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# PERTURBATIONS OF DISPERSION-MANAGED OPTICAL SOLITONS

**A. Biswas**

Department of Physics & Mathematics
Center of Excellence in ISEM
Tennessee State University
Nashville, TN 37209-1561, USA

**Abstract**—This paper studies the propagation of solitons through an optical fiber, with strong dispersion-management in presence of perturbation terms. The adiabatic parameter dynamics of the solitons in presence of such perturbation terms have been obtained by using the variational principle. In particular, the Gaussian and super-Gaussian pulses have been considered.

## 1. INTRODUCTION

The propagation of solitons through optical fibers has been a major area of research given its potential applicability in all optical communication systems. The field of telecommunications has undergone a substantial evolution in the last couple of decades due to the impressive progress in the development of optical fibers, optical amplifiers as well as transmitters and receivers. In a modern optical communication system, the transmission link is composed of optical fibers and amplifiers that replace the electrical regenerators. But the amplifiers introduce some noise and signal distortion that limit the system capacity. Presently the optical systems that show the best characteristics in terms of simplicity, cost and robustness against the degrading effects of a link are those based on intensity modulation with direct detection (IM-DD). Conventional IM-DD systems are based on non-return-to-zero (NRZ) format, but for transmission at higher data rate the return-to-zero (RZ) format is preferred. When the data rate is quite high, soliton transmission can be used. It allows the exploitation of the fiber capacity much more, but the NRZ signals offer very high potential especially in terms of simplicity.

There are limitations, however, on the performance of optical system due to several effects that are present in optical fibers and amplifiers. Signal propagation through optical fibers can be affected by group velocity dispersion (GVD), polarization mode dispersion (PMD) and the nonlinear effects. The chromatic dispersion that is essentially the GVD when waveguide dispersion is negligible, is a linear effect that introduces pulse broadening generates intersymbol interference. The PMD arises due the fact that optical fibers for telecommunications have have two polarization modes, in spite of the fact that they are called monomode fibers. These modes have two different group velocities that induce pulse broadening depending on the input signal state of polarization. The transmission impairment due to PMD looks similar to that of the GVD. However, PMD is a random process as compared to the GVD that is a deterministic process. So PMD cannot be controlled at the receiver. Newly installed optical fibers have quite low values of PMD that is about $0.1\,\mathrm{ps}/\sqrt{km}$.

The main nonlinear effects that arises in monomode fibers are the Brillouin scattering, Raman scattering and the Kerr effect. Brillouin is a backward scattering that arises from acoustic waves and can generate forward noise at the receiver. Raman scattering is a forward scattering from silica molecules. The Raman gain response is characterized by low gain and wide bandwidth namely about 5 THz. The Raman threshold in conventional fibers is of the order of 500 mW for copolarized pump

and Stokes' wave (that is about 1 W for random polarization), thus making Raman effect negligible for a single channel signal. However, it becomes important for multichannel wavelength-division-multiplexed (WDM) signal due to an extremely wide band of wide gain curve.

The Kerr effect of nonlinearity is due to the dependence of the fiber refractive index on the field intensity. This effect mainly manifests as a new frequency when an optical signal propagates through a fiber. In a single channel the Kerr effect induces a spectral broadening and the phase of the signal is modulated according to its power profile. This effect is called self-phase modulation (SPM). The SPM-induced chirp combines with the linear chirp generated by the chromatic dispersion. If the fiber dispersion coefficient is positive namely in the normal dispersion regime, linear and nonlinear chirps have the same sign while in the anomalous dispersion regime they are of opposite signs. In the former case, pulse broadening is enhanced by SPM while in the later case it is reduced. In the anomalous dispersion case the Kerr nonlinearity induces a chirp that can compensate the degradation induced by GVD. Such a compensation is total if soliton signals are used.

If multichannel WDM signals are considered, the Kerr effect can be more degrading since it induces nonlinear cross-talk among the channels that is known as the cross-phase modulation (XPM). In addition WDM generates new frequencies called the Four-Wave mixing (FWM). The other issue in the WDM system is the collision-induced timing jitter that is introduced due to the collision of solitons in different channels. The XPM causes further nonlinear chirp that interacts with the fiber GVD as in the case of SPM. The FWM is a parametric interaction among waves satisfying a particular relationship called phase-matching that lead to power transfer among different channels.

To limit the FWM effect in a WDM it is preferable to operate with a local high GVD that is periodically compensated by devices having an opposite sign of GVD. One such device is a simple optical fiber with opportune GVD and the method is commonly known as the dispersion management. With this approach the accumulated GVD can be very low and at the same time FWM effect is strongly limited. Through dispersion-management it is possible to achieve highest capacity for both RZ as well as NRZ signals. In that case the overall link dispersion has to be kept very close to zero, while a small amount of chromatic anomalous dispersion is useful for the efficient propagation of a soliton signal. It has been demonstrated that with soliton signals, the dispersion-management is very useful since it reduces the timing jitter [3] and also the pulse interactions. It thus

permits the achievement of higher capacities as compared to the link having constant chromatic dispersion.

In this paper, we are going to study the dynamics of dispersion-managed (DM) solitons propagating through an optical fiber in presence of perturbation terms. We shall consider both Gaussian and super-Gaussian type solitons for completeness.

## 2. GOVERNING EQUATIONS

The relevant equation for the propagation of solitons through an optical fiber in presence of damping and amplification is given by the nonlinear Schrodinger's equation (NLSE) [2, 13] namely:

$$iu_z + \frac{D(z)}{2}u_{tt} + |u|^2 u = -i\Gamma u + i\left[e^{\Gamma z_a} - 1\right]\sum_{n=1}^{N}\delta(z - nz_a)u \qquad (1)$$

Here $\Gamma$ is the normalized loss coefficient, $z_a$ is the normalized characteristic amplifier spacing while $z$ and $t$ represents the normalized propagation distance and the normalized time respectively that is expressed in the usual nondimensional units [2, 12]. Also, $D(z)$ is used to model dispersion-management. We decompose the fiber dispersion $D(z)$ into two components namely a path-averaged constant value $\delta_a$ and a term representing the large rapid variation due to the local values of dispersion [2, 3, 8]. Thus, we write

$$D(z) = \delta_a + \frac{1}{z_a}\Delta(\zeta) \qquad (2)$$

where $\zeta = z/z_a$. The function $\Delta(\zeta)$ is taken to have an average zero over an amplification period, namely

$$\langle\Delta\rangle = \frac{1}{z_a}\int_0^{z_a}\Delta\left(\frac{z}{z_a}\right)dz = 0 \qquad (3)$$

so that the path-averaged dispersion $D$ will have an average $\delta_a$ namely

$$\langle D\rangle = \frac{1}{z_a}\int_0^{z_a} D(z)dz = \delta_a \qquad (4)$$

The proportionality factor in front of $\Delta(\zeta)$, in (2) is chosen so that both $\delta_a$ and $\Delta(\zeta)$ are quantities of order one. In practical situations dispersion-management is often performed by concatenating two or more sections of given length with different values of fiber dispersion.

In the special case of a two-step dispersion map it is convenient to write the dispersion map as a periodic extension of [2, 3, 8]

$$\Delta(\zeta) = \begin{cases} \Delta_1 & : \quad 0 \le |\zeta| < \frac{\theta}{2} \\ \Delta_2 & : \quad \frac{\theta}{2} \le |\zeta| < \frac{1}{2} \end{cases} \tag{5}$$

where $\Delta_1$ and $\Delta_2$ are given by

$$\Delta_1 = \frac{2s}{\theta} \tag{6}$$

and

$$\Delta_2 = -\frac{2s}{1-\theta} \tag{7}$$

with the map strength $s$ defined as

$$s = \frac{\theta\Delta_1 - (1-\theta)\Delta_2}{4} \tag{8}$$

Conversely we have

$$s = \frac{\Delta_1\Delta_2}{4(\Delta_2 - \Delta_1)} \tag{9}$$

and

$$\theta = \frac{\Delta_2}{\Delta_2 - \Delta_1} \tag{10}$$

A typical dispersion map is shown in the following figure:

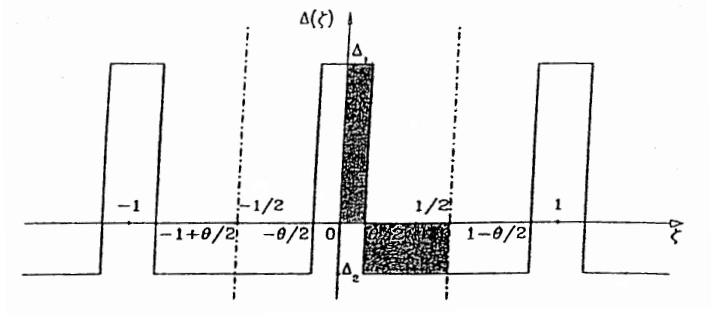


**Figure 1.** Schematic diagram of a two-step map.

We take into account the loss and amplification cycles by looking for a solution of (1) of the form $u(z,t) = Q(z)q(z,t)$ for real $Q$. Taking $Q$ to satisfy

$$Q_z + \Gamma Q - \left[e^{\Gamma z_a} - 1\right] \sum_{n=1}^{N} \delta(z - nz_a)Q = 0 \tag{11}$$

one can show that (1) transforms to

$$iq_z + \frac{D(z)}{2} q_{tt} + g(z)|q|^2 q = 0 \tag{12}$$

where we have

$$g(z) = Q^2(z) = a_0^2 e^{-2\Gamma(z - nz_a)} \tag{13}$$

for $z \in [nz_a, (n+1)z_a]$ and $n > 0$ and also

$$a_0 = \left[ \frac{2\Gamma z_a}{1 - e^{-2\Gamma z_a}} \right]^{\frac{1}{2}} \tag{14}$$

so that over each amplification period we have

$$\langle g(z) \rangle = \frac{1}{z_a} \int_0^{z_a} g(z) dz = 1 \tag{15}$$

Equation (12) is commonly known as the Dispersion-Managed Nonlinear Schrodinger's equation (DMNLSE) and it governs the propagation of a dispersion-managed soliton through a polarization preserving fiber with periodic damping and amplification. This equation is going to be the primary equation of our study in this paper.

In the following figures we have direct numerical simulations of (12). Figure 2 illustrates the profile of the pulse as the map strength($s$) varies from 0 to 16. However, Figures 3(a) and (b) are profiles of DM solitons in the linear and logarithmic scales respectively.

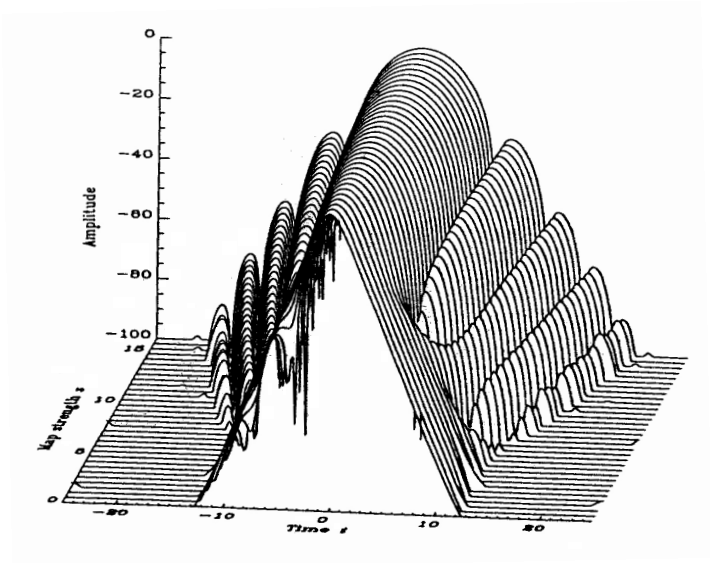


**Figure 2.** Pulse profile.

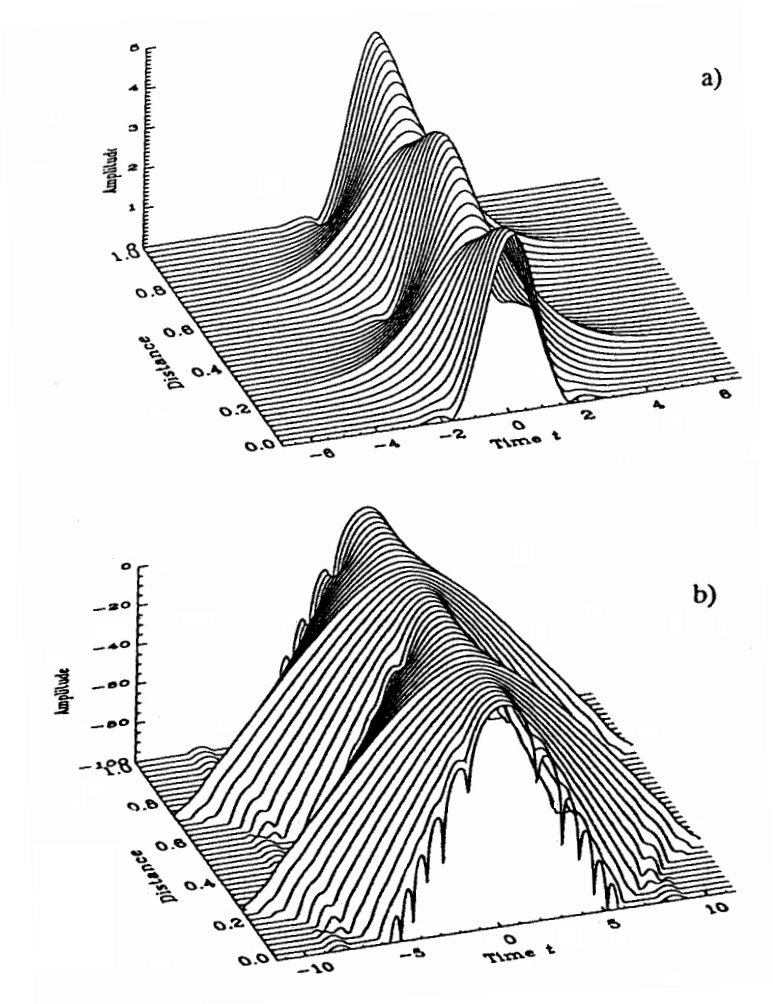


**Figure 3.** DM soliton profile (a) linear scale (b) logarithmic scale.

## 3. PULSE DYNAMICS

In (12), when we have $D(z) = g(z) = 1$, we get the NLSE. It is possible to integrate the NLSE by the method of Inverse Scattering Transform (IST) since NLSE belongs to the category of S-integrable partial differential equations. The IST is the nonlinear analog of Fourier transform that is used to solve linear partial differential equations. Here, (12) is a nonlinear parabolic type equation. Moreover, the NLSE

has an infinite number of conserved quantities. However, (12) as it appears is no longer integrable and it takes us away from the IST picture. Also, (12) does not contain an infinite numbers of integrals of motion either unless $D(z)$ and $g(z)$ are constants in which case one gets infinitely many conserved quantities. In fact, equation (12) has as few as two integrals of motion [6, 7]. They are the energy ($E$), also known as the $L_2$ norm and the linear momentum ($M$) that are respectively given by

$$E = \int_{-\infty}^{\infty} |q|^2 dt \tag{16}$$

and

$$M = \frac{i}{2} D(z) \int_{-\infty}^{\infty} (q q_t^* - q^* q_t) dt \tag{17}$$

The Hamiltonian ($H$) that is given by

$$H = \frac{1}{2} \int_{-\infty}^{\infty} \left( D(z)|q_t|^2 - g(z)|q|^4 \right) dt \tag{18}$$

is, however, not a constant of motion, in general. The case $D(z)$ and $g(z)$ a constant makes the Hamiltonaian a conserved quantity.

