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RIGOROUS AND FAST CONVERGENT ANALYSIS OF A RECTANGULAR WAVEGUIDE COUPLER SLOTTED IN COMMON WALL

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Abstract—Rigorous and fast convergent analysis of a coupler slotted in common wall between two dissimilar rectangular waveguides is described by a mode-matching method combined with Fourier transform technique and consideration of the singularity of electromagnetic field around edges. Comparing with a conventional mode-matching method, the present method has two advantages. One is that it can avoid the usage of the dyadic Green's function, the other is that it can overcome the relative convergence problem. The consideration of the field singularity has greatly improved the convergence and the calculated accuracy of a solution. This analysis is rigorous and the computer cost is very low.

1. INTRODUCTION

Slotted waveguides are wildly used in the design of microwave components, such as a directional coupler and a linear slot array. The classical theories found in standard antenna text [1] is presented by Bethe [2] and Stevenson [3] who developed the theory for very small apertures, and later Cohn [4] modified them to larger apertures of finite thickness. On the other hand, Levy [5] established the synthetic theory of multiaperture couplers based upon them. Besides these methods the various variational techniques [6, 7], the method based upon 'self-reaction' [8] and the moment method [9–14] have been presented.

The moment method is especially attractive, for it is applicable to the coupling problems with an arbitrary shaped aperture. If suitable basis functions can be found, it may take into account the effect of finite wall thickness and higher order modes of the coupling aperture. However it requires a formulation based on the dyadic Green's function. The mode-matching method combined with Fourier transform technique can avoid the usage of the dyadic Green's function, and it can overcome the relative convergence problem caused by a conventional mode-matching method. This method has successfully analyzed the various problems [15–17]. On the other hand, although most works were based on a narrow slot approximate condition, the problem of a wider slot coupler is very important, since a wider slot coupler can improve the bandwidth and the input power-handling capacity than a narrow slot coupler. Sinha [11] and Sangster etc. [12] have discussed a wider slot coupler.

In this paper, we describe a mode-matching method combined with Fourier transform technique and consideration of the singularity of electromagnetic field distribution to analyze a slot coupler between two dissimilar rectangular waveguides. This method can avoid the usage of the complex Green's functions, and can overcome the relative convergence problem than a normal mode-matching method. This method can greatly improve the convergence and the calculated accuracy of the solution. Although the formulation is derived for a longitudinal slot coupler, it is directly suitable for transverse and longitudinal/transverse ones by only exchanging the setting of the width and length of the coupling slot. Numerical examples have confirmed good convergence and high accuracy of the solution by the present method.

2. FORMULATION

The geometry of a coupler between two dissimilar rectangular waveguides through a slotted aperture in the common broad wall is shown in Fig. 1 and Fig. 2. The waveguide I is a main guide, and the waveguide II is a branch guide. The cross sectional dimensions of the waveguide I and II are assumed to be $2a^{\mathrm{I}} \times b^{\mathrm{I}}$ and $2a^{\mathrm{II}} \times b^{\mathrm{II}}$ respectively, and the dimension of the slot is $2l \times 2w \times 2t$. The incident wave fed from $y = -\infty$ in the waveguide I with a unit amplitude may be represented

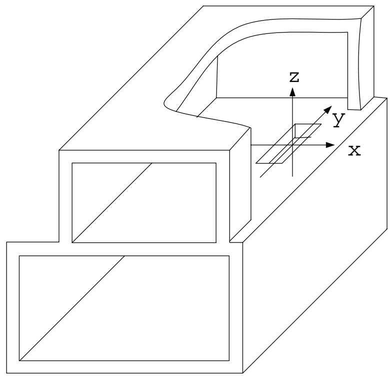


Figure 1. Geometry of a slot coupler between two parallel dissimilar rectangular waveguides.

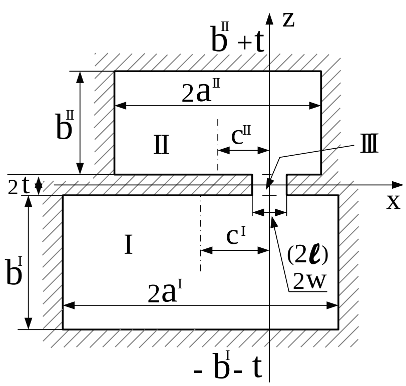


Figure 2. Cross-sectional view of the slot coupler shown in Fig. 1 at $y = 0$ plane cut.

by the magnetic Hertzian vector potential as follows:

$$\mathbf{\Pi}_h^i = \frac{\hat{y}}{k_0^2 Z_0} \cos \underline{a}_1^{\mathrm{I}} (x + a^{\mathrm{I}} + c^{\mathrm{I}}) e^{-j\eta_{10}^{\mathrm{I}} y} \tag{1}$$

where $\eta_{10}^{\mathrm{I}} = \sqrt{k_0^2 - \underline{a}_1^{\mathrm{I}\,2}}$, $\underline{a}_1^{\mathrm{I}} = \pi/2a^{\mathrm{I}}$, $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ and $Z_0 = \sqrt{\mu_0/\epsilon_0}$ are the wave number and intrinsic impedance in free space. The scattered electromagnetic fields due to the slot may be expressed by the Hertzian vectors in the waveguides I and II as follows:

$$\mathbf{\Pi}_e^\vartheta = \frac{\hat{y}}{k_0^2} \sum_{\mu=1}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} A_\mu^\vartheta(\eta) \sin \underline{a}_\mu^\vartheta \left(x + a^\vartheta + c^\vartheta\right)$$

$$\times\ \sin\zeta_\mu^\vartheta\left(z \pm b^\vartheta \pm t\right)e^{-j\eta y}d\eta \quad (2)$$

$$\mathbf{\Pi}_h^\vartheta = \frac{\hat{y}}{k_0^2 Z_0}\sum_{\mu=0}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}B_\mu^\vartheta(\eta)\cos\underline{a}_\mu^\vartheta\left(x + a^\vartheta + c^\vartheta\right)$$
$$\times\ \cos\zeta_\mu^\vartheta\left(z \pm b^\vartheta \pm t\right)e^{-j\eta y}d\eta \quad (3)$$
$$\text{for } \vartheta = \mathrm{I}, \mathrm{II}$$

where $\underline{a}_\mu^\vartheta = \mu\pi/2a^\vartheta, \zeta_\mu^\vartheta = \sqrt{k_0^2 - \underline{a}_\mu^{\vartheta\,2} - \eta^2}$, A_μ^ϑ and B_μ^ϑ are unknown spectral functions, and an infinitesimal small loss has been assumed in the wavenumber k_0 of free space, which is finally reduced to zero. The sign $\pm$ corresponds to $\vartheta = \mathrm{I}(\vartheta = \mathrm{II})$. Note that the electric fields derived from these Hertzian vectors (2) and (3) satisfy the boundary conditions on the conducting walls at $x = \pm a^\vartheta - c^\vartheta$ and $y = \mp(b^\vartheta + t)$. The fields in the slot region III can be derived by the following Hertzian vectors:

