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ELECTROMAGNETIC FIELD SOLUTION IN CONFORMAL STRUCTURES: THEORETICAL AND NUMERICAL ANALYSIS

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Abstract—A full-wave evaluation of the electromagnetic field in conformal structures with linear loading materials is presented in this paper. The analysis is performed considering at first conformal components with conventional isotropic and homogeneous media in the generalized orthogonal curvilinear reference system. In this first case, a summary of the possible analytical solutions of the vector wave equation obtainable through various factorization techniques is given. Then, the attention is focused on conformal structures involving non-conventional media (anisotropic, chiral, bianisotropic) and in this case the field solution is demanded to a new generalization of the transmission line approach. As an aside, exploiting a contravariant field formulation, which allows writing Maxwell's equations in the generalized reference system as in the Cartesian one, a useful relationship between the local curvature of the geometry and a suitable inhomogeneity of a related planar structure is presented. Finally, some results, obtained simulating the behavior of patch radiators mounted on curved bodies through the combined application of an extended Method of Line (MoL) numerical algorithm and the theoretical approach here derived, are presented.

1. INTRODUCTION

The popularity and the demand for conformal components in microwave applications has been growing in the last decades, in parallel with the interest in employing electromagnetic devices in spacecraft, satellites, aircraft and land vehicles, for which compactness and drag reduction are very important issues [1–13]. For this reason, extensive research efforts have been recently devoted to develop accurate and efficient analysis tools and design procedures for integrated circuits and radiative components mounted on curved surfaces, avoiding expensive and approximate experimental attempts. In the case of simple geometries (e.g., planar, cylindrical, spherical, etc.) and linear, isotropic and homogeneous dielectric materials, the full-wave theory is well established and analytical solutions are already available (see [14] and references therein). Therefore, nowadays the research in these cases is mainly devoted to find out new and fast design techniques.

When non-canonical surfaces are involved, instead, the analysis grows in complexity, due to the local curvature of the structure, and spending efforts on design procedures becomes a step ahead. The first aim in this case consists in the development of an accurate and efficient analysis tool, which may simulate the electromagnetic behavior of the conformal component varying the electrical and geometrical parameters to match the given requirements. For this purpose, a proper theoretical investigation is needed.

The main difficulty encountered when approaching the electromagnetic problem in curved structures is the analytical complexity of the involved equations, which very often cannot be solved in closed forms. These difficulties grow even more in presence of complex media, such as materials exhibiting the magneto-electric effect (chiral, bi-anisotropic, etc.), whose behavior in radiating and circuit components has been investigated in the past years [15–23] and has shown very promising features. The actual difficulties in this theoretical research field, thus, mainly depend on the complexity of the required mathematical background and on the fact that a well established and general theory on conformal structures has not been already carried out and published in the open technical literature. This paper is an attempt to present in a unified theory some new theoretical developments in the solution of the electromagnetic field in conformal structures. In the last part of the paper, then, the theory here presented is applied to analyze numerically some conformal components following an extended MoL numerical procedure as proposed by the authors in a recent work [24].

The structure of the present work is given in the following. In Section 2 a brief description of the generalized orthogonal curvilinear reference system and a detailed justification for its employment in the present analysis will be discussed. In Section 3 the solution of the electromagnetic field in presence of linear, isotropic, homogeneous media will be summarized and discussed, showing in which reference systems it is possible to find a closed-form analytical solution and in which others a numerical solution is unavoidable. In Section 4 the presence of complex materials will be addressed and a new set of transmission line equations in the generalized orthogonal curvilinear reference system will be derived, showing their utility in the analysis of conformal structures both for analytical and numerical tools. In Section 5, a useful relationship between the local curvature of the geometry and a suitable inhomogeneity introduced in an isomorphic planar structure is presented, which can be employed to simplify the analysis of electromagnetic components mounted on curved surfaces. Finally, in Section 6, some numerical results, obtained exploiting the generalized MoL algorithm described in [24] and based on the isomorphism proposed in Section 5, are presented.

2. GENERALIZED ORTHOGONAL CURVILINEAR REFERENCE SYSTEM

In the following we will refer to an integrated conformal structure mounted on a curved surface, such as the one depicted in Fig. 1. The related electromagnetic problem consists in a differential system (Maxwell's equations) with prescribed boundary conditions on the interfaces between the conformal slabs that compose the structure, on the conformal metallic surfaces and at infinity (the radiation condition).

This problem can be solved either analytically or numerically, but in both cases a suitable modeling of the curved surfaces is needed. Very often they are roughly represented by canonical surfaces, such as portions of spherical or cylindrical surfaces, but this approximation does not always yield correct results in the simulation process. The accuracy of the solution, in fact, obviously depends on the approximation of the real curvature of the structure. When this kind of solution is viable, however, the electromagnetic analysis is usually developed in the related reference system (i.e., cylindrical and spherical

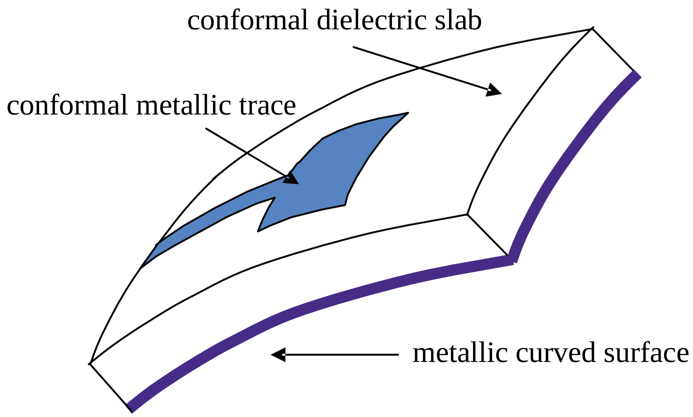


Figure 1. An example of an integrated conformal structure.

for structures approximated by cylinders or spheres, respectively), where the curved interfaces coincide with coordinate surfaces and the field components are tangential or orthogonal to the surface at each point. A direct consequence, in fact, is that the imposition of Dirichlèt or Neumann boundary conditions becomes straightforward since they do automatically factorize and refer only to one single field component.

On the other hand, when canonical geometries are not suitable to describe the interface surfaces, a more extensive theory is needed and the generalized orthogonal reference system may be successfully employed for this purpose. A conformal interface, in fact, can be very often expressed or approximated (generally in a more accurate way than in the case of the aforementioned canonical geometries) as a coordinate surface in an orthogonal reference system.

The orthogonal unit vectors $\hat{\boldsymbol{q}}_1, \hat{\boldsymbol{q}}_2, \hat{\boldsymbol{q}}_3$ generically describing the reference system are depicted in Fig. 2. If the associated spatial coordinates are expressed as q_1, q_2, q_3, the volume element (see Fig. 2) is given by $dV = h_1 h_2 h_3 dq_1 dq_2 dq_3$ [25], where h_1, h_2, h_3 are the so called metric factors (in general functions of the spatial coordinates). The reference system is univocally determined once the metric factors are fixed in every point of the space, since they are directly related to the local curvature of the coordinate lines and surfaces.

