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EIGENFUNCTIONAL REPRESENTATION OF DYADIC GREEN'S FUNCTIONS IN CYLINDRICALLY MULTILAYERED GYROELECTRIC CHIRAL MEDIA

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Abstract—This paper presents an eigenfunction expansion of the electric-type dyadic Green's functions for both a unbounded gyroelectric chiral medium and a cylindrically-multilayered gyroelectric chiral medium in terms of the cylindrical vector wave functions. The unbounded and scattering Green dyadics are formulated based on the principle of scattering superposition for the electromagnetic waves, namely, the direct wave and scattered waves. First, the unbounded dyadic Green's functions are correctly derived and some mistakes occurring in the literature are pointed out. Secondly, the scattering dyadic Green's functions are formulated and their coefficients are obtained from the boundary conditions at each interface. These coefficients are expressed in a compact form of recurrence matrices; coupling between TE and TM modes are considered and various wave modes are decomposed one from another. Finally, three cases, where the impressed current source are located in the first, the intermediate, and the last regions respectively, are taken into account in the mathematical manipulation of the coefficient recurrence matrices for the dyadic Green's functions.

1. INTRODUCTION

The dyadic Green's functions (DGFs) play an important role in electromagnetic theory and its applications to practical analysis of various electromagnetic boundary value problems. The eigenfunction expansion of the dyadic Green's functions has been well developed and applied over the last several decades [1–4], despite the ever increasing demand for numerical methods. The vector wave functions have found versatile applications in the formulation of the dyadic Green's functions, as can be seen from the work done [3, 5–16]. Although the DGFs in isotropic media have been well-studied in the last three decades, complete formulation of the DGFs in various anisotropic media using the eigenfunction expansion technique has not been achieved so far. Since 1970's, the dyadic Green's functions in anisotropic media have been derived [2, 17–30] using (1) the Fourier transform technique, (2) the method of angular spectrum expansion, and (3) the transmission matrix method. The DGFs and fields in gyroelectric media have also been formulated [31–36].

There have been some results for bianisotropic media or gyroelectric chiral media available nowadays, however most of them are basically valid for unbounded media only while some of them are not correct, for instance, the results in [36] commented by [37]. Thus, the motivation of this work is quite appearant. Different from the existing work, this paper aims at (1) the direct development of the unbounded dyadic Green's functions in an unbounded gyroelectric chiral medium where the cylindrical vector wave expansion technique is employed and mistakes in the existing work are pointed out; (2) the formulations of the scattering dyadic Green's functions and their coefficients in a cylindrically multilayered gyroelectric chiral medium where each layer is assumed to be the gyroelectric chiral medium and its results can be reduced to those of the isotropic media, and where the source is assumed to have an arbitrary 3-dimensional distribution and can be located anywhere while the field point can also be arbitrarily located in the multilayers; and (3) the rigorous derivation of the irrotational part of the dyadic Green's functions which were not always provided in the existing work. Due to the different geometries of the multilayered gyroelectric chiral media, the formulation of the dyadic Green's functions differs one from another. The work included in the present paper is a further extension of the previous work done in [38], where the dyadic Green's functions have been represented for the planar-multilayered gyroelectric chiral media.

In order to obtain a complete, general representation of the eigenfunction expansion of the dyadic Green's functions in a cylindrically multilayered medium, the mathematical derivation under the cylindrical coordinates is carefully conducted in this paper. Section 2 summarizes the normal series eigenfunction expansion in terms of the cylindrical vector wave functions, making the paper more complete and self-contained. The dyadic Green's function in cylindrical coordinates for an unbounded gyroelectric chiral medium is then obtained by a rigorous eigenfunction expansion in terms of the cylindrical vector wave functions. Most importantly, the irrotational part of the dyadic Green's function for the unbounded gyroelectric chiral medium is derived. The results obtained herein by the cylindrical vector wave function expansions are new formulas unavailable elsewhere. In Section 3, the principle of scattering superposition is applied to obtain the scattering dyadic Green's functions. These scattering coefficients of the dyadic Green's functions have, although coupled to each other, been obtained from the boundary conditions and have been represented by a set of compact recurrence matrices. Further analysis includes the derivation of the scattering coefficients of the DGFs for various cases where the current source located in the first, the intermediate and the last regions of the multilayered structure. Throughout the paper, a time dependence $e^{-i\omega t}$ is assumed and suppressed in the analysis.

2. DGFS FOR UNBOUNDED GYROELECTRIC CHIRAL MEDIA

A homogeneous gyroelectric chiral medium with the time harmonic excitation can be characterized by a set of constitutive relations [36]

$$\boldsymbol{D} = \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + i\xi_c \boldsymbol{B}, \tag{1a}$$

$$\boldsymbol{H} = i\xi_c \boldsymbol{E} + \boldsymbol{B}/\mu, \tag{1b}$$

where

$$\overline{\boldsymbol{\epsilon}} = \begin{bmatrix} \epsilon & -ig & 0 \\ ig & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{2}$$

Substituting (2) into the source-incorporated Maxwell's equations leads to

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - 2\omega\mu\xi_c \boldsymbol{\nabla} \times \boldsymbol{E} - \omega^2 \mu \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} = i\omega\mu \boldsymbol{J}. \tag{3}$$

2.1. General Formulation of Unbounded DGFs

The electric field can thus be expressed in terms of the DGF and electric source distribution as

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu \int_{V'} \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r'}) \cdot \boldsymbol{J}(\boldsymbol{r'})\, dV', \tag{4}$$

where V' denotes the volume occupied by the exciting current source. Similarly, substituting (4) into (3) leads to

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r'}) - 2\omega\mu\xi_c \boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r'}) - \omega^2 \mu \overline{\boldsymbol{\epsilon}} \cdot \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r'}) = \overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r'}), \tag{5}$$

where $\overline{\boldsymbol{I}}$ and $\delta(\boldsymbol{r} - \boldsymbol{r'})$ denotes the dyadic identity and Dirac delta function, respectively.

According to the well-known Ohm-Rayleigh method, the source term in (5) can be expanded in terms of the solenoidal and irrotational cylindrical vector wave functions in cylindrical coordinate system. Thus,

$$\begin{aligned} \overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r'}) = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} & [\boldsymbol{M}_n(h, \lambda) \boldsymbol{A}_n(h, \lambda) \\ & + \boldsymbol{N}_n(h, \lambda) \boldsymbol{B}_n(h, \lambda) + \boldsymbol{L}_n(h, \lambda) \boldsymbol{C}_n(h, \lambda)], \end{aligned} \tag{6}$$

where $\boldsymbol{M}_n(h, \lambda)$ & $\boldsymbol{N}_n(h, \lambda)$ are the solenoidal, and $\boldsymbol{L}_{n\lambda}(h)$ is the irrotational, cylindrical vector wave functions while λ & h are the spectral longitudinal and radial wave numbers, respectively. The solenoidal and irrotational cylindrical vector wave functions are defined as [36]

$$\boldsymbol{M}_n(h, \lambda) = \boldsymbol{\nabla} \times [\Psi_n(h, \lambda) \widehat{\boldsymbol{z}}], \tag{7a}$$

$$\boldsymbol{N}_n(h, \lambda) = \frac{1}{k_\lambda} \boldsymbol{\nabla} \times \boldsymbol{M}_n(h, \lambda), \tag{7b}$$

$$\boldsymbol{L}_n(h, \lambda) = \boldsymbol{\nabla} [\Psi_n(h, \lambda)], \tag{7c}$$

where $k_\lambda = \sqrt{\lambda^2 + h^2}$, and the generating function is given by

$$\Psi_n(h,\lambda) = J_n(\lambda\rho)e^{i(n\phi+hz)}. \tag{8}$$

The vector expansion coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$ in (6), are to be determined from the orthogonality relationships among the cylindrical vector wave functions which are given by:

$$\begin{aligned} &\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{M}_n(h,\lambda) \cdot \boldsymbol{M}_{-n'}(-h',-\lambda') \\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{N}_n(h,\lambda) \cdot \boldsymbol{N}_{-n'}(-h',-\lambda') \\ &= 4\pi^2 \lambda \delta(\lambda-\lambda')\delta(h-h')\delta_{nn'}, \end{aligned} \tag{9a}$$

$$\begin{aligned} &\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{L}_n(h,\lambda) \cdot \boldsymbol{L}_{-n'}(-h',-\lambda') \\ &= 4\pi^2 \frac{(\lambda^2+h^2)}{\lambda} \delta(\lambda-\lambda')\delta(h-h')\delta_{nn'}, \end{aligned} \tag{9b}$$

and

$$\begin{aligned} &\int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{M}_n(h,\lambda) \cdot \boldsymbol{N}_{-n'}(-h',-\lambda') \\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{N}_n(h,\lambda) \cdot \boldsymbol{L}_{-n'}(-h',-\lambda') \\ &= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \int_{-\infty}^{\infty} dz\, \boldsymbol{L}_n(h,\lambda) \cdot \boldsymbol{M}_{-n'}(-h',-\lambda') \\ &= 0. \end{aligned} \tag{9c}$$

Therefore, by taking the scalar product of (6) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$ each at a time, the vector expansion coefficients are given by:

$$\boldsymbol{A}_n(h,\lambda) = \frac{1}{4\pi^2\lambda} \boldsymbol{M}'_{-n}(-h,-\lambda), \tag{10a}$$

$$\boldsymbol{B}_n(h,\lambda) = \frac{1}{4\pi^2\lambda} \boldsymbol{N}'_{-n}(-h,-\lambda), \tag{10b}$$

$$\boldsymbol{C}_n(h,\lambda) = \frac{\lambda}{4\pi^2(\lambda^2+h^2)} \boldsymbol{L}'_{-n}(-h,-\lambda), \tag{10c}$$

where the prime notation of the cylindrical vector wave functions denotes the expressions at the source point $\boldsymbol{r}'$.