$$\mathbf{\Pi}_e^{\mathrm{III}} = \frac{\hat{y}}{k_0^2}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sin\underline{w}_m(x+w)\cos\underline{l}_n(y+l)$$
$$\times\left[A_{mn}^{\mathrm{III}(+)}e^{-jk_{mn}(z+t)} + A_{mn}^{\mathrm{III}(-)}e^{jk_{mn}(z-t)}\right] \quad (4)$$

$$\mathbf{\Pi}_h^{\mathrm{III}} = \frac{\hat{y}}{k_0^2 Z_0}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\cos\underline{w}_m(x+w)\sin\underline{l}_n(y+l)$$
$$\times\left[B_{mn}^{\mathrm{III}(+)}e^{-jk_{mn}(z+t)} + B_{mn}^{\mathrm{III}(-)}e^{jk_{mn}(z-t)}\right] \quad (5)$$

where $\underline{w}_m = m\pi/2w$, $\underline{l}_n = n\pi/2l$, $k_{mn} = \sqrt{k_0^2 - \underline{w}_m{}^2 - \underline{l}_n{}^2}$, $A_{mn}^{\mathrm{III}(+)}$, $A_{mn}^{\mathrm{III}(-)}$, $B_{mn}^{\mathrm{III}(+)}$ and $B_{mn}^{\mathrm{III}(-)}$ are unknown coefficients. The summation. Because there is 90° metallic edge on the aperture, the fields mast satisfy the edge-condition, and it is well known that the tangential components of the electric fields vanishes as $\rho^{-1/3}$, and the normal component as $\rho^{2/3}$ at a 90° metallic edge [18], then the fields on the aperture can be expanded as follows:

$$E_x^{\mathrm{III}}(x,y,\mp t) = \sum_{u,v=0}^{\infty}A_{uv}^\vartheta\left[1-\left(\frac{x}{w}\right)^2\right]^{-\frac{1}{3}}C_u^{1/6}\left(\frac{x}{w}\right)\left[1-\left(\frac{y}{l}\right)^2\right]^{\frac{2}{3}}C_v^{7/6}\left(\frac{y}{l}\right) \quad (6)$$

$$E_y^{\mathrm{III}}(x,y,\mp t) = \sum_{u,v=0}^{\infty}B_{uv}^\vartheta\left[1-\left(\frac{x}{w}\right)^2\right]^{\frac{2}{3}}C_u^{7/6}\left(\frac{x}{w}\right)\left[1-\left(\frac{y}{l}\right)^2\right]^{-\frac{1}{3}}C_v^{1/6}\left(\frac{y}{l}\right)$$

$$\text{for } \vartheta = \mathrm{I}, \mathrm{II} \tag{7}$$

where the functions of $C_u^{1/6}(\frac{x}{w})$ and $C_u^{7/6}(\frac{x}{w})$ are the Gegenbauer polynomials, A_{uv}^{Φ} and B_{uv}^{Φ} are unknown coefficients. Form table of integrals [19] the integrals are found.

$$\int_0^1 \left(1-t^2\right)^{\nu-\frac{1}{2}} C_{2n+1}^{\nu}(t)\sin(at)dt = (-1)^n \pi \frac{\Gamma(2n+2\nu+1)J_{2n+\nu+1}(a)}{(2n+1)!\Gamma(\nu)(2a)^{\nu}} \left[\mathrm{Re}\ \nu > -\frac{1}{2}, a>0\right] \tag{8}$$

$$\int_0^1 \left(1-t^2\right)^{\nu-\frac{1}{2}} C_{2n}^{\nu}(t)\cos(at)dt = (-1)^n \pi \frac{\Gamma(2n+2\nu)J_{2n+\nu}(a)}{(2n)!\Gamma(\nu)(2a)^{\nu}} \left[\mathrm{Re}\ \nu > -\frac{1}{2}, a>0\right] \tag{9}$$

The tangential electric fields derived from (4) and (5) are substituted into the left hand sides of (6) and (6). The resulting equations are multiplied by $\cos \underline{w}_m(x+w)\sin \underline{l}_n(y+l)$ and $\sin \underline{w}_m(x+w)\cos \underline{l}_n(y+l)$ and integrated over the aperture region, using the orthogonality of the trigonometric functions and the integral formula (8) and (9). After several manipulations, we have the following relations among the unknown coefficients:

$$A_{mn}^{\mathrm{III}(+)} = K_1 \sum_{u,v=0}^{\infty} \frac{k_0 k_{mn} \Theta_{mn}^{y}\left[B_{uv}^{\mathrm{I}} - B_{uv}^{\mathrm{II}} e^{-jk_{mn}t}\right]}{(k_0^2 - l_n^2)(1+\delta_{n0})} \tag{10}$$

$$A_{mn}^{\mathrm{III}(-)} = K_1 \sum_{u,v=0}^{\infty} \frac{k_0 k_{mn} \Theta_{mn}^{y}\left[B_{uv}^{\mathrm{II}} - B_{uv}^{\mathrm{I}} e^{-jk_{mn}t}\right]}{(k_0^2 - l_n^2)(1+\delta_{n0})} \tag{11}$$

$$B_{mn}^{\mathrm{III}(+)} = K_1 \sum_{u,v=0}^{\infty} \left\{ \frac{\Theta_{mn}^{x}\left[A_{uv}^{\mathrm{I}} - A_{uv}^{\mathrm{II}} e^{-j2k_{mn}t}\right]}{1+\delta_{m0}} + \frac{w_m l_n \Theta_{mn}^{y}\left[B_{uv}^{\mathrm{I}} - B_{uv}^{\mathrm{II}} e^{-j2k_{mn}t}\right]}{k_0^2 - l_n^2} \right\} \tag{12}$$

$$B_{mn}^{\mathrm{III}(-)} = K_1 \sum_{u,v=0}^{\infty} \left\{ \frac{\Theta_{mn}^{x}\left[A_{uv}^{\mathrm{II}} - A_{uv}^{\mathrm{I}} e^{-j2k_{mn}t}\right]}{1+\delta_{m0}} + \frac{w_m l_n \Theta_{mn}^{y}\left[B_{uv}^{\mathrm{II}} - B_{uv}^{\mathrm{I}} e^{-j2k_{mn}t}\right]}{k_0^2 - l_n^2} \right\} \tag{13}$$

