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Electromagnetic Retarded Potential Induced by Quantum Vacuum Polarization

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Abstract—Based on 1st order one-loop effective Lagrangian derived from the 2-point photon vertex in quantum electrodynamics, we obtaine a quantum modified Maxwell equations, and the classical expression of retarded potential is consequently modified by these equations. The results indicate that, due to the time-space non-locality of vacuum polarization, the vacuum polarization current is delayed relative to the field variation and induces a series of additional retarded potentials except for the classical part of retarded potential. Particularly, compared to the classical potential, these additional potentials are further retarded. Because the retarded potential is the base of theory of electromagnetic radiation, the results of this work are of great value to the studies of quantum effect in ultra-intense electromagnetic radiation.

1. INTRODUCTION

In the past five decades, the laser technology has acquired dramatic success. Terawatt and even Petawatt table-top laser equipments have already been employed [1]. It is believed that laser intensity can reach the Schwinger limit ($\sim 10^{29}\,\mathrm{W/cm^2}$ or $E_{cr} \sim 10^{16}\,\mathrm{V/m}$) [1, 2] in expectable future. Under the strong field brought by these anticipated lasers, vacuum is no longer linear but generally behaves as a nonlinear dielectric medium due to the quantum polarization effect [2–5]. This nonlinear effect has attracted great attention due to its laying outside classical Maxwell theory and even providing possibility of real electron-positron pairs production in the vacuum [6]. With the development of ultra-intense laser, the study of vacuum polarization becomes more and more important, and a lot of works have been reported [2, 7–10]. However, to our knowledge, the electromagnetic retarded potential with considering vacuum polarization has not yet been investigated. In this work, the quantum modification to the classical retarded potential is studied based on the first-order one-loop effective Lagrangian of electromagnetic field. Because the retarded potential theory is the base of electromagnetic radiation theory [11], this work is important to the future studies on the quantum effect in ultra-intense electromagnetic radiation.

To probing the nonlinear phenomena under strong electromagnetic field, in the 1930s, Heisenberg and Euler had already presented a quantum modification to Maxwell equations in the form of effective Lagrangian (Heisenberg-Euler Lagrangian) [3], but this Lagrangian abandons the non-local and absorption properties of vacuum polarization [12]. Later, in the 1950s, Schwinger proposed non-perturbation proper time technique [2] to obtain the effective Lagrangian of strong electromagnetic field. This is a popular method to explore the vacuum polarization, since it is a universal method in terms of formality to obtain the effective Lagrangian, but only for some simple field configuration, the exact expression of effective Lagrangian can be given. Schwinger's method is not workable in most cases. However, an approximate effective Lagrangian for arbitrary field configuration can be directly given

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by using the perturbation theory in QED [13]. Although it is not an accurate method when the field strength is beyond Schwinger limit, under which electron-positron pairs are produced with significant magnitude [2], it is still valid in the case of field lower than the Schwinger limit, and in particular, the non-locality of vacuum polarization is considered. Therefore, this work starts at this approximate Lagrangian, because we do not concern the electron-positron pair production in this work but focus on the vacuum polarization effects on retarded potentials.

In this work, based on this first-order one-loop effective Lagrangian derived from the 2-point photon vertex in quantum electrodynamics (QED), the modification of classical Maxwell equations is first obtained. Applying these equations, we get the corresponding retarded potential and find that except for the classical retarded potential, there exist a series of retarded potentials induced by polarization. These additional potentials are induced by the vacuum polarization, and due to the non-locality of vacuum polarization, they are further retarded relative to the classical part of retarded potential. This result indicates that the vacuum polarization effect can be separated from the strong classical background in the time and implies a way to explore the weak vacuum non-linear phenomena.

Because the retarded potential is the base of theory of electromagnetic radiation, the results of this work are of great value to the studies of quantum nonlinear effect in ultra-intense electromagnetic radiation.