The dyadic Green's function can thus be expanded as [36]:

$$\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \left[\boldsymbol{M}_n(h,\lambda)\,\boldsymbol{a}_n(h,\lambda) + \boldsymbol{N}_n(h,\lambda)\,\boldsymbol{b}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\,\boldsymbol{c}_n(h,\lambda)\right], \tag{11}$$

where the vector expansion coefficients $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ are obtained by substituting (11) and (6) into (5), which the dyadic Green's function must satisfy, and noting the instinct properties of the vector wave functions,

$$\boldsymbol{M}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{N}_n(h,\lambda), \tag{12a}$$

$$\boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{M}_n(h,\lambda), \tag{12b}$$

$$\boldsymbol{\nabla}\times\boldsymbol{L}_n(h,\lambda) = 0, \tag{12c}$$

we end up with

$$\begin{aligned}&\int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \Big\{\Big[k_\lambda^2\overline{\boldsymbol{I}} - \omega^2\mu\overline{\boldsymbol{\epsilon}}\Big]\cdot\left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda)\right.\\ &\quad\left.+\boldsymbol{N}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right] - 2k_\lambda\omega\mu\xi_c\left[\boldsymbol{N}_n(h,\lambda)\boldsymbol{a}_n(h,\lambda)\right.\\ &\quad\left.+\boldsymbol{M}_n(h,\lambda)\boldsymbol{b}_n(h,\lambda)\right] - \omega^2\mu\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{L}_n(h,\lambda)\boldsymbol{c}_n(h,\lambda)\Big\}\\ &= \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{A}_n(h,\lambda)\right.\\ &\quad\left.+\boldsymbol{N}_n(h,\lambda)\boldsymbol{B}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\boldsymbol{C}_n(h,\lambda)\right].\end{aligned} \tag{13}$$

The above approach shows an important fact that the afore-assumed unknowns, $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, and $\boldsymbol{c}_n(h,\lambda)$, may not be the same as those coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$ although they were assumed to be the same in [36].

By taking the anterior scalar product of (13) with the vector wave equations, respectively, and by performing the integration over the entire space, we can formulate the equations satisfied by the unknown vectors and the known scalar and vector parameters in a matrix form as given below:

$$[\boldsymbol{\Omega}][\boldsymbol{X}] = [\boldsymbol{\Theta}], \tag{14}$$

where $[\boldsymbol{\Omega}]$ is a 3×3 matrix given by

$$[\boldsymbol{\Omega}] = [\boldsymbol{\Omega}_1\boldsymbol{\Omega}_2\boldsymbol{\Omega}_3] \tag{15}$$

with

$$\boldsymbol{\Omega}_1 = \left[\begin{array}{c} k_\lambda^2 - \omega^2 \mu \epsilon \\ -\omega\mu \left(2\xi_c k_\lambda + \omega g \frac{h}{k_\lambda} \right) \\ -i\omega^2 \mu g \frac{\lambda^2}{k_\lambda^2} \end{array} \right], \tag{16a}$$

$$\boldsymbol{\Omega}_2 = \left[\begin{array}{c} -\omega\mu \left(2\xi_c k_\lambda + \omega g \frac{h}{k_\lambda} \right) \\ k_\lambda^2 - \frac{\omega^2\mu}{k_\lambda^2} (h^2\epsilon + \lambda^2 \epsilon_z) \\ -\frac{ih\lambda^2}{k_\lambda^3} \omega^2 \mu (\epsilon - \epsilon_z) \end{array} \right], \tag{16b}$$

$$\boldsymbol{\Omega}_3 = \left[\begin{array}{c} i\omega^2 \mu g \\ \frac{ih}{k_\lambda} \omega^2 \mu (\epsilon - \epsilon_z) \\ -\frac{\omega^2\mu}{k_\lambda^2} \left(\lambda^2 \epsilon + h^2 \epsilon_z \right) \end{array} \right], \tag{16c}$$

and $[\boldsymbol{X}]$ and $[\boldsymbol{\Theta}]$ are two column vectors given respectively by

$$[\boldsymbol{X}] = \left[\begin{array}{c} \boldsymbol{a}_n(h,\lambda) \\ \boldsymbol{b}_n(h,\lambda) \\ \boldsymbol{c}_n(h,\lambda) \end{array} \right], \tag{17a}$$

$$[\boldsymbol{\Theta}] = \left[\begin{array}{c} \boldsymbol{A}_n(h,\lambda) \\ \boldsymbol{B}_n(h,\lambda) \\ \boldsymbol{C}_n(h,\lambda) \end{array} \right]. \tag{17b}$$

Solving (14), we have the solutions for $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ as

$$\boldsymbol{a}_n(h,\lambda) = \frac{1}{\Gamma} \left[\alpha_1 \boldsymbol{A}_n(h,\lambda) + \beta_1 \boldsymbol{B}_n(h,\lambda) + \gamma_1 \boldsymbol{C}_n(h,\lambda) \right], \tag{18a}$$

$$\boldsymbol{b}_n(h,\lambda) = \frac{1}{\Gamma} \left[\alpha_2 \boldsymbol{A}_n(h,\lambda) + \beta_2 \boldsymbol{B}_n(h,\lambda) + \gamma_2 \boldsymbol{C}_n(h,\lambda) \right], \tag{18b}$$

$$\boldsymbol{c}_n(h,\lambda) = \frac{1}{\Gamma} \left[\alpha_3 \boldsymbol{A}_n(h,\lambda) + \beta_3 \boldsymbol{B}_n(h,\lambda) + \gamma_3 \boldsymbol{C}_n(h,\lambda) \right], \tag{18c}$$

where

$$\Gamma = k_\lambda^2 (h^2\epsilon_z + \epsilon\lambda^2) - \mu\omega^2 \left[2h^2 \epsilon\epsilon_z - \lambda^2 (g^2 - \epsilon^2 - \epsilon\epsilon_z) + 4\mu\xi^2 (h^2\epsilon_z + \epsilon\lambda^2) \right] \\ -4gh\epsilon_z \mu^2 \xi_c \omega^3 + \epsilon_z \mu^2 \omega^4 (\epsilon^2 - g^2) \tag{19}$$

and the coupling coefficients are

$$\alpha_1 = h^2\epsilon_z + \lambda^2\epsilon - \omega^2 \mu \epsilon \epsilon_z, \tag{20a}$$

$$\beta_1 = \alpha_2 = \frac{\omega\mu}{k_\lambda} \left[gh\epsilon_z \omega + 2\xi_c (h^2 \epsilon_z + \epsilon\lambda^2) \right], \tag{20b}$$

$$\beta_2 = \frac{1}{k_\lambda^2}\left[(k_\lambda^2 - \omega^2\mu\epsilon)(h^2\epsilon_z + \lambda^2\epsilon) + \omega^2\mu g^2\lambda^2\right], \tag{20c}$$

$$\gamma_1 = -\frac{k_\lambda^2}{\lambda^2}\alpha_3 = i\left[2\omega\mu h\xi_c(\epsilon - \epsilon_z) + g(k_\lambda^2 - \omega^2\mu\epsilon_z)\right], \tag{20d}$$

$$\begin{aligned}\gamma_2 &= -\frac{k_\lambda^2}{\lambda^2}\beta_3\\ &= i\frac{1}{k_\lambda}\left[h(k_\lambda^2 - \mu\omega^2\epsilon)(\epsilon - \epsilon_z) + \omega\mu g(2k_\lambda^2\xi_c + gh\omega\right],\end{aligned} \tag{20e}$$

$$\begin{aligned}\gamma_3 = \frac{1}{\omega^2\mu}&\left\{-k_\lambda^4 + \omega^2\mu\left[2h^2\epsilon + \lambda^2(\epsilon + \epsilon_z) + 4k_\lambda^2\mu\xi_c^2\right]\right.\\ &\left.+4gh\xi_c\mu^2\omega^3 + \frac{\omega^4\mu^2}{k_\lambda^2}\left[h^2(g^2 - \epsilon^2) - \epsilon\epsilon_z\lambda^2\right]\right\}.\end{aligned} \tag{20f}$$

It should be noted that the substitution of (11) and (6) into (5) to give (2.1) is based upon the condition that one can interchange the summation on n and the integrals on h, λ. This condition can be justified if one notes that the terms in the square brackets of (6) and (11) are continuous with respect to h and λ, simultaneously.

