Progress In Electromagnetics Research M, Vol. 46, 193–201, 2016

DOA and Power Estimation by Controlling the Roots of the Antenna Array Polynomial

Mohammad J. Mismar[1] and Taiseer H. Ismail[2, *]

Abstract—A new direction-of-arrival (DOA) and power estimation method of unknown number of source signals is proposed. The direction and power of coherent and/or noncoherent signals are estimated by controlling the roots of the array polynomial on the unit circle. The genetic algorithm is used to find the phases of the array polynomial roots that minimize the array output power. The pseudo-spectrum is obtained by phase rotation of the estimated roots, and the real power spectrum is derived from the pseudo-spectrum and the array factor. The results indicate that the direction of arrivals, power of the signals, and number of source signals are estimated from the real power spectrum.

1. INTRODUCTION

Direction-Of-Arrival (DOA) estimation is still considered a challenging research problem in wireless communication applications. Some of the applications in wireless communication include interference suppression, target tracking, navigation, and mobile communication. Practically, DOA estimation using antenna array processing is a very efficient technique that has been used for a long time. Many methods have been proposed to estimate the DOA such as traditional maximum entropy and maximum likelihood methods or eigen-structure methods which utilize orthogonality between signal subspace and noise subspace [1–5]. From the various methods that have been proposed, eigen-structure methods have received wide attention because of their relatively high resolution [1]. However, all eigen-structure algorithms need an exact number of sources to separate signal subspace and noise subspace. In most practical applications, the exact number of sources is unknown and must be estimated by any estimation method [6]. Therefore, it is recommended to develop the DOA estimation method without estimating the source number. Generally, beamforming techniques can avoid estimating the number of sources, but these methods cannot provide high resolution estimation. Also, the minimum variance distortionless response (MVDR) beamformer can estimate the DOA of source signals without knowing the number of source signals. However, the MVDR method does not have good resolution compared with other methods [1].

An interesting method applied to the DOA problem without estimating the number of sources is by combining the Pisarenko algorithm with the ASPECT method [7]. The drawback of this method is that it works only when the number of source signals is at most equal to half the number of array sensors. MUSIC-like method has been proposed to estimate the DOA without using the subspace decomposition, and the number of sources is not required for direction finding [8]. A method similar to SSMUSIC (Signal Subspace Scaled MUSIC) is used to estimate the DOA without knowing the number of sources [9]. In this method, a special spectrum is constructed using all information of the eigenvalues and eigenvectors of the array correlation matrix in low SNR.

Recently, adaptive smart antennas are used for increasing the capacity of the cellular system [10–12]. In practice, many transmitters are operating simultaneously and with each transmitter creating

Received 16 January 2016, Accepted 4 March 2016, Scheduled 12 March 2016
* Corresponding author: Taiseer Hasan Ismail (tghanim@zu.edu.jo).
[1] Department of Electrical Engineering, Princess Sumaya University for Technology, Amman, Jordan. [2] Department of Electrical Engineering, Zarqa University, Zarqa, Jordan.

many multipath components at the receiver. Therefore, it is required that the receiver must suppress the interfering signals which can be coherent and/or incoherent and also with an unknown number of sources [13, 14].

It is known that the genetic algorithms (GA) form a major group of nonlinear search algorithms which can be used to solve multiple parameter problems easily and search for all the parameter optimizers at the same time. Therefore, as the DOA estimation using adaptive arrays is a nonlinear problem, the GA can be used directly or with other methods to estimate the DOA of the signals [15, 16].

The proposed method of DOA estimation relies on the array polynomial representation of the array output power. With $M+1$ array elements, the array output power is represented as product of M roots. When a root of the polynomial is rotated to coincide with the angular location of a signal, the signal output power will be reduced by the power of that signal. If I signals are impinging on the array at the directions θ_i, $i = 1, \ldots, I$, then controlling I roots of the polynomial to coincide with the I angular locations of the received signals will reduce the output power to the noise power level. In this paper, the GA is used to estimate the roots of the array polynomial to coincide with the direction of the source signals. Generally, the solution space of each parameter in GA is usually unknown, and this will yield a larger number of iterations to find the optimal solution in this large solution space. However, the proposed method has a known solution space since the roots of array polynomial are on the unit circle, hence a fast convergence is expected. The pseudospectrum is obtained from the solution of the GA, and then the real power spectrum is derived using the steering vectors at the directions of the root locations. The results show that the DOA, power of the source signals, and number of signals are estimated accurately by controlling the roots of the array polynomial.