2. MODEL: FIRST ORDER ONE-LOOP EFFECTIVE LAGRANGIAN OF ELECTROMAGNETIC FIELD

In vacuum, the classical action of electromagnetic field as a functional of $W[A] = \int dx\mathcal{L}(A(x), \partial A(x))$, $A(x)$ is a four-potential vector of electromagnetic field, and $\mathcal{L}$ is Lagrangian of electromagnetic field and satisfies the least action principle: $\delta W[A]/\delta A_\mu(x) = j^\mu(x) = 0$. However, in QED, $j^\mu(x)$ is expressed as a vacuum expectation of $j^\mu(x) = \langle A|\hat{j}^\mu(x)|A\rangle$ (where $|A>$ denotes the vacuum state vector with a classical field $A_\mu(x)$), and particularly, $\langle A|\hat{j}^\mu(x)|A\rangle \neq 0$ due to the creation and the subsequent annihilation of virtual electron-positron pairs, which is so called vacuum polarization [12–15]. Thus, considering the vacuum polarization, it leads to a modified functional expression of Maxwell equations in vacuum:

$$\frac{\delta W[A]}{\delta A_\mu(x)} = \frac{\delta W^{(1)}[A]}{\delta A_\mu(x)} = \langle A|\,\hat{j}^\mu(x)\,|A\rangle \tag{1}$$

where the action functional $W^{(1)}[A]$ simulates the presence of electric current at purely classical level and describes the vacuum nonlinear effects [2, 3, 13]. In the first order approximation with the Feynman rules (the high order terms are neglected relative to the first order), $W^{(1)}[A]$ is given [13]:

$$W^{(1)}[A] \approx \int d^4x\mathcal{L}^{(1)} = \frac{1}{2}\int d^4x d^4y A^\mu(x)\Pi_{\mu\nu}(x,y)A^\nu(y) \tag{2}$$

$\mathcal{L}^{(1)}$ is nothing but the polarization modification to the classical Lagrangian. $\Pi_{\mu\nu}(xy)$ is the order-e^2 renormalized polarization tensor [14], in the momentum space which is given by:

$$\Pi_{\mu\nu}(k) = (g_{\mu\nu}k^2 - k_\mu k_\nu)\Pi(k^2) \tag{3}$$

$$\Pi(k^2) = -\frac{\alpha}{3\pi}k^2 \int\limits_{4m^2}^{\infty} \frac{d\tau}{\tau}g(\tau)\frac{1}{k^2+\tau-i\varepsilon} \tag{4}$$

$$g(t) = \left(1+\frac{2m^2}{\tau}\right)\left(1-\frac{4m^2}{\tau}\right)^{\frac{1}{2}} \tag{5}$$

$\alpha = e^2/4\pi$ is the fine-structure constant; m and $g_{\mu\nu}$ are the electron static mass and metric matrix component; ε is a positive infinitesimal.

Consequently, the effective Lagrangian $\mathcal{L}_{eff} = \mathcal{L} - \mathcal{L}^{(1)}$ ($W[A] = \int d^4x\mathcal{L}$) can be obtained, which still satisfies the classical least action principle. In addition, $\mathcal{L}$ is the classical Lagrangian:

$\mathcal{L} = -F_{\mu\nu}(x)F^{\mu\nu}(x)/4$. By Fourier transform of Equations (3)–(5) to the position space representation and together with Equation (2), the effect Lagrangian's explicit expression is obtained:

$$\mathcal{L}_{\mathit{eff}} = -\frac{1}{4}F_{\mu\nu}(x)\left[1+\frac{\alpha}{3\pi}\Box\int\limits_{4m^2}^{\infty}\frac{d\tau}{\tau}\frac{g(\tau)}{\tau-\Box}\right]F^{\mu\nu}(x) \tag{6}$$

where, $\Box$ is the D'Alembertian operator, and as usual, $F_{\mu\nu}(x) = \partial A_\nu(x)/x^\mu - \partial A_\mu(x)/x^\nu$ is the electromagnetic field tensor. In addition, the detailed calculation for the above is shown in [13].

3. VACUUM POLARIZATION MODIFICATION TO THE MAXWELL EQUATIONS

It can be seen that Equation (6) is not convenient to the concrete analysis. Thereupon, we calculate the field motion equations via Equation (6) and the least action principle:

$$\delta W_{\mathit{eff}} = \int d^4x\delta\mathcal{L}_{\mathit{eff}} = 0 \tag{7}$$

It first follows that:

$$\begin{aligned}\delta W_{\mathit{eff}} \;=\; &\int d^4x\left\{\left[1-\frac{\alpha}{3\pi}\int\limits_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\right]\frac{\partial F^{\mu\nu}(x)}{\partial x^v}\delta A_\mu(x)\right.\\ &\left.+\frac{\alpha}{6\pi}\int\limits_{4m^2}^{\infty}d\tau\left(\frac{\partial\frac{g(\tau)}{\tau-\Box}F^{\mu\nu}(x)}{\partial x^v}\delta A_\mu(x)-F^{\mu\nu}(x)\frac{g(\tau)}{\tau-\Box}\frac{\partial\delta A_\mu(x)}{\partial x^v}\right)\right\}\end{aligned} \tag{8}$$

The first term of right hand of Equation (8) is nothing but a classical part, but the second and third terms are still complicated, since there exists the operator $1/(\tau-\Box)$. It is the inverse of $\tau-\Box((\tau-\Box)\times(1/(\tau-\Box))f(x) = f(x))$.