Now, it becomes quite clear that each of the coefficients, $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, and $\boldsymbol{c}_n(h,\lambda)$, is actually a linear combination of those known coefficients, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$. In other words, the coupling of the TE and TM modes from source distribution, and to the field expression, exists. Thus, the results obtained in [36] are not correct at all because such coupling is absent there in the formulation. In terms of the cylindrical vector wave functions for the gyroelectric chiral media, this paper presents a correct form of the unbounded dyadic Green's functions.

Furthermore, the unbounded dyadic Green's function can be written as

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = \int_{-\infty}^{\infty}dh\int_0^{\infty}d\lambda\sum_{n=-\infty}^{\infty}\frac{1}{4\pi^2\lambda\Gamma}&\{\boldsymbol{M}_n(h,\lambda)[\alpha_1\boldsymbol{M}'_{-n}\\ &\times(-h,-\lambda) + \beta_1\boldsymbol{N}'_{-n}(-h,-\lambda) + \frac{\lambda^2}{k_\lambda^2}\gamma_1\boldsymbol{L}'_{-n}(-h,-\lambda)]\\ &+\boldsymbol{N}_n(h,\lambda)[\alpha_2\boldsymbol{M}'_{-n}(-h,-\lambda) + \beta_2\boldsymbol{N}'_{-n}(-h,-\lambda)\\ &+\frac{\lambda^2}{k_\lambda^2}\gamma_2\boldsymbol{L}'_{-n}(-h,-\lambda)] + \boldsymbol{L}_n(h,\lambda)[\alpha_3\boldsymbol{M}'_{-n}(-h,-\lambda)\\ &+\beta_3\boldsymbol{N}'_{-n}(-h,-\lambda) + \frac{\lambda^2}{k_\lambda^2}\gamma_3\boldsymbol{L}'_{-n}(-h,-\lambda)]\}.\end{aligned} \tag{21}$$

In this way, the dyadic Green's function in an unbounded gyroelectric chiral medium is explicitly represented in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions, as given in (21). However, for practical applications and interpretation to possible novel phenomena, mathematical simplification to (21) is necessary.

In order to apply the residue theorem to (21), we must first extract the part in (21) which does not satisfy the Jordan lemma [1]. To do so, we write

$$\boldsymbol{L}_n(h,\lambda) = \boldsymbol{L}_{nt}(h,\lambda) + \boldsymbol{L}_{nz}(h,\lambda), \tag{22a}$$

$$\boldsymbol{L}'_{-n}(-h,-\lambda) = \boldsymbol{L}'_{-nt}(-h,-\lambda) + \boldsymbol{L}'_{-nz}(-h,-\lambda), \tag{22b}$$

$$\boldsymbol{N}_n(h,\lambda) = \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda), \tag{22c}$$

$$\boldsymbol{N}'_{-n}(-h,-\lambda) = \boldsymbol{N}'_{-nt}(-h,-\lambda) + \boldsymbol{N}'_{-nz}(-h,-\lambda), \tag{22d}$$

where the subscript t and z denote the transverse vector components and the z-vector components respectively of the two functions $\boldsymbol{L}_n(h,\lambda)$ and $\boldsymbol{N}_n(h,\lambda)$. In terms of these functions, (21) can be rewritten in the form

$$\begin{aligned}
\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = & \int_{-\infty}^{\infty} dh \int_0^{\infty} d\lambda \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda\Gamma} \\
& \times \Big\{ (h^2\epsilon_z + \lambda^2\epsilon - \omega^2\mu\epsilon\epsilon_z) \boldsymbol{M}_n(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda) \\
& + \frac{k_\lambda}{h}\Big[g(\omega^2\mu\epsilon_z - \lambda^2) + 2h\epsilon_z\omega\mu\xi_c\Big] \\
& \times [\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda) + \boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)] \\
& + k_\lambda(gh + 2\epsilon\omega\mu\xi_c)[\boldsymbol{M}_n(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) \\
& + \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M}'_{-n}(-h,-\lambda)] \\
& + \frac{k_\lambda^2}{h^2\omega^2\mu}[\lambda^2(\omega^2\mu\epsilon + 4\omega^2\mu^2\xi_c^2 - k_\lambda^2) \\
& + \omega^2\mu\epsilon_z(k_\lambda^2 - \epsilon\omega^2\mu)]\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda) \\
& + \frac{k_\lambda^2}{h\omega^2\mu}\Big[h(k_\lambda^2 - \omega^2\mu\epsilon) - 2\omega^2\mu^2\xi_c(2h\xi_c + g\omega)\Big] \\
& \times [\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) + \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nt}(-h,-\lambda)] \\
& + \frac{k_\lambda^2}{\omega^2\mu\lambda^2}[-h^2k_\lambda^2 + \omega^2\mu\epsilon(2h^2 + \lambda^2) \\
& + 4h\omega^2\mu^2\xi_c(h\xi_c + g\omega) + \omega^4\mu^2(g^2 - \epsilon^2)] \\
& \times \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda)\Big\},
\end{aligned} \tag{23}$$

where we have expressed $\boldsymbol{L}_{nt}(h,\lambda)$ and $\boldsymbol{L}_{nz}(h,\lambda)$ in terms of $\boldsymbol{N}_{nt}(h,\lambda)$

and $\boldsymbol{N}_{nz}(h,\lambda)$, and similarly, for the primed functions; namely

$$\boldsymbol{L}_{nt}(h,\lambda) = -\frac{ik_\lambda}{h}\boldsymbol{N}_{nt}(h,\lambda), \tag{24a}$$

$$\boldsymbol{L}'_{-nt}(-h,-\lambda) = \frac{ik_\lambda}{h}\boldsymbol{N}'_{-nt}(-h,-\lambda); \tag{24b}$$

and

$$\boldsymbol{L}_{nz}(h,\lambda) = \frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda), \tag{24c}$$

$$\boldsymbol{L}'_{-nz}(-h,-\lambda) = -\frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}'_{-nz}(-h,-\lambda). \tag{24d}$$

2.2. Analytical Evaluation of the λ Integral

In this subsection, we will analytically evaluate the λ integrals for the dyadic Green's function arisen in (23). This effort is intended to make the results applicable in solving the source-incorporated boundary value problems of cylindrically multilayered structures consisting of gyroelectric chiral media.

By applying the idea given in [1] to obtain an exact expression of the irrotational dyadic Green's function, we have from (6)

$$\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r}') = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda). \tag{25}$$

Thus, the singular term in (23) is contained in the $\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}$ $(-h,-\lambda)$ dyad component.

From (19), we rewrite Γ into the following form in order to perform the λ integration,

$$\Gamma = \epsilon_z(k_\lambda^2 - k_1^2)(k_\lambda^2 - k_2^2) \tag{26}$$

where

$$k_{1,2}^2 = \frac{1}{2\epsilon}\left\{p_\lambda \pm \sqrt{p_\lambda^2 + q_\lambda}\right\} \tag{27}$$

and

$$p_\lambda = h^2(\epsilon - \epsilon_z) + \omega^2\mu\left[-g^2 + \epsilon(\epsilon + \epsilon_z + 4\mu\xi_c^2)\right]$$
$$q_\lambda = -4\epsilon\omega^2\mu[h^2(\epsilon-\epsilon_z)(\epsilon + 4\mu\xi_c^2) - 4gh\epsilon_z\omega\mu\xi_c + \epsilon^2\epsilon_z\omega^2\mu - g^2(h^2 + \epsilon_z\omega^2\mu)].$$

With simple algebraic manipulation, we can split (23) into

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = & -\int_{-\infty}^{\infty}dh\int_0^{\infty}d\lambda\sum_{n=-\infty}^{\infty}\frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\omega^2\mu\epsilon\lambda^2}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)\\ & +\int_{-\infty}^{\infty}dh\int_0^{\infty}d\lambda\sum_{n=-\infty}^{\infty}\frac{1}{4\pi^2\lambda}\frac{1}{\epsilon(k_\lambda^2-k_1^2)(k_\lambda^2-k_2^2)}\\ & \times\{(h^2\epsilon_z+\lambda^2\epsilon-\omega^2\mu\epsilon\epsilon_z)\boldsymbol{M}_n(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda)\\ & +\frac{k_\lambda}{h}\left[g(\omega^2\mu\epsilon_z-\lambda^2)+2h\epsilon_z\omega\mu\xi_c\right]\\ & \times[\boldsymbol{M}_n(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda)+\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda)]\\ & +k_\lambda(gh+2\epsilon\omega\mu\xi_c)[\boldsymbol{M}_n(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)\\ & +\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda)]\\ & +\frac{k_\lambda^2}{h^2\omega^2\mu}[\lambda^2(\omega^2\mu\epsilon+4\omega^2\mu^2\xi_c^2-k_\lambda^2)+\omega^2\mu\epsilon_z(k_\lambda^2-\epsilon\,\omega^2\mu)]\\ & \times\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda)\\ & +\frac{k_\lambda^2}{h\omega^2\mu}\left[h(k_\lambda^2-\omega^2\mu\epsilon)-2\omega^2\mu^2\xi_c(2h\xi_c+g\omega)\right]\\ & \times[\boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)+\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda)]\\ & +\frac{k_\lambda^2}{\lambda^2\epsilon\omega^2\mu}\{k_\lambda^2\left[k_\lambda^2\epsilon+h^2(\epsilon_z-2\epsilon)\right]+\omega^2\mu[g^2(k^2-h^2)\\ & +h^2(2\epsilon-\epsilon_z)(\epsilon+4\mu\xi_c^2)-k^2\epsilon(\epsilon_z+4\mu\xi_c^2)]\\ & +4gh\omega^3\mu^2\xi_c(\epsilon-\epsilon_z)+\omega^4\mu^2(g-\epsilon)(g+\epsilon)(\epsilon-\epsilon_z)\}\\ & \times\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)\}.\end{aligned}\tag{28}$$

In view of (25), the first integration term in (28) must be the contribution from the irrotational vector wave functions, given as follows:

$$-\frac{1}{\omega^2\mu\epsilon_z}\,\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}').\tag{29}$$

Obviously, this irrotational Green's dyadic of the gyroelectric chiral media can be simply reduced to that of an isotropic medium by letting $\epsilon_z=\epsilon$. It is observed that this irrotational term is only a function of the permittivity ϵ_z only. This is simply because the anisotropic medium has an axial direction of $\widehat{\boldsymbol{z}}$.

