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Principles of Ideal Wideband Reflectarray Antennas

Mohammad Khalaj-Amirhosseini*

Abstract—The principles of ideal wideband Rflecarray Antennas (RAAs) are determined through the idea of distortion-less radiation of a modulated pulse. Two conditions for the cells and one condition for the location of the feed are obtained. The conditions are discussed and clarified by some examples. Each cell requires its own phase at center frequency and its own phase derivative in the desired bandwidth. Some relations are obtained and discussed for the range of required phase derivative of the cells.

1. INTRODUCTION

Reflectarray Antennas (RAAs) are widely studied and used in recent years. In RAAs, the phases of reflection coefficient of their cells are adjusted so that the radiated wave becomes maximum at a specified direction [1]. Microstrip RAAs have some advantages such as low profile with respect to parabolic reflector antennas. However, RAAs have some drawbacks which the most important of them is narrow bandwidth performance. This drawback is due to lack of proper phase of all cells of a RAA at all frequencies inside the bandwidth.

So far, several solutions have been proposed to increase the bandwidth of RAAs, such as using a thick substrate, multiple stacked patches [2, 3], phase-delay lines [4], aperture-coupled patches to delay lines [5], an artificial impedance surface [6], and true time delay [7]. Almost all the proposed solutions have been based on linearization of the phase variation of the cells with respect to frequency. Even so, the broadening of bandwidth of RAAs is not so successful. In most of works such as in [8–14], 1-dB gain bandwidth is reported around 30% at most. This is because of ignoring this important fact that only having cells of linear phase response is not enough. What is important is that each cell must have its specific phase slope with respect to frequency. In other words, the phase slopes of all cells of a wideband RAA must not be identical but they should be different from each other. In references such as [8–14], the phase slopes of all cells are considered equal which limits broadening the bandwidth.

In this article, the principles of ideal wideband RAAs are determined through the idea of distortion-less radiation of a modulated pulse. Two conditions for the cells and one condition for the location of the feed of a wideband RAA are obtained. The conditions are discussed and clarified by some examples.

2. PHASES OF CELLS IN WIDEBAND RAAS

Figure 1 shows a typical configuration of an RRA in which a feed antenna located at the point (0, y_f, F) and illuminates a $D \times D$ aperture containing $N \times N$ cells of dimension d_0. One of the cells, the mn-th one, has a situation whose center is specified by x_{mn} and y_{mn} and has a distance R_{mn} from the feed as follows.

$$R_{mn} = \sqrt{x_{mn}^2 + (y_{mn} - y_f)^2 + F^2} \tag{1}$$

The mn-th cell reflects the illuminated wave from the feed with reflection coefficient of $\Gamma_{mn} \cong \exp(j\phi_{mn})$ in which ϕ_{mn} is phase of reflection coefficient of the mn-th cell.

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* Corresponding author: Mohammad Khalaj-Amirhosseini (khalaja@iust.ac.ir).
The author is with the Faculty of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran.

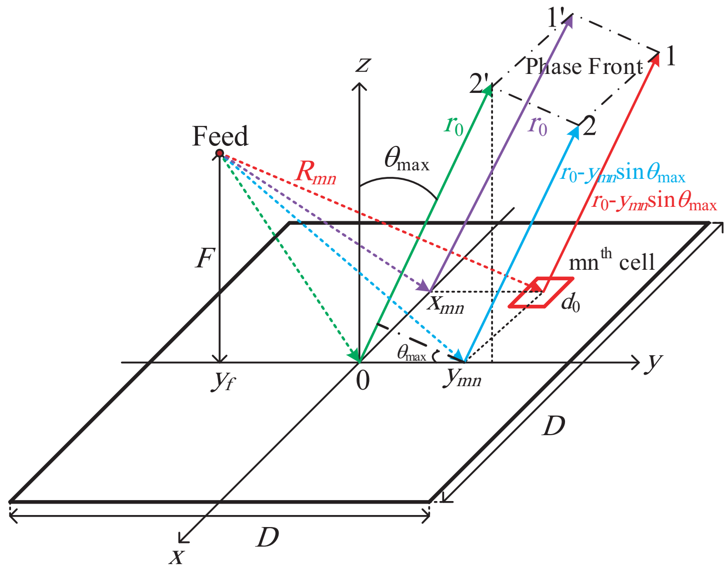

Figure 1. A typical configuration of reflectarray antennas.

