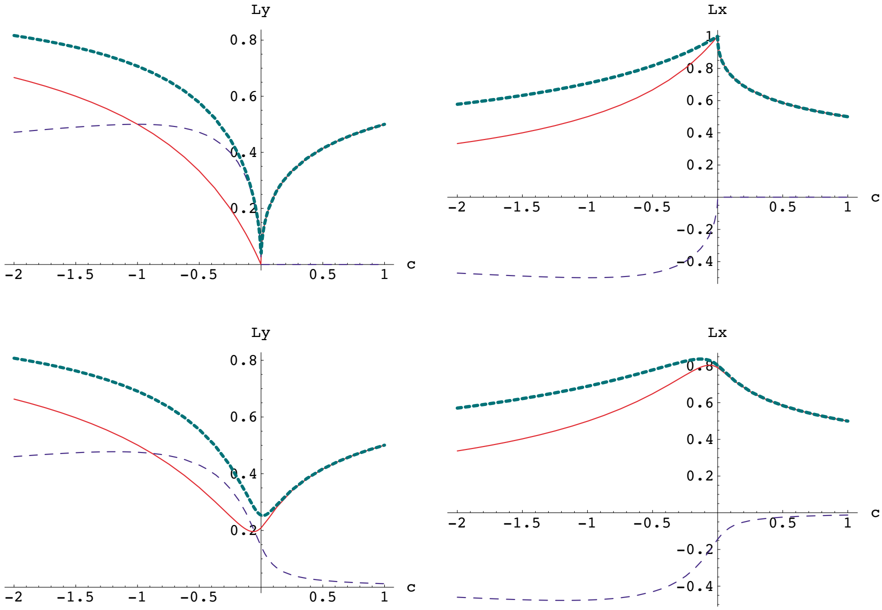

Figure 5. The depolarization factors for the two-dimensional case. Upper row: lossless. Lower row: with an imaginary part of $0.1 * \mathrm{i}$ added to c. (Solid line–real part, Dashed line–imaginary part, thick line–absolute value.)

(vii) The repolarization effect (negative real part of L_z) vanishes when the imaginary part of c increases.

(viii) Concerning the imaginary parts of the depolarization factors, it is important to note that the sign of $\mathrm{Im}\{L_z\}$ is the same as the sign of $\mathrm{Im}\{c\}$. Consequently, the imaginary part of L_t has the opposite sign of the imaginary part of c. In Figure 4, the imaginary part of c was taken to be positive; if that is inverted, the imaginary parts of L_z and L_t also change sign.

The behavior of the two-dimensional depolarization factors that corresponds to the illustration of Figure 4, when generalized into negative and complex values of c, can be seen in Figure 5.

The observations concerning 2D depolarization factors are very similar to those of the 3D case. Obviously we have to compare L_y with L_z in the 3D analysis, and L_x with L_t.

There is a similar downward "bump" in the real part of L_y as was was L_z when c crosses zero to become negative. However, that vanishes more rapidly when imaginary part is added to c. Perhaps the clearest qualitative difference from the 3D behavior is that there is no "repolarization effect" in 2D curves: the real part of L_y cannot become negative.

The imaginary part of L_y has the same sign as the imaginary part of c, and this is opposite to the imaginary part of L_x.

3.2. Repolarization Factor

Indeed, these are thought-provoking characteristics that can be observed from the curves in Figures 4 and 5. Perhaps the most intriguing is the fact that "repolarization factor" would seem a more suitable name than "depolarization factor" in certain indefinite mixing cases for these familiar parameters. It is very deeply rooted in the minds of electromagnetists that within an inclusion the field is *de*polarized and the amplitude of the opposing polarization is determined by the shape of the inclusion. Therefore this repolarization effect (a negative real part for the depolarization factor) is not only a new opportunity and challenge in engineering (to design novel composite structures by enhancing or suppressing some effects in a clever way by taking advantage of this effect) but also a reminder to us all to reformulate our electromagnetic mental models. In this vein, it is also certainly worth noting the fact that this repolarization effect is present in the three-dimensional case but not in two dimensions.