The second integration term can be evaluated by making use of the residue theorem in λ-plane (Appendix A). This term contributes from the solenoidal vector wave functions. Hence after some mathematical manipulations, we arrived at the final unbounded dyadic Green's function for a gyroelectric chiral medium which is suitable for further analysis of a cylindrically multilayered structure.

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) =& -\frac{1}{\omega^2\mu\epsilon_z}\,\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}')\\ &+\frac{i}{4\pi}\int_{-\infty}^{\infty}dh\sum_{n=-\infty}^{\infty}\frac{1}{2(k_1^2-k_2^2)}\sum_{j=1}^{2}\frac{(-1)^{j+1}}{\lambda_j^2}\\ &\times\left\{\left\{\begin{array}{c}\boldsymbol{M}^{(1)}_{n,h}(\lambda_j)\boldsymbol{P'}_{-n,-h}(-\lambda_j)\\ \boldsymbol{M}_{n,h}(-\lambda_j)\boldsymbol{P'}^{(1)}_{-n,-h}(\lambda_j)\end{array}\right\}\right.\\ &+\frac{k_{\lambda j}}{\epsilon}\left\{\begin{array}{c}\boldsymbol{Q}^{(1)}_{n,h}(\lambda_j)\boldsymbol{M'}_{-n,-h}(-\lambda_j)\\ \boldsymbol{Q}_{n,h}(-\lambda_j)\boldsymbol{M'}^{(1)}_{-n,-h}(\lambda_j)\end{array}\right\}\\ &+\frac{k_{\lambda j}^2}{h^2\omega^2\mu\epsilon}\left\{\begin{array}{c}\boldsymbol{U}^{(1)}_{n,h}(\lambda_j)\boldsymbol{N'}_{-nt,-h}(-\lambda_j)\\ \boldsymbol{U}_{n,h}(-\lambda_j)\boldsymbol{N'}^{(1)}_{-nt,-h}(\lambda_j)\end{array}\right\}\\ &\left.+\frac{k_{\lambda j}^2}{\lambda_j^2\omega^2\mu\epsilon}\left\{\begin{array}{c}\boldsymbol{V}^{(1)}_{n,h}(\lambda_j)\boldsymbol{N'}_{-nz,-h}(-\lambda_j)\\ \boldsymbol{V}_{n,h}(-\lambda_j)\boldsymbol{N'}^{(1)}_{-nz,-h}(\lambda_j)\end{array}\right\}\right\},\ \rho\gtrless\rho',\end{aligned}\tag{30}$$

where the superscript (1) of the vector wave functions denotes the first-kind cylindrical Hankel function $H_n^{(1)}(\lambda\rho)$. The vector wave functions $\boldsymbol{P'}_{-n,-h}(-\lambda_j)$, $\boldsymbol{Q}_{n,h}(\lambda_j)$, $\boldsymbol{U}_{n,h}(\lambda_j)$ and $\boldsymbol{V}_{n,h}(\lambda_j)$ are given respectively by

$$\begin{aligned}\boldsymbol{P'}_{-n,-h}(-\lambda_j) =& (\lambda_j^2+\frac{\epsilon_z}{\epsilon}h^2-\epsilon_z\omega^2\mu)\boldsymbol{M'}_{-n,-h}(-\lambda_j)\\ &+\frac{k_{\lambda j}}{\epsilon}\left[\frac{g}{h}(\epsilon_z\omega^2\mu-\lambda_j^2)+2\epsilon_z\omega\mu\xi_c\right]\boldsymbol{N'}_{-nt,-h}(-\lambda_j)\\ &+\frac{k_{\lambda j}}{\epsilon}(gh+2\epsilon\,\omega\mu\xi_c)\boldsymbol{N'}_{-nz,-h}(-\lambda_j),\end{aligned}\tag{31a}$$

$$\begin{aligned}\boldsymbol{Q}_{n,h}(\lambda_j) =& \left[\frac{g}{h}(\epsilon_z\omega^2\mu-\lambda_j^2)+2\epsilon_z\omega\mu\xi_c\right]\boldsymbol{N}_{nt,h}(\lambda_j)\\ &+(gh+2\epsilon\,\omega\mu\xi_c)\boldsymbol{N}_{nz,h}(\lambda_j),\end{aligned}\tag{31b}$$

$$\begin{aligned}\boldsymbol{U}_{n,h}(\lambda_j) =& \left[(k_{\lambda j}^2-\epsilon\,\omega^2\mu)(\epsilon_z\omega^2\mu-\lambda_j^2)+4\lambda_j^2\omega^2\mu^2\xi_c^2\right]\boldsymbol{N}_{nt,h}(\lambda_j)\\ &+h\left[h(k_j^2-\epsilon\,\omega^2\mu)-2\omega^2\mu^2\xi_c(2h\xi_c+g\omega)\right]\boldsymbol{N}_{nz,h}(\lambda_j),\end{aligned}\tag{31c}$$

$$\begin{aligned}\boldsymbol{V}_{n,h}(\lambda_j) = \lambda_j^2 &\left[(k_{\lambda j}^2 - \epsilon\,\omega^2\mu) - \frac{2\omega^2\mu^2\xi_c}{h}(2h\xi_c + g\omega)\right]\boldsymbol{N}_{nt,h}(\lambda_j)\\ &+ \frac{1}{\epsilon}\left\{k_{\lambda j}^2\left[\lambda_j^2\epsilon + h^2(\epsilon_z - \epsilon)\right] + \omega^2\mu\left[g^2\lambda_j^2\right.\right.\\ &\left. + h^2(2\epsilon - \epsilon_z)(\epsilon + 4\mu\xi_c^2) - k_{\lambda j}^2\epsilon(\epsilon_z + 4\mu\xi_c^2)\right]\\ &\left. + 4gh\omega^3\mu^2\xi_c(\epsilon - \epsilon_z) + \omega^4\mu^2(g^2 - \epsilon^2)(\epsilon - \epsilon_z)\right\}\boldsymbol{N}_{nz,h}(\lambda_j).\end{aligned} \tag{31d}$$

Now, we obtained the rigorous expression of the unbounded dyadic Green's functions for the gyroelectric chiral medium. This form differs from the existing representations of the dyadic Green's functions for the gyroelectric chiral medium or some more generalized media. The agreement can be obtained in no ways between the present form of DGFs and other forms given in the literature elsewhere. This is because (1) the dyadic Green's functions obtained elsewhere in the literature using different approaches take quite different forms as ours and the direct comparison among these theoretical results are almost impossible; (2) the dyadic Green's functions obtained using the same approach in [36] are incorrect as indicated and shown in [37]; and (3) the dyadic Green's functions given using the similar approach in [39–44] are not rigorously correct as the irrotational parts of the DGFs were missing in the presentations (where the mistakes are due to the ignorance of the Jordan lemma conditions [1]).

3. SCATTERING DGFS FOR CYLINDRICALLY MULTILAYERED GYROELECTRIC CHIRAL MEDIUM

In the following section, the scattering dyadic Green's function in a cylindrically multilayered gyroelectric chiral media is presented. The Green's dyadics are formulated based on the principle of superposition of the electromagnetic waves namely the direct wave and the scattered waves. We will solve for the source-incorporated boundary value problems of the cylindrically multilayered structures consisting of the gyroelectric chiral media.