The main advantage to adopt this general formulation is in the direct factorization of the boundary conditions at every interface, provided that such interfaces can be expressed, in the chosen reference system, as portions of coordinate surfaces. This assumption is very often satisfied, considering that the generalized orthogonal reference system can be referred to an infinite number of geometries, some of

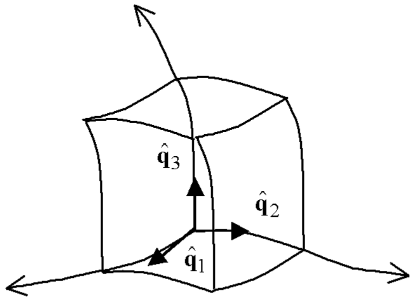


Figure 2. Unit vectors and volume element in the generalized orthogonal curvilinear reference system.

which are reported in Table 1.

The number of orthogonal systems for which this formulation can be applied is not limited to those reported in the table, but suitable reference systems may be synthesized, depending on necessity, through

Table 1. Some orthogonal reference systems, with related coordinate variables and metric factors.

Reference System	Spatial Variables			Metric Factors		
	q_1	q_2	q_3	h_1	h_2	h_3
Cartesian	x	y	z	1	1	1
Circular Cylindrical	ρ	ϕ	z	1	ρ	1
Parabolic Cylindrical	u	v	z	$\sqrt{u^2+v^2}$	$\sqrt{u^2+v^2}$	1
Elliptic Cylindrical	u	v	z	$a\sqrt{\sin^2 v+\sinh^2 u}$	$a\sqrt{\sin^2 v+\sinh^2 u}$	1
Spherical	r	θ	ϕ	1	r	$r\sin\theta$
Prolate Spheroidal	ξ	η	ϕ	$a\sqrt{\sin^2\eta+\sinh^2\xi}$	$a\sqrt{\sin^2\eta+\sinh^2\xi}$	$a\sin\eta+\sinh\xi$
Oblate Spheroidal	ξ	η	ϕ	$a\sqrt{\sin^2\xi+\sinh^2\eta}$	$a\sqrt{\sin^2\xi+\sinh^2\eta}$	$a\cos\xi+\cosh\eta$
Parabolic	u	v	ϕ	$\sqrt{u^2+v^2}$	$\sqrt{u^2+v^2}$	uv
Conic	λ	μ	v	1	$\frac{\sqrt{\mu^2-v^2}\lvert\lambda\rvert}{\sqrt{\mu^2-a^2}\sqrt{b^2-\mu^2}}$	$\frac{\sqrt{\mu^2-v^2}\lvert\lambda\rvert}{\sqrt{\mu^2-a^2}\sqrt{v^2-b^2}}$
Paraboloidal	λ	μ	v	$\frac{\sqrt{\mu-\lambda)(v-\lambda)}}{2\sqrt{a^2-\lambda}\sqrt{b^2-\lambda}}$	$\frac{\sqrt{(\lambda-\mu)(v-\mu)}}{2\sqrt{a^2-\mu}\sqrt{b^2-\mu}}$	$\frac{\sqrt{(\lambda-v)(\mu-v)}}{2\sqrt{a^2-v}\sqrt{b^2-v}}$
Ellipsoidal	λ	μ	v	$\frac{\sqrt{\mu-\lambda)(v-\lambda)}}{2\sqrt{a^2-\lambda}\sqrt{b^2-\lambda}\sqrt{c^2-\lambda}}$	$\frac{\sqrt{(\lambda-\mu)(v-\mu)}}{2\sqrt{a^2-\mu}\sqrt{b^2-\mu}\sqrt{c^2-\mu}}$	$\frac{\sqrt{(\lambda-v)(\mu-v)}}{2\sqrt{a^2-v}\sqrt{b^2-v}\sqrt{c^2-v}}$
Bipolar	u	v	z	$\frac{a}{\cosh v-\cos u}$	$\frac{a}{\cosh v-\cos u}$	1
Bispherical	u	v	ϕ	$\frac{a}{\cosh v-\cos u}$	$\frac{a}{\cosh v-\cos u}$	$\frac{a\sin u}{\cosh v-\cos u}$
Cardioid Cylinder	u	v	ϕ	$\frac{a}{\cosh u-\cos v}$	$\frac{a}{\cosh u-\cos v}$	$\frac{a\sin u}{\cosh u-\cos v}$
Cardioid Cylinder	μ	v	z	$(\mu^2+v^2)^{-3/2}$	$(\mu^2+v^2)^{-3/2}$	1
Tangent Sphere	μ	v	ψ	$\frac{1}{\mu^2+v^2}$	$\frac{1}{\mu^2+v^2}$	$\frac{\mu}{\mu^2+v^2}$

the use of conformal transformations in the complex plane, as shown in [26].

Spatial variables and scale factors associated to each reference system are also reported in the table. It has to be observed that some reference systems require a further assignment of some geometrical parameters.

3. FIELD SOLUTION IN CONFORMAL STRUCTURES WITH LINEAR, ISOTROPIC AND HOMOGENEOUS MATERIALS

In this section we briefly show a possible analytical solution of the electromagnetic field in conformal geometries when linear, isotropic and homogenous materials are considered. Assuming a time harmonic dependence $e^{j\omega t}$ and considering a source free region, the vector wave equations for the electric ($\boldsymbol{E}$) and magnetic ($\boldsymbol{H}$) fields are of the Helmholtz kind:

$$\begin{aligned} \nabla^2 \boldsymbol{E} - k^2 \boldsymbol{E} = 0 \\ \nabla^2 \boldsymbol{H} - k^2 \boldsymbol{H} = 0 \end{aligned} \tag{1}$$

where $k^2 = \omega^2 \mu \varepsilon$, being ω the angular frequency and μ, ε the permittivity and permeability of the medium, respectively.

The main goal is to solve analytically these vector equations, in order to express in a closed form the electromagnetic field excited in the conformal structure. To this end, the first problem to be considered is the reduction of the vector Helmholtz equation to scalar uncoupled differential equations. Then, these scalar equations should be solved, preferably in a closed form.

The problem is usually approached by directly projecting each vector equation on the three coordinate directions. Considering the generalized reference system, three coupled scalar differential equations are obtained and there is no way to reduce the system to scalar equations containing only one unknown each. Specifying the problem to a given reference system, on the other hand, it can be shown that uncoupled equations can be obtained in some cases: in Cartesian coordinates, as well known, the three scalar equations are all decoupled whereas in circular cylindrical, parabolic cylindrical, elliptic cylindrical, and bipolar systems only the scalar equation along the axial direction is decoupled. Provided that we can find an analytical solution for this equation, however, it can be shown that the whole spatial electromagnetic field can be related to this solution, and, thus, expressed analytically as well. In the spherical reference system, which does not belong to the previous class, it is also possible to reduce the vector equation to a scalar one by using in addition the divergence equations.