Figure 1 shows an arbitrary phase front composed of four special rays, radiating toward maximum radiation direction, i.e., $(\varphi = \pi/2,\, \theta = \theta_{\max})$. The distance from the feed to this phase front hitting the mn-th cell, (red ray ending point 1 in Fig. 1) is given by $[R_{mn} + r_0 - y_{mn} \sin\theta_{\max}]$, in which r_0 is an arbitrary distance to desired phase front.

It is known that the group delay between two points is equal to minus derivative of phase function relating to those points with respect to angular frequency. Therefore, the group delay from the feed to the phase front and reflecting from the mn-th cell, (red ray ending point 1 in Fig. 1), can be written as below.

$$T_g = \frac{1}{c}\left[R_{mn} + r_0 - y_{mn}\sin\theta_{\max}\right] - \frac{d\phi_{mn}}{d\omega} = \frac{r_0}{c} + \frac{1}{2\pi}k_{mn} - \frac{d\phi_{mn}}{d\omega} \tag{2}$$

where c is the velocity of the light, and k_{mn} is a frequency coefficient defined as follows.

$$k_{mn} = \frac{2\pi}{c}\left(R_{mn} - y_{mn}\sin\theta_{\max}\right) \tag{3}$$

It is seen from Fig. 1 that to have maximum radiation toward direction $(\varphi = \pi/2,\, \theta = \theta_{\max})$, at center frequency f_0, the required absolute phase of the mn-th cell will be

$$\phi_{mn} = k_{mn} f_0 + \phi_0 \pm 2n\pi; \quad n = 0,\, 1,\, 2,\, \ldots \tag{4}$$

where ϕ_0 is an arbitrary phase.

An ideal wideband RAA of a desired bandwidth must can radiate a modulated pulse of the same bandwidth without distortion. Therefore, it should have a constant group delay at all frequencies in that desired bandwidth. Therefore, according to Eq. (2), phase derivative of the mn-th cell with respect to frequency, in the desired frequency bandwidth as well as at the center frequency f_0, has to be a specific constant as follows.

$$\frac{d\phi_{mn}}{df}_{|f=f_0} = k_{mn} - 2\pi T_0 \tag{5}$$

where $T_0 = r_0/c - T_g$ is an arbitrary and constant group delay so that $\frac{d\phi_{mn}}{df}$ becomes negative. This is because the frequency slope of RAA cells, $\frac{d\phi_{mn}}{df}$, is inherently negative [2–14].

Figure 2 illustrates the required phase-frequency response of the mn-th cell of a wideband RRA with center frequency of f_0 and bandwidth from f_l to f_u. In fact, the cells of wideband RAAs must have two degrees of freedom so that their phases at center frequency meet two conditions in view of their locations;

1) To be equal to a specific value given by Eq. (4).
2) To be linear versus frequency with a slope given by Eq. (5), in the desired frequency bandwidth.

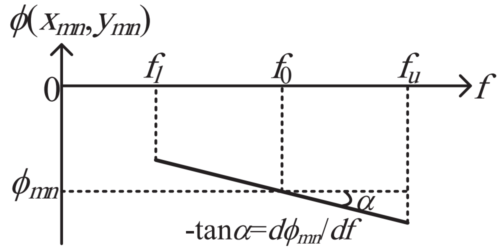

Figure 2. The phase-frequency response of cells of a wideband RAA.

3. THE RANGE OF PHASES AND PHASE DERIVATIVES

The conditions in Eqs. (4) and (5) have two arbitrary parameters ϕ_0 and T_0. Therefore, what is important to implement these two conditions is the relative (not absolute) phase and phase derivative of the cells with respect to each other.

Figure 3 shows the locations of minimums and maximums of the coefficient k_{mn}, for four cases. In this figure, $y_{\min}$ is a point at which $\frac{\partial k_{mn}}{\partial y_{mn}}{}_{|x=0} = 0$, given by

$$y_{\min} = y_f + F \tan\theta_{\max} \tag{6}$$

The minimum value of coefficient k_{mn} at point $(0, y_{\min})$ is given by $\frac{2\pi}{c}(F\cos\theta_{\max} - y_f \sin\theta_{\max})$.