2. PROBLEM FORMULATION USING THE ROOTS OF ARRAY POLYNOMIAL FOR DOA ESTIMATION

Consider an equispaced linear array of $M+1$ isotropic elements with inter-element spacing of d, which lies on x-axis with the first element at the origin. When I signals arrive from I directions, each element receives I signals plus additive white Gaussian noise (AWGN), i.e., the received signal of the mth element is

$$x_m(k) = \sum_{i=1}^{I} s_i(k) e^{-jm\frac{2\pi}{\lambda} d \sin(\theta_i)} + n_m(k), \quad m = 0, \ldots, M \tag{1}$$

where $s_i(k)$ represents the kth sample of the ith signal, and $n_m(k)$ represents the kth noise sample of the mth element of zero mean and σ^2 variance. Denoting $z = e^{-j\frac{2\pi}{\lambda} d \sin(\theta)}$, then the above equation can be written as

$$x_m(k) = \sum_{i=1}^{I} s_i(k) z_i^m + n_m(k), \quad m = 0, \ldots, M \tag{2}$$

where $z_i^m = e^{-jm\frac{2\pi}{\lambda} d \sin(\theta_i)}$. Let w_m denotes the coefficient of the mth element, then the output of the array is

$$y(k) = \sum_{m=0}^{M} w_m x_m(k) \tag{3}$$

$$y(k) = \sum_{m=0}^{M} w_m \left(\sum_{i=1}^{I} s_i(k) z_i^m + n_m(k) \right) \tag{4}$$

$$y(k) = \sum_{i=1}^{I} s_i(k) \sum_{m=0}^{M} w_m z_i^m + \sum_{m=0}^{M} w_m n_m(k) \tag{5}$$

The polynomial with $(M+1)$ coefficients can also be expressed in terms of M multiplicative terms as

$$\sum_{m=0}^{M} w_m z^m = \prod_{m=1}^{M} (z - zr_m) \tag{6}$$

where zr_m is the mth root of the array polynomial on the unit circle and $w_M = 1$. Using the above equation, the output of the array can be rewritten as

$$y(k) = \sum_{i=1}^{I} s_i(k) \prod_{m=1}^{M} (z_i - zr_m) + \sum_{m=0}^{M} w_m n_m(k) \tag{7}$$

where

$$z_i = e^{-j\frac{2\pi}{\lambda} d \sin(\theta_i)} \tag{8}$$

From Equation (7), if one root of the equation, zr_q, is rotated on the unit circle to coincide with the angular location of the pth signal, then the pth signal will be eliminated from the expression of the output signal. Let $\hat{z}r_q$ be the rotated qth root that coincides with the location of the pth signal direction, θ_p, then the rotated root can be written as

$$\hat{z}r_q = e^{-j\hat{\phi}_q} = z_p = e^{-j\frac{2\pi}{\lambda} d \sin(\theta_p)} \tag{9}$$

and hence the output signal can be expressed as

$$y(k) = \sum_{\substack{i=1 \\ i \neq p}}^{I} s_i(k) \left(z_i - e^{-j\frac{2\pi}{\lambda} d \sin(\theta_p)} \right) \prod_{\substack{m=1 \\ m \neq q}}^{M} (z_i - zr_m) + \sum_{m=0}^{M} w_m n_m(k) \tag{10}$$

In practice, the output power is used as a meaningful measurable quantity. Therefore, assuming that the signals are ergodic random processes, the array average output power can be calculated using a time-averaged correlation of K snapshots as

$$P_y = \frac{1}{K} \sum_{k=1}^{K} y(k) y^H(k) = \mathbf{w}^T \mathbf{R_{XX}} \mathbf{w}^H \tag{11}$$

where

$$\mathbf{R_{XX}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}(k)^H \tag{12}$$

$\mathbf{x}(k)$ and $\mathbf{w}$ are the vectors which contain data samples and coefficients of $M+1$ elements, respectively.

Thus, when the rotated root, $\hat{z}r_q$, coincides with the location of the pth signal direction, the output power will be reduced by the power of the pth signal, hence, the direction of the pth signal can be calculated from the expression

$$\theta_p = \sin^{-1} \left(\frac{\lambda}{2\pi\, d} \hat{\phi}_q \right) \tag{13}$$

where $\hat{\phi}_q$ is the phase of the rotated root on the unit circle.

