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MALVAR WAVELET BASED POCKLINGTON EQUATION SOLUTIONS TO THIN-WIRE ANTENNAS AND SCATTERERS

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Abstract—Malvar wavelets are often referred to as smooth local cosine (SLC) functions. In this paper the SLC functions are employed as the basis and testing functions in the Galerkin-based Method of Moments (MoM) for the Pocklington equation of thin-wire antennas and scatterers. The SLC system has rapid convergence and is particularly suitable to handle electrically large scatterers, where the integral kernel behaves in a highly oscillatory manner. Numerical examples demonstrate the scattering of electromagnetic waves from a thin-wire scatterer as well as wave radiation from the gull-shaped antenna. A comparison of the new approach versus the traditional MoM is provided.

1. INTRODUCTION

Recently, wavelets found their application in solving integral equations, resulting in sparse impedance matrices [1–4]. This is due to features of vanishing moments, orthogonality and multiresolution analysis in wavelets. Despite the above nice properties, standard wavelets are defined on the whole real line, while practical electromagnetic problems are often restricted to a finite interval or domain. To incorporate structures with physical constraints, modified wavelets, including periodic wavelets and intervallic wavelets, were introduced [5, 6]. In this article we use the Malvar wavelets, namely the SLC bases [7] for a thin-wire scatterer and antenna. The SLC basis has been previously employed in [8] for the modeling of 2D scatterers.

Smooth local trigonometric systems (SLT) were proposed by Malvar [9] and followed by Coifman and Meyer [10]. They are trigonometric functions multiplied by a smooth bell shaped window and form an orthogonal basis in the L^2. Similar to wavelets, the SLT system constructs its basis functions utilizing both translation and dilation of a single function. However, the construction is in a more flexible manner, thereby overcoming the inconvenience of conventional wavelets in handling the end points of non-periodic functions. The basic idea of SLT is to use smooth cutoff functions to split the function and to fold overlapping parts back into the intervals of interest, so that the orthogonality of the system is preserved. Moreover, by choosing the correct trigonometric basis, rapid convergence in the case of smooth functions is ensured. Intuitively, one may use a relatively small number of the SLT bases (in comparison to the number of pulse basis) to cover dominating spectral components of the unknown spatial current of the scatterer. In addition, the SLT allows the usage of the FFT-like fast numerical technique, e.g., the fast discrete cosine transform (DCT) for all numerical integrations. Hence, accurate and fast algorithm can be developed.

In this paper we apply the SLC functions to integral equations to solve thin-wire scattering and radiation problems. If scatterer or antenna consists of several segments, we divide the contour into pieces according to the geometric and physical nature of the problem. The SLC bases are allocated on each segment and overlapping with the SLC bases of the neighboring segments, so that the continuity of the solution is guaranteed.

Numerical examples of a thin-wire scatterer and antenna are presented. The results are compared with those obtained by using the standard pulse-based MoM [11]. It has been noticed, that the use of the fast discrete cosine transform DCT-IV [12] can drastically reduce

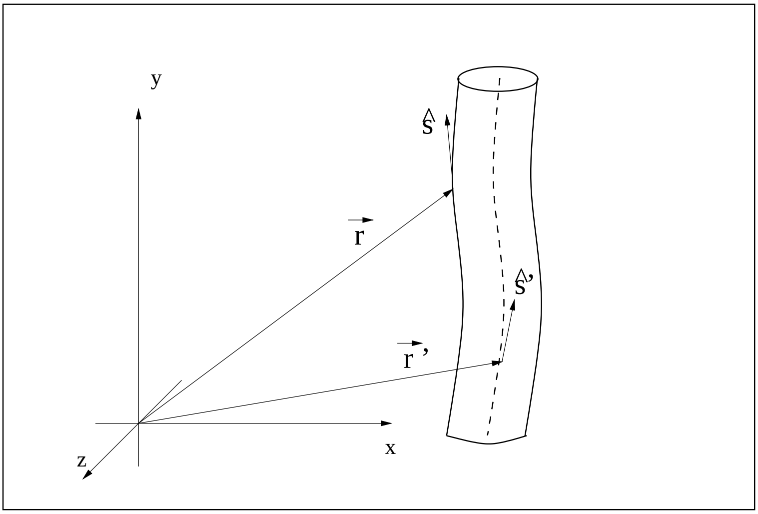

Figure 1. The thin-wire scatterer.

computational time and increase accuracy of numerical calculations.