Before the calculation, let us consider a special equation for $\tau-\Box$

$$(\tau-\Box)\int d^4yG(x-y|t)f(y) = f(x) \tag{9}$$

$$G(x-y|\tau) = \theta(x^0-y^0)G^+(x-y|\tau)+\theta(y^0-x^0)G^-(x-y|\tau) \tag{10}$$

where $G(x-y|\tau)$ is the Feynman Green's function satisfying: $(\tau-\Box)G(x-y|\tau) = \delta^4(x-y|\tau)$; $G^+(x-y|\tau)$ and $G^-(x-y|\tau)$ are positive and negative energy Green's functions, respectively; $\theta(x) = 0$, $x < 0$; $\theta(x) = 1$, $x \geq 0$. Then, it follows from Equations (9)–(10) that:

$$\frac{1}{(\tau-\Box)}f(x) = \int d^4yG(x-y|\tau)f(y) \tag{11}$$

Using Equation (11), Equation (8) becomes

$$\begin{aligned}\delta W_{\mathit{eff}} \;=\; &\int d^4x\left\{\left[1-\frac{\alpha}{3\pi}\int\limits_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\right]\frac{\partial F^{\mu\nu}(x)}{\partial x^v}\delta A_\mu(x)\right.\\ &+\frac{\alpha}{6\pi}\int d^4y\int\limits_{4m^2}^{\infty}d\tau g(\tau)\frac{\partial G(x-y|\tau)}{\partial x^v}F^{\mu\nu}(y)\delta A_\mu(x)\\ &\left.+\frac{\alpha}{6\pi}\int d^4y\int\limits_{4m^2}^{\infty}d\tau g(\tau)\frac{\partial G(x-y|\tau)}{\partial y^v}F^{\mu\nu}(x)\delta A_\mu(y)\right\} = 0\end{aligned} \tag{12}$$

Now, Equation (12) is convenient to the further calculation, beginning at which, we determine the Maxwell equations with vacuum polarization modification:

$$\begin{aligned}\frac{\delta W[A]}{\delta A_\mu(x)} &= \frac{\partial F^{\mu\nu}(x)}{\partial x^v} = \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\frac{\partial F^{\mu\nu}(x)}{\partial x^v} \\ &-\frac{\alpha}{6\pi}\int d^4y\int_{4m^2}^{\infty}d\tau g(\tau)[G(x-y|\tau)+G(y-x|\tau)]\frac{\partial F^{\mu\nu}(y)}{\partial y^v}\end{aligned} \tag{13}$$

Noting Equation (1) the right-hand part of Equation (13) is no other than the polarization current: $j^\mu(x)$. Furthermore, substituting Equation (10) into Equation (13) and swapping the positions of two negative energy Green's functions, $j^\mu(x)$ is written as:

$$\begin{aligned}j^\mu(x) &= \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\frac{\partial}{\partial x^v}F^{\mu\nu}(x) \\ &-\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(x^0-y^0)[G^+(x-y|\tau)+G^-(y-x|\tau)]\frac{\partial}{\partial y^v}F^{\mu\nu}(y) \\ &-\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(y^0-x^0)[G^+(y-x|\tau)+G^-(x-y|\tau)]\frac{\partial}{\partial y^v}F^{\mu\nu}(y)\end{aligned} \tag{14}$$

The first term of right hand of Equation (14) shows a general local polarization current. The second term implies the field in the past: y produces the future polarization at x. This polarization is obviously non-local: the field and polarization current are not at the same position. Here, x^0 and y^0 are the time coordinates of polarization source: $j^\mu(x)$ and field tensor: $F^{\mu\nu}(y)$, respectively. Considering the properties of θ function ($\theta(x)=0$, $x<0$; $\theta(x)=1$, $x\geq 0$), the function of $\theta(y^0-x^0)$ in the third term is not zero just when $y^0\geq x^0$, but it suggests that the future field $F^{\mu\nu}(y)$ produces polarization currents $j^\mu(x)$ in the past. Therefore, the third term of Equation (14) violates the law of causation although it is correct in mathematics. For this reason, this term is non-physical, and we eliminate it from Equation (14):