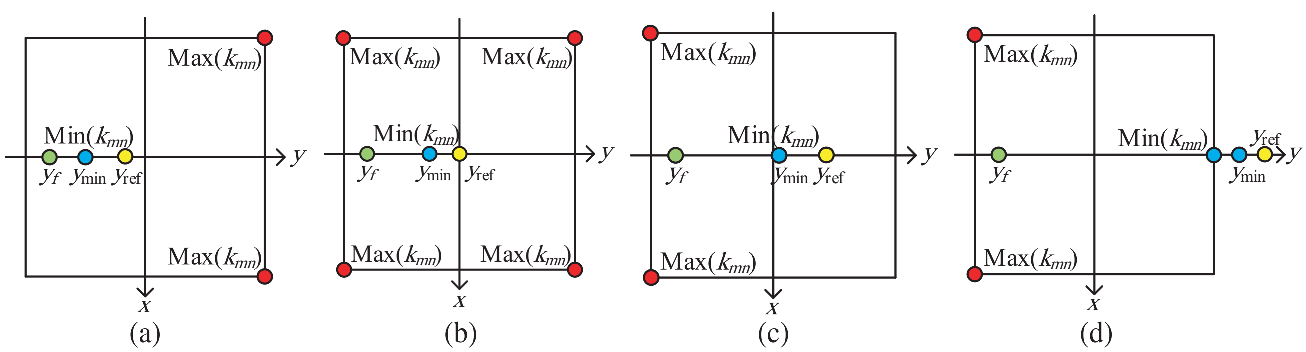

Figure 3. The location of minimum (blue point) and maximums (red points) of the coefficient *kmn*. (a) $y_{\text{ref}} < 0$ & $y_{\min} > -D/2$, (b) $y_{\text{ref}} = 0$, (c) $y_{\text{ref}} > 0$ & $y_{\min} < D/2$, (d) $y_{\min} > D/2$.

Also, y_{ref} is a determining quantity, called reference quantity, which is zero when Max(k_{mn}) existing at points $(\pm D/2, +D/2)$ is equal to Max(k_{mn}) existing at points $(\pm D/2, -D/2)$, i.e., left and right red points in Fig. 3. So, the following can be obtained after some mathematical manipulations,

$$y_{\text{ref}} = y_f + \sqrt{F^2 + 0.25D^2\left(1 + \cos^2\theta_{\max}\right)}\tan\theta_{\max} \tag{7}$$

According to Eqs. (4) and (5), the required range of phase and phase derivative of a wideband RAA at center frequency f_0 throughout its aperture will be given by

$$\Delta\frac{d\phi_{mn}}{df} = \Delta k_{mn} = \frac{2\pi}{c}DQ = 1200DQ \quad \text{[Degrees/GHz]} \tag{8}$$

$$\Delta\phi_{mn} = \Delta k_{mn} f_0 = 2\pi\frac{D}{\lambda_0}Q = 360\frac{D}{\lambda_0}Q \quad \text{[Degrees]} \tag{9}$$

where

$$Q=\begin{cases}\sqrt{0.25+(0.5-y_f/D)^2+(F/D)^2}-(F/D)\cos\theta_m-(0.5-y_f/D)\sin\theta_m; & y_{\rm ref}\leq 0\,\&\,y_{\min}\geq -D/2\\ \sqrt{0.25+(0.5+y_f/D)^2+(F/D)^2}-(F/D)\cos\theta_m+(0.5+y_f/D)\sin\theta_m; & y_{\rm ref}\geq 0\,\&\,y_{\min}\leq D/2\\ \sqrt{0.25+(0.5+y_f/D)^2+(F/D)^2}-\sqrt{(0.5-y_f/D)^2+(F/D)^2}+\sin\theta_m; & y_{\min}\geq D/2\end{cases} \quad (10)$$

It is seen from Eqs. (8) and (10) that the range of required phase derivative Δk_{mn} is dependent on three parameters F/D, y_f/D and $\theta_{\max}$ through the defined function Q. Fig. 4 depicts the function Q versus the normalized reference point, $y_{\rm ref}/D$. It is seen that the minimum value of Δk_{mn} occurs when $y_{\rm ref}$ is set to zero, and this minimum decreases as the parameter F/D increases. It is known that the parameter F/D is important for radiation efficiency and SLL of RAAs as well [15].