3.1. Scattering Dyadic Green's Functions

With the scattering superposition principle, it is assumed that

$$\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r}, \boldsymbol{r'}) = \overline{\boldsymbol{G}}_0(\boldsymbol{r}, \boldsymbol{r'})\delta_f^s + \overline{\boldsymbol{G}}_s^{(fs)}(\boldsymbol{r}, \boldsymbol{r'}), \tag{32}$$

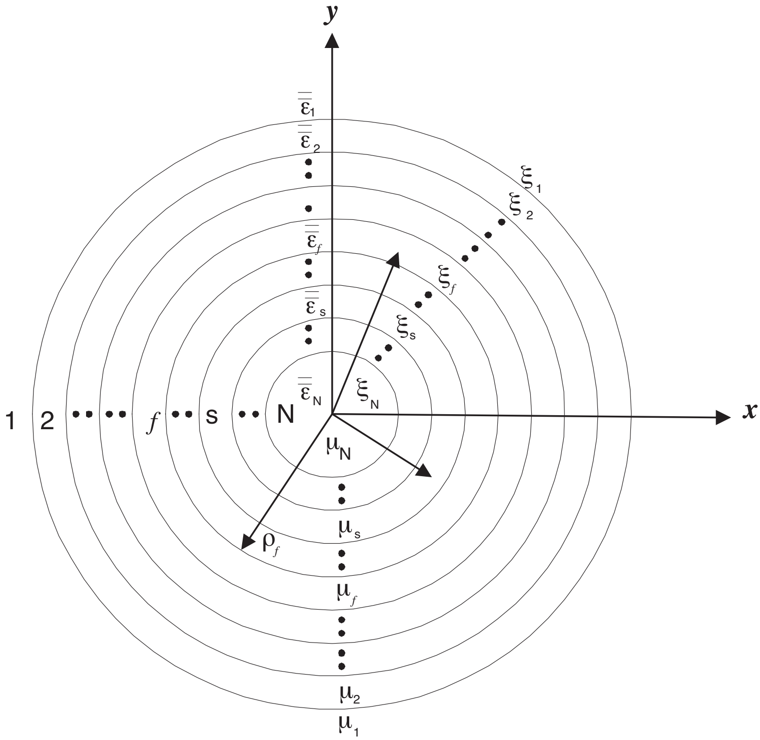

Figure 1. Geometry of a cylindrically-multilayered gyroelectric chiral medium.

where the representation of the scattered dyadic Green's function is given by:

$$\overline{\boldsymbol{G}}_s^{(fs)}(\boldsymbol{r}, \boldsymbol{r'}) = \overline{\boldsymbol{G_1}} + \overline{\boldsymbol{G_2}} \tag{33}$$

with the component dyadics given for $j = 1$ and 2 as follows:

$$\begin{aligned}\overline{\boldsymbol{G_j}} = \frac{i}{4\pi}\int_{-\infty}^{\infty}dh\sum_{n=-\infty}^{\infty}\frac{1}{(k_{1s}^2-k_{2s}^2)}\frac{(-1)^{j+1}}{\lambda_{js}^2}\Big\{(1-\delta_f^N)\boldsymbol{M}_{n,h}^{(1)}(\lambda_j^f)\\ \times\Big[(1-\delta_s^1)A_{Mj}^{fs}\boldsymbol{P'}_{-n,-h}(-\lambda_j^s)+(1-\delta_s^N)B_{Mj}^{fs}\boldsymbol{P'}^{(1)}_{-n,-h}(\lambda_j^s)\Big]\\ +(1-\delta_f^N)\frac{k_{\lambda js}}{\epsilon_s}\boldsymbol{Q}_{n,h}^{(1)}(\lambda_j^f)\Big[(1-\delta_s^1)A_{Qj}^{fs}\boldsymbol{M'}_{-n,-h}(-\lambda_j^s)\\ +(1-\delta_s^N)B_{Qj}^{fs}\boldsymbol{M'}^{(1)}_{-n,-h}(\lambda_j^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_s h^2}\boldsymbol{U}_{n,h}^{(1)}(\lambda_j^f)\end{aligned}$$

$$\Big[(1-\delta_s^1)A_{Uj}^{fs}\boldsymbol{N'}_{-nt,-\lambda}(-h_j^s)+(1-\delta_s^N)B_{Uj}^{fs}\boldsymbol{N'}^{(1)}_{-nt,-h}(\lambda_j^s)\Big]$$
$$+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\lambda_{js}^2\epsilon_{zs}\omega^2\mu_s}\boldsymbol{V}^{(1)}_{n,h}(\lambda_j^f)\Big[(1-\delta_s^1)A_{Vj}^{fs}\boldsymbol{N'}_{-nz,-h}(-\lambda_j^s)$$
$$+(1-\delta_s^N)B_{Vj}^{fs}\boldsymbol{N'}^{(1)}_{-nz,-h}(\lambda_j^s)\Big]+(1-\delta_f^N)\boldsymbol{M}_{n,h}(-\lambda_j^f)$$
$$\times\Big[(1-\delta_s^1)C_{Mj}^{fs}\boldsymbol{P'}_{-n,-h}(-\lambda_j^s)+(1-\delta_s^N)D_{Mj}^{fs}\boldsymbol{P'}^{(1)}_{-n,-h}(\lambda_j^s)\Big]$$
$$+(1-\delta_f^N)\frac{k_{\lambda js}}{\epsilon_s}\boldsymbol{Q}_{n,h}(-\lambda_j^f)\Big[(1-\delta_s^1)C_{Qj}^{fs}\boldsymbol{M'}_{-n,-h}(-\lambda_j^s)$$
$$+(1-\delta_s^N)D_{Qj}^{fs}\boldsymbol{M'}^{(1)}_{-n,-h}(\lambda_j^s)\Big]+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\epsilon_{zs}\omega^2\mu_s h^2}\boldsymbol{U}_{n,h}(-\lambda_j^f)$$
$$\times\Big[(1-\delta_s^1)C_{Uj}^{fs}\boldsymbol{N'}_{-nt,-\lambda}(-h_j^s)+(1-\delta_s^N)D_{Uj}^{fs}\boldsymbol{N'}^{(1)}_{-nt,-h}(\lambda_j^s)\Big]$$
$$+(1-\delta_f^N)\frac{k_{\lambda js}^2}{\lambda_{js}^2\epsilon_{zs}\omega^2\mu_s}\boldsymbol{V}_{n,h}(-\lambda_j^f)\Big[(1-\delta_s^1)C_{Vj}^{fs}\boldsymbol{N'}_{-nz,-h}(-\lambda_j^s)$$
$$+(1-\delta_s^N)D_{Vj}^{fs}\boldsymbol{N'}^{(1)}_{-nz,-h}(\lambda_j^s)\Big]\Big\}. \tag{34}$$

The construction of the dyadic Green's functions follows the similar considerations to those in [38]. In other words, we have taken the multiple transmissions and reflections into account when formulating the scattering dyadic Green's functions for the gyroelectric chiral cylinder of multiple layers. Although the scattering DGF can be reduced directly to that of the isotropic medium, it does differ in form from that of the isotropic medium. Therefore, it is necessary to formulate it in detail subsequently.

3.2. Boundary Condition Equations

The electric dyadic Green's function, $\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r},\boldsymbol{r'})$, satisfies the following boundary conditions at the cylindrical interfaces $\rho=\rho_j (j = 1, 2, ..., N-1)$:

$$\widehat{\boldsymbol{\rho}}\times\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r},\boldsymbol{r'})=\widehat{\boldsymbol{\rho}}\times\overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r},\boldsymbol{r'}), \tag{35a}$$

$$\widehat{\boldsymbol{\rho}}\times\left[\frac{1}{\mu_f}\boldsymbol{\nabla}\times\ \overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r},\boldsymbol{r'})-\omega\xi_{cf}\,\overline{\boldsymbol{G}}_e^{(fs)}(\boldsymbol{r},\boldsymbol{r'})\right]$$
$$=\widehat{\boldsymbol{\rho}}\times\left[\frac{1}{\mu_{f+1}}\boldsymbol{\nabla}\times\overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r},\boldsymbol{r'})-\omega\xi_{c(f+1)}\overline{\boldsymbol{G}}_e^{[(f+1)s]}(\boldsymbol{r},\boldsymbol{r'})\right]. \tag{35b}$$

To simplify the derivation of the general solution of the coefficients, we rewrite the boundary conditions (35a) and (35b) into the following matrix form.

3.3. Recurrence Formulae of Scattering DGFs' Coefficients

By using the boundary conditions, a set of linear equations of the scattering coefficients, which can be replaced by a series of compact matrices as done in [14], is obtained. The following compact recurrent equations are formulated:

$$\left[F_{lj(f+1)}\right]\cdot\left\{\left[\Upsilon_{lj(f+1)s}\right]+\delta_{f+1}^{s}\left[U_{(f+1)}\right]\right\}=\left[F_{ljf}\right]\cdot\left\{\left[\Upsilon_{ljfs}\right]+\delta_{f}^{s}\left[D_{f}\right]\right\} \tag{36}$$

where $j = 1, 2$ and $l = M, Q, U$ and V. These matrices are given by

$$[F_{Mjf}] = \begin{bmatrix} \partial\hbar_j & \partial\jmath_j \\ \frac{\lambda_{jf}^2}{\mu_f}\upsilon_{jf}\hbar_j - \omega\xi_{cf}\partial\hbar_j & \frac{\lambda_{jf}^2}{\mu_f}\upsilon_{jf}\jmath_j - \omega\xi_{cf}\partial\jmath_j \end{bmatrix}, \tag{37a}$$