$$\begin{aligned}j^\mu(x) &= \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\frac{\partial}{\partial x^v}F^{\mu\nu}(x) \\ &-\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(x^0-y^0)[G^+(x-y|\tau)+G^-(y-x|\tau)]\frac{\partial}{\partial x^v}F^{\mu\nu}(y)\end{aligned} \tag{15}$$

and as well, the modified Maxwell Equation (13) is further modified

$$\begin{aligned}\frac{\partial}{\partial x_v}F^{\mu\nu}(x) &= \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\frac{\partial}{\partial x^v}F^{\mu\nu}(x) \\ &-\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(x^0-y^0)[G^+(x-y|\tau)+G^-(y-x|\tau)]\frac{\partial}{\partial y^v}F^{\mu\nu}(y)\end{aligned} \tag{16}$$

Moreover, this non-local property implies that the vacuum polarization is a retarded propagating process.

4. RETARDED POTENTIALS WITH CONSIDERATION OF VACUUM POLARIZATION

Adding an external source current $j^{(ex)\mu}(x)$ to Equation (15), the modified Maxwell equations with a external four-current source are given:

$$\begin{aligned}\frac{\partial}{\partial x^v}F^{\mu\nu}(x) &= j^{(ex)\mu}(x) + j^{\mu}(x) = j^{(ex)\mu}(x) + \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\frac{\partial}{\partial x^v}F^{\mu\nu}(x) \\ &\quad -\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(x^0-y^0)[G^+(x-y|\tau)+G^-(y-x|\tau)]\frac{\partial}{\partial y^v}F^{\mu\nu}(y) \end{aligned} \tag{17}$$

Since the fine-structure constant α is very small, the first term in the right hand of Equation (17) is much larger than the last two, thus it is the major contribution to the right hand: $\partial F^{\mu\nu}(x)/\partial x^{\nu} \approx j^{(ex)\mu}(x)$. Substituting it into right hand of Equation (17) and imposing Lorentz condition $\partial A^{\nu}(x)/\partial x^{\mu} = 0$.

$$\begin{aligned}\Box A^{\mu}(x) &\cong j^{(ex)\mu}(x) + \frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)j^{(ex)\mu}(x) \\ &\quad -\frac{\alpha}{6\pi}\int_{4m^2}^{\infty}d\tau g(\tau)\int d^4y\theta(x^0-y^0)[G^+(x-y|\tau)+G^-(y-x|\tau)]j^{(ex)\mu}(y) \end{aligned} \tag{18}$$

Based on Equation (18) and as the process of deriving classical retarded potential [11], the retarded potential expression with considering vacuum polarization follows. (for convenience, the light velocity c is explicitly written; $c = 1$ in the natural unit system applied in our work and c is usually implicitly expressed.):

$$\begin{aligned}\vec{A}(t,\vec{r}) &= \frac{\vec{j}^{(ex)}(t-\frac{R}{c},\vec{r}')+\vec{j}(t-\frac{R}{c},\vec{r}')}{R} = \left[1+\frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\right]\int dV'\frac{\vec{j}^{(ex)}(t-\frac{R}{c},\vec{r}')}{R} \\ &\quad -\frac{\alpha}{6\pi}\int dV'\left\{\frac{1}{R}\int_{4m^2}^{\infty}d\tau g(\tau)\int dt''dV''\theta\left(\left(t-\frac{R}{c}\right)-t''\right)\left[G^+\left(\left(t-\frac{R}{c}\right)-t'',\vec{r}'-\vec{r}''\middle|\tau\right)\right.\right. \\ &\quad \left.\left.+G^-\left(t''-\left(t-\frac{R}{c}\right),\vec{r}''-\vec{r}'\middle|\tau\right)\right]\vec{j}^{(ex)}(t'',\vec{r}'')\right. \end{aligned} \tag{19}$$