In general, when the number of signals, I, is less or equal to the number of roots, M, I roots of the polynomial can be rotated on the unit circle to coincide with I directions of the signals. This will reduce the output power to only the value of output noise power, i.e.,

$$P_n = \mathbf{w}_0 \mathbf{R_{XX}} \mathbf{w}_0^H = P_y - \sum_{i=1}^{I} E\left[S_i^2\right] \tag{14}$$

where P_y is the output power calculated by Equation (11), P_n the array output power due to noise only, and $\mathbf{w}_0$ the coefficient vector which corresponds to the rotated roots on the unit circle that coincide with the direction of I signals. Without loss of generality, let the first I roots be rotated to coincide with the direction of I source signals, then the array coefficients can be expressed by the following relationship

$$\sum_{m=0}^{M} w_{0,m} z^m = \prod_{i=1}^{I} (z - z_i) \prod_{m=1+I}^{M} (z - zr_m) \tag{15}$$

where z_i is given by Equation (8). Therefore, from the locations of rotated roots that minimize the output power, the array coefficients, $\mathbf{w}_0$, can be calculated by Equation (15). From the previous discussion, the optimization problem can be formulated as follows:

$$\begin{aligned} &\underset{\mathbf{w}}{\text{Minimize}} \quad \mathbf{w}\mathbf{R}_{\mathbf{XX}}\mathbf{w}^H \\ &\text{Subject to} \quad \sum_{m=0}^{M} w_m z^m = \prod_{m=1}^{M}\left(z - e^{-j\phi_m}\right) \end{aligned} \tag{16}$$

where ϕ_m is the angle of the mth root on the unit circle. The solution is obtained when I roots of the polynomial will coincide with the direction of the I sources.

3. DOA ESTIMATION USING GENETIC ALGORITHM

In this paper the solution which minimizes the array output power is found using the genetic algorithm. The procedure can be explained as follows:

1. Use GA to solve Equation (16). Let the calculated roots of the array polynomial be denoted as $e^{-j\phi'_m}$ ($m = 1, 2, \ldots, M$). In fact, these estimated roots of the polynomial will yield minimum output power of the array. Then the corresponding new array coefficient, $\mathbf{w}_0$, can be calculated by the following relationship
$$\sum_{m=0}^{M} w_{0,m} z^m = \prod_{m=1}^{M}\left(z - e^{-j\phi'_m}\right) \tag{17}$$
2. Calculate the corresponding output power as
$$P_0 = \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0^H \tag{18}$$
3. For $m = 1, 2, \ldots, M$, set $\phi'_m = \phi'_m + \pi$ and calculate the corresponding coefficient vector $\mathbf{w}_m$ and the output power which is denoted as
$$P'_m = \mathbf{w}_m\mathbf{R}_{\mathbf{XX}}\mathbf{w}_m^H. \tag{19}$$
4. The pseudospectrum is obtained by plotting P'_m versus $\hat{\theta}_m$, where
$$\hat{\theta}_m = \sin^{-1}\left(\frac{\lambda}{2\pi d}\phi'_m\right) \tag{20}$$

As the number of source signals is unknown, only I roots of the calculated array polynomial are in the direction of the source signals. Therefore, to decide which roots of the estimated polynomial roots are in the directions of the source signals, a simple procedure is performed as explained in Section 4.

4. POWER ESTIMATION USING THE POLYNOMIAL ROOT METHOD

This section deals with estimating the power of the source signals from the pseudospectrum and the array factor at the directions of the root locations which yield the minimum power.

To obtain the power at the directions corresponding to the polynomial roots, assuming that the signals and noise are uncorrelated, the output power is calculated as

$$P_y = \mathbf{w}\mathbf{R}_{\mathbf{XX}}\mathbf{w}^H = \mathbf{w}\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H\mathbf{w}^H + \sigma^2\mathbf{w}\mathbf{w}^H \tag{21}$$

where σ^2 is the input noise power and $\mathbf{A}$ the $N \times M$ matrix of the steering vector. The matrix $\mathbf{A}$ is represented as

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \ldots & \mathbf{a}_m & \ldots & \mathbf{a}_M \end{bmatrix} \tag{22}$$

where $\mathbf{a}_m$ is the steering column vector in the direction of $\hat{\theta}_m$, i.e.,

$$\mathbf{a}_m = \begin{bmatrix} 1 & e^{-j\frac{2\pi}{\lambda}d\sin(\hat{\theta}_m)} & \ldots & e^{-jm\frac{2\pi}{\lambda}d\sin(\hat{\theta}_m)} & \ldots & e^{-jM\frac{2\pi}{\lambda}d\sin(\hat{\theta}_m)} \end{bmatrix}^T \tag{23}$$