where

$$K_1 = \frac{k_0}{k_{mn}(1 - e^{-j4k_{mn}t})} \tag{14}$$

$$\Theta^x_{mn} = (-1)^{[\frac{m+1}{2}]+[\frac{n}{2}]}\Lambda_{um}\left(\frac{1}{6}\right)\Lambda_{vn}\left(\frac{7}{6}\right)Q_{um}Q_{(v+1)n} \tag{15}$$

$$\Theta^y_{mn} = (-1)^{[\frac{m}{2}]+[\frac{n+1}{2}]}\Lambda_{um}\left(\frac{7}{6}\right)\Lambda_{vn}\left(\frac{1}{6}\right)Q_{(u+1)m}Q_{vn} \tag{16}$$

$$\Lambda_{um}(\varphi) = \frac{2(-1)^{[\frac{u}{2}]}\pi\Gamma(u+2\varphi)J_{u+\varphi}\left(\frac{m\pi}{2}\right)}{u!\Gamma(\varphi)(m\pi)^{\varphi}} \quad \text{for } \varphi = \frac{1}{6}, \frac{7}{6} \tag{17}$$

$$Q_{um} = \begin{cases} 1 & u + m \stackrel{\Delta}{=} even \\ 0 & u + m \stackrel{\Delta}{=} odd \end{cases} \tag{18}$$

where the square bracket in exponential function of (-1) is a integer operator, which indicates taking a integer number, the symbol δ_{n0} denotes the Kronecher's delta. The tangential electric and magnetic fields derived from (1)–(5) should be continuous across the boundary planes $z = \pm t$. These boundary conditions may be expressed as

$$E^{\mathrm{I}}_{x,y}(x, y, -t) = \begin{cases} E^{\mathrm{III}}_{x,y}(x, y, -t) & |x| < w, |y| < l \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

$$E^{\mathrm{II}}_{x,y}(x, y, t) = \begin{cases} E^{\mathrm{III}}_{x,y}(x, y, t) & |x| < w, |y| < l \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

$$H^{\mathrm{I}}_{x,y}(x, y, -t) + H^{i}_{x,y}(x, y, -t) = H^{\mathrm{III}}_{x,y}(x, y, -t) \quad |x| < w, |y| < l \tag{21}$$

$$H^{\mathrm{II}}_{x,y}(x, y, t) = H^{\mathrm{III}}_{x,y}(x, y, t) \quad |x| < w, |y| < l. \tag{22}$$

Note that the tangential components of electric fields of the incident wave is equal to zero on the plane $z = -t$. The boundary conditions for the electric fields are first applied. The E_y fields on the aperture of $z = -t$ can be derived from (2) and (4), and substitute (10) and (11) into (4), we have

$$E^{\mathrm{III}}_y(x, y, \mp t) = \sum_{u,v=0}^{\infty} B^{\vartheta}_{uv} \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{\Theta^y_{mn}}{1+\delta_{n0}} \sin \underline{w}_m(x+w) \cos \underline{l}_n(y+l) \tag{23}$$

$$E^{\vartheta}_y(x, y, \mp t) = \pm \sum_{\mu=1}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{k_0^2 - \eta^2}{k_0^2} A^{\vartheta}_{\mu}(\eta) \sin \underline{a}^{\vartheta}_{\mu}(x + a^{\vartheta} + c^{\vartheta}) \times \sin(\zeta^{\vartheta}_{\mu} b^{\vartheta}) e^{-j\eta y} d\eta. \tag{24}$$

Substituting (23) and (24) into the boundary condition (19) and (20), multiplying both sides by $\sin \underline{a}^{\vartheta}_{\mu}(x + a^{\vartheta} + c^{\vartheta})$ and integrating it from

$x = -(a^\vartheta + c^\vartheta)$ to $x = a^\vartheta - c^\vartheta$, finally Fourier transforms of the resulting expression are calculated with respect to y. This leads to a equation which relate the unknown spectral function $A_\mu^\vartheta(\eta)$ to the unknown coefficients B_{uv}^ϑ as follows:

$$A_\mu^\vartheta(\eta) = \frac{\mp j\eta k_0^2}{(k_0^2-\eta^2)a^\vartheta \sin(\zeta_\mu^\vartheta b^\vartheta)} \sum_{u,v=0}^{\infty} B_{uv}^\vartheta \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{\underline{w}_m \Theta_{mn}^y}{1+\delta_{n0}} V_{m\mu}^\vartheta U_n(\eta) \tag{25}$$

where

$$V_{m\mu}^\vartheta = \frac{1}{{\underline{a}_\mu^\vartheta}^2 - {\underline{w}_m}^2}\Big[(-1)^m \sin \underline{a}_\mu^\vartheta(a^\vartheta + c^\vartheta + w) - \sin \underline{a}_\mu^\vartheta (a^\vartheta + c^\vartheta - w)\Big] \tag{26}$$

$$U_n(\eta) = \frac{(-1)^n e^{j\eta l} - e^{-j\eta l}}{\eta^2 - {\underline{l}_n}^2} \tag{27}$$

Similarly, applying the boundary condition (19) and (20) to the E_x fields derived from (2)–(5) and using the relationship (25), we have

$$\begin{aligned} B_\mu^\vartheta(\eta) = {} & \frac{\pm j k_0 \underline{a}_\mu^\vartheta}{a^\vartheta \zeta_\mu^\vartheta \sin(\zeta_\mu^\vartheta b^\vartheta)} \sum_{u,v=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} V_{m\mu}^\vartheta U_n(\eta) \\ & \times \left[\frac{\eta^2 \underline{w}_m \Theta_{mn}^y}{(k_0^2-\eta^2)(1+\delta_{n0})} B_{uv}^\vartheta + \frac{\underline{l}_n \Theta_{mn}^x}{(1+\delta_{\mu 0})(1+\delta_{m0})} A_{uv}^\vartheta \right] \end{aligned} \tag{28}$$