2. FORMULATION OF PROBLEM

As an example of a scattering problem, which is solved using the SLC-based MoM, let us consider the scattering from a curved thin wire, shown in Figure 1. The problem under consideration is described in terms of the generalized Pocklington electric field integral equation (see [13]):

$$\sum_{l=1}^{L}\int_{C_l} J_l(\vec{r}\,')\left(\frac{\partial^2}{\partial s\partial s'}-\kappa^2\hat{s}\cdot\hat{s}'\right)\cdot G(\vec{r},\vec{r}\,')ds' = j\omega\epsilon\hat{s}\cdot\vec{E}^i(\vec{r}\,') \qquad (1)$$

where J_l is the current over a l-th wire. Parameter L denotes the number of thin wires, $\vec{E}^i$ is the incident excitation field, s and s' are the length variables. Unit vectors $\hat{s}$ and $\hat{s}'$ are tangent vectors of the wires at $\vec{r}$ and $\vec{r}\,'$, respectively. The function $G(\vec{r},\vec{r}\,')$ is the free-space Green's function given by

$$G(\vec{r},\vec{r}\,') = \frac{e^{-j\kappa|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|}. \qquad (2)$$

To avoid singularity of the Green's function during the impedance matrix calculation in the MoM, the observation point $\vec{r}$ is always assumed to be on the wire surfaces and the source point $\vec{r}\,'$ is on the wire axis as shown in Figure 1.

After one obtains the induced electric current J_l numerically over a thin-wire scatterer or antenna using (1) with a given excitation $\vec{E}^i(\vec{r})$, far-zone parameters, such as electromagnetic fields, radar cross section and antenna pattern, can be easily calculated.

3. SLC-BASED METHOD OF MOMENTS

The SLC-based MoM has been introduced in [8]. First, we map a curved scatterer or antenna onto a straight interval, where the unknown current J_s is defined. Then, a straight interval is split into subintervals I_j $(j = 0, 1, \ldots, M-1)$. We use N_j SLCs on each j-th subintervals to expand the unknown current J_s in the form

$$J_s(t) = \sum_{j=0}^{M-1} \sum_{k=0}^{N_j-1} q_{j,k}\psi_{j,k}(t), \tag{3}$$

where the basis functions $\{\psi_{j,k}(t)\}$ are defined as

$$\psi_{j,k}(t) = b_j(t)\sqrt{\frac{2}{|I_j|}}\cos\left[\frac{\pi}{|I_j|}\left(k+\frac{1}{2}\right)(t-\alpha_j)\right]. \tag{4}$$

with the same bell-shaped window $b_j(t)$ as in [8]. Figure 2 shows an example of two SLCs, defined on adjacent intervals.

The unknown coefficients $\{q_{j,k}\}$ are found using the Galerkin MoM [11]. The same functions as in (4) are chosen to be the testing functions for the MoM. The elements of the impedance matrix are formed by the double integration

$$A_{k+j\cdot N_j,k'+j'\cdot N_{j'}} = \iint K(t,t')\psi_{j',k'}(t')\psi_{j,k}(t)dt'dt, \tag{5}$$

where N_j denotes the number of frequency components used on the interval $I_j = [\alpha_j, \alpha_{j+1}]$ to approximate the unknown current, and $K(t,t')$ is the kernel of the integral equation (1).

The double integral (5) is evaluated by the 2D fast discrete cosine transform DCT-IV [12]. The fast DCT-IV provides us with the opportunity to generate the impedance matrix of the MoM very rapidly and accurately.

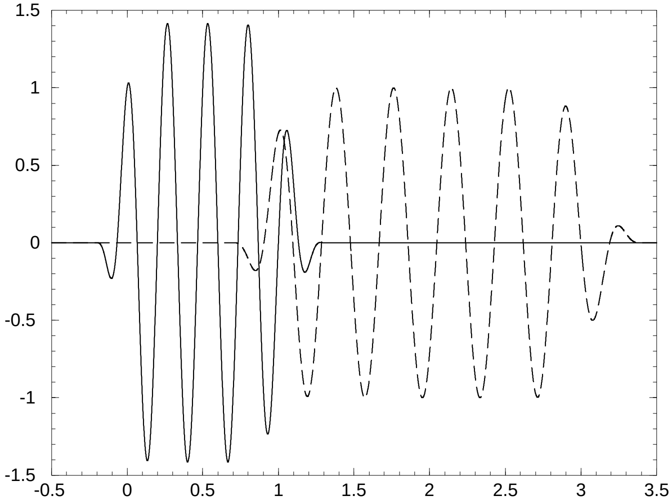

Figure 2. Two local cosine basis functions with different I_j and ϵ_j, defined on adjacent intervals.