In order to extend the family of orthogonal reference systems where this reduction is possible, we can operate introducing the Borgnis' potential functions U and V [27]. In those reference systems where the following two conditions hold:

$$h_3 = 1 \qquad \text{and} \qquad \frac{\partial}{\partial q_3}\left(\frac{h_1}{h_2}\right) = 0 \tag{2}$$

it can be proved that the overall electromagnetic field can be expressed as follows [27]:

$$\begin{cases} E_{q1}(q_1,q_2,q_3) = \dfrac{1}{h_1}\dfrac{\partial^2 U(q_1,q_2,q_3)}{\partial q_3 \partial q_1} - j\omega\mu\dfrac{1}{h_2}\dfrac{\partial V(q_1,q_2,q_3)}{\partial q_2} \\ E_{q2}(q_1,q_2,q_3) = \dfrac{1}{h_2}\dfrac{\partial^2 U(q_1,q_2,q_3)}{\partial q_2 \partial q_3} + j\omega\mu\dfrac{1}{h_1}\dfrac{\partial V(q_1,q_2,q_3)}{\partial q_1} \\ E_{q3}(q_1,q_2,q_3) = \dfrac{\partial^2 U(q_1,q_2,q_3)}{\partial q_3^2} + k^2 U(q_1,q_2,q_3) \end{cases} \tag{3}$$

$$\begin{cases} H_{q1}(q_1,q_2,q_3) = \dfrac{1}{h_1}\dfrac{\partial^2 V(q_1,q_2,q_3)}{\partial q_3 \partial q_1} + j\omega\varepsilon\dfrac{1}{h_2}\dfrac{\partial U(q_1,q_2,q_3)}{\partial q_2} \\ H_{q2}(q_1,q_2,q_3) = \dfrac{1}{h_2}\dfrac{\partial^2 V(q_1,q_2,q_3)}{\partial q_2 \partial q_3} - j\omega\varepsilon\dfrac{1}{h_1}\dfrac{\partial U(q_1,q_2,q_3)}{\partial q_1} \\ H_{q3}(q_1,q_2,q_3) = \dfrac{\partial^2 V(q_1,q_2,q_3)}{\partial q_3^2} + k^2 V(q_1,q_2,q_3) \end{cases} \tag{4}$$

while the Borgnis' potentials satisfy these two equations of the same kind:

$$\nabla_t^2 U(q_1,q_2,q_3) + \frac{\partial^2 U(q_1,q_2,q_3)}{\partial q_3^2} + k^2 U(q_1,q_2,q_3) = 0 \tag{5}$$

$$\nabla_t^2 V(q_1,q_2,q_3) + \frac{\partial^2 V(q_1,q_2,q_3)}{\partial q_3^2} + k^2 V(q_1,q_2,q_3) = 0 \tag{6}$$

being

$$\nabla_t^2 = \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2}{h_3}\frac{\partial}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1}{h_2}\frac{\partial}{\partial q_2}\right)\right].$$

Referring to Table 1, the reference systems satisfying (2), and for which, thus, scalar equations like (5), (6) can be derived, are the following seven: Cartesian, circular cylindrical, parabolic cylindrical, elliptic cylindrical, spherical, bipolar, and conical. We can observe that, apart from the conical case, equations (5), (6) are scalar Helmholtz equations in all these reference systems.

Once scalar uncoupled differential equations are obtained, the second problem to be considered is to find analytical solutions of these equations. A simple way to explore the possibility of a closed form solution for a partial differential equation is to check whether or not variable factorization can be applied to the scalar equations. A factorized analytical solution of the scalar Helmholtz equation can be directly found in eleven reference systems (Cartesian, circular cylindrical, parabolic cylindrical, elliptic cylindrical, spherical, prolate spheroidal, oblate spheroidal, parabolic, conical, paraboloidal, ellipsoidal) and it is expressed in terms of particular sets of orthogonal functions [26]. On the other hand, Borgnis' potential theory allows reducing the vector wave equation to a scalar differential one of the Helmholtz kind in six reference systems (Cartesian, circular cylindrical, parabolic cylindrical, elliptic cylindrical, spherical, bipolar), while equations (5) and (6) with factored analytical solutions can be found in the conical system. Eventually, we can conclude that among the orthogonal systems of Table 1, the following admit a scalar equation and a closed form solution: Cartesian, circular cylindrical, parabolic cylindrical, elliptic cylindrical, spherical and conical.

In the other reference systems, the solution of the electromagnetic field should be obtained numerically. For a convenient reference, the previous discussion is summarized in Table 2.

4. COMPLEX MEDIA AND THE CONTRAVARIANT APPROACH

In this section we present a different approach to solve the electromagnetic problem in conformal structures. Such an approach, based on an extension of the transmission line formulation, is effective also when "complex media" are involved. Here and in the following the expression "complex media" refers to materials whose constitutive relations involve anisotropic behaviors and/or magneto-electric coupling. As shown in the previous section, the analytical solution, even in the case of isotropic homogeneous media, is limited to some reference systems only and it is not possible in the general case. When considering complex media as substrates for integrated conformal structures, vector wave equations for $\boldsymbol{E}$ and $\boldsymbol{H}$ fields are no longer of the Helmholtz kind and the solution becomes more difficult even in the simplest reference systems. Limiting our study to linear

Table 2. Analytical solution possibility of the vector wave equation in linear, isotropic and homogeneous media for the reference systems reported in Table 1.

Reference System	1st Problem: Scalar Equation		2nd Problem: Analytical Solution	Final Result
	Direct Projection	Borgnis Potentials	Solution by Factorization	Analytical (A) Numerical (N)
Cartesian	YES	YES	YES	A
Circular Cylindrical	YES	YES	YES	A
Parabolic Cylindrical	YES	YES	YES	A
Elliptic Cylindrical	YES	YES	YES	A
Spherical	NO	YES	YES	A
Prolate Spheroidal	NO	NO	YES	N
Oblate Spheroidal	NO	NO	YES	N
Parabolic	NO	NO	YES	N
Conic	NO	YES	YES	A
Paraboloidal	NO	NO	YES	N
Ellipsoidal	NO	NO	YES	N
Bipolar	YES	YES	NO	N
Bispherical	NO	NO	NO	N
Toroidal	NO	NO	NO	N
Cardioid Cylinder	NO	NO	NO	N
Tangent Sphere	NO	YES	NO	N

materials, the constitutive relations can be written as:

$$\begin{cases} \boldsymbol{B} = \underline{\boldsymbol{\mu}}(q_1, q_2, q_3) \cdot \boldsymbol{H} + \underline{\boldsymbol{\beta}}(q_1, q_2, q_3) \cdot \boldsymbol{E} \\ \boldsymbol{D} = \underline{\boldsymbol{\alpha}}(q_1, q_2, q_3) \cdot \boldsymbol{H} + \underline{\boldsymbol{\varepsilon}}(q_1, q_2, q_3) \cdot \boldsymbol{E} \end{cases} \tag{7}$$

where the previous time-harmonic variation law has been assumed. $\underline{\boldsymbol{\varepsilon}}$ and $\underline{\boldsymbol{\mu}}$ are the permittivity and permeability tensors, respectively, while $\underline{\boldsymbol{\alpha}}$ and $\underline{\boldsymbol{\beta}}$ take into account the coupling effect between electric and magnetic fields (magneto-electric effect). They collapse to scalar quantities when anisotropic effects are not present. Moreover, their elements satisfy some physical constraints, as widely discussed in [28].