Next, the boundary conditions for the tangential magnetic fields are applied. The H_x fields on the plane of $z = -t$ can be derived from (1)–(5) as follows:

$$\begin{aligned} H_x^{\mathrm{I}}(x,y,-t) = {} & \frac{j}{Z_0}\sum_{\mu=0}^{\infty} \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[\frac{\eta \underline{a}_\mu^{\mathrm{I}}}{k_0^2} B_\mu^{\mathrm{I}}(\eta) - \frac{\zeta_\mu^{\mathrm{I}}}{k_0} A_\mu^{\mathrm{I}}(\eta)\right] \\ & \times \sin \underline{a}_\mu^{\mathrm{I}}(x + a^{\mathrm{I}} + c^{\mathrm{I}}) \cos(\zeta_\mu^{\mathrm{I}} b^{\mathrm{I}}) e^{-j\eta y} d\eta \end{aligned} \tag{29}$$

$$H_x^{in}(x,y,-t) = \frac{j\eta_{10}^{\mathrm{I}} \underline{a}_1^{\mathrm{I}}}{k_0^2 Z_0} \sin \underline{a}_1^{\mathrm{I}}(x + a^{\mathrm{I}} + c^{\mathrm{I}}) e^{-j\eta_{10}^{\mathrm{I}} y} \tag{30}$$

$$\begin{aligned} H_x^{\mathrm{III}}(x,y,-t) = {} & \frac{j}{Z_0 k_0}\sum_{u,v=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \Bigg\{ \frac{\underline{w}_m \underline{l}_n \Theta_{mn}^x}{k_{mn}} \\ & \times \Big[A_{uv}^{\mathrm{I}} \cot(2k_{mn}t) - A_{uv}^{\mathrm{II}} \csc(2k_{mn}t)\Big] + \frac{(k_0^2 - {\underline{w}_m}^2)\Theta_{uv}^y}{k_{mn}} \\ & \times \Big[B_{uv}^{\mathrm{I}} \cot(2k_{mn}t) - B_{uv}^{\mathrm{II}} \csc(2k_{mn}t)\Big] \Bigg\} \\ & \times \sin \underline{w}_m (x+w) \cos \underline{l}_n (y+l) \end{aligned} \tag{31}$$

First replacing the spectral functions of $A_\mu^{\mathrm{I}}(\eta)$ and $B_\mu^{\mathrm{I}}(\eta)$ in (29) by the correspondent expression of (25) and (28), and substituting (29)–(31) into the boundary condition (21), then we can obtain an equation about the unknown coefficients of A_{uv}^{I} to B_{uv}^{II}. Next, we integrate this equation over $|x| < w, |y| < l$ after multiplying both sides by the trigonometric functions $\sin \underline{w}_p(x+w) \cos \underline{l}_q(y+l)$, where the letters p and q are nonnegative integers. This leads to a set of linear equations for the unknown coefficients A_{uv}^{I} to B_{uv}^{II} as follows:

$$\begin{aligned}
&\sum_{u,v=0}^{\infty} \frac{\underline{l}_q w l a^{\mathrm{I}} \Theta_{pq}^{x}}{k_{pq}} \left[A_{uv}^{\mathrm{II}} \csc(2k_{pq}t) - A_{uv}^{\mathrm{I}} \cot(2k_{pq}t)\right] \\
&+ \sum_{u,v=0}^{\infty} \frac{\left(k_0^2 - \underline{w}_p{}^2\right) w l a^{\mathrm{I}} \Theta_{pq}^{y}}{k_{pq}\underline{w}_p} \left[B_{uv}^{\mathrm{II}} \csc(2k_{pq}t) - B_{uv}^{\mathrm{I}} \cot(2k_{pq}t)\right] \\
= &\sum_{u,v=0}^{\infty} A_{uv}^{\mathrm{I}} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\underline{l}_n I_1^{\mathrm{I}} \Theta_{mn}^{x}}{1+\delta_{m0}} + \sum_{u,v=0}^{\infty} B_{uv}^{\mathrm{I}} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{\underline{w}_m I_2^{\mathrm{I}} \Theta_{mn}^{y}}{1+\delta_{n0}} \\
&-j(\eta_{10}^{\mathrm{I}})^2 \underline{a}_1^{\mathrm{I}} a^{\mathrm{I}} k_0^{-1} V_{p1}^{\mathrm{I}} U_q(-\eta_{10}^{\mathrm{I}}) \quad \text{for } p = 1, 2, \cdots, q = 0, 1, \cdots
\end{aligned} \tag{32}$$

where

$$I_1^{\vartheta} = \sum_{\mu=0}^{\infty} \frac{\underline{a}_\mu^{\vartheta\,2}}{1+\delta_{\mu 0}} V_{m\mu}^{\vartheta} V_{p\mu}^{\vartheta} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\eta^2 U_n(\eta) U_q(-\eta)}{\zeta_\mu^{\vartheta} \tan(\zeta_\mu^{\vartheta} b^{\vartheta})} d\eta \tag{33}$$

$$I_2^{\vartheta} = \sum_{\mu=1}^{\infty} \left(k_0^2 - \underline{a}_\mu^{\vartheta\,2}\right) V_{m\mu}^{\vartheta} V_{p\mu}^{\vartheta} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\eta^2 U_n(\eta) U_q(-\eta)}{\zeta_\mu^{\vartheta} \tan(\zeta_\mu^{\vartheta} b^{\vartheta})} d\eta \tag{34}$$

$$\text{for } \vartheta = \mathrm{I}, \mathrm{II}$$

These integrals can be evaluated in closed form by residue-calculus [16, 17]. Likewise, we apply the continuous condition of the y-component of magnetic field across the plane of $z = -t$, another set of linear equations for the unknown coefficients A_{uv}^{I} to B_{uv}^{II} can be obtained.

$$\begin{aligned}
&\sum_{u,v=0}^{\infty} \frac{\left(k_0^2 - \underline{l}_q{}^2\right) w l a^{\mathrm{I}} \Theta_{pq}^{x}}{k_{pq}\underline{l}_q} \left[A_{uv}^{\mathrm{II}} \csc(2k_{pq}t) - A_{uv}^{\mathrm{I}} \cot(2k_{pq}t)\right] \\
&+ \sum_{u,v=0}^{\infty} \frac{\underline{w}_p w l a^{\mathrm{I}} \Theta_{pq}^{y}}{k_{pq}} \left[B_{uv}^{\mathrm{II}} \csc(2k_{pq}t) - B_{uv}^{\mathrm{I}} \cot(2k_{pq}t)\right] \\
= &\sum_{u,v=0}^{\infty} A_{uv}^{\mathrm{I}} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} 1+\delta_{m0} + \sum_{u,v=0}^{\infty} B_{uv}^{\mathrm{I}} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{\underline{w}_m I_1^{\mathrm{I}} \Theta_{mn}^{y}}{1+\delta_{n0}}
\end{aligned}$$

$$-j\underline{a}_1^{\text{I}\,3}a^{\text{I}}k_0^{-1}V_{p1}^{\text{I}}U_q(-\eta_{10}^{\text{I}}) \qquad \text{for } p=0,1,\cdots,q=1,2,\cdots \tag{35}$$

In the same way, applying the boundary condition of (22), we have the following equations.