4. OBSERVATIONS CONCERNING THE EFFECTIVE PERMITTIVITY COMPONENTS

Finally, let us study homogenization of indifinite media where the mixed signs in the permittivity components cause the depolarization factors to behave in the strange manner of Figures 4 and 5.

The fact that the depolarization factors in indefinite media may have imaginary parts of either sign should make us uneasy. The effect of the depolarization factors is strong on the effective permittivity values, and one might suspect that if the primary parameters are allowed to vary, it may happen that also the imaginary part of one of the effective permittivity components might also change sign, even if the two constituents are kept dissipative. And this is forbidden because a negative value for the imaginary part means that the medium is active. That cannot be possible since we are playing with passive components having positive (or zero) imaginary parts of their permittivities.

4.1. Lossless Components

Let us therefore make a closer look at the effective permittivity values as functions of the parameters in the problem.

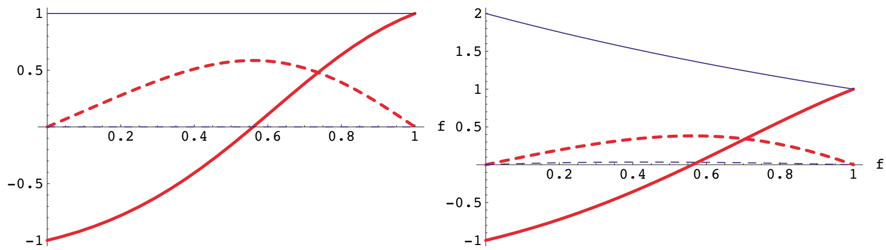

Figure 6. Effective permittivity components of a 3D mixture with $\epsilon_t = 1$, $\epsilon_z = -1$ (left) and $\epsilon_t = 2$, $\epsilon_z = -1$ (right). (Solid line–real part, Dashed line–imaginary part, Thick lines—$\epsilon_{\text{eff},z}$, Thin lines—$\epsilon_{\text{eff},t}$.) Note that the imaginary part of $\epsilon_{\text{eff},t}$ vanishes in the left-hand figure. The volume fraction of the inclusion phase is f.

4.1.1. 3D Case

Starting with the three-dimensional mixture, in Figure 6 the Maxwell Garnett prediction for the effective permittivity is shown for a mixture where isotropic inclusions (permittivity $\epsilon_i = 1$) occupy a volume fraction f in the indefinite, anisotropic background. Two parameter combinations are analyzed: the environment permittivity has negative value in the direction of the optical axis ($\epsilon_y = -1$) but the transversal permittivity of the environment is positive, either $\epsilon_t = 1$, as in the left-hand figure, or $\epsilon_t = 2$, as on the right side.

The effective permittivities are given as functions of the volume fraction, and the obvious results can be seen for the limiting cases: for $f = 0$ the effective parameters are those of the environment and for $f = 1$ they are those of the inclusions. And in between the curves go smoothly.

But a very important effect to be noted is that losses are present in the homogenized composite, even if both the guest and host phases were lossless. In other words, the effective permittivity components may have imaginary parts. And especially $\epsilon''_{\text{eff},z}$ is significant, in other words the permittivity in the direction of the optical axis (which is the direction with negative host permittivity component) may have a large imaginary part when the fractional division between the phases is close, but not exactly, to 50–50.

Another important fact to note is that if the transversal permittivity equals that of the inclusions ($\epsilon_t = \epsilon_i$), there is no effect of the z component on the transversal effective permittivity: $\epsilon_{\text{eff},t}$ remains unity, and real. However, if there is contrast between the transversal permittivities (right-hand figure), coupling appears, and $\epsilon_{\text{eff},t}$ attains an imaginary part, although a much smaller one than the imaginary part of the $\epsilon_{\text{eff},z}$ component.

4.1.2. 2D Case

Figure 7 shows the Maxwell Garnett prediction for the effective permittivity in the two-dimensional case.