Assuming that I roots of the estimated roots coincide with the direction of the I source signals, then the corresponding array factor with the weight vector $\mathbf{w}_0$ has nulls at the direction of signals, i.e.,

$$\mathbf{w}_0\mathbf{A} = [\ \mathbf{w}_0\mathbf{a}_1 \quad \mathbf{w}_0\mathbf{a}_2 \quad \dots \quad \mathbf{w}_0\mathbf{a}_m \quad \dots \quad \mathbf{w}_0\mathbf{a}_M\ ] = [\ 0 \quad 0 \quad \dots \quad 0 \quad \dots \quad 0\ ] \tag{24}$$

Substituting Equation (24) in (21), the output power is

$$P_0 = \mathbf{w}_0\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H\mathbf{w}_0^H + \sigma^2\mathbf{w}_0\mathbf{w}_0^H = \sigma^2\mathbf{w}_0\mathbf{w}_0^H \tag{25}$$

When the mth root, $\hat{z}r_m = e^{-j\hat{\phi}_m}$, is rotated on the unit circle by an angle of (π), i.e.,

$$\hat{z}r_m = e^{-j\hat{\phi}_m} \quad \Rightarrow \quad e^{-j(\hat{\phi}_m+\pi)}$$

then the output power of Equation (21) with the new array coefficients $\mathbf{w}_m$ is expressed as

$$P'_m = \mathbf{w}_m\mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H\mathbf{w}_m^H + \sigma^2\mathbf{w}_m\mathbf{w}_m^H \tag{26}$$

In fact, the array factor with weight vector $\mathbf{w}_m$ increases at the direction corresponding to $\hat{\varphi}_m$ and has nulls at the direction of all other signals. Consequently, the array factor at the direction of the roots is

$$\mathbf{w}_m\mathbf{A} = [\ 0 \quad 0 \quad \dots \quad \mathbf{w}_m\mathbf{a}_m \quad \dots \quad 0\ ] \tag{27}$$

Thus, substituting Equation (27) in Equation (26), the output power corresponding to this new weight vector, $\mathbf{w}_m$, is

$$P'_m = \hat{P}_m\mathbf{w}_m\mathbf{a}_m\mathbf{a}_m^H\mathbf{w}_m^H + \sigma^2\mathbf{w}_m\mathbf{w}_m^H \tag{28}$$

where $\hat{P}_m$ denotes the estimated power at the direction of the mth root. Consequently, the estimated power is

$$\hat{P}_m = \frac{P_m - \sigma^2\mathbf{w}_m\mathbf{w}_m^H}{\mathbf{w}_m\mathbf{a}_m\mathbf{a}_m^H\mathbf{w}_m^H} \tag{29}$$

Finally, combining Equations (25) and (29), the estimated power at the mth root direction on the unit circle is

$$\hat{P}_m = \frac{\mathbf{w}_m\mathbf{R}_{\mathbf{XX}}\mathbf{w}_m^H - \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0^H\dfrac{\mathbf{w}_m\mathbf{w}_m^H}{\mathbf{w}_0\mathbf{w}_0^H}}{\mathbf{w}_m\mathbf{a}_m\mathbf{a}_m^H\mathbf{w}_m^H}, \quad m = 1, 2, \dots, M \tag{30}$$

where $P'_m = \mathbf{w}_m\mathbf{R}_{\mathbf{XX}}\mathbf{w}_m^H$ and $P_0 = \mathbf{w}_0\mathbf{R}_{\mathbf{XX}}\mathbf{w}_0^H$. From Equation (30), only the autocorrelation matrix and the solution of Equation (16) are needed to compute the power at the direction of roots on the unit circle.

The estimated real power spectrum is obtained by plotting $\hat{P}_m$ versus θ_m found by the GA and expressed in Equation (20). The power value larger than a certain small margin (ideally equal to zero) is considered to be a real source signal. Consequently, the number of source signals can be determined from the estimated real power spectrum.