$$\begin{aligned}
&\sum_{u,v=0}^{\infty}\frac{\underline{l}_q wla^{\text{II}}\Theta_{pq}^x}{k_{pq}}\left[A_{uv}^{\text{I}}\csc(2k_{pq}t)-A_{uv}^{\text{II}}\cot(2k_{pq}t)\right]\\
&+\sum_{u,v=0}^{\infty}\frac{\left(k_0^2-\underline{w}_p{}^2\right)wla^{\text{II}}\Theta_{pq}^y}{k_{pq}\underline{w}_p}\left[B_{uv}^{\text{I}}\csc(2k_{pq}t)-B_{uv}^{\text{II}}\cot(2k_{pq}t)\right]\\
=\;&\sum_{u,v=0}^{\infty}A_{uv}^{\text{II}}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{\underline{l}_n I_1^{\text{II}}\Theta_{mn}^x}{1+\delta_{m0}}+\sum_{u,v=0}^{\infty}B_{uv}^{\text{II}}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{\underline{w}_m I_2^{\text{II}}\Theta_{mn}^y}{1+\delta_{n0}}
\end{aligned} \tag{36}$$

for $p=1,2,\cdots,q=0,1,\cdots$

$$\begin{aligned}
&\sum_{u,v=0}^{\infty}\frac{\left(k_0^2-\underline{l}_q{}^2\right)wla^{\text{II}}\Theta_{pq}^x}{k_{pq}\underline{l}_q}\left[A_{uv}^{\text{I}}\csc(2k_{pq}t)-A_{uv}^{\text{II}}\cot(2k_{pq}t)\right]\\
&+\sum_{u,v=0}^{\infty}\frac{\underline{w}_p wla^{\text{II}}\Theta_{pq}^y}{k_{pq}}\left[B_{uv}^{\text{I}}\csc(2k_{pq}t)-B_{uv}^{\text{II}}\cot(2k_{pq}t)\right]\\
=\;&\sum_{u,v=0}^{\infty}A_{uv}^{\text{II}}\sum_{m,n'=0}^{\infty}\frac{\underline{l}_n I_1^{\text{II}}\Theta_{mn}^x}{1+\delta_{m0}}+\sum_{u,v=0}^{\infty}B_{uv}^{\text{II}}\sum_{m',n=0}^{\infty}\frac{\underline{w}_m I_1^{\text{II}}\Theta_{mn}^y}{1+\delta_{n0}}
\end{aligned} \tag{37}$$

for $p=0,1,\cdots,q=1,2,\cdots$

Equations (32), (35), (36) and (37) are solved to obtain the unknown expansion coefficients A_{uv}^{I} to B_{uv}^{II} for the scattered waves into waveguide I and II, after truncating the modal expansion up to $u=v=U$. The results are used in (25) and (28) to determine the unknown spectral functions $A_{\mu}^{\text{I}}(\eta)$ to $B_{\mu}^{\text{II}}(\eta)$. When the coupler is excited by TE_{10} mode fed from waveguide I, the scattering parameters are calculated in terms of expansion coefficients rooted by (32), (35), (36) and (37) as follows

$$s_{11}=-\sum_{u,v=0}^{U}\sum_{m,n=0}^{\infty}\left\{\frac{\underline{a}_1^{\text{I}}\underline{l}_n\Theta_{mn}^x}{\eta_{10}^{\text{I}}(1+\delta_{m0})}A_{uv}^{\text{I}}+\frac{\eta_{10}^{\text{I}}\underline{w}_m\Theta_{mn}^y}{\underline{a}_1^{\text{I}}(1+\delta_{n0})}B_{uv}^{\text{I}}\right\}\Psi^{\text{I}}(-\eta_{10}^{\text{I}}) \tag{38}$$

$$s_{21}=-\sum_{u,v=0}^{U}\sum_{m,n=0}^{\infty}\left\{\frac{\underline{a}_1^{\text{I}}\underline{l}_n\Theta_{mn}^x}{\eta_{10}^{\text{I}}(1+\delta_{m0})}A_{uv}^{\text{I}}+\frac{\eta_{10}^{\text{I}}\underline{w}_m\Theta_{mn}^y}{\underline{a}_1^{\text{I}}(1+\delta_{n0})}B_{uv}^{\text{I}}\right\}\Psi^{\text{I}}(\eta_{10}^{\text{I}})+1 \tag{39}$$

$$s_{31} = K_2 \sum_{u,v=0}^{U} \sum_{m,n=0}^{\infty} \left\{ \frac{\underline{a}_1^{\text{II}} \underline{l}_n \Theta_{mn}^x}{\eta_{10}^{\text{II}}(1+\delta_{m0})} A_{uv}^{\text{II}} + \frac{\eta_{10}^{\text{II}} \underline{w}_m \Theta_{mn}^y}{\underline{a}_1^{\text{II}}(1+\delta_{n0})} B_{uv}^{\text{II}} \right\} \Psi^{\text{II}}(-\eta_{10}^{\text{II}}) \tag{40}$$

$$s_{41} = K_2 \sum_{u,v=0}^{U} \sum_{m,n=0}^{\infty} \left\{ \frac{\underline{a}_1^{\text{II}} \underline{l}_n \Theta_{mn}^x}{\eta_{10}^{\text{II}}(1+\delta_{m0})} A_{uv}^{\text{II}} + \frac{\eta_{10}^{\text{II}} \underline{w}_m \Theta_{mn}^y}{\underline{a}_1^{\text{II}}(1+\delta_{n0})} B_{uv}^{\text{II}} \right\} \Psi^{\text{II}}(\eta_{10}^{\text{II}}) \tag{41}$$

where

$$\Psi^{\vartheta}(\eta_{10}^{\vartheta}) = \frac{k_0 V_{m1}^{\vartheta} U_n(\eta_{10}^{\vartheta})}{2a^{\vartheta} b^{\vartheta}} \qquad \text{for } \vartheta = \text{I}, \text{II} \tag{42}$$

$$K_2 = \sqrt{\frac{\underline{a}_1^{\text{II}} b^{\text{II}} \eta_{10}^{\text{II}}}{\underline{a}_1^{\text{I}} b^{\text{I}} \eta_{10}^{\text{I}}}} \tag{43}$$

The other elements of scattering matrix are obtained by changing the port of initial excitation.

3. NUMERICAL RESULTS AND DISCUSSION

The proposed method has been applied to a longitudinal slot coupler in the common broad wall and a rectangular T-junction. Although the set of linear equations (32) and (35)–(37) is derived for a longitudinal slot coupler, these linear equations are directly suitable for transverse and longitudinal/transverse problems by only exchanging the setting of the width an length of the coupling slot. In order to check the validity of the present method, we first consider a rectangular T-junction with a reduced width side waveguide. This problem can be solved using a set of linear equations (32) and (35), where the coefficients in terms of A_{uv}^{II} and B_{uv}^{II} in (32) and (35) are set to be zero. The reflection and transmission coefficients of $|S_{11}|^2$ and $|S_{21}|^2$ are plotted in Fig. 3 as function of the normalized wavelength $\lambda/(2a)$ with $2a^{\text{I}} = 34.85\,\text{mm}$, $b^{\text{I}} = 15.799\,\text{mm}$, $2w = 22.86\,\text{mm}$, $2l = 10.16\,\text{mm}$, $c^{\text{I}} = 0$, and $t = \infty$. The solid lines, dash lines, and symbols denote the results of the present method with truncation number $U = 5$, the mode-matching method with the generalized admittance matrix [20], and the measured results by the literature [21], respectively. The present results are much closer to the measured data than a conventional mode-matching method. This example shows that the present method is very available to improve calculated accuracy.