$$\begin{aligned}\phi(t,\vec{r}) &= \frac{j^{(ex)0}(t-\frac{R}{c},\vec{r}')+j^0(t-\frac{R}{c},\vec{r}')}{R} = \left[1+\frac{\alpha}{3\pi}\int_{4m^2}^{\infty}\frac{d\tau}{\tau}g(\tau)\right]\int dV'\frac{j^{(ex)0}(t-\frac{R}{c},\vec{r}')}{R} \\ &\quad -\frac{\alpha}{6\pi}\int dV'\left\{\frac{1}{R}\int_{4m^2}^{\infty}d\tau g(\tau)\int dt''dV''\theta\left(\left(t-\frac{R}{c}\right)-t''\right)\left[G^+\left(\left(t-\frac{R}{c}\right)-t'',\vec{r}'-\vec{r}''\middle|\tau\right)\right.\right. \\ &\quad \left.\left.+G^-\left(t''-\left(t-\frac{R}{c}\right),\vec{r}''-\vec{r}'\middle|\tau\right)\right]j^{(ex)0}(t'',\vec{r}'')\right. \end{aligned} \tag{20}$$

where $R = |\mathbf{r}-\mathbf{r}'|$, $dV' = dr'^1dr'^2dr'^3$ and $dV'' = dr''^1dr''^2dr''^3$.

Before further analyzing Equations (19)–(20), we determine the explicit expression of Green functions: $G^{\pm}(x-y|\tau)$. Noting that $\tau-\Box = -(\Box - m_{\tau}^2)$, after substituting $\tau = m_{\tau}^2$, the operator $\tau-\Box$ is nothing but a minus Klein-Gordon (KG) operator [15] with mass m_{τ}. Thus $G^{\pm}(x-y|\tau) = -D^{\pm}(x-y|m_{\tau})$, where $D^{\pm}(x-y|m_{\tau})$ is the positive and negative energy KG Feynman Green's functions with particle mass m_τ. Via the explicit expression of KG Green's function [15], we can get the explicit expression of $G^{\pm}(x-y|\tau)$:

$$\begin{aligned} G^{\pm}(x-x'|\tau) &= -D^{\pm}(x-x'|m_\tau) = -\theta(\pm(x^0-x'^0))D(x-x'|m_\tau) \\ &= \theta(\pm(x^0-x'^0))\left\{\frac{1}{4\pi}\delta((x-x')^2) - \frac{m_\tau\theta((x-x')^2)}{8\pi\sqrt{(x-x')^2}}H_1^{(2)}\left(m_\tau\sqrt{(x-x')^2}\right)\right. \\ &\left. -\frac{m_\tau\theta(-(x-x')^2)}{8\pi\sqrt{-(x-x')^2}}H_1^{(2)}\left(-im_\tau\sqrt{-(x-x')^2}\right)\right\} \end{aligned} \tag{21}$$

$H_1^{(2)}(z)$ is the complex Hankel function (the third kind Bessel function). By the properties of Hankel function, asymptotic behavior of $G^{\pm}(x,x'|\tau)$ can be deduced:

$$G^{\pm}(x-x'|\tau) \rightarrow \begin{cases} \text{const.}\theta(\pm(x^0-x'^0))((x-x')^2)^{-3/4}\exp(-im\sqrt{(x-x')^2}), ((x-x')^2 \rightarrow +\infty) \\ \text{const.}\theta(\pm(x^0-x'^0))((x-x')^2)^{-3/4}\exp(-m\sqrt{|(x-x')^2|}), ((x-x')^2 \rightarrow -\infty) \end{cases} \tag{22}$$

Equation (22) implies that for the time-like distance ($(x-x')^2 > 0$, velocity lower than c), $G^{\pm}(x-x'|\tau)$ has a oscillating behavior and decreases in amplitude owing to the power-law factor. On the other hand, for the space-like distance ($(x-x')^2 < 0$, velocity being over c), it rapidly falls to the zero with the decreasing of $(x-x')^2$. It suggests the well-known result that particles can propagate over light velocity due to quantum tunneling effect [16], but the probability is ultra-small. Thus, the velocity of major propagating process represented by $G^{\pm}(x-x'|\tau)$ is regarded lower than c or generally, only when $(x-x')^2 > 0$, $G^{\pm}(x-x'|\tau) \neq 0$.

Now, let us turn back to Equations (19)–(20). Ignoring the tiny value variation owing to the local vacuum polarization, the first terms of Equations (19)–(20) represent the classical retarded potential, which is exactly the classical expression of electromagnetic retarded potentials [11]. It is more interesting that the second term of Equations (19)–(20), which implies that there exists an additional retarded potential if the quantum vacuum polarization is taken into consideration. This retarded potential does not lay in any classical theories and is purely a quantum effect. Particularly, they are further retarded compared with the classical part according to the follow analysis.