Since in general the curl expression is quite awkward in reference systems different from the canonical ones, curl Maxwell's equations

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega(\underline{\boldsymbol{\mu}} \cdot \boldsymbol{H} + \underline{\boldsymbol{\beta}} \cdot \boldsymbol{E}) \\ \nabla \times \boldsymbol{H} = j\omega(\underline{\boldsymbol{\alpha}} \cdot \boldsymbol{H} + \underline{\boldsymbol{\varepsilon}} \cdot \boldsymbol{E}) \end{cases} \tag{8}$$

should be properly manipulated in order to get a more convenient formulation. Adopting the following change of variables and normalizations, in fact:

$$\begin{cases} \overline{\boldsymbol{E}} = \underline{\boldsymbol{K}} \cdot \boldsymbol{E} \\ \overline{\boldsymbol{H}} = \underline{\boldsymbol{K}} \cdot \boldsymbol{H} \end{cases}, \quad \underline{\nabla}_r = \frac{1}{jk_0}\underline{\nabla}, \quad \begin{cases} \underline{\boldsymbol{\varepsilon}}_r = \varepsilon_0^{-1}\underline{\boldsymbol{\varepsilon}} \\ \underline{\boldsymbol{\mu}}_r = \mu_0^{-1}\underline{\boldsymbol{\mu}} \end{cases}, \quad \begin{cases} \underline{\boldsymbol{\alpha}}_r = c_0\underline{\boldsymbol{\alpha}} \\ \underline{\boldsymbol{\beta}}_r = c_0\underline{\boldsymbol{\beta}} \end{cases}$$

where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$, and

$$\underline{\boldsymbol{K}} = \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix} \qquad \underline{\nabla} = \begin{pmatrix} 0 & -\partial/\partial q_3 & \partial/\partial q_2 \\ \partial/\partial q_3 & 0 & -\partial/\partial q_1 \\ -\partial/\partial q_2 & \partial/\partial q_1 & 0 \end{pmatrix}$$

curl Maxwell's equations can be rewritten in the following form:

$$\begin{cases} \left(\underline{\nabla}_r + \overline{\underline{\boldsymbol{\beta}}}_r\right) \cdot \overline{\boldsymbol{E}} = -\overline{\underline{\boldsymbol{\mu}}}_r \cdot Z_0\overline{\boldsymbol{H}} \\ \left(\underline{\nabla}_r - \overline{\underline{\boldsymbol{\alpha}}}_r\right) \cdot Z_0\overline{\boldsymbol{H}} = \overline{\underline{\boldsymbol{\varepsilon}}}_r \cdot \overline{\boldsymbol{E}} \end{cases} \tag{9}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ and $\overline{\underline{\boldsymbol{v}}} = h_1h_2h_3\underline{\boldsymbol{K}}^{-1} \cdot \underline{\boldsymbol{v}} \cdot \underline{\boldsymbol{K}}^{-1}$, being $\underline{\boldsymbol{v}} = \underline{\boldsymbol{\varepsilon}}_r, \underline{\boldsymbol{\mu}}_r, \underline{\boldsymbol{\alpha}}_r, \underline{\boldsymbol{\beta}}_r$.

It is worth to underline that, since the obtained equations in the contravariant fields $\overline{\boldsymbol{E}}$ and $\overline{\boldsymbol{H}}$ [29] are formally equivalent to the vector equations obtainable in the Cartesian geometry, the formalism here derived allows handling Maxwell equations in every orthogonal reference system in the same way as in the Cartesian case. This property leads to important simplifications and may be employed, as will be shown in the next section, even in a more extensive way to find out analytical solutions of the electromagnetic field sustained by conformal structures. Basically, the relation between (9) and (8) is straightforward: the complexity related to the locally varying curvature in the geometry (in (8)) has been suitably transferred in a variation of the medium inhomogeneity with the spatial coordinates (in (9)), following the transformation $\overline{\underline{\boldsymbol{v}}} = h_1h_2h_3\underline{\boldsymbol{K}}^{-1} \cdot \underline{\boldsymbol{v}} \cdot \underline{\boldsymbol{K}}^{-1}$.

Exploiting the formalism introduced by (9), the derivation of a generalized transmission line system of equations relating the field components becomes a straightforward task, as in the Cartesian case [30]. Starting from (9), eliminating the longitudinal components of the fields (with respect to $\hat{\boldsymbol{q}}_3$ direction), the following telegraphers' equations for the transverse fields $\overline{\boldsymbol{E}}_t$ and $\overline{\boldsymbol{H}}_t$ can be derived:

$$\begin{cases} \dfrac{\partial\overline{\boldsymbol{E}}_t}{\partial\overline{q}_3} = \overline{\underline{\boldsymbol{A}}} \cdot \overline{\boldsymbol{E}}_t + \overline{\underline{\boldsymbol{Z}}} \cdot \overline{\boldsymbol{H}}_t \\ \dfrac{\partial\overline{\boldsymbol{H}}_t}{\partial\overline{q}_3} = \overline{\underline{\boldsymbol{Y}}} \cdot \overline{\boldsymbol{E}}_t + \overline{\underline{\boldsymbol{B}}} \cdot \overline{\boldsymbol{H}}_t \end{cases} \tag{10}$$

where $\overline{q}_i = jk_0 q_i$ $(i = 1, 2, 3)$.

The matrices $\underline{\overline{\boldsymbol{A}}}, \underline{\overline{\boldsymbol{Z}}}, \underline{\overline{\boldsymbol{Y}}}, \underline{\overline{\boldsymbol{B}}}$ are operatorial matrices, applied to the transverse field components. Their elements have, in the general case, a complicated form and depend on the constitutive parameters, on the scale factors and on the derivatives with respect to the transverse variables. The elements of $\underline{\overline{\boldsymbol{A}}}$ and $\underline{\overline{\boldsymbol{Z}}}$ are reported in the following, while those ones of the two other matrices $\underline{\overline{\boldsymbol{Y}}}$ and $\underline{\overline{\boldsymbol{B}}}$ can be easily derived by using the duality principle:

$$\begin{cases} A_{11} = -\overline{\beta}_{21} - \left(\dfrac{\partial}{\partial \overline{q}_1} - \overline{\beta}_{23}\right)\left(\dfrac{\overline{\alpha}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_2} - \overline{\beta}_{31}\right) + \dfrac{\overline{\mu}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{31}\right) \\ \qquad -\overline{\mu}_{23}\left(\dfrac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_2} - \overline{\beta}_{31}\right) + \dfrac{\overline{\beta}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{31}\right) \\ A_{12} = -\overline{\beta}_{22} + \left(\dfrac{\partial}{\partial \overline{q}_1} - \overline{\beta}_{23}\right)\left(\dfrac{\overline{\alpha}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_1} + \overline{\beta}_{32}\right) - \dfrac{\overline{\mu}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{32}\right) \\ \qquad +\overline{\mu}_{23}\left(\dfrac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_1} + \overline{\beta}_{32}\right) - \dfrac{\overline{\beta}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{32}\right) \\ A_{21} = \overline{\beta}_{11} - \left(\dfrac{\partial}{\partial \overline{q}_2} + \overline{\beta}_{13}\right)\left(\dfrac{\overline{\alpha}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_2} - \overline{\beta}_{31}\right) + \dfrac{\overline{\mu}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{31}\right) \\ \qquad +\overline{\mu}_{13}\left(\dfrac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_2} - \overline{\beta}_{31}\right) + \dfrac{\overline{\beta}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{31}\right) \\ A_{22} = \overline{\beta}_{12} + \left(\dfrac{\partial}{\partial \overline{q}_2} + \overline{\beta}_{13}\right)\left(\dfrac{\overline{\alpha}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_1} + \overline{\beta}_{32}\right) - \dfrac{\overline{\mu}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{32}\right) \\ \qquad -\overline{\mu}_{13}\left(\dfrac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\left(\dfrac{\partial}{\partial \overline{q}_1} + \overline{\beta}_{32}\right) - \dfrac{\overline{\beta}_{33}}{\overline{\Delta}}\overline{\varepsilon}_{32}\right) \end{cases}$$