Next, we shall discuss the effect of the singularity of fields around the edges of the slot on the calculated results. For convenience, we consider a narrow slot coupler, in which the electromagnetic fields in

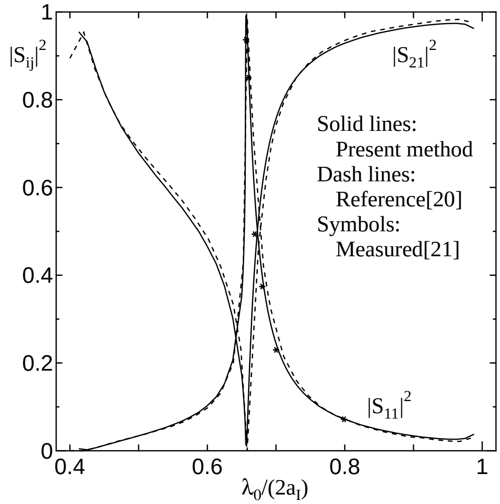


Figure 3. Comparison of the scattering parameters $|s_{11}|^2$ and $|s_{21}|^2$ between calculated results and measurements for a WR-137 rectangular waveguide T-junction with a reduced width side waveguide with $2a^{\mathrm{I}} = 34.85\,\mathrm{mm}$, $b^{\mathrm{I}} = 15.799\,\mathrm{mm}$, $2w = 22.86\,\mathrm{mm}$, $2t = 10.16\,\mathrm{mm}$, and $c^{\mathrm{I}} = 0$.

slot region are well approximated with the superposition of only TE^y_{0n} modes. Fig. 4 shows the convergence behavior of the scattering parameters s_{11} and s_{21} as function of the truncation number U by the present method for a slot coupler with $2a^{\mathrm{I}} = 2a^{\mathrm{II}} = 7.10\,\mathrm{mm}$, $b^{\mathrm{I}} = b^{\mathrm{II}} = 3.55\,\mathrm{mm}$, $c^{\mathrm{I}} = c^{\mathrm{II}} = 2.5\,\mathrm{mm}$, $2l = 4.70\,\mathrm{mm}$, $2w = 0.5\,\mathrm{mm}$, $2t = 1.0\,\mathrm{mm}$, and $f = 32\,\mathrm{GHz}$. We can see that the convergence of s-parameters is very good. Fig. 5 shows the convergence of the same scattering parameters as function of the truncated number N without consideration of the singularity of electromagnetic fields. If the singularity is not taken into account, (6)–(18) will be omitted from the analysis, so that the unknowns become the coefficients ($A^{\mathrm{III}(\pm)}_{mn}$ and $B^{\mathrm{III}(\pm)}_{mn}$) for the full modes expansion, and the coefficients ($B^{\mathrm{III}(\pm)}_{0n}$ for the TE^y_{0n} modes expansion. Form these figures, the results are also convergent well. However, comparing these figures with Fig. 4, we can find that the convergent values are quite different from those obtained by considering the field's singularity. This phenomenon can be explained form calculating the distribution of the magnetic fields. Fig. 6 is the plot of the distribution of magnetic fields along the line of $x = 0$ and $z = -t$ as function of y with various truncation number.

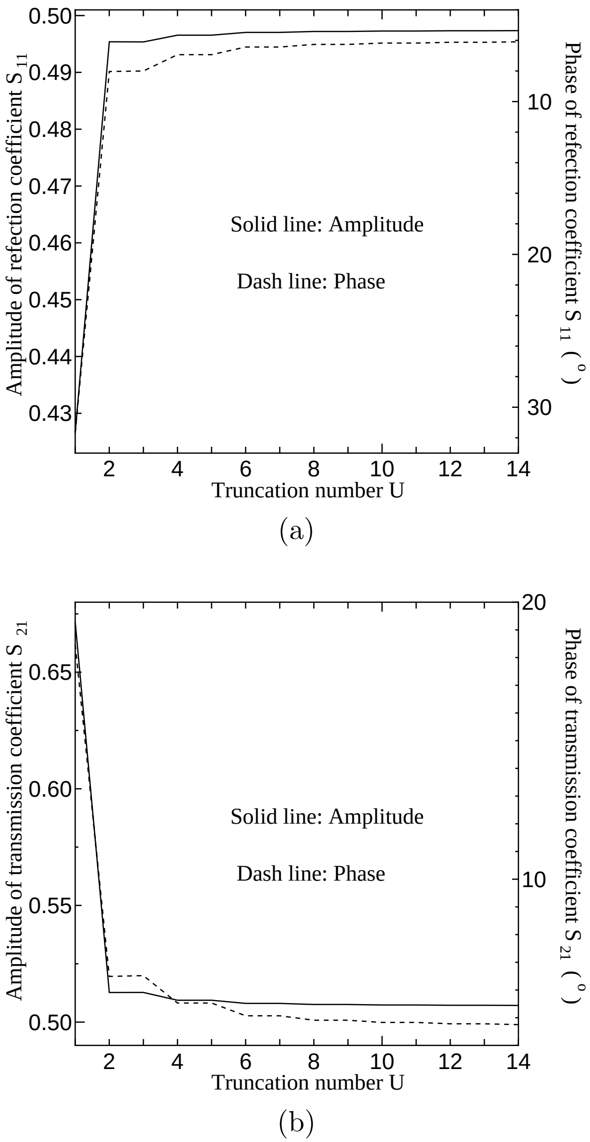
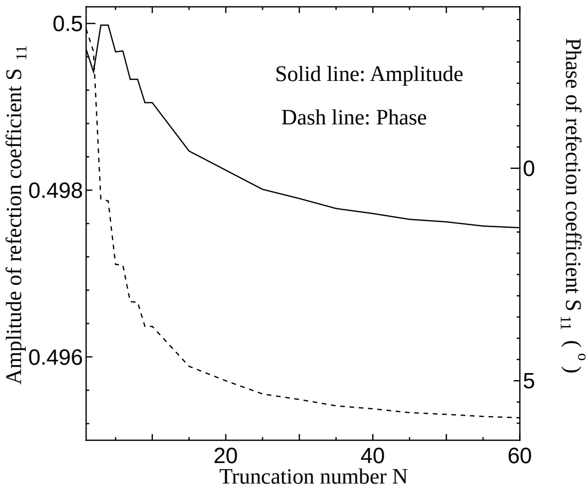
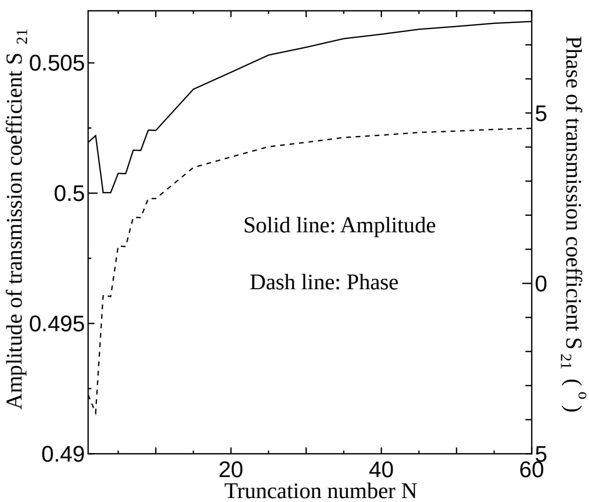