5. COMPUTER SIMULATION AND RESULT DISCUSSIONS

The new method of controlling the roots of the array polynomial to estimate the DOA of the source signals is demonstrated using six equispaced linear array elements of a half-wave inter-element spacing. Therefore, five roots are used to estimate at most the DOA of five source signals. The GA is used to search for the angles of arrival by minimizing the output received power. The angles of arrival are estimated by Equation (20) while the power of the source signals is calculated by Equation (30).

To validate the proposed method using the polynomial representation, let two uncorrelated and closely spaced signals impinging at angles 20° and 25° with 10.1 dBm and 6.1 dBm power levels, respectively. Figure 1 shows the estimated angles and power of the two signals using the roots of array polynomial method with 100 snapshots and 0 dBm input noise level (σ^2). From the figure, the estimated angles are 20.17° and 25.28°, and the estimated power levels are 10.3 dBm and 5.5 dBm, respectively. As a result, the proposed method using the roots of the array polynomial resolves the angles of the two signals and estimates the power of the two signals accurately with no previous assumption about

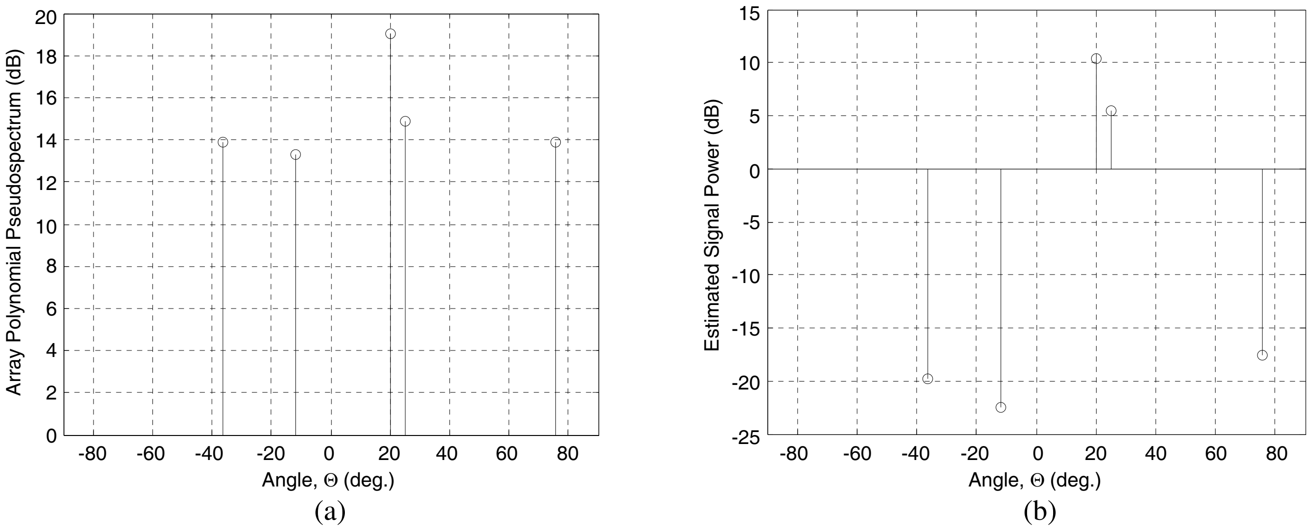


Figure 1. (a) The pseudospectrum and (b) the estimated real power spectrum at the direction of the roots that yield minimum power when two uncorrelated signals are impinging at the angles 20° and 25° with 10.1 dBm and 6.1 dBm power levels. (No. of array elements = 6, $\sigma^2 = 0$ dBm and 100 snapshots).

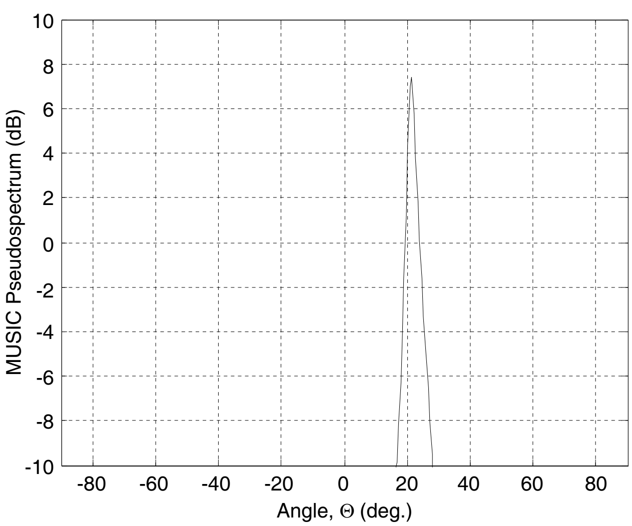


Figure 2. The power pseudospectrum of the MUSIC algorithm when two uncorrelated signals are impinging at the angles 20° and 25° with 10.1 dBm and 6.1 dBm power levels. (No. of array elements = 6, $\sigma^2 = 0$ dBm and 100 snapshots).