$$\begin{cases} Z_{11} = -\overline{\mu}_{21} - \left(\frac{\partial}{\partial \overline{q}_1} - \overline{\beta}_{23}\right)\left(\frac{\overline{\mu}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_2} + \overline{\alpha}_{31}\right) - \frac{\overline{\alpha}_{33}}{\overline{\Delta}}\overline{\mu}_{31}\right) \\ \qquad -\overline{\mu}_{23}\left(\frac{\overline{\beta}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_2} + \overline{\alpha}_{31}\right) - \frac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\overline{\mu}_{31}\right) \\ Z_{12} = -\overline{\mu}_{22} + \left(\frac{\partial}{\partial \overline{q}_1} - \overline{\beta}_{23}\right)\left(\frac{\overline{\mu}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_1} - \overline{\alpha}_{32}\right) + \frac{\overline{\alpha}_{33}}{\overline{\Delta}}\overline{\mu}_{32}\right) \\ \qquad +\overline{\mu}_{23}\left(\frac{\overline{\beta}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_1} - \overline{\alpha}_{32}\right) + \frac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\overline{\mu}_{32}\right) \\ Z_{21} = \overline{\mu}_{11} - \left(\frac{\partial}{\partial \overline{q}_2} + \overline{\beta}_{13}\right)\left(\frac{\overline{\mu}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_2} + \overline{\alpha}_{31}\right) - \frac{\overline{\alpha}_{33}}{\overline{\Delta}}\overline{\mu}_{31}\right) \\ \qquad +\overline{\mu}_{13}\left(\frac{\overline{\beta}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_2} + \overline{\alpha}_{31}\right) - \frac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\overline{\mu}_{31}\right) \\ Z_{22} = \overline{\mu}_{12} + \left(\frac{\partial}{\partial \overline{q}_2} + \overline{\beta}_{13}\right)\left(\frac{\overline{\mu}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_1} - \overline{\alpha}_{32}\right) + \frac{\overline{\alpha}_{33}}{\overline{\Delta}}\overline{\mu}_{32}\right) \\ \qquad -\overline{\mu}_{13}\left(\frac{\overline{\beta}_{33}}{\overline{\Delta}}\left(\frac{\partial}{\partial \overline{q}_1} - \overline{\alpha}_{32}\right) + \frac{\overline{\varepsilon}_{33}}{\overline{\Delta}}\overline{\mu}_{32}\right) \end{cases}$$

being $\overline{\Delta} = \overline{\mu}_{33}\overline{\varepsilon}_{33} - \overline{\alpha}_{33}\overline{\beta}_{33}$ and $\overline{v}_{ij}$ the elements of tensor $\underline{\overline{\boldsymbol{v}}}$.

The system here found is still analytical and does not contain any numerical approximation. Its exact solution is very difficult in the general case, and solutions are not known except for very special cases. However, the structure of these equations is very suitable for numerical applications and in particular for a MoL procedure [31–36]. In this case, as extensively discussed in [24], the four operatorial matrices in (10) can be made algebraic after a proper 2-D discretization on the transverse coordinate surface q_3=constant and a total-derivative system can be easily obtained, analytically solvable along the third direction ($\hat{\boldsymbol{q}}_3$). This procedure, described more thoroughly in [24], extends the MoL algorithm [31–36] to conformal components involving complex media and represents an easy and efficient way to study numerically these structures.

Moreover, (10) are commonly used in electromagnetic theory as a starting point to derive the spectral dyadic Green's function of integrated structures with a prescribed coordinate stratification axis (in this case $\hat{\boldsymbol{q}}_3$). Applying a suitable 2D spectral transform, the transverse derivatives appearing in $\underline{\overline{\boldsymbol{A}}}$, $\underline{\overline{\boldsymbol{Z}}}$, $\underline{\overline{\boldsymbol{Y}}}$ and $\underline{\overline{\boldsymbol{B}}}$ become algebraic as well and it is possible to look for analytical solutions of the equations in the remaining spatial variable q_3. For canonical reference systems the proper spectral transform is already known (e.g., Fourier transform, Hankel transform, Legendre transform) and the transverse derivatives are replaced by proper combinations of transverse wave-numbers, as it can be readily recognized for isotropic materials. In this case, the Green's function derivation from (10) is straightforward, while for other reference systems the preliminary step of finding a proper spectral transform has to be performed and is out of the scopes of the present paper.

5. ISOMORPHISM

As already pointed out in the previous section, the formalism presented above rewrites Maxwell's equations in the generalized reference system in the same form as in the Cartesian case. In the following, we will show that this implies the possibility of associating a suitable planar structure to the conformal one under analysis or, in other words, to find out an isomorphism between the sets of conformal and planar components.

Let's consider a stratified conformal component as the one depicted in Fig. 3a. The interfaces between two adjacent conformal slabs are assumed to be properly represented in a given reference system (in this case the cylindrical one) as coordinate surfaces q_i=constant. In addition, each i-th slab is characterized by the constitutive tensors $\underline{\boldsymbol{\varepsilon}}_i$, $\underline{\boldsymbol{\mu}}_i$, $\underline{\boldsymbol{\alpha}}_i$, $\underline{\boldsymbol{\beta}}_i$, in general depending on the spatial coordinates. Let us assume that $\boldsymbol{E}$ and $\boldsymbol{H}$are the electromagnetic field solutions of Maxwell's equations in every conformal slab and that the proper boundary conditions are satisfied. As stated before, an isomorphic planar structure (Fig. 3b) can be synthesized in the Cartesian reference system $(\tilde{x}, \tilde{y}, \tilde{z})$, whose interface surfaces are obtained through the simple transformations:

$$
\begin{aligned}
q_1 &= \text{constant}_1 \rightarrow \tilde{x} = \text{constant}_1 \\
q_2 &= \text{constant}_2 \rightarrow \tilde{y} = \text{constant}_2 \\
q_3 &= \text{constant}_3 \rightarrow \tilde{z} = \text{constant}_3
\end{aligned}
$$

and whose planar slabs are filled up by materials with transformed constitutive tensors expressed in the form:

$$
\begin{aligned}
\underline{\tilde{\boldsymbol{v}}}(\tilde{x}, \tilde{y}, \tilde{z}) \;=\; & h_1(\tilde{x}, \tilde{y}, \tilde{z}) h_2(\tilde{x}, \tilde{y}, \tilde{z}) h_3(\tilde{x}, \tilde{y}, \tilde{z}) \underline{\boldsymbol{K}}^{-1}(\tilde{x}, \tilde{y}, \tilde{z}) \cdot \underline{\boldsymbol{v}}(\tilde{x}, \tilde{y}, \tilde{z}) \\
& \cdot \underline{\boldsymbol{K}}^{-1}(\tilde{x}, \tilde{y}, \tilde{z})
\end{aligned}
$$