4. NUMERICAL RESULTS

Example 1. Straight-wire scatterer

As the first example of a thin-wire scatterer we consider the simple straight wire scatterer of length $l = \lambda$ and radius $r = 0.01348\lambda$. The scatterer is excited by a plane wave which has $45°$ angle of incidence. This example has been taken from the well-known book [11].

Figure 3 shows the induced current distribution. It has been found that we need 70 pulse functions to achieve stable numerical solution. At the same time, only 20 SLCs has been used. First approach runs for 15.7 seconds on the standard PC computer with the AMD 400 MHz CPU, 256 Mb memory and Linux operational system. The new technique needs only 1.05 seconds to compute solution. The gain in the computational time is approximately $15.7/1.05 \approx 15$.

Example 2. Thin-wire scatterer

In this example we consider scattering from the thin-wire scatterer, shown in Figure 4. This example is taken from [3]. The thin-wire scatterer consists of two elliptic arc wires of radius 0.01λ with the following parameters: $a = 1.6\lambda$, $b = 0.8\lambda$. The scatterer is excited by a plane wave. The integral equation (1) is solved using pulse- and SLC-based Galerkin MoM.

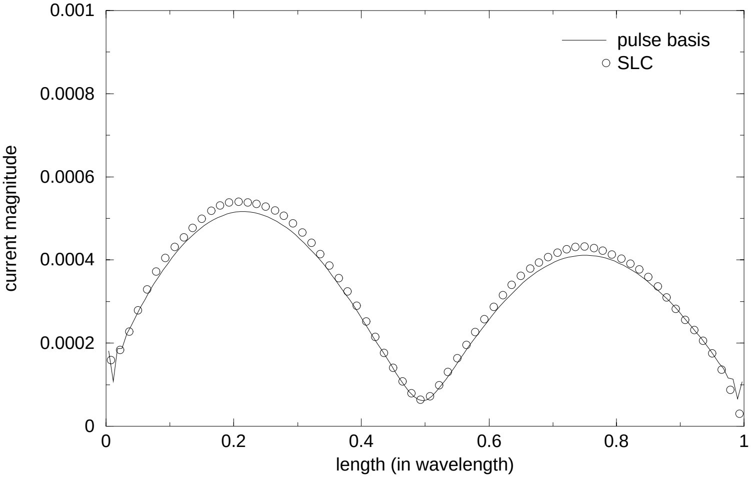

Figure 3. The normalized current magnitude: straight-wire scatterer.

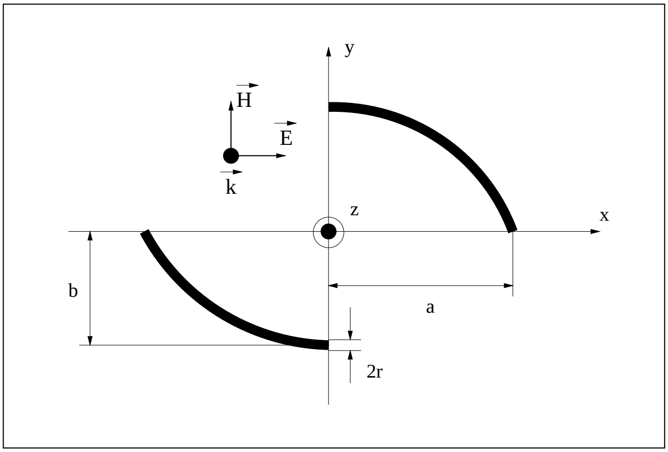

Figure 4. The thin-wire scatterer.

In Figure 5 we plot results using pulse and SLC bases for the MoM. Due to the symmetry, the induced current is depicted versus the normalized arclength only for one wire. The normalized arclength starts at the major axis and stops at the minor axis.