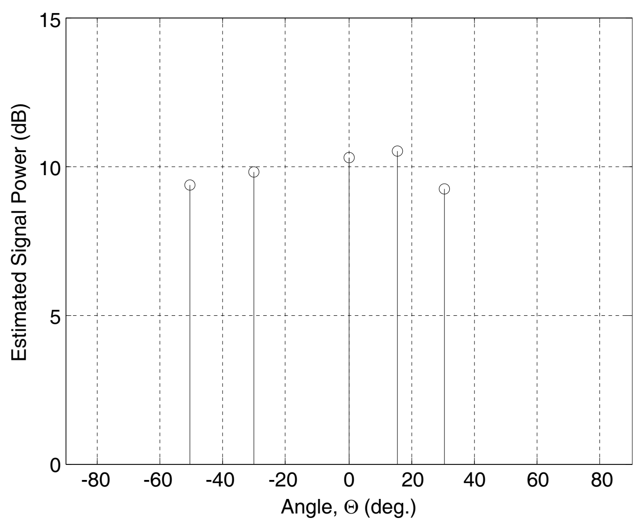


Figure 3. The estimated real power spectrum by controlling the roots of the array polynomial when five uncorrelated signals are impinging at the angles −50°, −30°, 0°, 15°, and 30° with power level around 10 dBm for each signal. (No. of array elements = 6, $\sigma^2 = 0$ dBm and 100 snapshots).

the number of source signals. For comparison purpose, Figure 2 shows the pseudo-spectrum using the MUSIC algorithm when the same two signals are assumed. The two signals are not resolved with the MUSIC algorithm as the figure shows only one peak.

Let five uncorrelated source signals impinging at the array with angles −50°, −30°, 0°, 15°, and 30° when the signal-to-noise ratio (SNR) is around 10 dB for each of the five signals. Figure 3 shows the estimated angles and power of the five signals by controlling the roots of the array polynomial

Table 1. The estimated angles, $\hat{\theta}_i$, and the estimated power level, $\hat{P}_i$, by controlling the roots of the array polynomial for five uncorrelated signals. (No. of array elements = 6, $\sigma^2 = 0$ dBm and 100 snapshots).

θ_i	P_i (dBm)	$\hat{\theta}_i$	$\hat{P}_i$ (dBm)
$-50°$	9.54	$-50.40°$	9.39
$-30°$	9.71	$-30.16°$	9.80
$0°$	10.20	$0.23°$	10.30
$15°$	10.53	$15.63°$	10.49
$30°$	9.51	$30.41°$	9.23