There are two steps for producing this additional retarded potential. At first, the source current $j^{(ex)}$ at $x^{(\mathrm{S})} = (t'', \mathbf{r}'')$ excites polarization current at $x^{(P)} = (t-R/x, \mathbf{r}')$. As the discussion of Eq. (22), the major contributors to the polarization are the source currents whose distance to the polarization current is time-like: $(t-R/c)-t'' > |\mathbf{r}-\mathbf{r}''|$; $(t-R/c) > t''$ is major contributor to the polarization. Subsequently, the polarization current at $x^{(P)}$ produces field at $x^{(F)} = (t, \mathbf{r})$. The distance between the polarization currents and their excited fields is light-like as the classical retarded potential.

Furthermore, using the above time-space distance relations, we can deduce the property of distance between $x^{(S)}$ and $x^{(F)}$:

$$(t-R/c)-t'' > \left|\vec{r}' - \vec{r}''\right| \Rightarrow (t-t'')^2 - (r-\vec{r}'')^2 > 0 \tag{23}$$

Equation (23) shows that for the additional quantum retarded potential, the distance between field and its source current is mainly time-like, or the quantum retarded potentials propagate with velocity lower than c. In other words, relative to the classical part of retarded potential, the additional quantum retarded potential is further retarded.

5. CONCLUSIONS

In conclusion, based on the first order one-loop effective Lagrangian of electromagnetic field, we have obtained the corresponding quantum modification to the Maxwell equations and classical retarded potentials. It is found that there exists an additional quantum retarded potential besides the classical retarded potential, and more importantly, this additional potential propagates more slowly than the classical one. In other words, it is further retarded relative to the classical part of potential. Furthermore, we determine that this effect is induced by the non-locality of vacuum polarization.

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Original Contribution

The classical Maxwell equations are modified, based on the 1st-order one-loop effective Lagrangian of electromagnetic field. A quantum modified retarded electromagnetic potential expression is obtained in this work.

REFERENCES

1. Mourou, G. A., T. Tajima, and S. V. Bulanov, "Optics in the relativistic regime," *Rev. Mod. Phys.*, Vol. 78, No. 2, 309, 2006.
2. Schwinger, J., "On Gauge invariance and vacuum polarization," *Phys. Rev.*, Vol. 82, No. 5, 664–679, 1951.
3. Heisenberg, W. and H. Euler, "Folgerungen aus der diracschen theorie des positrons," *Z. Für Phys.*, Vol. 98, No. 11–12, 714–732, 1936.
4. King, B. and A. D. Piazza, "Investigating the QED vacuum with ultra-intense laser fields," *Eur. Phys. J. Spec. Top.*, Vol. 223, No. 6, 1063–1068, 2014.
5. King, B. and T. Heinzl, "Measuring vacuum polarization with high-power lasers," *High Power Laser Sci. Eng.*, Vol. 4, e5, 2016.
6. Dittrich, W. and H. Gies, *Probing the Quantum Vacuum*, Springer, Berlin, 2000.
7. Tommasini, D., D. Novoa, and L. Roso, "Quantum vacuum polarization searches with high power lasers below the pair production regime," *Progress in Ultrafast Intense Laser Science*, 137–153, 2014.
8. Shearer, J. W., J. Garrison, J. Wong, and J. E. Swain, "Pair production by relativistic electrons from an intense laser focus," *Phys. Rev. A*, Vol. 8, No. 3, 1582–1588, 1973.
9. Bulanov, S. S., "Pair production by a circularly polarized electromagnetic wave in a plasma," *Phys. Rev. E*, Vol. 69, No. 3, 036408, 2004.
10. Di Piazza, A., K. Z. Hatsagortsyan, and C. H. Keitel, "Nonperturbative vacuum-polarization effects in proton-laser collisions," *Phys. Rev. Lett.*, Vol. 100, No. 1, 010403, 2008.
11. Landau, L. D. and E. M. Lifshitz, *The Classical Theory of Fields*, Elsevier, Oxford, 2013.
12. Itzykson, C. and J.-B. Zuber, *Quantum Field Theory*, Dover Publications, New York, 2006.
13. Dittrich, W. and M. Reuter, *Effective Lagrangians in Quantum Electrodynamics*, Springer-Verlag, Berlin, 1985.
14. Pokorski, S., *Gauge Field Theories*, Cambridge University Press, Cambridge, 2000.
15. Greiner, W. and J. Reinhardt, *Quantum Electrodynamics*, Springer Science & Business Media, Berlin, 2013.