It has been found numerically, that we need at least 128 pulses per wire to obtain a stable solution, which is in a good agreement with the results published in [3]. The corresponding computation time is

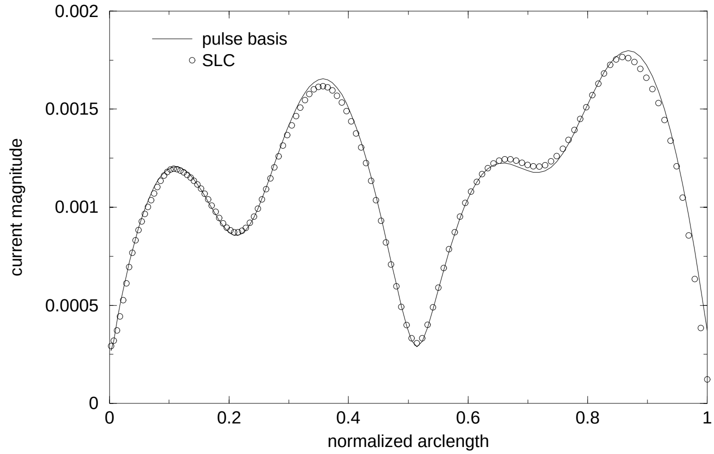

Figure 5. The normalized current magnitude versus normalized arclength: thin-wire scatterer.

193.31 seconds. At the same time, we need only 24 SLCs per wire to reach an accurate numerical result (see Figure 5). The corresponding CPU time here is only 6.94 seconds. This makes the SLC based MoM $193.31/6.94 \approx 27$ times faster than the standard Galerkin MoM with pulses. In addition to that, we need approximately 5 times less unknowns.

Example 3. Straight-wire antenna

As the third example we consider the simple straight-wire antenna of length $l = 2\lambda$ and radius $r = 0.01348\lambda$. This simple example has been taken from [11].

Figure 6 shows the normalized current magnitude for this straight-wire antenna. Due to symmetry, only half of the current distribution is shown.

In this example, we found that we need at least 140 pulses to obtain numerically stable solution. At the same time we observed, that the minimum number of required SLCs is equal to 60. The computation time for the pulse-based MoM is 64.9 seconds and for the SLC's based MoM is 9 seconds. Therefore, we conclude that here the new approach is $64.9/9 \approx 7$ faster than the standard pulse-based MoM.

Example 4. Gull-shaped antenna

In the last example, we will model the gull-shaped piecewise linear antenna as shown in Figure 7. Antenna has the following parameters:

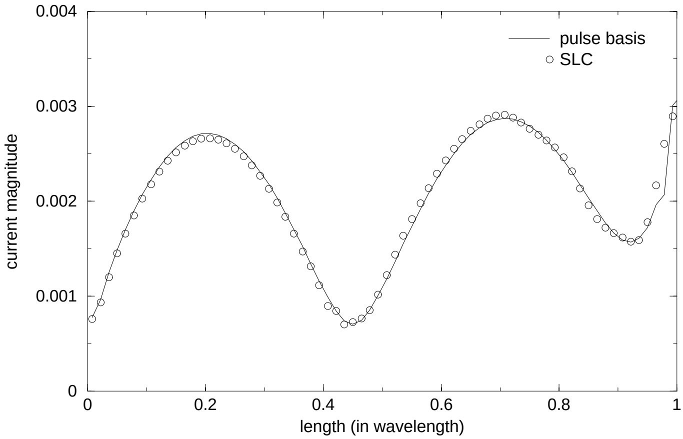

Figure 6. The normalized current magnitude: straight-wire antenna.

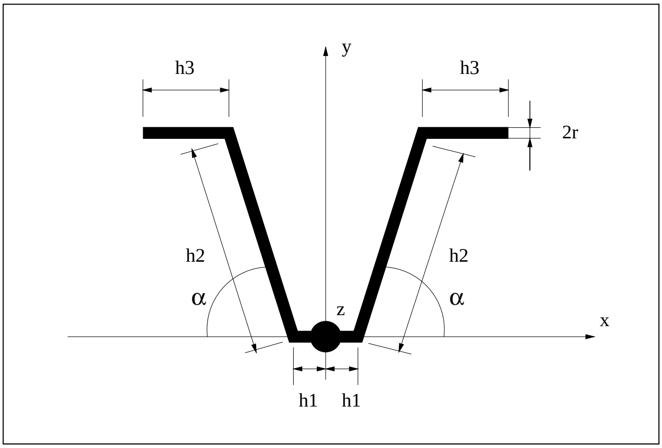

Figure 7. Geometry of the gull-shaped piecewise linear antenna.