We shall now study (12) based on the observation that it supports well-defined chirped soliton solution whose shape is close to that of a Gaussian [15–17]. These pulses deviate from a classical soliton. However, Gaussian pulses have relatively broad leading and trailing edges. As one may expect that dispersion-induced broadening is sensitive to steepness of soliton edges. In general, a soliton with leading and trailing edges broadens more rapidly as it propagates since such a pulse has a wider spectrum to start with. Pulses emitted by directly modulated semiconductor lasers fall in this category and cannot generally be approximated by a Gaussian soliton. A hyper-Gaussian, also known as a super-Gaussian (SG) soliton can be used to model the effects of steep leading and trailing edges on dispersion-induced pulse broadening [4]. It is to be noted here that these pulses are solitary waves and are not strictly solitons as it is not yet established whether they regain their form after interaction. Henceforth, we shall call these solitary waves as simply pulses.

Now, we assume that the solution of (12) is given by a chirped pulse of the form [3, 6, 7, 15–17, 19]

$$\begin{aligned} q(z,t) \;=\; & A(z) f[B(z)\{t - \bar{t}(z)\}] \\ & \exp\left[ iC(z)\{t - \bar{t}(z)\}^2 - i\kappa(z)\{t - \bar{t}(z)\} + i\theta(z) \right] \end{aligned} \tag{19}$$

where $f$ represents the shape of the pulse. It could be a Gaussian type or a SG type pulse. Also, here the parameters

$A(z)$, $B(z)$, $C(z)$, $\kappa(z)$, $\bar{t}(z)$ and $\theta(z)$ respectively represent the soliton amplitude, the inverse width of the pulse, chirp, frequency, the center of the pulse and the phase of the pulse. We shall now derive a set of evolution equations for the pulse parameters. We note that, our approach in this paper is only approximate and does not account for characteristics such as energy loss due to continuum radiation, damping of the amplitude oscillations and changing of the pulse shape. For convenience, we shall now define the following integrals

$$I_{a,b,c,l,m} = \int_{-\infty}^{\infty} \tau^a f^b(\tau) \left(\frac{df}{d\tau}\right)^c \left(\frac{d^2f}{d\tau^2}\right)^l \left(\frac{d^3f}{d\tau^3}\right)^m d\tau \tag{20}$$

$$J_{a,b,c} = \int_{-\infty}^{\infty} \tau^a f^b(\tau) \left(\frac{df}{d\tau}\right)^c \left(\int_{-\infty}^{\tau} f^2(s)ds\right) d\tau \tag{21}$$

where $a$, $b$, $c$, $l$ and $m$ are nonnegative integers. For such a pulse form given by (19), we have the integrals of motion as

$$E = \int_{-\infty}^{\infty} |q|^2 dt = \frac{A^2}{B} I_{0,2,0,0,0} \tag{22}$$

$$M = \frac{i}{2} D(z) \int_{-\infty}^{\infty} (q^* q_t - q q_t^*) dt = -\kappa D(z) \frac{A^2}{B} I_{0,2,0,0,0} \tag{23}$$

while the Hamiltonian is given by

$$\begin{aligned} H &= \frac{1}{2} \int_{-\infty}^{\infty} \left[ D(z)|q_t|^2 - g(z)|q|^4 \right] dt \\ &= \frac{D(z)}{2} \left( A^2 B I_{0,0,2,0,0} + 4\frac{A^2C^2}{B^3} I_{2,2,0,0,0} + \frac{\kappa^2 A^2}{B} I_{0,2,0,0,0} \right) \\ &\quad - \frac{g(z)}{2} \frac{A^4}{B} I_{0,4,0,0,0} \end{aligned} \tag{24}$$

## 4. VARIATIONAL PRINCIPLE

For a finite dimensional problem of a single particle, the temporal development of its position is given by the Hamilton's principle of least action [12]. It states that the action given by the time integral of the Lagrangian is an extremum, namely

$$\delta \int_{t_1}^{t_2} L(x, \dot{x}) dt = 0 \tag{25}$$

where $x$ is the position of the particle and $\dot{x} = dx/dt$. The variational problem (25) then leads to the familiar Euler-Lagrange's (EL) equation

[12]

$$\frac{\partial L}{\partial p} - \frac{d}{dt}\left(\frac{\partial L}{\partial p_z}\right) = 0 \tag{26}$$

where $p$ is one of the six soliton parameters. Here, for (12), the Lagrangian is given by

$$L = \frac{1}{2}\int_{-\infty}^{\infty}\left[i(q^*q_z - qq_z^*) - D(z)|q_t|^2 + g(z)|q|^4\right]dt \tag{27}$$

Now, using (19), the Lagrangian given by (27), reduces to

$$\begin{aligned} L = & -D(z)A^2\left(\frac{B}{2}I_{0,0,2,0,0} + 2\frac{C^2}{B^3}I_{2,2,0,0,0} + \frac{\kappa^2}{2B}I_{0,2,0,0,0}\right) \\ & + \frac{g(z)}{2}\frac{A^4}{B}I_{0,4,0,0,0} - \frac{A^2}{B^3}I_{2,2,0,0,0}\frac{dC}{dz} + \frac{A^2}{B}I_{0,2,0,0,0}\left(\bar{t}\frac{d\kappa}{dz} - \frac{d\theta}{dz}\right) \end{aligned} \tag{28}$$

Substituting $A$, $B$, $C$, $\kappa$, $\bar{t}$ and $\theta$ for $p$ in (26) we arrive at the following set of equations

$$\frac{dA}{dz} = -ACD(z) \tag{29}$$

$$\frac{dB}{dz} = -2BCD(z) \tag{30}$$

$$\frac{dC}{dz} = \left(\frac{B^4}{2}\frac{I_{0,0,2,0,0}}{I_{2,2,0,0,0}} - 2C^2\right)D(z) - \frac{g(z)}{4}A^2B^2\frac{I_{0,4,0,0,0}}{I_{2,2,0,0,0}} \tag{31}$$

$$\frac{d\kappa}{dz} = 0 \tag{32}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z) \tag{33}$$

$$\frac{d\theta}{dz} = \left(\frac{\kappa^2}{2} - \frac{I_{0,0,2,0,0}}{I_{0,2,0,0,0}}B^2\right)D(z) + g(z)\frac{5A^2}{4}\frac{I_{0,4,0,0,0}}{I_{0,2,0,0,0}} \tag{34}$$

Now, from (29) and (30) we conclude that $A = K\sqrt{B}$ where the constant $K$ is proportional to the square root of the energy as seen from (22). So, the number of parameters reduces by one. Thus, (29) through (34), respectively, modify to

$$\frac{dB}{dz} = -2BCD(z) \tag{35}$$

$$\frac{dC}{dz} = \left(\frac{B^4}{2}\frac{I_{0,0,2,0,0}}{I_{2,2,0,0,0}} - 2C^2\right)D(z) - \frac{K^2gB^3}{4}\frac{I_{0,4,0,0,0}}{I_{2,2,0,0,0}} \tag{36}$$

$$\frac{d\kappa}{dz} = 0 \tag{37}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z) \tag{38}$$

$$\frac{d\theta}{dz} = \left(\frac{\kappa^2}{2} - \frac{I_{0,0,2,0,0}}{I_{0,2,0,0,0}}B^2\right) D(z) + g(z)\frac{5KB}{4}\frac{I_{0,4,0,0,0}}{I_{0,2,0,0,0}} \tag{39}$$

### 4.1. Gaussian Pulses

For a pulse of Gaussian type, we substitute $f(\tau) = e^{-\frac{\tau^2}{2}}$. So, the conserved quantities respectively reduce to

$$E = \int_{-\infty}^{\infty} |q|^2 dt = \frac{A^2}{B}\sqrt{\frac{\pi}{2}} = K^2\sqrt{\frac{\pi}{2}} \tag{40}$$

$$\begin{aligned} M &= \frac{i}{2}D(z)\int_{-\infty}^{\infty}(q^*q_t - qq_t^*)dt \\ &= -\kappa D(z)\frac{A^2}{B}\sqrt{\frac{\pi}{2}} = -\kappa D(z)K^2\sqrt{\frac{\pi}{2}} \end{aligned} \tag{41}$$

while the Hamiltonian is

$$\begin{aligned} H &= \frac{1}{2}\int_{-\infty}^{\infty}\left[D(z)|q_t|^2 - g(z)|q|^4\right]dt \\ &= \sqrt{\frac{\pi}{2}}\frac{D(z)}{2}\left(A^2B + \frac{A^2C^2}{B^3} + \frac{\kappa^2A^2}{B}\right) - \frac{\sqrt{\pi}}{4}g(z)\frac{A^4}{B} \\ &= \sqrt{\frac{\pi}{2}}\frac{D(z)}{2}K^2\left(B^2 + \frac{C^2}{B^2} + \kappa^2\right) - \frac{\sqrt{\pi}}{4}g(z)\frac{K^4}{B^3} \end{aligned} \tag{42}$$

Also, the parameter dynamics given by (35) through (39) respectively are

$$\frac{dB}{dz} = -2BCD(z) \tag{43}$$

$$\frac{dC}{dz} = \frac{D(z)}{4}(B^4 - 8C^2) - \frac{g(z)}{4\sqrt{2}}g(z)A^2B^2 \tag{44}$$

$$\frac{d\kappa}{dz} = 0 \tag{45}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z) \tag{46}$$

$$\frac{d\theta}{dz} = \frac{D(z)}{2}(\kappa^2 - B^2) + \frac{5\sqrt{2}}{8}g(z)A^2 \tag{47}$$

Equations (43) to (47) represent the evolution equations of the parameters of a Gaussian soliton propagating through an optical fiber. These evolution equations can be used to study various issues including the pulse interaction.

### 4.2. Super-Gaussian Pulses

For SG pulses we choose $f(\tau) = e^{-\frac{\tau^{2m}}{2}}$ with $m \geq 1$ where the parameter $m$ controls the degree of edge sharpness. With $m = 1$, we recover the case of a chirped Gaussian pulse while for larger values of $m$ the pulse gradually becomes square shaped with sharper laeding and trailing edges [4]. In Figure 4 below, one can see the shapes of the pulses as the parameter $m$ varies.