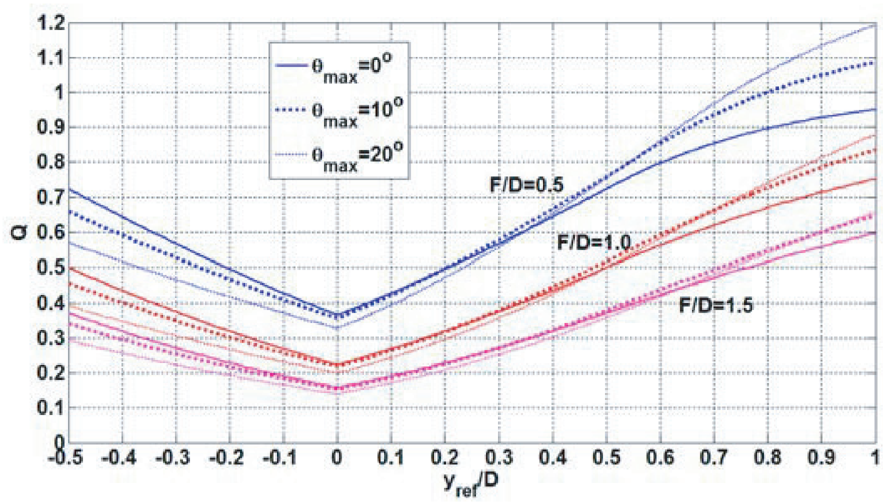

Figure 4. The function Q versus the normalized reference point $y_{\rm ref}/D$.

Moreover, the range of required phase derivative is proportional to absolute dimension (not relative to the wavelength) of the aperture D. Therefore, as the size of aperture increases for the purpose of increasing the gain of a wideband RAA, the required range of phase derivative of the cells increases.

4. TO DESIGN WIDEBAND RAAS

According to the aforesaid issues, one can consider two following vital points to design a practical wideband RAAs.

1) To select a proper substrate to cause the possibility of linear variation of phase of cells in a desired bandwidth. For example, electric permittivity as low as possible and thickness as large as possible for single layer substrates are necessary.
2) The selected shape of cells must have two degrees of freedom to control both the values of the phase and phase derivative at center frequency.
3) To have minimum range of required phase and phase derivatives throughout the aperture, it is better to select feed point so that the reference point $y_{\rm ref}$ in Eq. (7) becomes zero or near zero as possible.

It is interesting to note that jumping in cell shapes due to the need for phase variation more than 360° on the aperture cannot limit the bandwidth of RAA on condition that two above requirements are met for cells situated before and after the jump.

5. AN EXAMPLE AND DISCUSSION

Here we wish to design a wideband RAA at center frequency of $f_0 = 10\,\text{GHz}$, with gain of $G = 30\,\text{dB}$ over $\Delta f = 4\,\text{GHz}$ bandwidth, supposing $\theta_{\max} = 10°$ and $F/D = 1.5$. Assuming aperture efficiency equal to 50%, a square aperture of length $D = 390\,\text{mm} = 13\lambda_0$ consisting of $N \times N = 31 \times 31$ cells of dimension $d_0 = 0.42\lambda_0$ ($0.5\lambda_0$ at upper frequency $f_u = 12\,\text{GHz}$) is needed.

We choose three cases: 1) $y_f = -0.462D$, 2) $-0.292D$ and 3) 0 to set $y_{\text{ref}} = -0.17D$, 0 and $0.292D$, respectively. According to Eqs. (7) and (8), there is need to have

1) $\Delta k_{mn} = 92.0$ [degrees/GHz] and $\Delta\phi_{mn} = 920$ [degrees],
2) $\Delta k_{mn} = 67.8$ [degrees/GHz] and $\Delta\phi_{mn} = 678$ [degrees] and
3) $\Delta k_{mn} = 119.6$ [degrees/GHz] and $\Delta\phi_{mn} = 1196$ [degrees],

for the cells in respective cases.

Figures 5–7 show the required phase of the cells in three chosen cases at center frequency, relative to those of the cell located at point $(0, y_{\min})$ on which k_{mn} is minimum. Also, Figs. 8–10 show the required phase derivative of the cells in three chosen cases in the desired bandwidth, relative to those of the cell located at point $(0, y_{\min})$ on which k_{mn} is minimum.