$$[F_{ljf}] = \begin{bmatrix} \frac{\chi_{ljf}}{k_{\lambda jf}}\hbar_j & \frac{1}{k_{\lambda jf}}\left[\frac{w_{ltj}h^2+w_{lzj}\lambda_{jf}^2}{\mu_f}\partial\hbar_j - \omega\xi_{cf}\chi_{ljf}\hbar_j\right] \\ \frac{\chi_{ljf}}{k_{\lambda jf}}\jmath_j & \frac{1}{k_{\lambda jf}}\left[\frac{w_{ltj}h^2+w_{lzj}\lambda_{jf}^2}{\mu_f}\partial\jmath_j - \omega\xi_{cf}\chi_{ljf}\jmath_j\right] \end{bmatrix}^T, \tag{37b}$$

$$\hbar_j = H_n^{(1)}(\lambda_{jf}\rho_f), \tag{37c}$$

$$\jmath_j = J_n(\lambda_{jf}\rho_f), \tag{37d}$$

$$\partial\hbar_j = \frac{d\left[H_n^{(1)}(\lambda_{jf}\rho)\right]}{d\rho}\Big|_{\rho=\rho_f}, \tag{37e}$$

$$\partial\jmath_j = \frac{d\left[J_n(\lambda_{jf}\rho)\right]}{d\rho}\Big|_{\rho=\rho_f}, \tag{37f}$$

$$\upsilon_{jf} = \frac{nh}{\lambda_{jf}^2\rho_f} + 1, \tag{37g}$$

$$\chi_{ljf} = \frac{1}{k_{\lambda jf}}\left[w_{ltj}\frac{nh}{\rho_f} + w_{lzj}\lambda_{jf}^2\right]. \tag{37h}$$

The terms w_{ltj} and w_{lzj} are the weighting factors associated with the scattering coefficients A_{lj}^{fs}, B_{lj}^{fs}, C_{lj}^{fs} and D_{lj}^{fs}. They are expressed as

$$w_{qtj} = \frac{g_s}{h}(\epsilon_{zs}\omega^2\mu_s - \lambda_{js}^2) + 2\epsilon_{zs}\omega\mu_s\xi_{cs}, \tag{38a}$$

$$w_{qzj} = g_s h_{js} + 2\epsilon_s \omega \mu_s \xi_{cs}, \tag{38b}$$

$$w_{utj} = (k_{\lambda js}^2 - \epsilon_s \omega^2 \mu_s)(\epsilon_{zs}\omega^2 \mu_s - \lambda_{js}^2) + 4\lambda_{js}^2 \omega^2 \mu_s^2 \xi_{cs}^2, \tag{38c}$$

$$w_{uzj} = h\left[h(k_\lambda^2 - \epsilon_s \omega^2 \mu_s) - 2\omega^2 \mu_s^2 \xi_{cs}(2h\xi_{cs} + g_s\omega)\right], \tag{38d}$$

$$w_{vtj} = \lambda_{js}^2 k_{\lambda js}^2 - \epsilon_s \omega^2 \mu_s - \frac{2\omega^2 \mu_s^2 \xi_{cs}}{h}(2h\xi_{cs} + g_s\omega), \tag{38e}$$

$$\begin{aligned} w_{vzj} = \frac{1}{\epsilon_s}\Big\{ & k_{\lambda js}^2\left[\lambda_{js}^2 \epsilon_s + h^2(\epsilon_{zs} - \epsilon_s)\right] + \omega^2 \mu_s \Big[g_s^2 \lambda_{js}^2 \\ & + h^2(2\epsilon_s - \epsilon_{zs})(\epsilon_s + 4\mu_s \xi_{cs}^2) - k_{\lambda js}^2 \epsilon_s(\epsilon_{zs} + 4\mu_s \xi_{cs}^2)\Big] \\ & + 4g_s h(\epsilon_s - \epsilon_{zs})\omega^3 \mu_s^2 \xi_{cs} + \omega^4 \mu_s (g_s - \epsilon_s)(g_s + \epsilon_s)(\epsilon_s - \epsilon_{zs})\Big\}. \end{aligned} \tag{38f}$$

The following matrices are also given as

$$[\Upsilon_{ljfs}] = \begin{bmatrix} A_{lj}^{fs} & B_{lj}^{fs} \\ C_{lj}^{fs} & D_{lj}^{fs} \end{bmatrix}, \tag{39a}$$

$$[U_f] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{39b}$$

$$[D_f] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{39c}$$

Defining the following transmission T-matrix:

$$[T_{ljf}] = \left[F_{lj(f+1)f}\right]^{-1} \cdot [F_{ljff}] \tag{40}$$

where $\left[F_{lj(f+1)f}\right]^{-1}$ is the inverse matrix of $\left[F_{lj(f+1)f}\right]$, we rewrite the linear equation into the following form:

$$\left[\Upsilon_{lj(f+1)s}\right] = [T_{ljf}] \cdot \left\{[\Upsilon_{ljfs}] + \delta_f^s [D_f]\right\} - \delta_{f+1}^s \left[U_{(f+1)}\right]. \tag{41}$$

We also introduce:

$$\begin{aligned} \left[T_{lj}^K\right]_{2\times 2} &= [T_{lj,N-1}]\,[T_{lj,N-2}]\cdots[T_{lj,K+1}]\,[T_{lj,K}] \\ &= \begin{bmatrix} T_{lj,11}^K & T_{lj,12}^K \\ T_{lj,21}^K & T_{lj,22}^K \end{bmatrix}. \end{aligned} \tag{42}$$

It should be noted that the coefficients matrices of the first and the last layers have the following relations:

$$[\Upsilon_{lj1s}] = \begin{bmatrix} A_{lj}^{1s} & B_{lj}^{1s} \\ 0 & 0 \end{bmatrix}, \tag{43a}$$

$$[\Upsilon_{ljNs}] = \begin{bmatrix} 0 & 0 \\ C_{lj}^{Ns} & D_{lj}^{Ns} \end{bmatrix}. \tag{43b}$$

It should be pointed out that the current formulation for the gyroelectric chiral media is more generalized as compared to those for isotropic and chiral media in [1] and [14]. Also, it is realized from the current programming exercises that the explicit symbolic derivations of the scattering coefficients of the dyadic Green's functions for an arbitrarily large number of planar layers is not realistic so far, although possible in principle. It is basically due to the restrictions of computer platforms themselves and symbolic features of the software packages such as Mathematica. Therefore, the current procedure of formulating some of the intermediates is necessary.

3.4. Three Specific Cases

Although general, the previously obtained coefficients can be significantly reduced in form, respectively, for the following three cases where the source point is located in the first, the second, and the third regions.

3.4.1. Source in the First Layer

When the current source is located in the first layer (i.e., $s = 1$), the first term containing $(1-\delta_s^1)$ in (34) vanishes. These will further reduce the coefficient matrices in (39a) and (3.3) to:

$$[\Upsilon_{lj,11}] = \begin{bmatrix} 0 & B_{lj}^{11} \\ 0 & 0 \end{bmatrix}, \tag{44a}$$

$$[\Upsilon_{lj,m1}] = \begin{bmatrix} 0 & B_{lj}^{m1} \\ 0 & D_{lj}^{m1} \end{bmatrix}, \tag{44b}$$

$$[\Upsilon_{lj,N1}] = \begin{bmatrix} 0 & 0 \\ 0 & D_{lj}^{N1} \end{bmatrix}, \tag{44c}$$

where $m = 2, 3, \cdots, N-1$. It can be seen that only four coefficients for the first layer or the last layer, and 8 coefficients for each of the remaining layers need to be solved for. Following (41), the recurrence relations in the f^{th} layer become:

$$[\Upsilon_{lj,f1}] = [T_{lj,f-1}] \cdots [T_{lj,1}] \left\{ [\Upsilon_{lj,11}] + [D_1] \right\}. \tag{45}$$

With $f = N$ in (45), a matrix equation satisfied by the coefficient matrices in (3.4.1) can be obtained. The coefficients for the first layer where the source is (i.e., $s = 1$) is given by:

$$B_{lj}^{11} = -\frac{T_{lj,12}^{(1)}}{T_{lj,11}^{(1)}}. \tag{46}$$

The coefficients for the last layer can be derived in terms of the coefficients for the first layer given by:

$$D_{lj}^{N1} = T_{lj,21}^{(1)} B_{lj}^{11} + T_{lj,22}^{(1)}. \tag{47}$$

The coefficients for the intermediate layers can be then obtained by substituting the coefficients for the first layer in (46) to (45). Thus, all the coefficients can be obtained by these procedures.