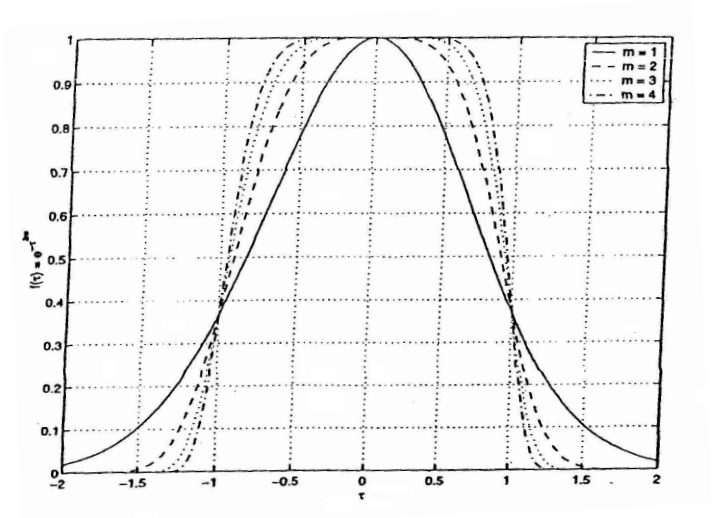


**Figure 4.** SG pulse with the variation of the parameter $m$.

For a SG pulse the integrals of motion respectively are

$$E = \int_{-\infty}^{\infty} |q|^2 dt = \frac{A^2}{B} \frac{1}{m2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) = \frac{K^2}{m2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) \tag{48}$$

$$\begin{aligned} M &= \frac{i}{2} D(z) \int_{-\infty}^{\infty} (q^* q_t - q q_t^*) dt \\ &= -\kappa D(z) \frac{A^2}{B} \frac{1}{m2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) = -\frac{\kappa D(z) K^2}{m2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) \end{aligned} \tag{49}$$

while the Hamiltonian is

$$\begin{aligned} H &= \frac{1}{2}\int_{-\infty}^{\infty}\left[D(z)|q_t|^2 - g(z)|q|^4\right]dt \\ &= D(z)\left[\frac{m}{2^{\frac{2m-1}{2m}}}A^2B\Gamma\left(\frac{4m-1}{2m}\right) + \frac{2^{\frac{2m-3}{2m}}}{m}\frac{A^2C^2}{B^3}\Gamma\left(\frac{3}{2m}\right)\right. \\ &\quad \left. + \frac{1}{2^{\frac{2m+1}{2m}}}\frac{\kappa^2A^2}{B}\Gamma\left(\frac{1}{2m}\right)\right] - g(z)\frac{1}{2^{\frac{m+1}{m}}}\frac{A^4}{B}\Gamma\left(\frac{1}{2m}\right) \\ &= D(z)K^2\left[\frac{m}{2^{\frac{1-2m}{2m}}}B^2\Gamma\left(\frac{4m-1}{2m}\right) + \frac{2^{\frac{2m-3}{2m}}}{m}\frac{C^2}{B^3}\Gamma\left(\frac{3}{2m}\right)\right. \\ &\quad \left. + \frac{1}{2^{\frac{2m+1}{2m}}}\kappa^2\Gamma\left(\frac{1}{2m}\right)\right] - g(z)\frac{1}{2^{\frac{m+1}{m}}}\frac{K^4}{B^3}\Gamma\left(\frac{1}{2m}\right) \end{aligned} \tag{50}$$

Here, $\Gamma(x)$ is the usual gamma function. Also, we have our evolution equations for the pulse parameters (35)–(39) respectively reduce to

$$\frac{dB}{dz} = -2BCD(z) \tag{51}$$

$$\begin{aligned} \frac{dC}{dz} &= \frac{D(z)}{8}\left\{m(2m-1)B^4\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} - 16C^2\right\} \\ &\quad -\frac{g(z)}{2^{\frac{4m+1}{2m}}}A^2B^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \end{aligned} \tag{52}$$

$$\frac{d\kappa}{dz} = 0 \tag{53}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z) \tag{54}$$

$$\begin{aligned} \frac{d\theta}{dz} &= \frac{D(z)}{2}\left\{\kappa^2 - m(2m-1)B^2\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}\right\} \\ &\quad +\frac{5}{2^{\frac{4m+1}{2m}}}g(z)A^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \end{aligned} \tag{55}$$

We note, here, that for $m = 1$, (51)–(55) reduce to (43)–(47) respectively for Gaussian pulses.

## 5. PERTURBATION TERMS

We shall now consider the DM-NLSE along with its perturbation terms that is given by

$$iq_z + \frac{D(z)}{2}q_{tt} + g(z)|q|^2 q = i\epsilon R[q, q^*] \tag{56}$$

Here $R$ is a spatio-differential operator and $\epsilon$ is a perturbation parameter with $0 < \epsilon \ll 1$ and is called the relative width of the spectrum that arises due to quasi-monochromaticity [12]. In presence of the perturbation terms we have the EL equation modify to [6, 17]

$$\frac{\partial L}{\partial p} - \frac{d}{dz}\left(\frac{\partial L}{\partial p_z}\right) = i\epsilon \int_{-\infty}^{\infty} \left(R\frac{\partial q^*}{\partial p} - R^*\frac{\partial q}{\partial p}\right) dt \tag{57}$$

where $p$ represents the six soliton parameters. Once again, substituting $A$, $B$, $C$, $\kappa, \bar{t}$ and $\theta$ for $p$ in (57) we arrive at the following adiabatic evolution equations. This leads to the following adiabatic evolution of the soliton parameters in presence of the perturbation terms.

$$\begin{aligned}
\frac{dA}{dz} &= -ACD(z) + \frac{\epsilon B}{4}\frac{1}{I_{0,2,0,0,0}I_{2,2,0,0,0}} \\
&\quad \int_{-\infty}^{\infty} \left(\tau^2 I_{0,2,0,0,0} - 3I_{2,2,0,0,0}\right)(q^*R + qR^*)dt \qquad (58) \\
\frac{dB}{dz} &= -2BCD(z) + \frac{\epsilon B^2}{2A}\frac{1}{I_{0,2,0,0,0}I_{2,2,0,0,0}} \\
&\quad \int_{-\infty}^{\infty} \left(\tau^2 I_{0,2,0,0,0} - I_{2,2,0,0,0}\right)(q^*R + qR^*)dt \qquad (59) \\
\frac{dC}{dz} &= \left(\frac{B^4}{2}\frac{I_{0,0,2,0,0}}{I_{2,2,0,0,0}} - 2C^2\right) D(z) - g(z)\frac{A^2B^2}{4}\frac{I_{0,4,0,0,0}}{I_{2,2,0,0,0}} \\
&\quad -\frac{i\epsilon}{4}\frac{1}{A}\frac{1}{I_{2,2,0,0,0}}\int_{-\infty}^{\infty}\left[B(qR^* - q^*R) + 2\tau(q_tR^* - q_t^*R)\right]dt \qquad (60) \\
\frac{d\kappa}{dz} &= \frac{\epsilon}{A}\frac{1}{I_{0,2,0,0,0}}\int_{-\infty}^{\infty}[iB(q_tR^* - q_t^*R) - 2\tau C(q^*R + qR^*)]dt \qquad (61) \\
\frac{d\bar{t}}{dz} &= -\kappa D(z) + \frac{\epsilon}{A}\frac{1}{I_{0,2,0,0,0}}\int_{-\infty}^{\infty}\tau(q^*R + qR^*)dt \qquad (62) \\
\frac{d\theta}{dz} &= \left(\frac{\kappa^2}{2} - \frac{I_{0,0,2,0,0}}{I_{0,2,0,0,0}}B^2\right) D(z) + \frac{5g}{4}A^2\frac{I_{0,4,0,0,0}}{I_{0,2,0,0,0}} \\
&\quad +\frac{\epsilon}{2}\frac{1}{A}\frac{1}{I_{0,2,0,0,0}}\int_{-\infty}^{\infty}[3iB(qR^* - q^*R) + 2i\tau(q_tR^* - q_t^*R)
\end{aligned}$$

$$+4\kappa\tau(q^*R + qR^*)]dt \tag{63}$$

We, now, note that equations (58)–(63) can also be rewritten in the following alternative format

$$\begin{aligned}
\frac{dA}{dz} &= -ACD(z) - \frac{\epsilon}{AI_{0,2,0,0,0}I_{2,2,0,0,0}} \\
&\int_{-\infty}^{\infty} \Re[Re^{-i\phi}] \left(I_{0,2,0,0,0}\tau^2 - 3I_{2,2,0,0,0}\right) f(\tau)d\tau \qquad (64) \\
\frac{dB}{dz} &= -2BCD(z) - \frac{\epsilon B}{AI_{0,2,0,0,0}I_{2,2,0,0,0}} \\
&\int_{-\infty}^{\infty} \Re[Re^{-i\phi}] \left(I_{0,2,0,0,0}\tau^2 - I_{2,2,0,0,0}\right) f(\tau)d\tau \qquad (65) \\
\frac{dC}{dz} &= \left(\frac{B^4}{2}\frac{I_{0,0,2,0,0}}{I_{2,2,0,0,0}} - 2C^2\right) D(z) - \frac{gA^2B^2}{4}\frac{I_{0,4,0,0,0}}{I_{2,2,0,0,0}} \\
&-\frac{\epsilon B^2}{2AI_{2,2,0,0,0}} \int_{-\infty}^{\infty} \Im[Re^{-i\phi}] \left(f(\tau) + 2\tau\frac{df}{d\tau}\right) d\tau \qquad (66) \\
\frac{d\kappa}{dz} &= \frac{2\epsilon}{ABI_{0,2,0,0,0}} \int_{-\infty}^{\infty} \left\{B^2\Im[Re^{-i\phi}]\frac{df}{d\tau} - 2C\Re[Re^{-i\phi}]\tau f(\tau)\right\} d\tau \qquad (67) \\
\frac{d\bar{t}}{dz} &= -\kappa D(z) + \frac{2\epsilon}{ABI_{0,2,0,0,0}} \int_{-\infty}^{\infty} \Re[Re^{-i\phi}]\tau f(\tau)d\tau \qquad (68) \\
\frac{d\theta}{dz} &= \left(\frac{\kappa^2}{2} - \frac{I_{0,0,2,0,0}}{I_{0,2,0,0,0}}B^2\right) D(z) + \frac{5gA^2}{4}\frac{I_{0,4,0,0,0}}{I_{0,2,0,0,0}} + \frac{\epsilon}{2ABI_{0,2,0,0,0}} \\
&\int_{-\infty}^{\infty} \left\{B\Im[Re^{-i\phi}] \left(3f(\tau) + 2\tau\frac{df}{d\tau}\right) + 4\kappa\Re[Re^{-i\phi}]\tau f(\tau)\right\} d\tau \qquad (69)
\end{aligned}$$

where we have used the notations

$$\tau = B(z)(t - \bar{t}(z))$$

and

$$\phi = C(z)\left\{t - \bar{t}(z)\right\}^2 - \kappa(z)\left\{t - \bar{t}(z)\right\} + \theta(z)$$

Also $\Re$ and $\Im$ represent the real and imaginary parts respectively. In the following two subsections, we shall obtain the adiabatic dynamics of the soliton parameters due to the Gaussian and SG pulses in presence of the perturbations.

### 5.1. Gaussian Pulses

For Gaussian pulses, we have (63)–(68) respectively simplicify to

$$\frac{dA}{dz} = -ACD(z) - \frac{4\epsilon}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-\frac{3}{2}\tau^2}\Re[Re^{-i\phi}]d\tau \tag{70}$$

$$\begin{aligned}\frac{dB}{dz} &= -2BCD(z) + \frac{8\epsilon}{\sqrt{\pi}}\frac{B}{A}\int_{-\infty}^{\infty} e^{-\frac{3}{2}\tau^2}\Re[Re^{-i\phi}]d\tau \\ &\quad + \frac{2\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{B}{A}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\tau^2}\Re[Re^{-i\phi}]d\tau\end{aligned} \tag{71}$$

$$\begin{aligned}\frac{dC}{dz} &= \frac{D(z)}{4}\left(B^4 - 8C^2\right) - \frac{g(z)}{4\sqrt{2}}g(z)A^2B^2 + \frac{4\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{1}{AB} \\ &\quad \int_{-\infty}^{\infty} te^{-\frac{1}{2}\tau^2}\left\{\tau\Im[Re^{-i\phi}] + B(\tau BC - \kappa)\Re[Re^{-i\phi}]\right\}d\tau \\ &\quad -\frac{2\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{1}{A}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\tau^2}\Im[Re^{-i\phi}]d\tau\end{aligned} \tag{72}$$

$$\begin{aligned}\frac{d\kappa}{dz} &= -\frac{2\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{B}{A}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\tau^2}\Re[Re^{-i\phi}]d\tau + \frac{2\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{1}{AB} \\ &\quad \int_{-\infty}^{\infty} e^{-\frac{1}{2}\tau^2}\left\{\tau\Im[Re^{-i\phi}] + B(\tau BC - \kappa)\Re[Re^{-i\phi}]\right\}d\tau\end{aligned} \tag{73}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z) + \frac{2\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{B}{A}\int_{-\infty}^{\infty} te^{-\frac{1}{2}\tau^2}\Re[Re^{-i\phi}]d\tau \tag{74}$$

$$\frac{d\theta}{dz} = \frac{D(z)}{2}\left(\kappa^2 - B^2\right) + \frac{5\sqrt{2}}{8}g(z)A^2 + \frac{\epsilon\sqrt{2}}{\sqrt{\pi}}\frac{1}{A}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\tau^2}\Im[Re^{-i\phi}]d\tau \tag{75}$$

These equations now represent the evolution equations for the parameters of a Gaussian pulse ropagating through an optical fiber in presence of the perturbation terms.

### 5.2. Super-Gaussian Pulses

For SG pulses (64)–(69) respectively modify to

$$\frac{dA}{dz} = -ACD(z) + \epsilon\frac{m2^{\frac{m+1}{m}}}{\Gamma\left(\frac{1}{2m}\right)}\int_{-\infty}^{\infty} e^{-\frac{3}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau \tag{76}$$

$$\frac{dB}{dz} = -2BCD(z) + \epsilon\frac{m2^{\frac{2m+1}{m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{B}{A}\int_{-\infty}^{\infty} e^{-\frac{3}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau$$

$$+\epsilon\frac{m2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{B}{A}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau \tag{77}$$

$$\frac{dC}{dz} = \frac{D(z)}{8}\left\{m(2m-1)B^4\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}-16C^2\right\}$$
$$-\frac{g(z)}{2^{\frac{4m+1}{2m}}}A^2B^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}+\epsilon\frac{m2^{\frac{3}{2m}}}{\Gamma\left(\frac{3}{2m}\right)}\frac{1}{AB}$$
$$\int_{-\infty}^{\infty}te^{-\frac{1}{2}\tau^{2m}}\left\{m\tau^{2m-1}\Im[Re^{-i\phi}]+B(\tau BC-\kappa)\Re[Re^{-i\phi}]\right\}d\tau$$
$$-\epsilon\frac{m2^{\frac{3-2m}{2m}}}{\Gamma\left(\frac{3}{2m}\right)}\frac{1}{A}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\tau^{2m}}\Im[Re^{-i\phi}]d\tau \tag{78}$$

$$\frac{d\kappa}{dz} = -\epsilon\frac{m2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{B}{A}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau+\epsilon\frac{m2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{1}{AB}$$
$$\int_{-\infty}^{\infty}e^{-\frac{1}{2}\tau^{2m}}\left\{m\tau^{2m-1}\Im[Re^{-i\phi}]+B(\tau BC-\kappa)\Re[Re^{-i\phi}]\right\}d\tau \tag{79}$$

$$\frac{d\bar{t}}{dz} = -\kappa D(z)+\epsilon\frac{m2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{1}{A}\int_{-\infty}^{\infty}te^{-\frac{1}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau \tag{80}$$

$$\frac{d\theta}{dz} = \frac{D(z)}{2}\left\{\kappa^2-m(2m-1)B^2\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}\right\}+\frac{5}{2^{\frac{4m+1}{2m}}}g(z)A^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}$$
$$+\epsilon\frac{m2^{\frac{1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)}\frac{1}{A}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\tau^{2m}}\Re[Re^{-i\phi}]d\tau \tag{81}$$

So, now, these are the adiabatic evolution of the soliton parameters for a SG pulse in presence of the perturbation terms.