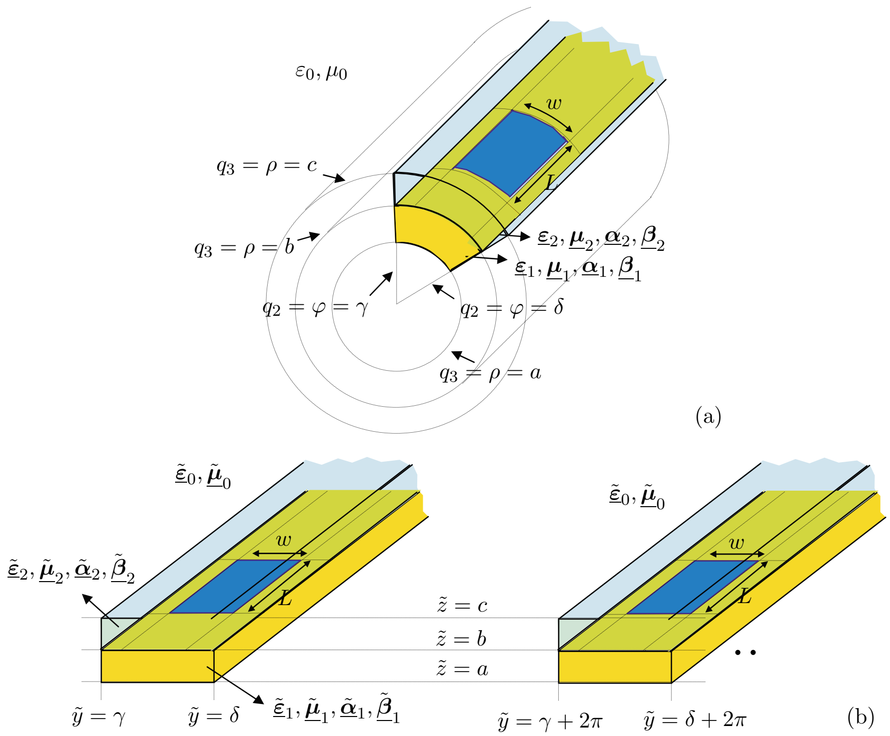


Figure 3. (a) Conformal metallic patch mounted on a cylindrical support and (b) its isomorphic geometry. The associated reference systems are: $\hat{\boldsymbol{q}}_1 = \hat{\boldsymbol{z}},\ \hat{\boldsymbol{q}}_2 = \hat{\boldsymbol{\varphi}},\ \hat{\boldsymbol{q}}_3 = \hat{\boldsymbol{\rho}};\ h_1 = h_3 = 1,\ h_2 = \rho$ for Fig. 3a and $\hat{\boldsymbol{q}}_1 = \hat{\tilde{\boldsymbol{x}}},\ \hat{\boldsymbol{q}}_2 = \hat{\tilde{\boldsymbol{y}}},\ \hat{\boldsymbol{q}}_3 = \hat{\tilde{\boldsymbol{z}}}$ for Fig. 3b.

$$= \begin{pmatrix} \dfrac{h_2 h_3}{h_1} v_{11} & h_3 v_{12} & h_2 v_{13} \\ h_3 v_{21} & \dfrac{h_1 h_3}{h_2} v_{22} & h_1 v_{23} \\ h_2 v_{31} & h_1 v_{32} & \dfrac{h_1 h_2}{h_3} v_{33} \end{pmatrix} \tag{11}$$

Following (9), the electromagnetic field $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}}$ excited in this isomorphic planar structure is contravariant with respect to the one excited in the conformal structure, i.e.,

$$\begin{cases} \tilde{\boldsymbol{E}} = \underline{\boldsymbol{K}}(\tilde{x}, \tilde{y}, \tilde{z}) \cdot \boldsymbol{E} \\ \tilde{\boldsymbol{H}} = \underline{\boldsymbol{K}}(\tilde{x}, \tilde{y}, \tilde{z}) \cdot \boldsymbol{H} \end{cases} \tag{12}$$

and therefore solving one of the two electromagnetic problems implies solution of the second one. The equivalence of the two differential problems satisfied respectively by $\boldsymbol{E}, \boldsymbol{H}$ and $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}}$ is granted, following [28], by:

- the same system of equations: Maxwell's equations are invariant with respect to the transformation here introduced, as shown in the previous section;
- the same set of boundary conditions on the interfaces separating the stratified slabs: on coordinate surfaces the boundary conditions are factorized and refer to the tangential components of the field;
- the same radiation condition at infinity.

In the case of closed structures, the equivalence is straightforward and can be easily employed to model conformal waveguides. In the general case, however, particular attention should be paid to:

- special values of the spatial variables for which the coordinate surfaces are singular (i.e., they reduce to lines or points, as it happens for instance for the coordinate surface $\rho = 0$ in the circular cylindrical system of Fig. 3): in this case, the isomorphic planar equivalent remains a plane, but on this surface some constitutive elements go to infinity, as visible from (11). These singularities in the constitutive tensors, though not physical, ensure the right behavior of the field on the planes isomorphic to the singular surfaces, as discussed later.
- open structures: the empty space (i.e., air surrounding the conformal component) is transformed into an inhomogeneous medium, whose constitutive tensors are:

$$\underline{\tilde{\boldsymbol{\varepsilon}}}_0 = \begin{pmatrix} \dfrac{h_2 h_3}{h_1}\varepsilon_0 & 0 & 0 \\ 0 & \dfrac{h_1 h_3}{h_2}\varepsilon_0 & 0 \\ 0 & 0 & \dfrac{h_1 h_2}{h_3}\varepsilon_0 \end{pmatrix}$$

$$\underline{\tilde{\boldsymbol{\mu}}}_0 = \begin{pmatrix} \dfrac{h_2 h_3}{h_1}\mu_0 & 0 & 0 \\ 0 & \dfrac{h_1 h_3}{h_2}\mu_0 & 0 \\ 0 & 0 & \dfrac{h_1 h_2}{h_3}\mu_0 \end{pmatrix}.$$

- geometries with periodic coordinates: the planar equivalent structure will follow the same periodicity and, since also the excitation will have a periodic nature, the whole electromagnetic field maintains for symmetry the same periodicity and may be studied only in one spatial period.

As an example involving also the particular cases now cited, Fig. 3a represents a conformal cylindrical patch with a homogeneous substrate and a superstrate. Its planar equivalent is shown in Fig. 3b, which depicts an infinite array of rectangular patches, periodical along $\tilde{y}$, loaded by inhomogeneous media. The equations satisfied by the field $\boldsymbol{E}, \boldsymbol{H}$ in the cylindrical structure are the same satisfied in the planar structure by $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{H}}$ and the two fields are linked by relation (12). At the interfaces the boundary conditions are of the same form and are related to the same field components in the two isomorphic reference systems. Also the radiation condition is expressed in an equivalent form: the fields decay for $q_3 = \rho \to +\infty$, $\tilde{z} \to +\infty$ in both the structures. Referring to singular surfaces, when $q_3 = \rho \to 0^+$, the coordinate surface becomes a line. In the equivalent planar structure on the plane $\tilde{z} \to 0^+$ the constitutive tensors become singular:

$$\underline{\tilde{\boldsymbol{v}}}_0 \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where

$$\underline{\tilde{\boldsymbol{v}}}_0 = v_0 \begin{pmatrix} \rho & 0 & 0 \\ 0 & 1/\rho & 0 \\ 0 & 0 & \rho \end{pmatrix}$$

with $v_0 = \varepsilon_0,\ \mu_0,\ \underline{\tilde{\boldsymbol{v}}}_0 = \underline{\tilde{\boldsymbol{\varepsilon}}}_0,\ \underline{\tilde{\boldsymbol{\mu}}}_0$.

Notice that, though not physical, such an expression for the constitutive tensors avoids radiation of power towards the region $\tilde{z} < 0$, whose points are not mapped in the cylindrical structure (for which $q_3 = \rho \geq 0$). Examples like the previous one, or even more complex, can be analyzed in more detail, showing the complete correspondence of the two field solutions.