3.4.2. Source in the Intermediate Layers

When the current source is located in an intermediate layer, (i.e. $s \neq 1, N$), only the terms containing $(1 - \delta_f^1)$ for the first layer and $(1 - \delta_f^N)$ for the last layer vanishes in (34). We thus have:

$$[\Upsilon_{lj,1s}] = \begin{bmatrix} A_{lj}^{1s} & B_{lj}^{1s} \\ 0 & 0 \end{bmatrix}, \tag{48a}$$

$$[\Upsilon_{lj,ms}] = \begin{bmatrix} A_{lj}^{ms} & B_{lj}^{ms} \\ C_{lj}^{ms} & D_{lj}^{ms} \end{bmatrix}, \tag{48b}$$

$$[\Upsilon_{lj,Ns}] = \begin{bmatrix} 0 & 0 \\ C_{lj}^{Ns} & D_{lj}^{Ns} \end{bmatrix}. \tag{48c}$$

From (41), the recurrence equation becomes:

$$\begin{aligned} [\Upsilon_{lj,fs}] &= [T_{lj,f-1}] \cdots [T_{lj,s}] \left\{ [T_{lj,s-1}] \cdots [T_{lj,1}] [\Upsilon_{lj,1s}] \right. \\ &\quad \left. + u(f-s-1) [D_s] - u(f-s) [U_s] \right\}, \end{aligned} \tag{49}$$

where $u(x - x_0)$ is the unit step function. For $f = N$, the coefficients for the first layer are given by:

$$A_{lj}^{1s} = \frac{T_{lj,11}^{(s)}}{T_{lj,11}^{(1)}}, \tag{50a}$$

$$B_{lj}^{1s} = -\frac{T_{lj,12}^{(s)}}{T_{lj,11}^{(1)}}. \tag{50b}$$

For the last layer,

$$C_{lj}^{Ns} = T_{lj,21}^{(1)} A_{lj}^{1s} - T_{lj,21}^{(s)}, \tag{51a}$$

$$D_{lj}^{Ns} = T_{lj,21}^{(1)} B_{lj}^{(s)} + T_{lj,22}^{(s)}. \tag{51b}$$

Substituting (3.4.2) into (49), the rest of the coefficients can be obtained for the dyadic Green's function.

3.4.3. Source in the Last Layer

For the source to be located in the last layer (i.e., $S = N$), the coefficients are:

$$[\Upsilon_{lj,1N}] = \begin{bmatrix} A_{lj}^{1N} & 0 \\ 0 & 0 \end{bmatrix}, \tag{52a}$$

$$[\Upsilon_{lj,mN}] = \begin{bmatrix} A_{lj}^{mN} & 0 \\ C_{lj}^{mN} & 0 \end{bmatrix}, \tag{52b}$$

$$[\Upsilon_{lj,NN}] = \begin{bmatrix} 0 & 0 \\ C_{lj}^{NN} & 0 \end{bmatrix}. \tag{52c}$$

From the recurrence equation (41), similarly we have,

$$[\Upsilon_{lj,fN}] = [T_{lj,f-1}] \cdots [T_{lj,1}] [\Upsilon_{lj,1N}] - u(f-N) [U_N]. \tag{53}$$

By letting $f = N$,

$$A_{lj}^{1N} = \frac{1}{T_{lj,11}^{(1)}}. \tag{54}$$

And for the last layer,

$$C_{lj}^{NN} = T_{lj,21}^{(1)} A_{lj}^{1N}. \tag{55}$$

Similarly, we can obtain the rest of the coefficients.

Thus, we have obtained a complete set of scattering dyadic Green's functions in a gyroelectric chiral medium in terms of the cylindrical vector wave functions. Reduction can be made for formulating the dyadic Green's function in a less complex medium of specific cylindrical geometries.

4. CONCLUSION

A complete eigenfunction expansion of the dyadic Green's functions for a unbounded gyroelectric chiral medium and a cylindrically multilayered gyroelectric chiral medium is presented in this paper. The unbounded dyadic Green's function in the gyroelectric chiral medium is first obtained, based on the Ohm-Raleigh method. The scattered dyadics are constructed with the principle of scattering superposition for a multilayered medium. By the use of the boundary conditions at each interface, the scattering coefficients of the dyadic Green's functions are represented in the form of compact recurrence matrices. Further analysis is performed for three cases, i.e., the source excitation located in the first, the intermediate and the last regions, respectively. From the formulation of the generalized dyadic Green's functions, it is seen that (1) the general form of DGFs for the cylindrically multilayered gyroelectric chiral medium can be reduced to those DGFs for less complex media, such as chiral media, anisotropic media, and isotropic media; (2) the wave mode splitting is observed from the formulation of the DGFs; and (3) as a result, the dyadic Green's functions can be easily decomposed using the aforementioned modes. In summary, the present paper contributes to (1) a correct formulation of the unbounded dyadic Green's function in a gyroelectric chiral medium as compared with the published work [36] and [37], (2) a detailed formulation of the irrotational part of the dyadic Green's functions which was quite often ignored in the recent publications such as in [36], (3) a formulation of the dyadic Green's functions in a cylindrically multilayered gyroelectric chiral medium, and (4) the compact matrix expression of the scattering coefficients of the dyadic Green's functions. Application of the present work can be made to problems of electromagnetic wave propagation through and scattering by, and antenna radiation in, cylindrically multilayered gyroelectric chiral media.

APPENDIX A. INTEGRATION OF λ

To this end, we write

$$\boldsymbol{M}_n(h,\lambda) = (\boldsymbol{\nabla}\times\hat{\boldsymbol{z}})\Psi_n(h,\lambda), \tag{A1a}$$

$$\boldsymbol{M}'_{-n}(-h,-\lambda) = (\boldsymbol{\nabla}'\times\hat{\boldsymbol{z}})\Psi'_{-n}(-h,-\lambda), \tag{A1b}$$

$$\begin{aligned}\boldsymbol{N}_n(h,\lambda) &= \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda)\\ &= (\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{\boldsymbol{z}})\frac{1}{k_\lambda}\Psi_n(h,\lambda).\end{aligned} \tag{A1c}$$

Noting that $\nabla\times\nabla\times\hat{z} = ih\nabla - \nabla^2\hat{z} = ih\nabla_t - \nabla_t^2\hat{z}$ where the subscript t denotes the transverse gradient operator. Then,

$$\boldsymbol{N}_{nt}(h,\lambda) = (ih\nabla_t)\frac{1}{k_\lambda}\Psi_n(h,\lambda), \tag{A2a}$$

and

$$\boldsymbol{N}_{nz}(h,\lambda) = (-\nabla_t^2\hat{z})\frac{1}{k_\lambda}\Psi_n(h,\lambda). \tag{A2b}$$

Similarly,

$$\boldsymbol{N}'_{-nt}(-h,-\lambda) = (-ih\nabla'_t)\frac{1}{k_\lambda}\Psi'_{-n}(-h,-\lambda), \tag{A3a}$$

and

$$\boldsymbol{N}'_{-nz}(-h,-\lambda) = (-\nabla_t'^2\hat{z})\frac{1}{k_\lambda}\Psi_{-n}(-h,-\lambda), \tag{A3b}$$

where $\Psi_n(h,\lambda)$ is given by (8). In actuality, the differentiations are performed before the integration. But in this case, it may be simpler to perform the $d\lambda$ integration before taking the derivative operations inside $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_{nt}(h,\lambda)$ and $\boldsymbol{N}_{nz}(h,\lambda)$. Hence after exchanging the order of $d\lambda$ integration and differentiation, typical integrals involving $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_{nt}(h,\lambda)$ and $\boldsymbol{N}_{nz}(h,\lambda)$ terms in (28) are of the form

$$I_2 = \int_0^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)J_{-n}(-\lambda\rho')}{\lambda(k_\lambda^2 - k_1^2)(k_\lambda^2 - k_2^2)}. \tag{A4}$$

With

$$J_n(\lambda\rho) = \frac{1}{2}\left[H_n^{(1)}(\lambda\rho) + H_n^{(2)}(\lambda\rho)\right],$$

we thus have

$$I_2 = \frac{1}{2}\lim_{\delta\to 0}\left[\int_\delta^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_{-n}^{(1)}(-\lambda\rho')}{\lambda(k_\lambda^2 - k_1^2)(k_\lambda^2 - k_2^2)} + \int_\delta^\infty d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_{-n}^{(2)}(-\lambda\rho')}{\lambda(k_\lambda^2 - k_1^2)(k_\lambda^2 - k_2^2)}\right]. \tag{A5}$$

Here, the limit is introduced because now, a pole at $\lambda = 0$ exists in each of the integrands due to the Hankel functions. Furthermore, by letting $\lambda = e^{-i\pi}\lambda'$, and using the reflection formulas $H_n^{(2)}(e^{-i\pi}\lambda\rho) = (-1)^n H_n^{(1)}(\lambda\rho)$ and $J_n(-\lambda\rho) = (-1)^n J_n(\lambda\rho)$,

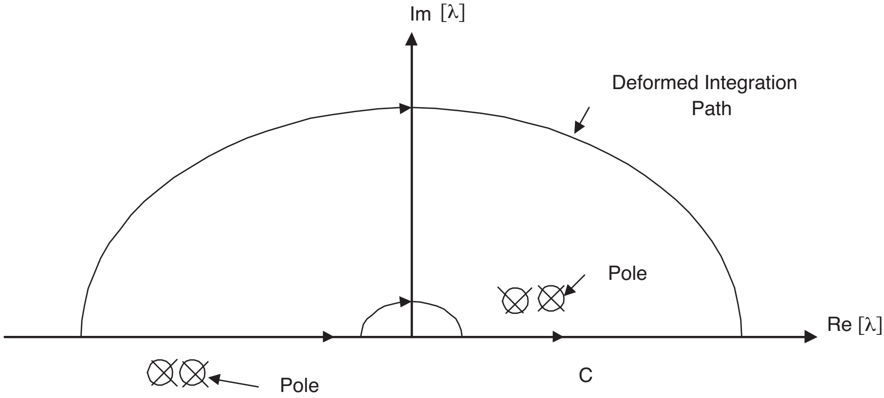

Figure A1. The contour C and the deformed path of integration on the complex λ plane.

$$
\begin{aligned}
I_2 &= \frac{1}{2}\lim_{\delta\to 0}\left[\int_{\delta}^{\infty} d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(k_\lambda^2-k_1^2)(k_\lambda^2-k_2^2)} + \int_{-\infty}^{-\delta} d\lambda' \frac{f(\lambda)J_n(\lambda'\rho)H_n^{(1)}(\lambda'\rho')}{\lambda'(k_\lambda^2-k_1^2)(k_\lambda^2-k_2^2)}\right] \\
&= \frac{1}{2} P.V. \int_{-\infty}^{\infty} d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(k_\lambda^2-k_1^2)(k_\lambda^2-k_2^2)}, \qquad \text{(A6)}
\end{aligned}
$$

where P.V. represents a principal value integral. Notice that in (A6), poles exist at $\lambda = \pm\sqrt{k_{1,2}^2 - h^2}$ which may be on the real axis. But again with the introduction of some loss, these Figure A1 and the integral in (A6) is well defined.