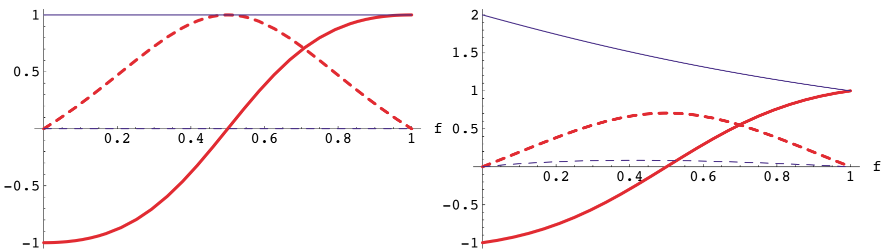

Figure 7. Effective permittivity components of a 2D mixture with $\epsilon_x = 1$, $\epsilon_y = -1$ (left) and $\epsilon_x = 2$, $\epsilon_y = -1$ (right). (Solid line–real part, Dashed line–imaginary part, Thick lines–$\epsilon_{\text{eff},y}$, Thin lines–$\epsilon_{\text{eff},x}$.) Note that the imaginary part of $\epsilon_{\text{eff},x}$ vanishes in the left-hand figure. The volume fraction of inclusions is f.

As expected, the results resemble to some extent those in Figure 6. The soft transition from the host parameters to those of the inclusions takes place in the ordinary Maxwell Garnett manner. Losses appear in the $\epsilon_{\text{eff},y}$ component similarly as in $\epsilon_{\text{eff},z}$ in the 3D case. Also a small loss effect can be observed in $\epsilon_{\text{eff},x}$, given that there is contrast in the x-component of the inclusion and environment permittivities.

Some differences in the character between the two- and three-dimensional mixtures are worth noting. The behavior of the 2D effective parameters is more "symmetric" than in the 3D case. The maximum of $\epsilon''_{\text{eff},y}$ and the associated condition $\epsilon'_{\text{eff},y} = 0$ happens at exactly $f = 0.5$, whereas in the 3D case the $\epsilon'_{\text{eff},z} = 0$ condition was at higher loadings of the inclusion phase. Also it is visible from the curves that the losses in the effective medium are higher than with the same parameters in the 3D case. Figure 5 shows that the maximum imaginary part in $\epsilon_{\text{eff},y}$ is 1.

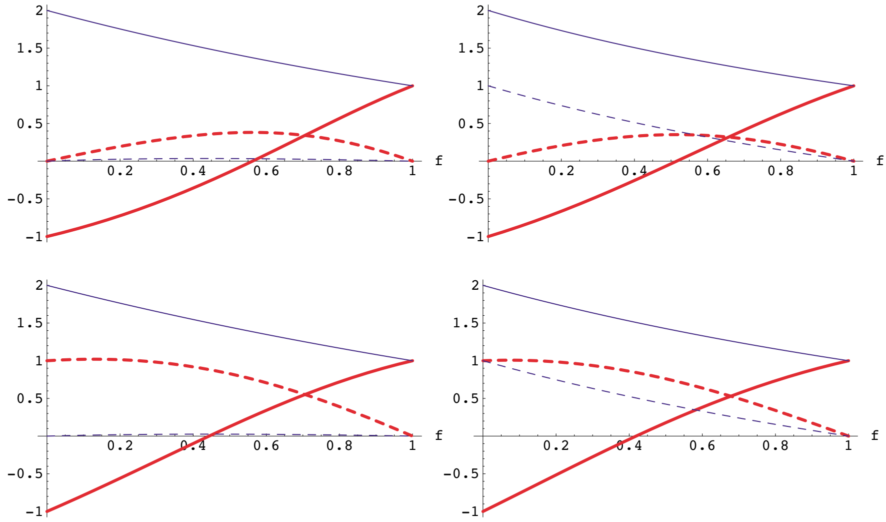

Figure 8. Effective permittivity components of a 3D mixture with real parts $\epsilon_t = 2$, $\epsilon_z = -1$: both real (upper left), $\text{Im}\{\epsilon_t\} = -1$ (upper right), $\text{Im}\{\epsilon_z\} = -1$ (lower left), and $\text{Im}\{\epsilon_t\} = \text{Im}\{\epsilon_z\} = -1$ (lower right). (Solid line–real part, Dashed line–imaginary part, Thick lines–$\epsilon_{\text{eff},z}$, Thin lines–$\epsilon_{\text{eff},t}$.) The parameter f is the volume fraction of the inclusion phase.