As already pointed out, it is worth mentioning that the proposed isomorphism essentially transfers the complexity related to the local curvature of the conformal geometry into a proper inhomogeneity of the media characterizing the equivalent planar component. As a consequence of this property, the analysis of a conformal component can be equivalently performed by studying its isomorphic planar structure and then obtaining the electromagnetic field of interest through (12). It has to be noted, however, that the planar analysis maintains the same mathematical difficulties if one is interested in a rigorous analytical solution of the field equations (which is obvious since the resulting equations in the two reference systems are the same). Nevertheless, the introduction of the isomorphic planar structure makes the numerical analysis of conformal components viable through Finite Difference Time Domain (FDTD), Finite Element (FE), MoL and other well established numerical methods that can take into account possible medium inhomogeneities in planar structures. In this direction, the numerical results presented in [24], based on a proper extension of the MoL to study conformal components, may be revised as a straightforward application of the theoretical approach developed in this paper. Some details about this concept will be addressed in the following section.

As a final remark concerning the isomorphism here presented, it is worth to notice that the complete mathematical equivalence between the conformal problem and the related planar one retains also the related physical phenomena. For instance, creepy waves or mutual coupling are still present in the equivalent planar mapping and their contribution to the radiation process may still be considered and weighted. This is due to the fact that the dispersion relation associated to the transformed inhomogeneous planar structure remains unchanged and the solutions corresponding to the curved structure are indeed fully preserved.

6. NUMERICAL RESULTS

As previously remarked, the generalized transmission line formulation derived in section IV can be applied to develop a full-wave numerical tool based on the MoL to simulate the behavior of conformal structures. As shown by the authors in [24], the application of the MoL when complex media and conformal structures are involved is not a straightforward matter and a proper extension of the method is needed.

The details of the extended method may be found in [24] where some numerical results, showing the capabilities of the numerical tool developed following the MoL approach, are also presented. Those numerical results already validate the theoretical formulation presented in this paper. In addition, in this section we propose a revised reading of some of them, based on the application of the approach presented in Section 5.

The first example we consider here is related to the simple test structure reported in Fig. 4a. Such a structure has been already analyzed through the extended MoL approach in the cylindrical reference system [24] and the numerical results there obtained agree very well with the ones reported in the literature [37]. In this case we have performed a new simulation of the wraparound cylindrical

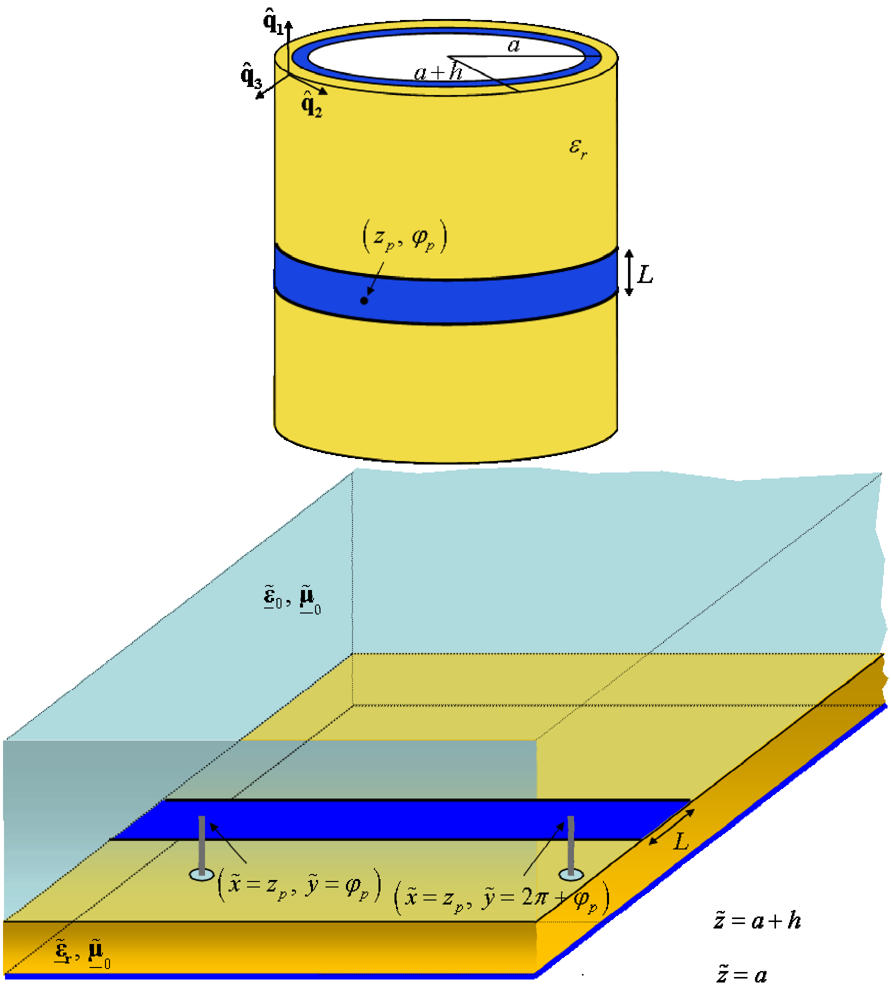


Figure 4. (a) Wrap-around cylindrical patch antenna. The geometrical and electrical parameters are: $L = 80\,\text{mm}$, $h = 1.6\,\text{mm}$, $a = 20\,\text{mm}$, $z_p =$ upper patch edge position, $\varphi_p = 0$, $\varepsilon_r = 4.2 + j0.02$. (b) Isomorphic planar structure.