Figure 4. Convergence of the scattering parameters s_{11} and s_{21} as function of truncation number U for a slot coupler with $2a^{\mathrm{I}} = 2a^{\mathrm{II}} = 7.10\,\mathrm{mm}$, $b^{\mathrm{I}} = b^{\mathrm{II}} = 3.55\,\mathrm{mm}$, $c^{\mathrm{I}} = c^{\mathrm{II}} = 2.5\,\mathrm{mm}$, $2l = 4.70\,\mathrm{mm}$, $2w = 0.5\,\mathrm{mm}$, $2t = 1.0\,\mathrm{mm}$, and $f = 32\,\mathrm{GHz}$. (a) s_{11}. (b) s_{21}.

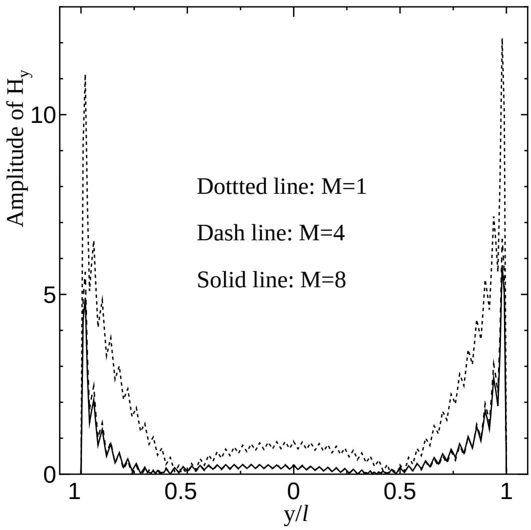


(a)

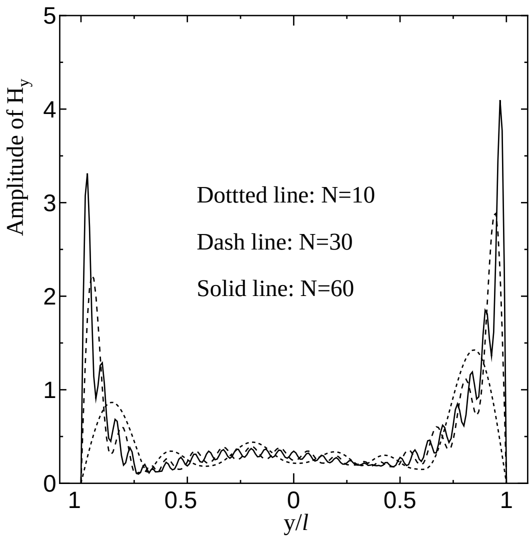


(b)

Figure 5. Convergence of the scattering parameters s_{11} and s_{21} as function of truncation number N for the same coupler as shown in Fig.3. (a) s_{11}. (b) s_{21}.



(a)



(b)

Figure 6. Distribution of magnetic fields on the line of $x = 0$ and $z = -t$ as function of y with various truncation number. (a) With the consideration of fields singularity. (b) Without the consideration of fields singularity.