The imaginary parts in the effective permittivity that emerge in the mixing process for lossless host and guest components are always positive.[‖] In other words, the dissipative character of the mixture is preserved, as one should expect from a passive constellation.

4.2. Effect of Losses in Components

The interesting question remains what happens to the prediction for the effective permittivity when losses are admitted in the components. We have now seen that losses are generated by the very mixing process. But if real material losses enter the picture, what is the result? The effect of losses can be seen from the calculated curves which are depicted in Figure 8 (3D case) and in Figure 9 (2D case).

‖ Caveat: in the numerical evaluation of the results, depending on the software, it may happen that the sign of the imaginary part may become undefined. The results shown here are checked in the manner that first a very small loss was added to the component permittivities and then the amplitude of this loss was gradually decreased and the results recalculated so many times that there no further numerical effect could be observed in the curves.

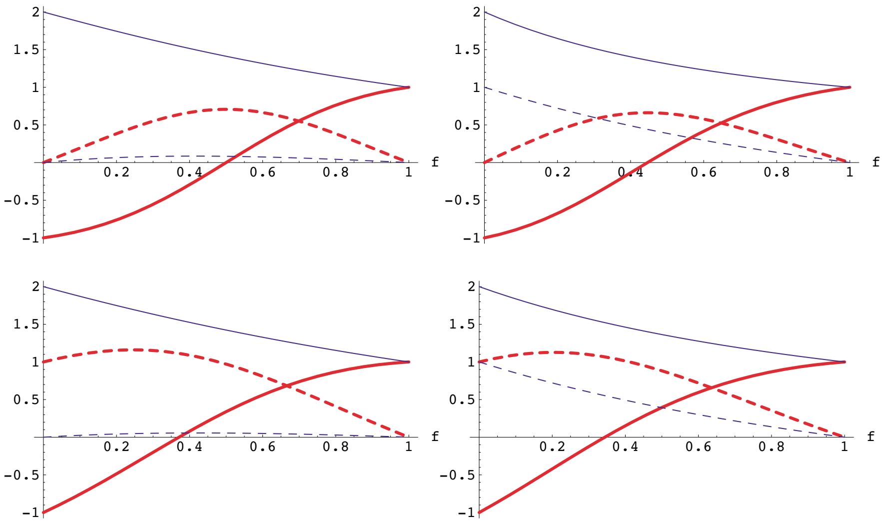

Figure 9. Effective permittivity components of a 2D mixture with real parts $\epsilon_x = 2$, $\epsilon_y = -1$: both real (upper left), $\mathrm{Im}\{\epsilon_x\} = -1$ (upper right), $\mathrm{Im}\{\epsilon_y\} = -1$ (lower left), and $\mathrm{Im}\{\epsilon_x\} = \mathrm{Im}\{\epsilon_y\} = -1$ (lower right). (Solid line–real part, Dashed line–imaginary part, Thick lines–$\epsilon_{\mathrm{eff},y}$, Thin lines–$\epsilon_{\mathrm{eff},x}$.) The parameter f is the volume fraction of the inclusion phase.

The inclusion permittivity is assumed to be $\epsilon_i = 1$, and the real parts of the environment permittivities are 2 and -1. An imaginary part of $+1$ is added separately to the two environment permittivities, and also in the last case to both of them. As can be seen clearly from the figures, the effect of losses is to increase the losses in both components of the homogenized material. There is no risk of violating the passivity requirement.

5. CONCLUSION

Combination of metamaterials, in particular, indefinite metamaterials, and homogenization may lead to extremely interesting macroscopic material behavior. Even in the simplest geometry, spherical inclusions in homogeneous background, the effective parameters show unexpected details which call for physical interpretation. One example was the repolarization effect that was discussed in Section 3. Speculative predictions call for tests and checks that the results obey limitations dictated by physics, and it was indeed encouraging to see that the calculations were in accordance with the restriction of dissipative character of passive mixtures.

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