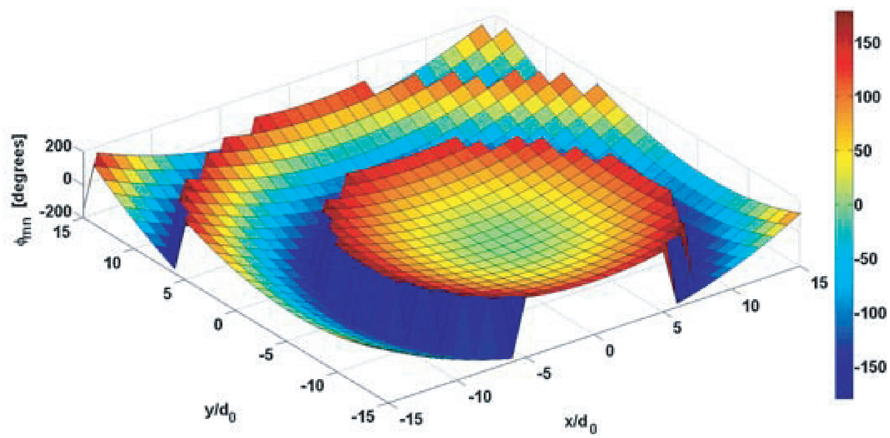

Figure 5. The required relative phase of cells at center frequency, for $y_f = -0.462D$ ($y_{\text{ref}} = -0.17D$).

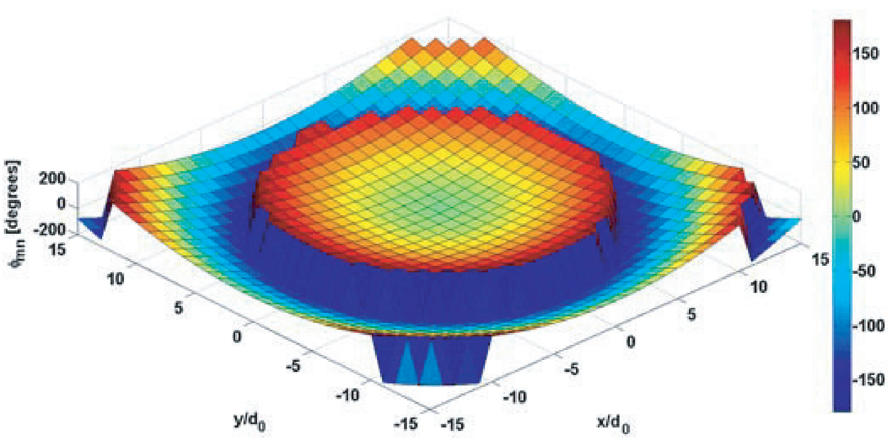

Figure 6. The required relative phase of cells at center frequency, for $y_f = -0.292D$ ($y_{\text{ref}} = 0$).

It is seen that each cell requires its individual relative phase and phase derivative at center frequency and in the desired bandwidth, respectively. Also, when the feed is located at the special point $y_f = -0.292D$ in which y_{ref} is zero, the range of required phase derivative becomes minimum, i.e., 67.8 [degrees/GHz].

The shape of cells should be designed so that their phase derivatives do not deviate significantly from their specific values from the lower to the upper frequencies of the desired bandwidth (8 and 12 GHz in this example). This is a very important matter which is out of aim and scope of this manuscript, of course.

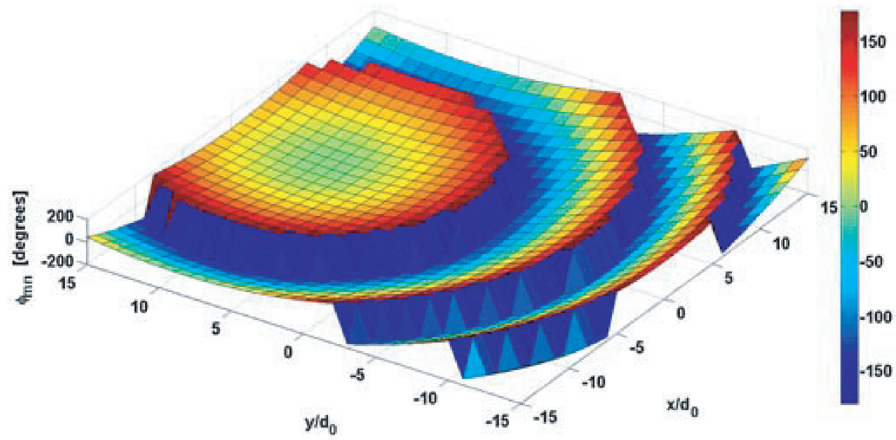

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Figure 7. The required relative phase of cells at center frequency, for $y_f = 0$ ($y_{\text{ref}} = 0.292D$).