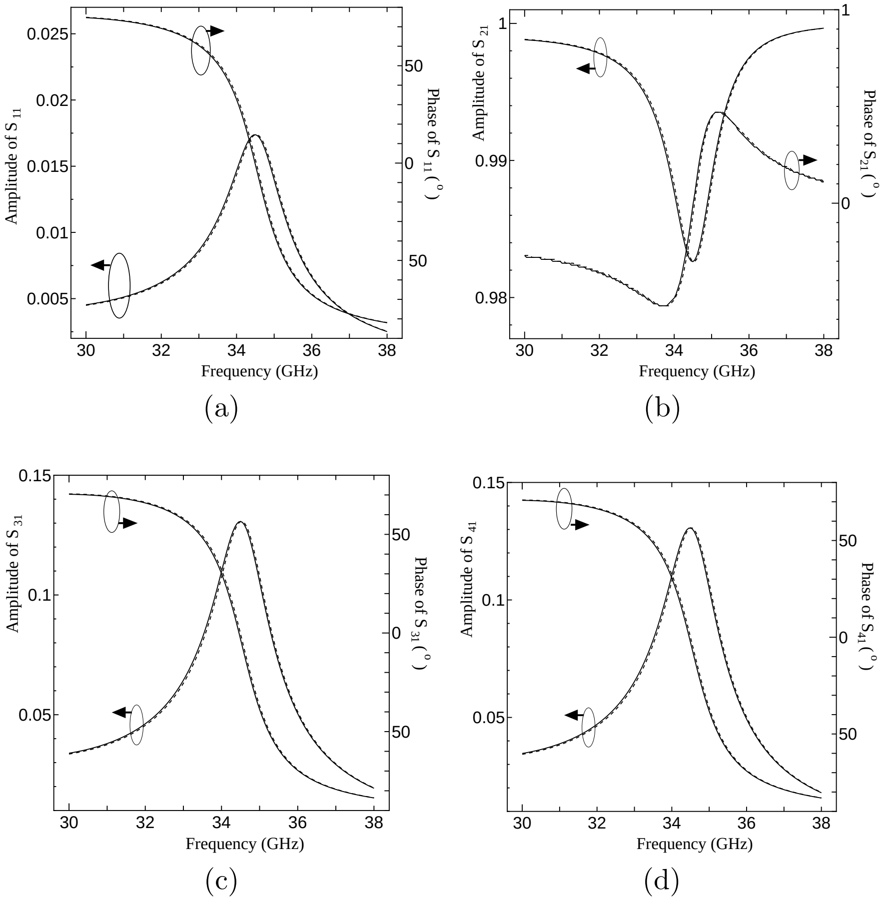


Figure 7. Comparison of the scattering parameter between the present solution and the approximate results with a narrow slot condition for a slot coupler with $2a^{\mathrm{I}} = 7.112\,\mathrm{mm}$, $b^{\mathrm{I}} = 3.556\,\mathrm{mm}$, $2a^{\mathrm{II}} = 5.70\,\mathrm{mm}$, $b^{\mathrm{II}} = 2.85\,\mathrm{mm}$, $c^{\mathrm{I}} = 0.5\,\mathrm{mm}$, $c^{\mathrm{II}} = 2.35\,\mathrm{mm}$ $2l = 4.30\,\mathrm{mm}$, $2w = 0.5\,\mathrm{mm}$, and $2t = 1.0\,\mathrm{mm}$. (a) s_{11}, (b) s_{21}, (c) s_{31}, (d) s_{41}.

Fig. 6a shows the results obtained with consideration of the fields singularity, whereas Fig. 6b shows the results obtained without taking the field's singularity into account. When the singularity is taken into account, the magnetic field is convergent over the whole region, as the expansion terms are truncated at only $U = 4$. However, the calculated magnetic fields do not converge yet without consideration of the field singularity, even if the truncation number is increased to $N = 60$.

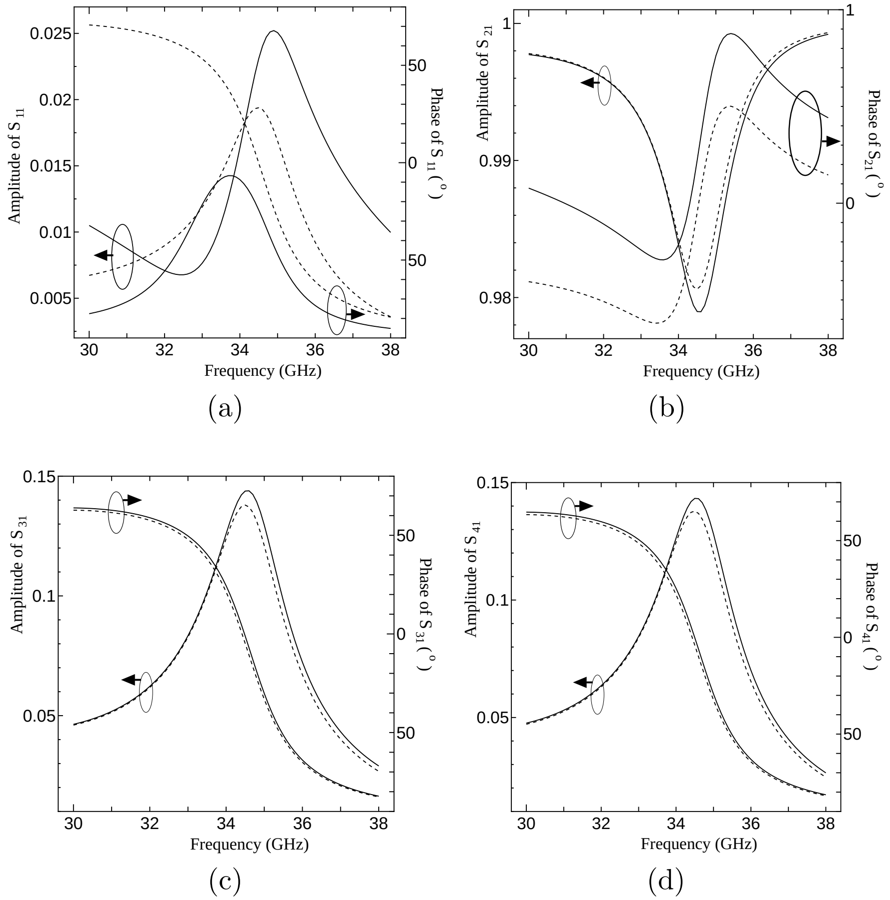


Figure 8. Comparison of the scattering parameter between the present solutions and the approximate results with a narrow slot condition for a slot coupler with the same parameters given in Fig. 7 except for $2w = 1.0\,\mathrm{mm}$. (a) s_{11}, (b) s_{21}, (c) s_{31}, (d) s_{41}.

Thus, the consideration of the field singularity is indispensability, in order to obtain a complete convergent solution. It is noticed that the effect of the singularity is decreasing with increasing the aperture size, since the ratio of electromagnetic energy around edges over the whole one in the aperture region is in declined tendency, so that the mode-matching method combined only with Fourier transform technique can also get practicable results for s-parameters, when the aperture size is relative larger.

Finally, we discuss the accuracy of the calculated results by a narrow slot approximation. Fig. 7 shows comparison of scattering parameters between the present solution and the approximate results with a narrow slot condition of a slot coupler with $2a^{\mathrm{I}} = 7.112\,\mathrm{mm}$, $b^{\mathrm{I}} = 3.556\,\mathrm{mm}$, $2a^{\mathrm{II}} = 5.70\,\mathrm{mm}$, $b^{\mathrm{II}} = 2.85\,\mathrm{mm}$, $c^{\mathrm{I}} = 0.5\,\mathrm{mm}$, $c^{\mathrm{II}} = 2.35\,\mathrm{mm}$, $2l = 4.3\,\mathrm{mm}$, $2w = 0.5\,\mathrm{mm}$, $2t = 1.0\,\mathrm{mm}$. The dash lines denote the approximate results with TE^y_{0n} modes expansion. The solid lines denote the present results. We can see that the difference between the two methods is very small. On the other hand, Fig. 8 shows comparison of scattering parameters between the present solution and the approximate results with a narrow slot condition of a slot coupler with the same parameters given in Fig. 7 except for $2w = 1.0\,\mathrm{mm}$. In this case, there is a noticeable difference between two methods. As is well known, the zero point of s_{11} phase have particular value to define the resonant frequency by some definitions. We can see that the calculation with a narrow slot approximation is not suitable for this case, so that it is careful enough to use the narrow slot condition.

4. CONCLUSION

A rigorous and fast convergent method for analyzing slot couplers has been presented, which is based on a mode-matching method combined with Fourier transform technique and the consideration of the field singularity. The fields in the waveguide region expressed by the Fourier integrals are evaluated in closed form by a simple residue-calculus, and the fields on the apertures are expanded by the Gegenbauer polynomials weighted functions for describing the edge conditions. The present method can avoid the dyadic Green's function, and can overcome the problem of relative convergence. Numerical tests have confirmed very good convergence and high accuracy of the solution by the present method.

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