## 6. OBSERVATIONS

In this section, we shall take a look into some particular type of perturbations of the GNLSE, namely

$$R = -i(\delta_1+i\delta_2)|q|^{2N}q-i(\alpha_1+i\alpha_2)q_t$$
$$+\beta q_{tt}-i(\gamma_1+i\gamma_2)q_{ttt}-i(\lambda_1+i\lambda_2)\left(|q|^2q\right)_t$$

$$-i(\nu_1+i\nu_2)\left(|q|^2\right)_t q-i\sigma\left(q^2q_t^*\right)_t-i\chi q_t^2q^*-i\omega q^*\left(q^2\right)_{tt}$$
$$-i(P_1+iP_2)q\int_{-\infty}^{t}|q|^2ds-i(Q_1+iQ_2)q_t\int_{-\infty}^{t}|q|^2ds \qquad (82)$$

Here, in (82), we have $\delta_j$, for $j=1,2$, as the coefficient of nonlinear damping or amplification depending on whether $\delta_j<0$ or $>0$. Also, $N$ represents the degree of nonlinear damping (amplification) [7, 18]. For $N=0$, we have $\delta$ is the linear gain or attenuation. For $N=1$, $\delta$ represents a two-photon absorption, or nonlinear gain, while for $N=2$, $\delta$ represents the higher order correction (saturation or loss) to the nonlinear amplification-absorption. The perturbation coefficients of $P_j$ and $Q_j$ for $j=1,2$ are of nonlocal type and represents the gain (loss) saturation [7, 9–11].

In addition to these, the coefficients of $\sigma$, $\chi$ and $\omega$ arise in the context of quasi-solitons [20]. Moreover, $\nu_1$ represents the Raman scattering term while $\nu_2$ is the coefficient of nonlinear dispersion. The necessity of higher order dispersion term arises when the group velocity dispersion [12, 14] is small and also we need this term for performance enhancement for trans-oceanic distances. Thus, the coefficients of $\gamma_1$ and $\gamma_2$ terms are necessary. The terms with $\lambda_1$ and $\lambda_2$ are for self-steepening and the nonlinear dispersion. Also, $\alpha_1$ and $\alpha_2$ are for the dispersion terms too [12].

For these type of perturbations in (82), we have, on using (64)–(69), The adiabatic parameter dynamics of the soliton as

$$\begin{aligned}\frac{dA}{dz}=&-ACD(z)\\
&+\frac{\epsilon}{4}\frac{A^2}{B^2}\frac{1}{I_{2,2,0,0,0}}\Big[2\delta_2A^{2N}B^2I_{2,2N+2,0,0,0}-2\alpha_1\kappa B^2I_{2,2,0,0,0}\\
&-\gamma_1\left\{\left(4\kappa B^2-4\kappa B^4\right)I_{2,0,2,0,0}+\left(4\kappa B^2-2\kappa B^4\right)I_{2,1,0,2,0}\right\}\\
&+\gamma_2\left\{\left(8\kappa C+4\kappa B^2C\right)I_{2,2,0,0,0}+\left(16C+8\kappa B^2C\right)I_{3,1,1,0,0}\right.\\
&\left.+2\kappa^3B^2I_{2,2,0,0,0}+24\kappa C^2I_{4,2,0,0,0}\right\}-2\lambda_1\kappa A^2B^2I_{2,4,0,0,0}\\
&+\beta\left(2B^4I_{2,1,0,1,0}-8C^2I_{4,2,0,0,0}-2\kappa^2B^2I_{2,2,0,0,0}\right)\\
&+8A^2B^2C(\chi-\sigma+3\omega)I_{3,3,1,0,0}+4A^2B^2C(\sigma+2\omega)I_{2,4,0,0,0}\\
&+2P_2A^2BJ_{2,2,0}+4Q_1A^2CJ_{3,2,0}-2Q_1\kappa A^2BJ_{2,2,0}\\
&+2Q_2A^2B^2J_{2,1,1}\Big]-\frac{3\epsilon}{4}\frac{A^2}{B^2}\frac{1}{I_{0,2,0,0,0}}\\
&\cdot\Big[2\delta_2A^{2N}B^2I_{0,2N+2,0,0,0}-2\alpha_1\kappa B^2I_{0,2,0,0,0}\end{aligned}$$

$$
\begin{aligned}
&-\gamma_1\left\{\left(4\kappa B^2-4\kappa B^4\right)I_{0,0,2,0,0,0}+\left(4\kappa B^2-2\kappa B^4\right)I_{0,1,0,2,0}\right\}\\
&+\gamma_2\left\{\left(8\kappa C+4\kappa B^2C\right)I_{0,2,0,0,0,0}+\left(16C+8\kappa B^2C\right)I_{1,1,1,0,0}\right.\\
&\left.+\,2\kappa^3B^2I_{0,2,0,0,0,0}+24\kappa C^2I_{2,2,0,0,0,0}\right\}-2\lambda_1\kappa A^2B^2I_{0,4,0,0,0}\\
&+\beta\left(2B^4I_{0,1,0,1,0}-8C^2I_{2,2,0,0,0,0}-2\kappa^2B^2I_{0,2,0,0,0,0}\right)\\
&+8A^2B^2C(\chi-\sigma+3\omega)I_{1,3,1,0,0}+4A^2B^2C(\sigma+2\omega)I_{0,4,0,0,0}\\
&+2P_2A^2BJ_{0,2,0}+4Q_1A^2CJ_{1,2,0}-2Q_1\kappa A^2BJ_{0,2,0}+2Q_2A^2B^2J_{0,1,1}\Big]
\end{aligned}
\tag{83}
$$

$$
\begin{aligned}
\frac{dB}{dz}=&-2BCD(z)\\
&+\frac{\epsilon}{2}\frac{A}{B}\frac{1}{I_{2,2,0,0,0}}\Big[2\delta_2A^{2N}B^2I_{2,2N+2,0,0,0}-2\alpha_1\kappa B^2I_{2,2,0,0,0}\\
&-\gamma_1\left\{\left(4\kappa B^2-4\kappa B^4\right)I_{2,0,2,0,0}+\left(4\kappa B^2-2\kappa B^4\right)I_{2,1,0,2,0}\right\}\\
&+\gamma_2\left\{\left(8\kappa C+4\kappa B^2C\right)I_{2,2,0,0,0}+\left(16C+8\kappa B^2C\right)I_{3,1,1,0,0}\right.\\
&\left.+\,2\kappa^3B^2I_{2,2,0,0,0}+24\kappa C^2I_{4,2,0,0,0}\right\}-2\lambda_1\kappa A^2B^2I_{2,4,0,0,0}\\
&+\beta\left(2B^4I_{2,1,0,1,0}-8C^2I_{4,2,0,0,0}-2\kappa^2B^2I_{2,2,0,0,0}\right)\\
&+8A^2B^2C(\chi-\sigma+3\omega)I_{3,3,1,0,0}+4A^2B^2C(\sigma+2\omega)I_{2,4,0,0,0}\\
&+2P_2A^2BJ_{2,2,0}+4Q_1A^2CI_{3,2,0}-2Q_1\kappa A^2BJ_{2,2,0}+2Q_2A^2B^2J_{2,1,1}\Big]\\
&-\frac{\epsilon}{2}\frac{A^2}{B^2}\frac{1}{I_{0,2,0,0,0}}\Big[2\delta_2A^{2N}B^2I_{0,2N+2,0,0,0}-2\alpha_1\kappa B^2I_{0,2,0,0,0}\\
&-\gamma_1\left\{\left(4\kappa B^2-4\kappa B^4\right)I_{0,0,2,0,0}+\left(4\kappa B^2-2\kappa B^4\right)I_{0,1,0,2,0}\right\}\\
&+\gamma_2\left\{\left(8\kappa C+4\kappa B^2C\right)I_{0,2,0,0,0}+\left(16C+8\kappa B^2C\right)I_{1,1,1,0,0}\right.\\
&\left.+\,2\kappa^3B^2I_{0,2,0,0,0}+24\kappa C^2I_{2,2,0,0,0}\right\}-2\lambda_1\kappa A^2B^2I_{0,4,0,0,0}\\
&+\beta\left(2B^4I_{0,1,0,1,0}-8C^2I_{2,2,0,0,0}-2\kappa^2B^2I_{0,2,0,0,0}\right)\\
&+8A^2B^2C(\chi-\sigma+3\omega)I_{1,3,1,0,0}+4A^2B^2C(\sigma+2\omega)I_{0,4,0,0,0}\\
&+2P_2A^2BJ_{0,2,0}+4Q_1A^2CJ_{1,2,0}-2Q_1\kappa A^2BJ_{0,2,0}+2Q_2A^2B^2J_{0,1,1}\Big]
\end{aligned}
\tag{84}
$$

$$
\frac{dC}{dz}=\left(\frac{B^4}{2}\frac{I_{0,0,2,0,0}}{I_{2,2,0,0,0}}-2C^2\right)D(z)-\frac{g(z)}{4}A^2B^2\frac{I_{0,4,0,0,0}}{I_{2,2,0,0,0}}
$$

$$
\begin{aligned}
&+\frac{\epsilon}{4}\frac{A}{B^2}\frac{1}{I_{2,2,0,0,0}}\Big[2\delta_1 A^{2N}B^2 I_{0,2N+2,0,0,0}-2\alpha_2\kappa A^2B^2I_{0,4,0,0,0}\\
&+\gamma_1\Big\{\Big(8\kappa C+4\kappa B^2C\Big)I_{0,2,0,0,0}+\Big(16\kappa C+8\kappa B^2C\Big)I_{1,1,1,0,0}\\
&+2\kappa^3B^2I_{0,2,0,0,0}+24\kappa C^2I_{2,2,0,0,0}\Big\}\\
&+\gamma_2\Big\{\Big(4\kappa B^2+2\kappa B^4\Big)I_{0,1,0,1,0}+\Big(4\kappa B^2+4\kappa B^4\Big)I_{0,0,2,0,0}\Big\}\\
&+2\lambda_2\kappa A^2B^2I_{0,4,0,0,0}-4\beta\Big(2B^2CI_{1,1,1,0}+B^2CI_{0,2,0,0,0}\Big)\\
&+\sigma\Big(2A^2B^4I_{0,3,0,1,0}+8A^2C^2I_{2,4,0,0,0}+2\kappa^2A^2B^2I_{0,4,0,0,0}\Big)\\
&+\chi\Big\{\Big(1-2\kappa^2\Big)A^2B^2I_{0,4,0,0,0}-8A^2C^2I_{2,4,0,0,0}-2A^2B^4I_{0,0,2,0,0}\Big\}\\
&-2\omega\Big(24C^2I_{2,4,0,0,0}+6\kappa^2A^2B^2I_{0,4,0,0,0}+A^2B^2I_{0,4,0,0,0}\\
&-2A^2B^4I_{0,2,2,0,0}\Big)+2\Big(P_1A^2BJ_{2,2,0}+Q_1A^2B^2J_{0,1,1}\\
&-2Q_2A^2CJ_{1,2,0}+2Q_2\kappa A^2BJ_{0,2,0}\Big)\Big]\\
&-\frac{\epsilon}{2}\frac{A}{B^4}\frac{1}{I_{3,2,0,0,0}}\Big[4\delta_2A^{2N}B^2CI_{2,2N+2,0,0,0}-8\alpha_1\kappa B^2CI_{2,2,0,0,0}\\
&-\gamma_1\Big(8\kappa B^4CI_{2,0,2,0,0}+24\kappa B^4CI_{2,1,0,1,0}\\
&+8\kappa B^4CI_{1,1,1,0,0}-16\kappa C^3I_{4,2,0,0,0}-16\kappa^3B^2CI_{2,2,0,0,0}\Big)\\
&+2\gamma_2\kappa B^6I_{1,1,0,0,1}-8\lambda_1\kappa A^2B^2CI_{2,4,0,0,0}+8A^4C(\nu_2+\lambda_2)I_{2,3,1,0,0}\\
&-2\beta\Big(2B^4CI_{2,1,0,1}+4B^4CI_{2,0,2,0,0}-2B^4CI_{1,1,1,0,0}\Big)\\
&+\sigma\Big(2A^2B^6I_{1,2,1,1,0}-8A^2B^2C^2I_{2,4,0,0,0}+4A^2B^6I_{1,1,3,0,0}\\
&-16\kappa A^2B^3CI_{2,3,1,0,0}\Big)\\
&2\chi\Big(A^2B^6I_{1,1,3,0,0}+4\kappa A^2B^3CI_{2,3,1,0,0}\Big)-8\omega\kappa A^2B^2CI_{2,4,0,0,0}\\
&+2\Big(P_1A^3B^3J_{1,1,1}+2P_2A^2BCJ_{2,2,0}-P_2\kappa A^2B^2J_{1,2,0}\\
&+Q_1A^2B^4J_{1,0,2}+4Q_1A^2C^2J_{3,2,0}+Q_1\kappa^2A^2B^2J_{1,2,0}\\
&-4Q_2\kappa A^2BCJ_{2,2,0}\Big)\Big) \qquad (85)\\
\frac{d\kappa}{dz}=&-\epsilon\frac{1}{AB^4}\frac{1}{I_{0,2,0,0,0}}\Big\{2\delta_1A^{2N+2}B^5I_{0,2N+1,1,0,0}\\
&-2\delta_2\kappa A^{2N+2}B^4I_{0,2N+2,0,0,0}+\gamma_1\Big\{2A^2B^8I_{0,0,1,0,0}
\end{aligned}
$$