$h_1 = 0.0714\lambda,\ h_2 = 0.4286\lambda,\ h_3 = 0.25\lambda,\ r = 0.005\lambda,\ \alpha = 50^\circ$. This example was taken from [14].

Figure 8 shows the normalized current magnitude for the gull-shaped antenna. Due to symmetry, current is shown only for half of the antenna. We used 150 pulses and only 20 SLCs to obtain numerical results, presented in Figure 8. The computation time for the pulse-based MoM is 159.15 seconds. At the same time, SLC-based MoM runs only 15.13 seconds. This gives us a factor of $159.15/15.13 \approx 10$

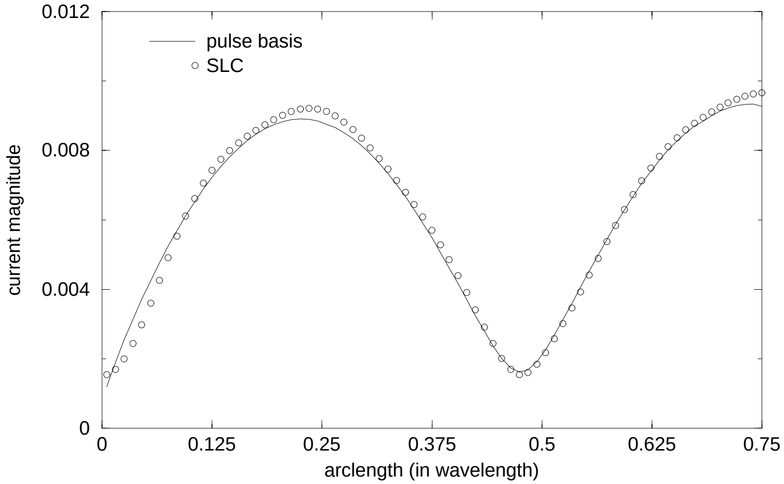

Figure 8. The normalized current magnitude versus arclength: gull-shaped antenna.

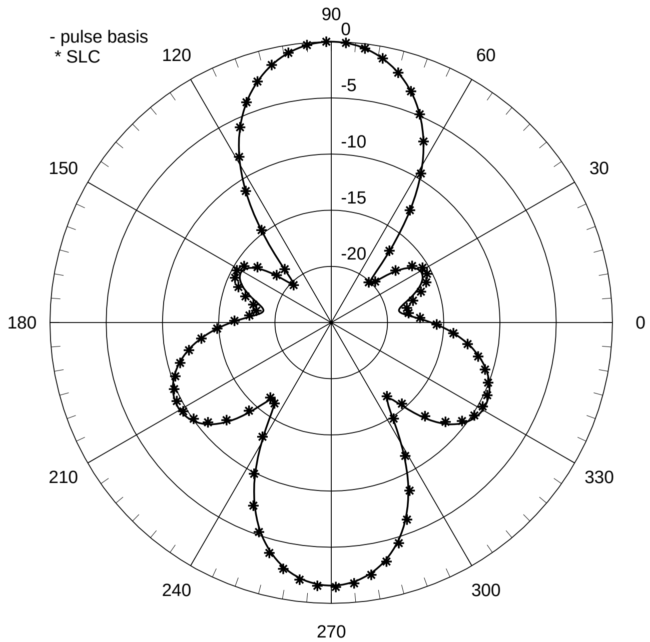

Figure 9. Radiation pattern of the gull-shaped antenna in the xy plane.

the CPU time savings. Finally, Figure 9 presents radiation pattern of the gull-shaped antenna in the xy plane.

5. CONCLUSIONS AND DISCUSSIONS

In this paper, we have applied the SLC basis to the MoM for thin-wire scattering and radiation problems. The SLC forms an orthogonal system which is more suitable to approximate unknown functions in a restricted interval or domain numerically. Because of the smooth window function, rapid convergence of the expansion, and a fast computational algorithm can be achieved. The SLC is particularly useful when the integral kernel behaves in a highly oscillatory manner. Examples of the use of the SLC-based MoM are presented and compared with the standard pulse-based MoM approach. These examples demonstrate that the SLC works effectively in terms of computational time and numerical accuracy.

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