Moreover, a residue contribution can be added to (A6) at the origin to make it a complete contour integral. In other words,

$$
\begin{aligned}
I_2 = \frac{1}{2}\int_C d\lambda \frac{f(\lambda)J_n(\lambda\rho)H_n^{(1)}(\lambda\rho')}{\lambda(k_\lambda^2-k_1^2)(k_\lambda^2-k_2^2)} \\
- \frac{1}{2|n|} f(0)\left(\frac{\rho^<}{\rho^>}\right)^{|n|}\left[\frac{1}{(h^2-k_1^2)(h^2-k_2^2)}\right], \qquad \text{(A7)}
\end{aligned}
$$

where the following relation has been utilized:

$$
\lim_{\lambda\to 0}\left[J_n(\lambda\rho^<)H_n^{(1)}(\lambda\rho^>)\right] = -\frac{i}{|n|\pi}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}.
$$

In (A7), the last term is the residue contribution which has been included in the first term to make C a continuous contour.

Thus, we have for $\rho > \rho'$ the following formula:

$$I_2 = \pi i \sum_{j=1}^{2} \frac{(-1)^{j+1} f(\lambda_j) J_n(\lambda_j \rho') H_n^{(1)}(\lambda_j \rho)}{2\lambda_j^2 (k_1^2 - k_2^2)} - \frac{1}{2|n|} f(0) \left(\frac{\rho'}{\rho}\right)^{|n|} \left[\frac{1}{(h^2 - k_1^2)(h^2 - k_2^2)}\right]. \tag{A8}$$

A similar operation on $\rho < \rho'$ will result in:

$$\begin{aligned} I_2 \;&=\; \pi i \sum_{j=1}^{2} (-1)^{j+1} \frac{f(\lambda_j) J_n(-\lambda_j \rho) H_n^{(1)}(-\lambda_j \rho')}{2\lambda_j^2 (k_1^2 - k_2^2)} \\ &\quad - \frac{1}{2|n|} f(0) (\frac{\rho}{\rho'})^{|n|} \left[\frac{1}{(h^2 - k_1^2)(h^2 - k_2^2)}\right], \end{aligned} \tag{A9}$$

since $f(\lambda) = f(-\lambda)$.

The term due to the residue contribution from the origin $\lambda = 0$ in (28) is given by

$$\begin{aligned} I_0 = &-\frac{1}{8\pi^2 |n|} \frac{1}{\epsilon(h^2 - k_1^2)(h^2 - k_2^2)} \Big\{ (h^2 \epsilon_z - \omega^2 \mu \epsilon \epsilon_z) \\ &\times (\nabla_t \times \hat{z})(\nabla_t' \times \hat{z}) + \frac{k_\lambda}{h} \left[g\omega^2 \mu \epsilon_z + 2h \epsilon_z \omega \mu \xi_c \right] \\ &\times [(\nabla_t \times \hat{z})(-ih \nabla_t') + (ih \nabla_t)(\nabla_t' \times \hat{z})] \\ &+ k_\lambda (gh + 2\epsilon \omega \mu \xi_c) [(\nabla_t \times \hat{z})(-\nabla_t'^2 \hat{z}) - (\nabla_t^2 \hat{z})(\nabla_t' \times \hat{z})] \\ &+ \frac{k_\lambda^2}{h^2 \omega^2 \mu} \left[\omega^2 \mu \epsilon_z (k_\lambda^2 - \epsilon\, \omega^2 \mu) \right] (ih \nabla_t)(-ih \nabla_t') \\ &+ \frac{k_\lambda^2}{h \omega^2 \mu} [h(k_\lambda^2 - \omega^2 \mu \epsilon) - 2\omega^2 \mu^2 \xi_c (2h \xi_c + g\omega)] \\ &\times [(ih \nabla_t)(-\nabla_t'^2 \hat{z}) + (\nabla_t^2 \hat{z})(ih \nabla_t)] + \frac{k_\lambda^2}{\lambda^2 \epsilon \omega^2 \mu} \{ k_\lambda^2 [k_\lambda^2 \epsilon + h^2 (\epsilon_z - 2\epsilon)] \\ &+ \omega^2 \mu [g^2 (k^2 - h^2) + h^2 (2\epsilon - \epsilon_z)(\epsilon + 4\mu \xi_c^2) - k^2 \epsilon (\epsilon_z + 4\mu \xi_c^2)] \\ &+ 4gh\omega^3 \mu^2 \xi_c (\epsilon - \epsilon_z) + \omega^4 \mu^2 (g - \epsilon)(g + \epsilon)(\epsilon - \epsilon_z) \} \psi_n(h, \lambda) \\ &\times \psi_{-n}'(-h, -\lambda)(-\nabla_t^2 \hat{z})(-\nabla_t'^2 \hat{z}) \Big\} \psi_n(h, \lambda) \psi_{-n}'(-h, -\lambda) \left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \end{aligned} \tag{A10}$$

where all the intermediates inside should be replaced by themselves after taking $\lambda = 0$.

This term I_0 tends to vanish as a consequence of

$$(\boldsymbol{\nabla}_t\times\widehat{\boldsymbol{z}})(\boldsymbol{\nabla}_t'\times\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=(i\widehat{\boldsymbol{\rho}}-\widehat{\boldsymbol{\phi}})(\widehat{\boldsymbol{\phi}}-i\widehat{\boldsymbol{\rho}})\frac{n^2}{\rho^>\rho^<}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \quad \text{(A11a)}$$

$$(\boldsymbol{\nabla}_t\times\widehat{\boldsymbol{z}})(-ih\boldsymbol{\nabla}_t')\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=ih(i\widehat{\boldsymbol{\rho}}-\widehat{\boldsymbol{\phi}})(\widehat{\boldsymbol{\rho}}+i\widehat{\boldsymbol{\phi}})\frac{n^2}{\rho^>\rho^<}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \quad \text{(A11b)}$$

$$(ih\boldsymbol{\nabla}_t)(\boldsymbol{\nabla}_t'\times\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=ih(\widehat{\boldsymbol{\rho}}+i\widehat{\boldsymbol{\phi}})(-i\widehat{\boldsymbol{\rho}}+\widehat{\boldsymbol{\phi}})\frac{n^2}{\rho^>\rho^<}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \quad \text{(A11c)}$$

$$(\boldsymbol{\nabla}_t\times\widehat{\boldsymbol{z}})(-\boldsymbol{\nabla}_t'^2\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0, \quad \text{(A11d)}$$

$$(-\boldsymbol{\nabla}_t^2\widehat{\boldsymbol{z}})(\boldsymbol{\nabla}_t'\times\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0, \quad \text{(A11e)}$$

$$(ih\boldsymbol{\nabla}_t)(-ih\boldsymbol{\nabla}_t')\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=-h^2(\widehat{\boldsymbol{\rho}}+i\widehat{\boldsymbol{\phi}})(\widehat{\boldsymbol{\rho}}+i\widehat{\boldsymbol{\phi}})\frac{n^2}{\rho^>\rho^<}\left(\frac{\rho^<}{\rho^>}\right)^{|n|}, \quad \text{(A11f)}$$

$$(ih\boldsymbol{\nabla}_t)(-\boldsymbol{\nabla}_t'^2\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0, \quad \text{(A11g)}$$

$$(-\boldsymbol{\nabla}_t^2\widehat{\boldsymbol{z}})(-ih\boldsymbol{\nabla}_t)\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0, \quad \text{(A11h)}$$

$$(-\boldsymbol{\nabla}_t^2\widehat{\boldsymbol{z}})(-\boldsymbol{\nabla}_t'^2\widehat{\boldsymbol{z}})\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0, \quad \text{(A11i)}$$

where we assume that $\boldsymbol{\nabla}_t^2\left(\frac{\rho^<}{\rho^>}\right)^{|n|}=0$ for $\rho\neq\rho'$. $I_0=0$ is expected on physical grounds since this is an unphysical field with $\lambda=0$, $h\neq0$. In fact, this field does not satisfy the dispersion relationship.

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