$$
\begin{aligned}
&+8A^2B^4C^2I_{2,0,2,0,0}+2\kappa^2A^2B^6I_{0,0,2,0,0}+24A^2B^4C^2I_{2,1,0,1,0}\\
&+2\kappa^2A^2B^6I_{0,1,0,1,0}+16A^2B^4C^2I_{1,1,1,0,0}+\left(8A^2C^2-\kappa^4\right)B^4I_{0,2,0,0,0}\\
&-32A^2C^2I_{4,2,0,0,0}-24\kappa^2A^2B^2C^2I_{2,2,0,0,0}\Big\}-\gamma_2\Big(8A^2B^6CI_{0,1,0,1,0}\\
&+20A^2B^6CI_{1,0,1,1,0}+4A^2B^6CI_{1,1,0,0,1}+12A^2B^6CI_{0,2,0,0,0}\Big)\\
&+2\lambda_1\left(4A^2B^2C^2I_{2,4,0,0,0}+\kappa^2A^4B^4I_{0,4,0,0,0}+3A^4B^6I_{0,2,2,0,0}\right)\\
&-4\lambda_2A^4B^4CI_{1,3,1,0,0}+4\nu_1A^4B^6I_{0,2,2,0,0}+8\nu_2A^4B^4CI_{1,3,1,0,0}\\
&+8\chi\kappa A^4B^4CI_{1,3,1,0,0}+2\beta\left(\kappa A^2B^6I_{0,1,0,2,0}+2\kappa A^2B^6I_{0,0,2,0,0}\right)\\
&+\sigma\left(4\kappa A^4B^4CI_{0,4,0,0,0}-16\kappa A^4B^4CI_{1,3,1,0,0}\right)\\
&+2\omega\left(A^4B^6I_{0,2,2,0,0}+4A^4B^2C^2I_{2,4,0,0,0}+\kappa^2A^4B^4I_{0,4,0,0,0}\right)\\
&+2\Big(P_1A^4B^4J_{0,1,1}+2P_2A^4B^2CJ_{1,2,0}\\
&-P_2\kappa A^4B^3J_{0,2,0}+Q_1A^4B^5J_{0,0,2}+4Q_1A^4BC^2J_{2,2,0}\\
&+Q_1\kappa^2A^4B^3J_{0,2,0}-4Q_1\kappa A^4B^2CJ_{1,2,0}\Big)\Big]\\
&-2\epsilon\frac{AC}{B^4}\frac{1}{I_{0,2,0,0,0}}\Big[4\alpha_1B^2CI_{2,2,0,0,0}+4\alpha_2B^4I_{1,1,1,0,0}\\
&+\gamma_1\Big\{\left(16B^2C-4B^4C\right)I_{1,1,1,0,0}+\left(8B^2C-8B^4C\right)I_{2,0,2,0}\\
&+\left(8B^2C-4B^4C\right)I_{2,1,0,1,0}\Big\}+\gamma_2\Big\{2B^4I_{1,1,0,0,0}\\
&+\left(2B^4-2B^6\right)I_{1,0,1,1,0}-\left(4\kappa^2B^2+2\kappa^2B^4\right)I_{1,1,1,0,0}\\
&-\left(16C^2+8B^2C^2\right)I_{2,2,0,0,0}-\left(16C^2+8B^2C^2\right)I_{3,1,1,0,0}\\
&-12\kappa^2B^2CI_{2,2,0,0,0}-16C^2I_{4,2,0,0,0}\Big\}+4\lambda_1A^2B^2CI_{2,4,0,0,0}\\
&+8\beta\kappa B^2CI_{2,2,0,0,0}+(4\lambda_2+4\nu_2+4\sigma\kappa-4\chi\kappa-16\omega\kappa)\\
&\cdot A^2B^4I_{1,3,1,0,0}+\Big(2P_2A^2B^2J_{1,2,0}+4Q_1A^2B^2CJ_{2,2,0}\\
&-2Q_1\kappa A^2B^2J_{1,2,0}+2Q_2A^2B^3J_{1,1,1}\Big)\Big]
\end{aligned}
\tag{86}
$$

$$
\begin{aligned}
\frac{d\bar{t}}{dz}&=-\kappa D(z)\\
&+\epsilon\frac{A}{B^4}\frac{1}{I_{0,2,0,0,0}}\Big[4\alpha_1B^2CI_{2,2,0,0,0}+4\alpha_2B^4I_{1,1,1,0,0}\\
&\gamma_1\Big\{\left(16B^2C-8B^4C\right)I_{1,1,1,0,0}+\left(8B^2C-8B^4C\right)I_{2,0,2,0,0}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(8B^2C-4B^4C\right)I_{2,1,0,1,0}\Big\}\gamma_2\Big\{2B^4I_{1,1,0,0,0}\\
&+\left(2B^4-2B^6\right)I_{1,0,1,1,0}-\left(4\kappa^2B^2+2\kappa^2B^4\right)I_{1,1,1,0,0}\\
&-\left(16C^2+8B^2C^2\right)(I_{2,2,0,0,0}+I_{3,1,1,0,0})-12\kappa^2B^2CI_{2,2,0,0,0}\\
&-16C^2I_{4,2,0,0,0}\Big\}+4\lambda_1A^2B^2CI_{2,4,0,0,0}+8\beta\kappa B^2CI_{2,2,0,0,0}\\
&+(4\lambda_2+4\nu_2+4\sigma\kappa-4\chi\kappa-16\omega\kappa)A^2B^4I_{1,3,1,0,0}\\
&+\left(2P_2A^2B^4J_{1,2,0}+4Q_1CJ_{2,2,0}-2Q_1\kappa A^2B^2J_{1,2,0}\right.\\
&+\left.2Q_2A^2B^3J_{1,2,1}\right)\Big] \qquad (87)\\
\frac{d\theta}{dz}=&\left(\frac{\kappa^2}{2}-\frac{I_{0,0,2,0,0}}{I_{0,2,0,0,0}}B^2\right)D(z)+\frac{5}{4}g(z)A^2\frac{I_{0,4,0,0,0}}{I_{0,2,0,0,0}}\\
&-\epsilon\frac{3}{2AB}\frac{1}{I_{0,2,0,0,0}}\Big[2\delta_1A^{2N+2}B^2I_{0,2N+2,0,0,0}+2\alpha_2\kappa A^4B^2I_{0,4,0,0,0}\\
&+\gamma_1\Big\{2\kappa^3A^2B^2I_{0,2,0,0,0}+24\kappa A^2C^2I_{2,2,0,0,0}\\
&+\left(8\kappa A^2C+4\kappa A^2B^2C\right)I_{0,2,0,0,0}\\
&+\left(16\kappa A^2C+8\kappa A^2B^2C\right)I_{1,1,1,0,0}\Big\}\\
&+\gamma_2\Big\{\left(4\kappa A^2B^2+2\kappa A^2B^4\right)I_{0,1,0,1,0}\\
&+\left(4\kappa A^2B^2+4\kappa A^2B^4\right)I_{0,0,2,0,0}\Big\}\\
&+2\lambda_2\kappa A^4B^2I_{0,4,0,0,0}-4\beta\left(2A^2B^2CI_{1,1,1,0,0}+A^2B^2CI_{0,2,0,0,0}\right)\\
&-\sigma\left(2A^4B^4I_{0,3,0,2,0}+8A^4C^2I_{2,4,0,0,0}+2\kappa^2A^4B^2I_{0,4,0,0,0}\right)\\
&+\chi\left(A^4B^2I_{0,4,0,0,0}-2A^4B^4I_{0,2,2,0,0}-8A^4C^2I_{2,4,0,0,0}\right.\\
&-\left.2\kappa^2A^4B^2I_{0,4,0,0,0}\right)+2\omega\left(A^4B^4I_{0,3,0,2,0}-24A^4C^2I_{2,4,0,0,0}\right.\\
&-6\kappa^2A^4B^2I_{0,4,0,0,0}+A^4B^2I_{0,4,0,0,0}-\left.2A^4B^4I_{0,2,2,0,0}\right)\\
&+2\left(2P_1A^4BJ_{0,2,0}+Q_1A^4B^2J_{0,1,1}-2Q_2A^4CJ_{1,2,0}\right.\\
&+\left.Q_2\kappa A^4BJ_{0,2,0}\right)\Big]-\epsilon\frac{1}{AB^3}\frac{1}{I_{0,2,0,0,0}}\Big[4\delta_2A^{2N+2}B^2CI_{2,2N+2,0,0,0}\\
&-8\alpha_1\kappa A^2B^2I_{2,2,0,0,0}-\gamma_1\Big(8\kappa A^2B^4CI_{2,2,0,0,0}+24\kappa A^2B^4CI_{2,1,1,2,0}\\
&+8\kappa A^2B^4CI_{1,1,1,0,0}-16A^2C^3I_{4,2,0,0,0}-16\kappa^3A^2B^2CI_{2,2,0,0,0}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+2\gamma_2 A^2B^6(\kappa I_{1,1,0,0,1}+5\kappa I_{1,0,1,1})-8\lambda_1\kappa A^4B^2CI_{2,4,0,0,0}\\
&-4\lambda_2\kappa A^4B^4I_{1,3,1,0,0}-4\nu_2\kappa A^4B^4I_{1,3,1,0,0}\\
&-4\beta A^2B^4(CI_{2,1,0,2,0}+2\kappa CI_{2,2,0,0,0}+\kappa CI_{1,1,1,0,0})\\
&+2\sigma A^2B^2\Big(A^2B^4I_{1,2,1,1,0}-2\kappa^2B^2C^2I_{1,3,1,0,0}\\
&-4A^2C^2I_{2,4,0,0,0}+2A^2B^4I_{1,1,3,0,0}+8A^2C^2I_{3,3,1,0,0}\Big)\\
&+2\chi\Big(A^4B^6I_{1,1,3,0,0}-4A^4B^2C^2I_{3,3,1,0,0}-\kappa^2A^4B^4I_{1,3,1,0,0}\Big)\\
&-2\omega\Big(4\kappa A^2CI_{1,4,0,0,0}-2A^4B^4I_{1,3,1,0,0}\Big)\\
&+2\Big(P_1A^4B^3J_{1,1,1}+2P_2A^4BCJ_{1,2,0}-P_2\kappa A^4B^2J_{1,2,0}\\
&+Q_1A^4B^4J_{1,0,2}+4Q_1A^4C^2J_{2,2,0}+Q_1\kappa^2A^4B^2J_{1,3,0}\\
&-4Q_1\kappa A^4BCJ_{2,2,0}\Big)\Big]+\epsilon\frac{2\kappa}{AB^2}\frac{1}{I_{0,2,0,0,0}}\Big[4\alpha_1A^2BCI_{2,2,0,0,0}\\
&+4\alpha_2A^2B^4I_{1,1,1,0,0}+\gamma_1\Big\{\Big(16A^2B^2C-4A^2B^4C\Big)I_{2,2,0,0,0}\\
&+\Big(8A^2B^2C-8A^2B^4C\Big)I_{2,0,2,0,0}\\
&+\Big(8A^2B^2C-4A^2B^4C\Big)I_{2,1,0,2,0}\Big\}+\gamma_2\Big\{2A^2B^4I_{1,1,0,0,1}\\
&-12\kappa^2A^2B^2CI_{2,2,0,0,0}-16A^2C^2I_{4,2,0,0,0}\\
&+\Big(2A^2B^4C-2A^3B^6\Big)I_{1,0,1,1,0}-\Big(4\kappa^2A^2B^2+2\kappa^2A^2B^4\Big)I_{1,1,1,0,0}\\
&-\Big(16A^2C^2+8A^2B^2C^2\Big)(I_{2,2,0,0,0}+I_{3,1,1,0,0})\Big\}\\
&+4\lambda_1A^4B^2CI_{2,4,0,0,0}+8\beta\kappa A^2B^2CI_{2,2,0,0,0}\\
&+(4\lambda_2+4\sigma\kappa+4\nu_2-4\chi\kappa-16\omega\kappa)A^4B^4I_{1,3,1,0,0}\\
&+2P_2A^4B^2J_{1,2,0}+4Q_1A^4BCJ_{2,2,0}\\
&-2Q_1\kappa A^4B^2J_{1,2,0}+2Q_2A^4B^3J_{1,1,1}\Big]
\end{aligned}
\tag{88}
$$