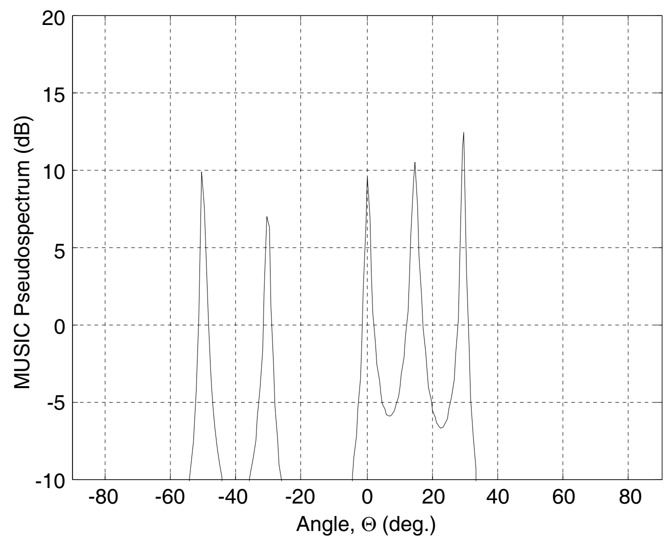
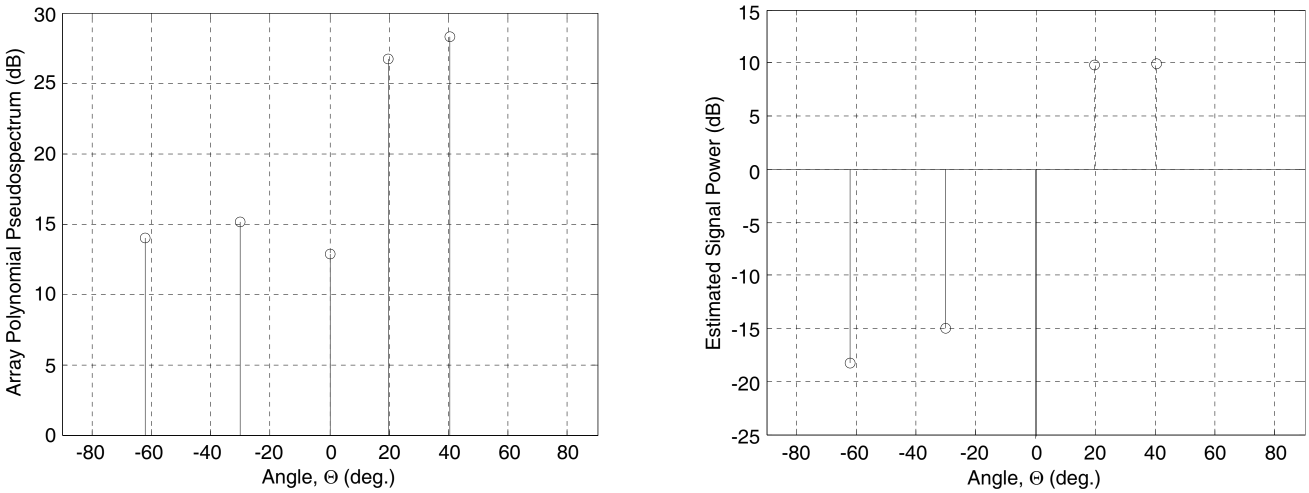


Figure 4. The power pseudospectrum of the MUSIC algorithm when five uncorrelated signals are impinging at the angles $-50°$, $-30°$, $0°$, $15°$, and $30°$ with power level around 10 dBm for each signal. (No. of array elements = 6, $\sigma^2 = 0$ dBm and 100 snapshots).

method with only 100 snapshots, and Table 1 gives the estimated angles of arrival and the estimated power levels for the five sources. On the other hand, Figure 4 shows the pseudospectrum for the same five uncorrelated signals impinging at the same angles of Figure 3 using the MUSIC algorithm with 100 snapshots. Comparing the power spectrum results shown by Figures 3 and 4, the estimated angles using the proposed method are more accurate than the estimated angles using the MUSIC algorithm. Moreover, the estimated values of the power signals are almost as accurate as shown in Figure 3 and given in Table 1.

To examine estimating coherent signals, let two coherent source signals impinging at the angles $20°$ and $40°$ with 10 dB SNR for each signal. Figure 5 shows the power spectrum of the two coherent signals using the proposed method with 100 snapshots. From the figure, the estimated angles of the two coherent signals are $19.90°$ and $40.52°$, and the estimated power levels of the two coherent signals are 9.83 dBm and 9.87 dBm. The MUSIC algorithm failed to estimate the angle of coherent source signals.

To find the effect of the signal-to-noise ratio (SNR) on the performance of estimating the DOA and the power level by controlling the roots of the array polynomial, one source signal impinging at $30°$ is simulated. Table 2 gives the estimated angle, $\hat{\theta}_1$, and the estimated power level, $\hat{P}_1$, for different SNRs with 100 snapshots. From the table, the estimated DOA of the signal is nearly insensitive to SNR variation, and the estimated power level of the signal, $\hat{P}_1$, is less accurate when the SNR value is low.



(a) (b)

Figure 5. (a) The pseudospectrum and (b) the estimated real power spectrum at the direction of the roots that yield minimum power when two coherent signals are impinging at the angles 20° and 40° with power level of 10 dBm for each signal. (No. of array elements = 6, $\sigma^2 = 0\,\text{dBm}$ and 100 snapshots).

Table 2. The estimated angle, $\hat{\theta}_1$, and the estimated power level of the signal source, $\hat{P}_1$, against different SNR values, when only one signal is impinging at the angle 30°. (No. of array elements = 6, $\sigma^2 = 0\,\text{dBm}$ and 100 snapshots).