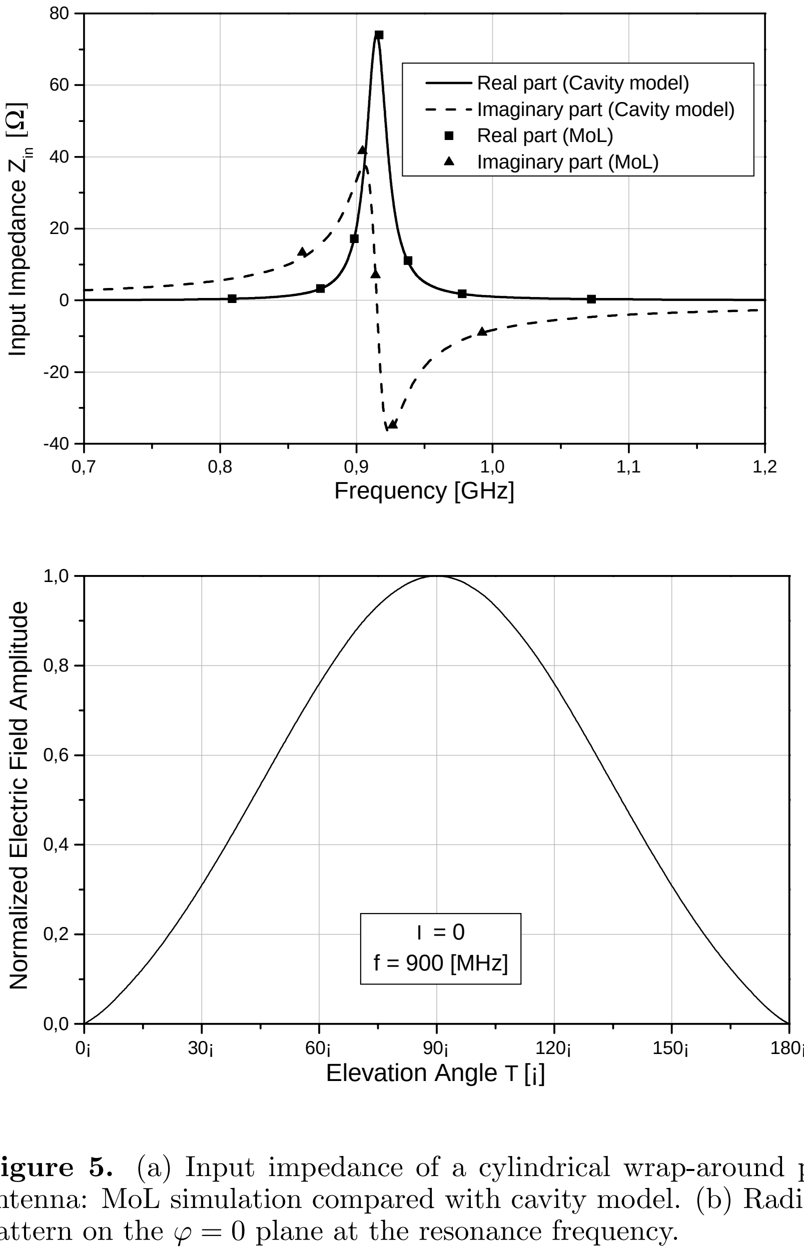


Figure 5. (a) Input impedance of a cylindrical wrap-around patch antenna: MoL simulation compared with cavity model. (b) Radiation pattern on the $\varphi = 0$ plane at the resonance frequency.

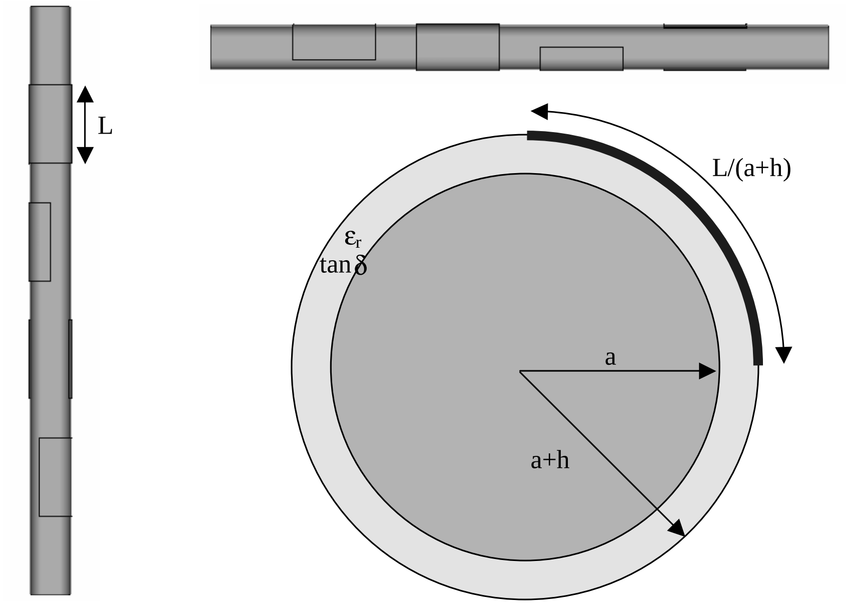


Figure 6. Geometry of an array of four patches on a cylindrical substrate with a cylindrical ground plane. The geometrical and electrical parameters of the structure are: $L = 0.155\,\mathrm{m}$; $a = 0.05\,\mathrm{m}$; $h = 0.0016$; $\varepsilon_r = 1.15$; $\tan\delta = 0.001$.

antenna applying the approach developed in Section 5.

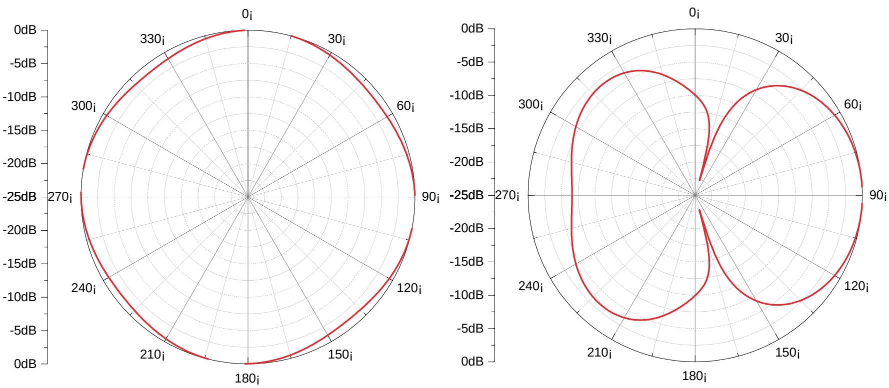
The isomorphic planar structure is depicted in Fig. 4b and consists of an infinitely long microstrip line loaded by inhomogeneous media and fed periodically by in-phase excitation probes.

The excited field in the isomorphic structure is periodical as well and can be obtained through a MoL procedure which takes into account both the inhomogeneities of the media and the structure periodicity.

The structure has been discretized in two dimensions with 8×20 lines, adopting periodical boundary conditions [38] in the y direction and absorbing boundary conditions and a perfect magnetic plane (placed at the center of the metallic strip to exploit the structure symmetry) in the x direction. The antenna input impedance is plotted as a function of frequency in Fig. 5a and compared with the cavity model reported in [37]. Fig. 5b, instead, plots the radiation pattern on the principal plane at the resonant frequency.

The results agree very well with those obtained through a standard MoL algorithm developed in the cylindrical coordinates [24].

A similar analysis, closer to a real application, has also been



(a) (b)

Figure 7. Radiation patterns of the array of Fig. 6, (a) horizontal plane, (b) vertical plane.

performed considering the more complicated cylindrical structure depicted in Fig. 6. In this case the antenna consists of four metallic patches located on a cylindrical substrate and the structure has been tested again exploiting the isomorphic planar structure obtained applying the transformation rules presented in Section 5.

The patches have been designed to resonate at 900 MHz and to work with a double linear polarization. Their position should grant an omni-directional radiation pattern for a whole array, for a typical radiobase station for mobile communications.

In Fig. 7, the radiation patterns on the horizontal (Fig. 7a) and vertical (Fig. 7b) planes are depicted, as obtained from our simulation. The directivity is around 3.8 dB.

7. CONCLUSIONS

This paper has the aim to present an extensive formulation for the treatment of Maxwell's equations in the generalized orthogonal reference system. The theoretical approach starts with an overview of the possible analytical solutions of the field wave equations in the case of isotropic and homogeneous media, showing the limited number of reference systems in which a complete solution is viable. Then, a general formulation valid for any orthogonal reference system and any linear material has been presented, showing its advantage of handling Maxwell's equations formally as in the Cartesian reference system. A first result is the generalization of the transmission line equations, relating the transverse components of the electromagnetic field with respect to a generic coordinate reference direction. This system of equations is suitably written both for the application of a numerical MoL procedure, which have been already developed by the authors [24], and for an analytical approach in the spectral domain.

Moreover, by exploiting the same mathematical formalism an interesting relation between conformal components and equivalent planar structures has been presented. This result essentially transfers the complexities related to the varying local curvature of the component into a suitable variation of the medium inhomogeneities and may be successfully employed to simplify the numerical analysis of closed and open microwave conformal components. Some numerical examples, based on the application of the MoL, have been finally presented to show the potential impact of the proposed isomorphism in the simulation of complex conformal structures.

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