### 6.1. Gaussian Pulses

For Gaussian pulses, we have the adiabatic parameter dynamics of the soliton parameters, given by (83)–(88), as follows

$$
\begin{aligned}
\frac{dA}{dz}=&-ACD(z)\\
&+\frac{\epsilon}{2}\frac{A^2}{B^2}\Big[\delta_2\frac{A^{2N}B^2}{(N+1)^{\frac{3}{2}}}-\alpha_1\kappa B^2
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\gamma_1}{54}\left\{(81+4\sqrt{6})\kappa B^2-(81+8\sqrt{6})\kappa B^4\right\}\\
&+\gamma_2\left(4\kappa C-\kappa B^2C+\kappa^3B^2+9\kappa C^2-6C\right)\\
&-\frac{\sqrt{2}}{4}\lambda_1\kappa A^2B^2-\frac{\beta}{4}\left(B^4+12C^2+4\kappa^2B^2\right)\\
&-\frac{\sqrt{2}}{4}A^2B^2C(3\chi-5\sigma+5\omega)\\
&+\sqrt{2}\left(P_2A^2B-Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&+\sqrt{2}\left(2Q_1A^2C-Q_2\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau^3e^{-\tau^2}\{1+erf(\tau)\}d\tau\Big]\\
&-\frac{3\epsilon}{4}\frac{A^2}{B^2}\Bigg[2\delta_2\frac{A^{2N}B^2}{\sqrt{N+1}}-2\alpha_1\kappa B^2\\
&-\frac{\gamma_1}{9}\left\{(18+8\sqrt{6})\kappa B^2-(18+4\sqrt{6})\kappa B^4\right\}\\
&+2\gamma_2\left(4\kappa C+\kappa^3B^2C+6\kappa C^2-4C\right)\\
&-\beta\left(B^4+4C^2+2\kappa^2B^2\right)-\sqrt{2}A^2B^2C(\chi-3\sigma-\omega)\\
&+\sqrt{2}\left(P_2A^2B-Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&+\sqrt{2}\left(2Q_1A^2C-Q_2A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau\Big] \qquad (89)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dB}{dz}=&-2BCD(z)\\
&+\epsilon\frac{A}{B}\Bigg[\delta_2\frac{A^{2N}B^2}{(N+1)^{\frac{3}{2}}}-\alpha_1\kappa B^2\\
&-\frac{\gamma_1}{54}\left\{(81+24\sqrt{6})\kappa B^2-(81+12\sqrt{6})\kappa B^4\right\}\\
&+\gamma_2\left(4\kappa C-2\kappa B^2C+\kappa^3B^2+9\kappa C^2-6C\right)\\
&-\frac{\beta}{4}\left(B^4+12C^2+4\kappa^2B^2\right)-\frac{\sqrt{2}}{4}A^2B^2C(3\chi-5\sigma-\omega)\\
&+\sqrt{2}\left(P_2A^2B-Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&+\sqrt{2}\left(2Q_1A^2C-Q_2A^2B^2\right)\int_{-\infty}^{\infty}\tau^3e^{-\tau^2}\{1+erf(\tau)\}d\tau\Big]
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\epsilon}{2}\frac{A^2}{B^2}\left[2\delta_2\frac{A^{2N}B^2}{\sqrt{N+1}}-2\alpha_1\kappa B^2\right.\\
&-\frac{\gamma_1}{9}\left\{(18+8\sqrt{6})\kappa B^2-(18+4\sqrt{6})\kappa B^4\right\}\\
&+2\gamma_2\left(4\kappa C+\kappa^3B^2+6\kappa C^2-4C\right)-\sqrt{2}\lambda_1\kappa A^2B^2\\
&-\beta\left(B^4+4C^2+2\kappa^2B^2\right)-\sqrt{2}A^2B^2C(\chi-3\sigma-\omega)\\
&+\sqrt{2}\left(P_2A^2B-Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&\left.+\sqrt{2}\left(2Q_1A^2C-Q_2A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau\right] \qquad (90)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dC}{dz}=&\frac{D(z)}{4}\left(B^4-8C^2\right)-\frac{g(z)}{4\sqrt{2}}g(z)A^2B^2\\
&+\frac{\epsilon}{2}\frac{A}{B^2}\left[2\delta_1\frac{A^{2N}B^2}{\sqrt{N+1}}-\sqrt{2}\alpha_2\kappa A^2B^2\right.\\
&+\gamma_1\left(4\kappa C+2\kappa^3B^2+12\kappa C^2\right)+\gamma_2\kappa B^4+\sqrt{2}\lambda_2\kappa A^2B^2\\
&-\frac{\sigma\sqrt{2}}{4}\left(3A^2B^4-4A^2C^2-4\kappa^2A^2B^2\right)\\
&+\frac{\chi\sqrt{2}}{2}\left\{\left(1-2\kappa^2\right)A^2B^2-2A^2C^2-A^2B^4\right\}\\
&-\frac{\omega\sqrt{2}}{2}\left(12C^2+12\kappa^2A^2B^2+2A^2B^2-A^2B^4\right)\\
&+\frac{1}{2}\left(2\sqrt{2}P_1A^2B+\sqrt{2}Q_2\kappa A^2B\right)\int_{-\infty}^{\infty}e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&\left.-\sqrt{2}\left(Q_1A^2B^2+Q_2A^2C\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau\right]\\
&-\epsilon\frac{A}{B^4}\left[2\delta_2\frac{A^{2N}B^2C}{(N+1)^{\frac{3}{2}}}-4\alpha_1\kappa B^2C\right.\\
&+2\gamma_1\left(2\kappa B^4C+3\kappa C^3+4\kappa^3B^2C\right)+\frac{3}{2}\gamma_2\kappa B^6\\
&-\sqrt{2}\lambda_1\kappa A^2B^2C-\frac{9}{2}\beta B^4C-\frac{3\sqrt{2}}{16}\chi A^2B^6-\sqrt{2}\omega\kappa A^2B^2C\\
&+2\sqrt{2}\left(P_1A^2BC+P_2A^2BC-2Q_2\kappa A^2BC\right)\\
&\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau
\end{aligned}
$$

$$-\sqrt{2}\left(P_2\kappa A^2B^2-Q_1A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau$$
$$+4\sqrt{2}Q_1A^2C^2\int_{-\infty}^{\infty}\tau^3e^{-\tau^2}\{1+erf(\tau)\}d\tau\Bigg] \quad (91)$$

$$\frac{d\kappa}{dz}=\frac{\epsilon}{AB^4}\Bigg[2\delta_2\frac{A^{2N+2}B^4}{\sqrt{N+1}}$$
$$-\alpha_1\left(A^2B^6+2\kappa^2A^2B^4+4A^2B^2C^2\right)$$
$$+\frac{\gamma_1}{2}\left(3A^2B^8-2\kappa^2B^4-24A^2C^2-24\kappa^2A^2B^2C^2\right)$$
$$-\frac{\gamma_2}{2}\left\{8A^2B^6-24\kappa^2A^2B^2C^2-3\left(\sqrt{2}+2\right)A^2B^6C\right\}$$
$$-\frac{\lambda_1\sqrt{2}}{8}\left(4A^4B^2C^2+4\kappa^2A^4B^4+3A^4B^6\right)$$
$$-\frac{\sqrt{2}}{2}\lambda_2A^4B^4C-\frac{\sqrt{2}}{2}\nu_1A^4B^6+\sqrt{2}\nu_2A^4B^4C$$
$$+\sqrt{2}\chi\kappa A^4B^4-\frac{2\beta}{9}(2\sqrt{6}+9)\kappa A^2B^6$$
$$-\frac{\omega\sqrt{2}}{4}\left(A^4B^6+4A^4B^2C^2+8\kappa^2A^4B^4\right)$$
$$-\sqrt{2}\left(P_1A^4B^4+2P_2A^4B^2C-4Q_1\kappa A^4B^2C\right)$$
$$\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau$$
$$+\sqrt{2}\left(P_2\kappa A^4B^3-Q_1\kappa^2A^4B^3\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau$$
$$-\sqrt{2}\left(Q_1A^4B^5+4Q_1A^2BC^2\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\Bigg]$$
$$-2\epsilon\frac{AC}{B^4}\Bigg[2\alpha_1B^2C-2\alpha_2B^4-\frac{\gamma_1}{2}\left(12B^2C+B^4C\right)$$
$$+\frac{\gamma_2}{8}\left(5B^4-2B^6+16\kappa^2B^2+8\kappa^2B^4-64C^2-8B^2C^2-48\kappa^2B^2C\right)$$
$$+\frac{\sqrt{2}}{2}\lambda_1A^2B^2C+4\beta\kappa B^2C-\frac{\sqrt{2}}{2}(\lambda_2+\nu_2+\sigma\kappa-\chi\kappa-4\omega\kappa)A^2B^4$$
$$+\sqrt{2}\left(P_2A^2B^2C-Q_1\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau$$
$$+\sqrt{2}\left(2Q_1A^2B^2C-Q_2A^2B^3\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\Bigg] \quad (92)$$

$$\begin{aligned}
\frac{d\bar{t}}{dz} &= -\kappa D(z) \\
&\quad +\epsilon\frac{A}{B^4}\left[2\alpha_1 B^2C - 2\alpha_2 B^4 - \frac{3\gamma_1}{2}\left(4B^2C - B^4C\right)\right. \\
&\quad +\frac{\gamma_2}{4}\left(B^4 - B^6 + 8\kappa^2B^2 + 4\kappa^2B^4 - 32C^2 - 4B^2C^2 - 24\kappa^2B^2C^2\right) \\
&\quad +\frac{\sqrt{2}}{2}\lambda_1 A^2B^2C + 4\beta\kappa^2C - \frac{\sqrt{2}}{2}(\lambda_2 + \nu_2 + \sigma\kappa - \chi\kappa - 4\omega\kappa)A^2B^4 \\
&\quad +\sqrt{2}\left(P_2A^4B^4 - Q_1\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1 + erf(\tau)\}d\tau \\
&\quad +2\sqrt{2}Q_1C\int_{-\infty}^{\infty}\tau^2 e^{-\tau^2}\{1 + erf(\tau)\}d\tau \\
&\quad \left. -2Q_2A^2B^3\int_{-\infty}^{\infty}\tau^2 e^{-\frac{3}{2}\tau^2}\{1 + erf(\tau)\}d\tau\right] \qquad (93) \\
\frac{d\theta}{dz} &= \frac{D(z)}{2}\left(\kappa^2 - B^2\right) + \frac{5\sqrt{2}}{8}g(z)A^2 \\
&\quad -\frac{3\epsilon}{2}\frac{1}{AB}\left[2\delta_2\frac{A^{2N+2}B^4}{\sqrt{N+1}} - \sqrt{2}\alpha_2A^4B^2\right. \\
&\quad +\gamma_1\left(2\kappa^3A^2B^2 + 12\kappa A^2C^2\right) + \gamma_2\kappa A^2B^4 \\
&\quad -\frac{\sigma\sqrt{2}}{125}\left(36\sqrt{5}A^2B^4 + 125A^4 + 125\kappa^2A^4B^2\right) \\
&\quad +\frac{\chi\sqrt{2}}{4}\left(2A^4B^2 - A^4B^4 - 4A^4C^2\right) \\
&\quad +\frac{\omega\sqrt{2}}{250}\left\{\left(144\sqrt{5} - 125\right)A^4B^4 - 12A^4C^2 - 12\kappa^2A^4B^2 + 2A^4B^2\right\} \\
&\quad +\sqrt{2}\left(P_1A^4B + Q_2\kappa A^4B\right)\int_{-\infty}^{\infty}e^{-\tau^2}\{1 + erf(\tau)\}d\tau \\
&\quad \left. -2\sqrt{2}\left(Q_1A^4B^2 - 2Q_2A^4B^2C\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1 + erf(\tau)\}d\tau\right] \\
&\quad -\frac{\epsilon}{AB^3}\left[2\delta_2\frac{A^{2N+2}B^2C}{(N+1)^{\frac{3}{2}}} - 4\alpha_1\kappa A^2B^2C\right. \\
&\quad +\gamma_1\left(\kappa A^2B^4C + 6\kappa A^2C^3 + 8\kappa^3A^2B^2C\right) + \frac{11\gamma_2}{4}\kappa A^2B^6 \\
&\quad \sqrt{2}\lambda_1\kappa A^4B^2C + \frac{\sqrt{2}}{2}\kappa A^4B^4 + \frac{\sqrt{2}}{2}\nu\kappa A^4B^4 - \frac{2\beta}{27}(4\sqrt{6} + 27)A^2B^4C
\end{aligned}$$

$$
\begin{aligned}
&-\frac{\sigma\sqrt{2}}{16}\left(4\kappa^2A^4B^4-40A^4B^2C^2-7A^4B^6\right)\\
&+\frac{\chi\sqrt{2}}{16}\left(12A^4B^2C^2-3A^4B^6+4\kappa^2A^4B^4\right)\\
&+\frac{\omega\sqrt{2}}{250}\left\{\left(144\sqrt{5}-125\right)A^4B^4-12A^4C^2-12\kappa^2A^4B^2+2A^4B^2\right\}\\
&+\sqrt{2}\left(P_2A^4BC-P_2\kappa A^4B^2+Q_1\kappa^2A^4B^2\right)\\
&\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&+\sqrt{2}\left(2P_2A^4BC-P_2\kappa A^4B^2+Q_1\kappa^2A^4B^2\right)\\
&\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau\Big]\\
&+2\epsilon\frac{\kappa}{AB^3}\Big[2\alpha_1A^2B^2C-2\alpha_2A^2B^4\\
&+\frac{\gamma_1}{27}\left\{(16\sqrt{6}-135)A^2B^2C-(8\sqrt{6}+27)A^2B^4C\right\}\\
&+\frac{\gamma_2}{4}\left(7A^2B^4-56A^2C^2-A^3B^6\right.\\
&\left.+\,8\kappa^2A^2B^2+4\kappa^2A^2B^4-24\kappa^2A^2B^2C-16A^2B^2C^2\right)\\
&+\frac{\sqrt{2}}{2}\lambda_1A^2B^4C+4\beta\kappa A^2B^2C\\
&-\frac{\sqrt{2}}{2}(\lambda_2+\sigma\kappa+\nu_2-\chi\kappa-4\omega\kappa)A^4B^4\\
&+\sqrt{2}\left(P_2A^4B^2-Q_1\kappa^2A^4B^2\right)\int_{-\infty}^{\infty}\tau e^{-\tau^2}\{1+erf(\tau)\}d\tau\\
&+\sqrt{2}\left(2Q_1A^4BC-Q_2A^4B^3\right)\int_{-\infty}^{\infty}\tau^2e^{-\tau^2}\{1+erf(\tau)\}d\tau\Big] \quad (94)
\end{aligned}
$$

where $erf(x)$ stands for the error function of $x$.