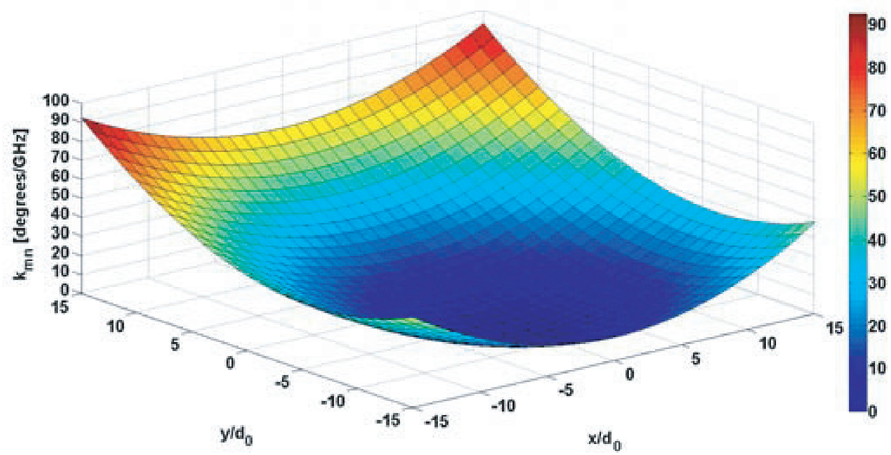

Figure 8. The required relative phase derivative of cells at center frequency, for $y_f = -0.462D$ ($y_{\text{ref}} = -0.17D$).

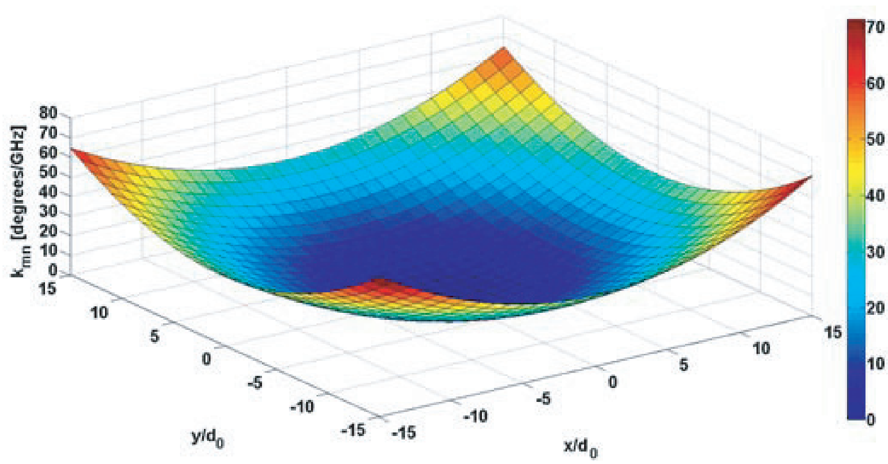

Figure 9. The required relative phase derivative of cells at center frequency, for $y_f = -0.292D$ ($y_{\text{ref}} = 0$).

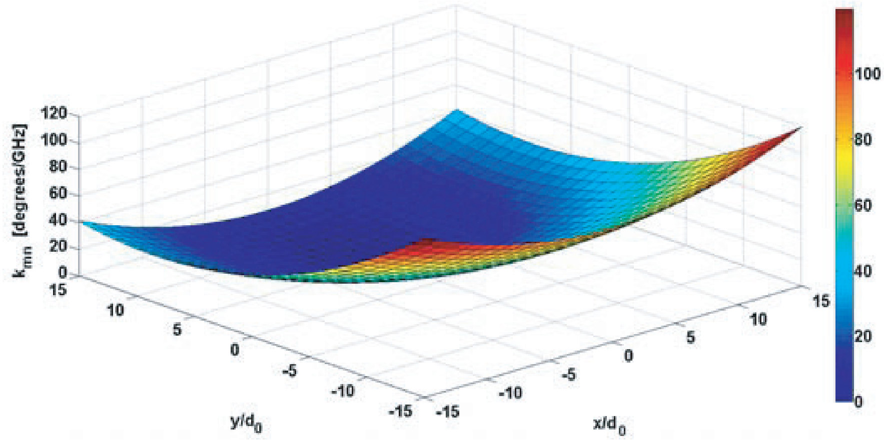

Figure 10. The required relative phase derivative of cells at center frequency, for $y_f = 0$ ($y_{\text{ref}} = 0.292D$).

6. CONCLUSION

The principles of ideal wideband RAAs were determined. Two conditions for the cells and one condition for the location of the feed were obtained and discussed. Each cell requires its own phase at center frequency and its own phase derivative in the desired bandwidth. Some relations were obtained for the range of required phase derivative of the cells. This range is proportional to the size of RAA aperture and becomes minimum when a particular point, y_{ref}, is set to zero. Also, it decreases as the parameter F/D increases.

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