SNR (dB)	$\hat{\theta}_i$	$\hat{P}_i$ (dBm)
10.27	30.15°	10.33
8.60	30.40°	8.68
6.23	30.00°	6.31
2.94	30.50°	2.76
1.14	29.93°	0.99
−2.38	30.19°	−2.12

6. CONCLUSIONS

In this work, the DOA and power of the source signals are estimated using the array polynomial representation. On the unit circle, the roots of array polynomial are rotated to coincide with the direction of the signals so that the output power is reduced to the value of noise power only. The real power spectrum is derived using the pseudospectrum, the steering vectors at the directions of the root locations which yield the minimum power, and the minimum power value. From the estimated real power spectrum, number of source signals, direction of arrivals, and power of the signals are estimated when the number of source signals is less than the number of array elements. The results indicate that the proposed method accurately estimates both the angle of arrivals and the power of the source signals even when the source signals are coherent.

REFERENCES

1. Godara, L., "Application of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival considerations," *Proceedings of the IEEE*, Vol. 85, No. 8, 1195–1245, 1997.
2. Schmidt, R. O., "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, Vol. 34, No. 3, 276–280, 1986.

3. Wong, K. M., J. P. Reilly, Q. Wu, and S. Qiao, "Estimation of the direction of arrival of signals in unknown correlated noise, Part I: The MAP approach and its implementation," *IEEE Trans. Signal Processing*, Vol. 40, 2007–2017, 1992.
4. Ariananda, D. D. and G. Leus, "Direction of arrival estimation of correlated signals using a dynamic linear array," *Conference on Signals, Systems and Computers (ASILOMAR)*, 2028–2035, Nov. 4–7, 2012.
5. Weiss, A. J., B. Friedlander, and P. Stoica, "Direction-of-arrival estimation using MODE with interpolated arrays," *IEEE Trans. Signal Processing*, Vol. 43, 296–300, 1995.
6. Zhu, W., J. Hu, X. Liu, Z. Liu, and M. Zhu, "Source number estimation using eigenspace in direction of arrival (DOA) estimate," *OCEANS 2009 — Europe*, 1–6, Bremen, May 11–14, 2009.
7. Qi, C. Y., Y. S. Zhang, Y. Han, and X. H. Chen, "An algorithm on high resolution DOA estimation with unknown number of signal sources," *4th International Conference on Microwave and Millimeter Wave Technology*, 227–230, 2004.
8. Zhang, Y. and B. P. Ng, "MUSIC-Like DOA estimation without estimating the number of sources," *IEEE Trans. on Signal Processing*, Vol. 58, No. 3, 1668–1676, 2010.
9. Zhou, Q.-C., H. Gao, and F. Wang, "A high resolution DOA estimation method without estimating the number of sources," *Progress In Electromagnetics Research C*, Vol. 25, 233–247, 2012.
10. Chiang, C. T. and A. C. Chang, "DOA estimation in the asynchronous DS-CDMA system," *IEEE Transactions on Antennas Propagation*, Vol. 51, No. 1, 40–47, 2003.
11. Alamouti, S. M., "A simple transmit diversity technique for wireless communications," *IEEE J. Selec. Areas Commun.*, Vol. 16, 1451–1458, 1998.
12. Hottinen, A., O. Tirkkonen, and R. Wichman, *Multi-antenna Transceiver Techniques for 3G and Beyond*, John Wiley and Sons, New York, 2003.
13. Naderi Shahi, S., M. Emadi, and K. H. Sadeghi, "High resolution DOA estimation in fully coherent environments," *Progress In Electromagnetics Research C*, Vol. 5, 135–148, 2008.
14. Sha, Z.-C., Z. Liu, Z. Huang, and Y. Zhou, "Direction estimation of correlated/coherent signals by sparsely representing the signal-subspace eigenvectors," *Progress In Electromagnetics Research C*, Vol. 40, 37–52, 2013.
15. Arikan, F., O. Koroglu, S. Fidan, O. Arikan, and M. B. Guldogan, "Multipath separation-direction of arrival (MS-DOA) with genetic search algorithm for HF channels," *Advances in Space Research*, Vol. 44, 641–652, 2009.
16. Mismar, M. J. and T. H. Ismail, "DOA estimation by controlling the nulls of the antenna array factor," *Proceedings of the International Conference on Electrical and Electronic Engineering, Telecommunication Engineering, and Mechatronics (EEETEM 2015)*, Kuala Lumpur, Malaysia, Sep. 8–10, 2015.