### 6.2. Super-Gaussian Pulses

For SG pulses we have the adiabatic parameter dynamics of the parameters as

$$\frac{dA}{dz}=-ACD(z)$$

$$
\begin{aligned}
&+\frac{\epsilon}{4}\frac{A^2}{B^2}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\left[\frac{2\delta_2 A^{2N}B^2}{(N+1)^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right)-2\alpha_1\kappa B^2\Gamma\left(\frac{3}{2m}\right)\right.\\
&-\gamma_1\left\{(2m+1)\left(\kappa B^2-\kappa B^4\right)\Gamma\left(\frac{1}{2m}\right)\right.\\
&\left.+\left(\frac{2}{3}\right)^{\frac{8m-1}{2m}}m(2m-1)(6m^2-5m+2)\left(2\kappa B^2-\kappa B^4\right)\Gamma\left(\frac{2m-1}{2m}\right)\right\}\\
&+\gamma_2\left\{\left(8\kappa C-8\kappa B^2C-24C+2\kappa^3B^2\right)\Gamma\left(\frac{3}{2m}\right)+24\kappa C^2\Gamma\left(\frac{5}{2m}\right)\right\}\\
&-2^{\frac{2m-3}{2m}}\lambda_1\kappa A^2B^2\Gamma\left(\frac{3}{2m}\right)\\
&-\frac{\beta}{2}\left\{(2m-3)B^4\Gamma\left(\frac{1}{2m}\right)+16C^2\Gamma\left(\frac{5}{2m}\right)+4\kappa^2B^2\Gamma\left(\frac{3}{2m}\right)\right\}\\
&-\frac{3}{2^{\frac{1-6m}{2m}}}A^2B^2C(\chi-\sigma+3\omega)\Gamma\left(\frac{3}{2m}\right)\\
&+\frac{1}{2^{\frac{3-4m}{2m}}}A^2B^2C(\sigma+2\omega)\Gamma\left(\frac{3}{2m}\right)\\
&+2\left(P_2A^2B-2Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1A^2C\int_{-\infty}^{\infty}\tau^3e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-2m^2Q_2A^2B^2\int_{-\infty}^{\infty}\tau^{2m+1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg]\\
&+\frac{\epsilon}{4}\frac{A^2}{B^2}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\left[\frac{2\delta_2 A^{2N}B^2}{(N+1)^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right)-2\alpha_1\kappa B^2\Gamma\left(\frac{3}{2m}\right)\right.\\
&-\gamma_1\left\{(2m+1)\left(\kappa B^2-\kappa B^4\right)\Gamma\left(\frac{1}{2m}\right)\right.\\
&\left.+\left(\frac{2}{3}\right)^{\frac{8m-1}{2m}}m(2m-1)(6m^2-5m+2)(2\kappa B^2-\kappa B^4)\Gamma\left(\frac{2m-1}{2m}\right)\right\}\\
&+\gamma_2\left\{\left(8\kappa C-8\kappa B^2C-24C+2\kappa^3B^2\right)\Gamma\left(\frac{3}{2m}\right)+24\kappa C^2\Gamma\left(\frac{5}{2m}\right)\right\}\\
&-2^{\frac{2m-3}{2m}}\lambda_1\kappa A^2B^2\Gamma\left(\frac{3}{2m}\right)\\
&-\frac{\beta}{2}\left\{(2m-3)B^4\Gamma\left(\frac{1}{2m}\right)+16C^2\Gamma\left(\frac{5}{2m}\right)+4\kappa^2B^2\Gamma\left(\frac{3}{2m}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{3}{2^{\frac{1-6m}{2m}}}A^2B^2C(\chi-\sigma+3\omega)\Gamma\left(\frac{3}{2m}\right)\\
&+\frac{1}{2^{\frac{3-4m}{2m}}}A^2B^2C(\sigma+2\omega)\Gamma\left(\frac{3}{2m}\right)\\
&+2m\left(P_2A^2B-2Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1A^2C\int_{-\infty}^{\infty}\tau^3e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-2m^2Q_2A^2B^2\int_{-\infty}^{\infty}\tau^{2m+1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg] \qquad (95)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dB}{dz}=&-2BCD(z)\\
&\frac{\epsilon}{2}\frac{A}{B}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\left[\frac{2\delta_2A^{2N}B^2}{(N+1)^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right)-2\alpha_1\kappa B^2\Gamma\left(\frac{3}{2m}\right)\right.\\
&-\gamma_1\left\{(2m+1)\left(\kappa B^2-\kappa B^4\right)\Gamma\left(\frac{1}{2m}\right)\right.\\
&\left.+\left(\frac{2}{3}\right)^{\frac{8m-1}{2m}}m(2m-1)(6m^2-5m+2)\left(2\kappa B^2-\kappa B^4\right)\Gamma\left(\frac{2m-1}{2m}\right)\right\}\\
&+\gamma_2\left\{\left(8\kappa C-8\kappa B^2C-24C+2\kappa^3B^2\right)\Gamma\left(\frac{3}{2m}\right)+24\kappa C^2\Gamma\left(\frac{5}{2m}\right)\right\}\\
&-2^{\frac{2m-3}{2m}}\lambda_1\kappa A^2B^2\Gamma\left(\frac{3}{2m}\right)-\frac{\beta}{2}\left\{(2m-3)B^4\Gamma\left(\frac{1}{2m}\right)\right.\\
&\left.+16C^2\Gamma\left(\frac{5}{2m}\right)+2\kappa^2B^2\Gamma\left(\frac{3}{2m}\right)\right\}\\
&-\frac{3}{2^{\frac{1-6m}{2m}}}A^2B^2C(\chi-\sigma+3\omega)\Gamma\left(\frac{3}{2m}\right)+\frac{1}{2^{\frac{3-4m}{2m}}}A^2B^2C(\sigma+2\omega)\Gamma\left(\frac{3}{2m}\right)\\
&+2m\left(P_2A^2B-2Q_1\kappa A^2B\right)\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1A^2C\int_{-\infty}^{\infty}\tau^3e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-2m^2Q_2A^2B^2\int_{-\infty}^{\infty}\tau^{2m+1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg]\\
&-\epsilon\frac{A}{B}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\left[\frac{2\delta_2A^{2N}B^2}{(N+1)^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-2\alpha_1\kappa B^2\Gamma\left(\frac{1}{2m}\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\gamma_1}{18}\left\{18(2m+1)\left(4\kappa B^2-4\kappa B^4\right)\Gamma\left(\frac{2m-1}{2m}\right)\right.\\
&\left.+\left(\frac{2}{3}\right)^{\frac{6m-3}{2m}} m(2m-1)(2m-3)(38m-15)(2\kappa B^2-\kappa B^4)\Gamma\left(\frac{2m-3}{2m}\right)\right\}\\
&+\gamma_2\left\{\left(8\kappa C+2\kappa^3B^2-16C\right)\Gamma\left(\frac{1}{2m}\right)+24\kappa C^2\Gamma\left(\frac{3}{2m}\right)\right\}\\
&-2^{\frac{2m-1}{2m}}\lambda_1\kappa A^2B^2\Gamma\left(\frac{1}{2m}\right)-\beta\left\{(2m-1)B^4\Gamma\left(\frac{2m-1}{2m}\right)\right.\\
&\left.+\,8C^2\Gamma\left(\frac{3}{2m}\right)+2\kappa^2B^2\Gamma\left(\frac{1}{2m}\right)\right\}\\
&+\frac{1}{2^{\frac{1-2m}{2m}}}A^2B^2C(\chi+3\sigma+\omega)\Gamma\left(\frac{1}{2m}\right)\\
&+2m\left(P_2A^2B-2Q_1\kappa A^2B\right)\int_{-\infty}^{\infty} ee^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau} e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1A^2C\int_{-\infty}^{\infty}\tau e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau} e^{-s^{2m}}ds\right)d\tau\\
&-\,2m^2Q_2A^2B^2\int_{-\infty}^{\infty}\tau^{2m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau} e^{-s^{2m}}ds\right)d\tau\Big] \qquad (96)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dC}{dz}=&\frac{D(z)}{8}\left\{m(2m-1)B^4\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}-16C^2\right\}-\frac{g(z)}{2^{\frac{4m+1}{2m}}}A^2B^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}\\
&+\frac{\epsilon}{4}\frac{A}{B^2}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\left[\frac{2\delta_1A^{2N}B^2}{(N+1)^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{\alpha_2}{2^{\frac{1-2m}{2m}}}\kappa A^2B^2\Gamma\left(\frac{1}{2m}\right)\right.\\
&+\gamma_1\left\{2\kappa^2B^2\Gamma\left(\frac{1}{2m}\right)+24\kappa C^2\Gamma\left(\frac{3}{2m}\right)\right\}\\
&+\gamma_2(2m-1)\kappa B^4\Gamma\left(\frac{2m-1}{2m}\right)+\frac{\lambda_2\kappa A^2B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&-\sigma\left\{\frac{2m-1}{2^{\frac{2m-1}{2m}}}A^2B^4\Gamma\left(\frac{2m-1}{2m}\right)-\frac{A^2C^2}{2^{\frac{3-8m}{2m}}}\Gamma\left(\frac{3}{2m}\right)-\frac{\kappa^2A^2B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\right\}\\
&+\chi\left\{\frac{1-2\kappa^2}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{A^2C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right.\\
&\left.\left.+\,m(2m-1)A^4B^4\Gamma\left(\frac{2m-1}{2m}\right)\right\}-6\omega\left\{\frac{C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right.
\end{aligned}
$$

$$+\frac{\kappa^2A^2B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{A^2B^2}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{2m-1}{2^{\frac{4m-1}{2m}}}A^2B^4\Gamma\left(\frac{1}{2m}\right)\Bigg\}$$
$$+m\left(2P_1A^2B-Q_2\kappa A^2B\right)\int_{-\infty}^{\infty}e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$-2mQ_2A^2C\int_{-\infty}^{\infty}\tau e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$-2m^2Q_1A^2B^2\int_{-\infty}^{\infty}\tau^{2m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg]$$
$$-\frac{\epsilon}{2}\frac{A}{B^4}\frac{1}{\Gamma\left(\frac{3}{2m}\right)}\Bigg[\frac{4\delta_2A^{2N}B^2C}{(N+1)^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right)-8\alpha_1\kappa B^2C\Gamma\left(\frac{3}{2m}\right)$$
$$+\gamma_1\Bigg\{2(4m-7)\kappa B^4C\Gamma\left(\frac{1}{2m}\right)+16\kappa C^3\Gamma\left(\frac{5}{2m}\right)+16\kappa^3B^2C\Gamma\left(\frac{3}{2m}\right)\Bigg\}$$
$$+\frac{3\gamma_2}{8}(2m-1)^2\Gamma\left(\frac{2m-1}{2m}\right)-\frac{\lambda_1\kappa A^2B^2C}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)$$
$$-\beta(5m+3)B^4C\Gamma\left(\frac{3}{2m}\right)-\sigma\left\{\frac{(2m-1)(4m-3)}{2^{\frac{8m-1}{2m}}}A^2B^6\Gamma\left(\frac{2m-1}{2m}\right)\right.$$
$$\left.+\frac{(2m-1)(4m-1)}{2^{\frac{2m-1}{2m}}}A^2B^6\Gamma\left(\frac{2m-1}{2m}\right)+\frac{A^2B^2C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right\}$$
$$-\chi\frac{(2m-1)(4m-1)}{2^{\frac{8m-1}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right)-\omega\frac{\kappa A^2B^2C}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)$$
$$+2m\left(P_2\kappa A^2B^2-2Q_1\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$+4m\left(P_2A^2BC-2Q_2\kappa A^2BC\right)\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$+4mQ_1A^2C^2\int_{-\infty}^{\infty}\tau^3e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$-2m^2P_1A^2B^3\int_{-\infty}^{\infty}\tau^{2m}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$+2m^3Q_1A^2B^4\int_{-\infty}^{\infty}\tau^{4m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg] \tag{97}$$

$$\frac{d\kappa}{dz}=\epsilon\frac{1}{AB^4}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\Bigg[\frac{2\delta_2\kappa A^{2N+2}B^4}{(N+1)^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)$$
$$\alpha_1\Bigg\{(2m-1)A^2B^6\Gamma\left(\frac{2m-1}{2m}\right)+2\kappa^2A^2B^4\Gamma\left(\frac{1}{2m}\right)$$

$$
\begin{aligned}
&+ 8A^2B^2C^2\Gamma\left(\frac{3}{2m}\right)\Big\} + \frac{\gamma_1}{8}\Big\{A^2B^8(2m-1)(2m-3)(4m-1)\\
&\Gamma\left(\frac{2m-3}{2m}\right) - 4(2m-5)A^2B^4C^2\Gamma\left(\frac{1}{2m}\right)\\
&- \kappa^2B^4\Gamma\left(\frac{1}{2m}\right) - 32A^2C^2\Gamma\left(\frac{5}{2m}\right) - 24\kappa^2A^2B^2C^2\Gamma\left(\frac{3}{2m}\right)\Big\}\\
&+\gamma_2(2m-1)(4m+1)A^2B^6C\Gamma\left(\frac{2m-1}{2m}\right)\\
&+2\lambda_1\left\{\frac{A^2B^2C^2}{2^{\frac{3-4m}{2m}}}\Gamma\left(\frac{3}{2m}\right) + \frac{\kappa^2A^4B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)\right.\\
&\left.+ 3A^4B^6\frac{2m-1}{2^{\frac{6m-1}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right)\right\} - \lambda_1\frac{A^4B^4C}{2^{\frac{2m+1}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&+\nu_1A^4B^6\frac{2m-1}{2^{\frac{2m-1}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right) - \nu_2\frac{A^4B^4C}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&+\frac{\beta}{27}\kappa A^2B^6\left\{\left(\frac{2}{3}\right)^{\frac{4m-3}{2m}}(2m-3)(38m-15)\Gamma\left(\frac{2m-3}{2m}\right)\right.\\
&\left.+ \Gamma\left(\frac{2m-1}{2m}\right)\right\} - \sigma\frac{\kappa A^4B^4C}{2^{\frac{1-6m}{2m}}}\Gamma\left(\frac{1}{2m}\right) - \chi\frac{\kappa A^4B^4C}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&+2\omega\left\{A^4B^6\frac{m(2m-1)}{2^{\frac{6m-1}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right) + \frac{A^4B^2C^2}{2^{\frac{3-4m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right.\\
&\left.+ \frac{\kappa^2A^2B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)\right\}\\
&-2m\left(2P_2\kappa A^4B^3 - Q_1\kappa^2A^4B^3\right)\int_{-\infty}^{\infty}e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4m\left(P_2A^4B^2C - Q_1\kappa A^4B^2C\right)\int_{-\infty}^{\infty}\tau e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+8mQ_1A^2BC^2\int_{-\infty}^{\infty}\tau^2e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-2m^2P_1A^4B^4\int_{-\infty}^{\infty}\tau^{2m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+ 2m^3Q_1A^4B^5\int_{-\infty}^{\infty}\tau^{4m-2}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Big]\\
&-2\epsilon\frac{AC}{B^4}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\left[4\alpha_1B^2C\Gamma\left(\frac{3}{2m}\right) - 2\alpha_2B^4\Gamma\left(\frac{1}{2m}\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
&-\gamma_1\left\{\left(8B^2C-2B^4C\right)-(2m+1)\left(2B^2C-2B^4C\right)\right.\\
&\left.+\,(2m-3)\left(2B^2C-B^4C\right)\right\}\Gamma\left(\frac{1}{2m}\right)\\
&+\frac{\gamma_2}{2}\left\{m(2m-1)(4m-3)B^4\Gamma\left(\frac{2m-1}{2m}\right)\right.\\
&+m(2m-1)(4m-3)B^6\Gamma\left(\frac{2m-1}{2m}\right)-12\kappa^2B^2\Gamma\left(\frac{2m-1}{2m}\right)\\
&+\left(8C^2+4B^2C^2\right)\Gamma\left(\frac{3}{2m}\right)+\left(2\kappa^2B^2+\kappa^2B^4\right)\Gamma\left(\frac{1}{2m}\right)\\
&\left.-16C^2\Gamma\left(\frac{1}{2m}\right)\right\}-\lambda_1\frac{A^2B^2C}{2^{\frac{3-4m}{2m}}}\Gamma\left(\frac{3}{2m}\right)+8\beta\kappa B^2C\Gamma\left(\frac{3}{2m}\right)\\
&+(\lambda_2+\nu+\sigma\kappa-\chi\kappa-4\omega\kappa)\frac{A^2B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{3}{2m}\right)\\
&+2m\left(P_2A^2B^2C-Q_1\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1A^2B^2C\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-\,2m^2Q_2A^2B^3\int_{-\infty}^{\infty}\tau^{2m}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Big] \qquad (98)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\bar{t}}{dz}=&-\kappa D(z)\\
&+\epsilon\frac{A}{B^4}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\left[4\alpha_1B^2C\Gamma\left(\frac{1}{2m}\right)-2\alpha_2B^4\Gamma\left(\frac{1}{2m}\right)\right.\\
&+\gamma_1\left\{\left(8B^2C-4B^4C\right)-(2m+1)\left(2B^2C-2B^4C\right)\right.\\
&\left.-\,(2m-3)B^2C\right\}\Gamma\left(\frac{1}{2m}\right)\\
&+\frac{\gamma_2}{2}\left\{3(2m-1)(4m-3)B^4\Gamma\left(\frac{2m-1}{2m}\right)\right.\\
&+m(2m-1)B^6\Gamma\left(\frac{2m-1}{2m}\right)+\left(2\kappa^2B^2+\kappa^2B^4\right)\Gamma\left(\frac{1}{2m}\right)\\
&\left.+\left(8C^2+4B^2C-12\kappa^2B^2C\right)\Gamma\left(\frac{3}{2m}\right)-16C^2\Gamma\left(\frac{5}{2m}\right)\right\}\\
&+\lambda_1\frac{A^2B^2C}{2^{\frac{3-4m}{2m}}}\Gamma\left(\frac{3}{2m}\right)+8\beta\kappa B^2C\Gamma\left(\frac{3}{2m}\right)
\end{aligned}
$$

$$
\begin{aligned}
&-A^2B^4(\lambda_2+\nu_2+\sigma\kappa-\chi\kappa-4\omega\kappa)\frac{1}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&+2m\left(P_2A^2B^4-Q_1\kappa A^2B^2\right)\int_{-\infty}^{\infty}\tau e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&+4mQ_1C\int_{-\infty}^{\infty}\tau^2e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\\
&-2m^2Q_2A^2B^3\int_{-\infty}^{\infty}\tau^{2m}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg] \qquad (99)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\theta}{dz}=&\frac{D(z)}{2}\left\{\kappa^2-m(2m-1)B^2\frac{\Gamma\left(\frac{2m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)}\right\}+\frac{5}{2^{\frac{4m+1}{2m}}}g(z)A^2\frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}\\
&-\frac{3\epsilon}{2}\frac{1}{AB}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\Bigg[\frac{2\delta_1A^{2N+2}B^2}{(N+1)^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{\alpha_2}{2^{\frac{1-2m}{2m}}}\kappa A^4B^2\Gamma\left(\frac{1}{2m}\right)\\
&+\gamma_1\left\{2\kappa^3A^2B^2\Gamma\left(\frac{1}{2m}\right)+24\kappa A^2C^2\Gamma\left(\frac{3}{2m}\right)\right\}\\
&+\gamma_2(2m-1)\kappa A^2B^4\Gamma\left(\frac{2m-1}{2m}\right)+\lambda_2\frac{\kappa A^4B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&+\sigma\left\{\frac{A^2B^4}{25}\left(\frac{2}{5}\right)^{\frac{4m-3}{2m}}m(2m-1)(2m-3)(3m+4)\Gamma\left(\frac{2m-3}{2m}\right)\right.\\
&\left.-\frac{A^4C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)+\frac{\kappa^2A^4B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\right\}\\
&+\chi\left\{\frac{A^4B^2}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{m(2m-1)}{2^{\frac{4m-1}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right)\right.\\
&\left.-\frac{A^4C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)-\frac{\kappa^2A^4B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{2m-1}{2m}\right)\right\}\\
&-2\omega\left\{\frac{A^2B^4}{25}\left(\frac{2}{5}\right)^{\frac{4m-3}{2m}}m(2m-1)(2m-3)(3m+4)\Gamma\left(\frac{2m-3}{2m}\right)\right.\\
&+\frac{3A^4C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)-3\frac{3\kappa^2A^4B^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{1}{2m}\right)\\
&\left.-\frac{A^4B^2}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)-\frac{2m-1}{2^{\frac{4m-1}{2m}}}A^4B^4\Gamma\left(\frac{2m-1}{2m}\right)\right\}\\
&+m\left(2P_1A^4B-Q_2\kappa A^4B\right)\int_{-\infty}^{\infty}e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau
\end{aligned}
$$

$$+4mQ_2A^4C\int_{-\infty}^{\infty}\tau e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$-2m^2Q_1A^4B^2\int_{-\infty}^{\infty}\tau^{2m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg]$$
$$-\epsilon\frac{1}{AB^3}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\left[\frac{4\delta_2A^{2N+2}B^2C}{(N+1)^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right)-8\alpha_1\kappa A^2B^2C\Gamma\left(\frac{3}{2m}\right)\right.$$
$$-\gamma_1\left\{2(4m-1)\kappa A^2B^4C\Gamma\left(\frac{1}{2m}\right)-16\kappa^3A^2B^2C\Gamma\left(\frac{3}{2m}\right)\right.$$
$$\left.-16\kappa A^2C^3\Gamma\left(\frac{5}{2m}\right)\right\}-\gamma_2(2m-1)(4m-3)\kappa A^2B^6\Gamma\left(\frac{2m-1}{2m}\right)$$
$$-\lambda_1\frac{\kappa A^4B^2C}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)+\lambda_2\frac{\kappa A^4B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)+\nu_2\frac{\kappa A^4B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)$$
$$-2\beta\left\{\left(\frac{2}{3}\right)^{\frac{8m-1}{2m}}m(2m-1)\left(6m^2-5m+2\right)A^4B^4C\Gamma\left(\frac{2m-3}{2m}\right)\right.$$
$$\left.+3A^4B^4C\Gamma\left(\frac{3}{2m}\right)\right\}-\sigma\left\{\frac{(2m-1)(4m-3)}{2^{\frac{8m-1}{2m}}}A^4B^6\Gamma\left(\frac{8m-1}{2m}\right)\right.$$
$$-\frac{\kappa^2A^4B^4}{2^{\frac{2m+1}{2m}}}A^2B^6\Gamma\left(\frac{1}{2m}\right)+\frac{A^4B^2C^2}{2^{\frac{3-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)$$
$$\left.+\frac{(2m-1)(4m-1)}{2^{\frac{6m-1}{2m}}}A^4B^6\Gamma\left(\frac{2m-1}{2m}\right)+\frac{3A^4B^2C^2}{2^{\frac{1-6m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right\}$$
$$-2\chi\left\{\frac{(2m-1)(4m-1)}{2^{\frac{10m-1}{2m}}}A^4B^6\Gamma\left(\frac{2m-1}{2m}\right)-\frac{3A^4B^2C^2}{2^{\frac{1-2m}{2m}}}\Gamma\left(\frac{3}{2m}\right)\right.$$
$$\left.+\frac{\kappa^2A^4B^4}{2^{\frac{4m+1}{2m}}}\Gamma\left(\frac{1}{2m}\right)\right\}-\omega\frac{A^4B^4}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right)$$
$$+2m\left(2P_2A^4BC-P_2\kappa A^4B^2+2Q_1\kappa^2A^4B^2\right)$$
$$\int_{-\infty}^{\infty}\tau e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$+8m\left(Q_1A^4C^2-Q_1\kappa A^4BC\right)\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau$$
$$+2m^3\left(P_1A^4B^3+Q_1A^4B^4\right)\int_{-\infty}^{\infty}\tau^{4m-1}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg]$$
$$+2\epsilon\frac{\kappa}{AB^3}\frac{1}{\Gamma\left(\frac{1}{2m}\right)}\left[4\alpha_1A^2B^2C\Gamma\left(\frac{3}{2m}\right)-2\alpha_2A^2B^4\Gamma\left(\frac{1}{2m}\right)\right.$$

$$
\begin{aligned}
&-\gamma_1 \left\{ \left(8A^2B^2C - 2A^2B^4C\right) \Gamma\left(\frac{1}{2m}\right) \right. \\
&-(2m+1)\left(2A^2B^2C-2A^2B^4C\right)\Gamma\left(\frac{1}{2m}\right)+\left(\frac{2}{3}\right)^{\frac{8m-1}{2m}} m(2m-1) \\
&\left.\left(6m^2-5m+2\right)\left(4A^2B^2C-2A^2B^4C\right)\Gamma\left(\frac{2m-1}{2m}\right)\right\} \\
&-2\gamma_2 \left\{ m(2m-1)(4m-3)A^2B^4\Gamma\left(\frac{2m-1}{2m}\right) \right. \\
&+2\left(2\kappa^2A^2B^2+\kappa^2A^2B^4\right)\Gamma\left(\frac{1}{2m}\right) \\
&-4\left(2A^2C^2+2A^2B^2C^2+6\kappa^2A^2B^2C^2\right)\Gamma\left(\frac{3}{2m}\right) \\
&\left.+\ 32A^2C^2\Gamma\left(\frac{5}{2m}\right)\right\} + \lambda_1 \frac{A^4B^2C}{2^{\frac{3-4m}{2m}}}\Gamma\left(\frac{3}{2m}\right) + 8\beta\kappa A^2B^2C\Gamma\left(\frac{3}{2m}\right) \\
&-(\lambda_2+\kappa\sigma+\nu_2-\chi\kappa-4\omega\kappa)A^4B^4\frac{1}{2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right) \\
&+2m\left(P_2A^4B^2-Q_1\kappa A^4B^2\right)\int_{-\infty}^{\infty}\tau e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau \\
&+4mQ_1A^4BC\int_{-\infty}^{\infty}\tau^2e^{-2\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau \\
&-\ 2m^2Q_2A^4B^3\int_{-\infty}^{\infty}\tau^{2m}e^{-\tau^{2m}}\left(\int_{-\infty}^{\tau}e^{-s^{2m}}ds\right)d\tau\Bigg] \qquad (100)
\end{aligned}
$$

## 7. CONCLUSIONS

In this paper, we have studied the perturbations of solitons propagating through an optical fiber with strong dispersion-management. The adiabatic dynamics of the soliton parameters in presence of these perturbations are obtained. In particular, we have considered both the Gaussian as well as the SG type pulses. One can use these adiabatic parameter dynamics to study number of aspects of dynamics of optical solitons propagating through an optical fiber namely the four-wave mixing, the collision induced frequency and timing jitter just to name a few.

In reality, besides the solitons, one obtains the small amplitude dispersive waves commonly known as radiations. The mathematical expressions for radiations due to the type of perturbations, considered in this paper, have not been obtained. However, such studies are under way and the results will be reported in a future publication.

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**Anjan Biswas** obtained his B.Sc. (Honors) degree in Mathematics from St. Xavier's College, Calcutta, India in 1986. After that, he got his M.Sc. and M.Phil. in Applied Mathematics from the University of Calcutta, India in 1988 and 1989 respectively. Subsequently, he obtained MA and Ph.D. in Applied Mathematics from the University of New Mexico in Albuquerque, New Mexico, USA in 1993 and 1998 respectively. Then, he was an Instructor in the Department of Mathematics at Valley City State University in North Dakota from 1998–1999. During 1999–2001 he was a Research Associate in the Department of Applied Mathematics at the University of Colorado, Boulder, Colorado. At present, he is an Assistant Professor in the Department of Physics and Mathematics of Tennessee State University in Nashville, Tennessee. His current research interest is on optical solitons, non-Kerr law solitons, theory of solitons, solitary waves, quasilinear pulses and the theory of compactons.