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A NEW OPEN CAVITY AT MILLIMETER WAVE BAND FOR PERMITTIVITY MEASUREMENT OF DIELECTRICS

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Abstract—In this paper, an open cavity is proposed to measure the permittivity of dielectrics. The cavity consists of an ellipsoidal mirror and two planar mirrors. The relationship between the parameters of the beam in the open cavity and the cavity geometrical parameters is presented. The transcendental equation of dielectric loaded cavity is presented, from which the permittivity of the dielectric can be solved. The resonance frequencies of the vacuum cavity and loaded cavity are computed by the resonance frequency formula and the transcendental equation. They are compared to the results from FDTD simulation. The results from two methods are almost same with each other. The advantage of the proposed open cavity over the conventional open cavity composed of spherical mirror and planar mirror is demonstrated.

1. INTRODUCTION

Open cavity technique is an important technique, as a high Q structure, and it may be used in high stable oscillator and power combining oscillator at millimeter wavelengths [1–3]. Meanwhile, open cavity is also a power tool for measuring accurately complex permittivity of dielectric materials at millimeter and sub-millimeter wavelengths. How to use this technique to measure the complex permittivity of dielectric material accurately have been investigated in detail [4–20]. All of the open cavities investigated are based on spherical mirrors or spherical and planar mirrors. For these open cavities, the small coupling holes of the input and output are placed on the spherical mirror and the material to be measured is placed on the planar mirror. When the electromagnetic power is fed into the cavity through the small hole,

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some of the power will leak to the out of the cavity due to the diffraction of the wave.

It is known that the power of input wave has to be greater than that of loss in the open cavity including the power absorbed by the cavity wall and the power leaked out of the cavity, and the resonance in the cavity can be established. Weak coupling can gain higher Q value, meanwhile, weak coupling also reduce the incident power. If we want to excite a field in the cavity, more incident power is needed for the weak coupling. However, at short millimeter wavelength (wavelength is less than 3 mm) or sub-millimeter wavelength, to produce enough incident power is a difficult task. In this paper, we propose a new open cavity to solve this problem. Using the open cavity composed of an ellipsoidal mirror and two planar mirrors, the leak of power can be avoided or reduced much, and then low incident power can excite a resonance field in the cavity because the power absorbed by the cavity wall is very little.

2. OPEN CAVITY COMPOSED OF AN ELLIPSOIDAL MIRROR AND TWO PLANAR MIRRORS

Ellipsoid that can be used as reflective focusing element has been described in [1]. Here we use it and two planar mirrors to compose an open cavity. The open cavity configuration is depicted in Fig. 1. The ellipsoid is rotationally symmetrical about its major axis. The major axis has length $2a$, and the minor axis has length $2b$. The distances from the two focal points F_1 and F_2 to a point P on the surface of the ellipsoid are denoted by R_1 and R_2, respectively. And they are related

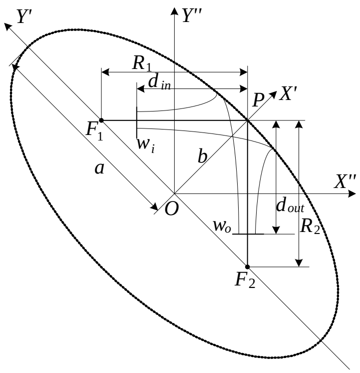

Figure 1. Geometry of new open cavity composed of ellipsoidal mirror and planar mirrors.

by

$$R_1 + R_2 = 2a \tag{1}$$

The distance from input beam waist w_i and output beam waist w_o to the point P are denoted by d_{in} and d_{out}. The focal length of the section of the ellipsoid is given in terms of the distances to the foci.

$$f = \frac{R_1 R_2}{R_1 + R_2} \tag{2}$$

A special case with $R_1 = R_2 = R$, then $f = R/2$ and $\theta_i = \pi/4$ is chosen so that we are using a section centered on the minor axis of the ellipsoid with $a = R$ and $b = R\cos\theta_i$. This geometry introduces minimal beam distortion effects. Two planar mirrors are placed at the position of w_i and w_o, respectively, so an open cavity is obtained, as shown in Fig. 1.

For this open cavity, an $ABCD$ matrix can be used to describe its properties, from which we can get the condition of the open cavity resonate stably as follows.

$$0 < (1 - d_{in}/f)(1 - d_{out}/f) < 1 \tag{3}$$

It is interesting to compare the new cavity with the cavity composed of spherical mirrors. For the open cavity of spherical mirrors, the condition of cavity resonate stably is,

$$0 < (1 - l/R_A)(1 - l/R_B) < 1 \tag{4}$$

where l is cavity length, and R_A and R_B are the curvature radii of spherical mirrors A and B, respectively.

If $d_{in} = d_{out} = d$, then

$$z_0^2 = \frac{d^2[1 + (1 - d/f)]^2}{[1 - (1 - d/f)^2]} = d(2f - d) \tag{5}$$

In this situation $w_i = w_o = w_0$.

And for the open cavity with spherical mirror and planar mirror, we have

$$z_0^2 = d_0(R_0 - d_0) \tag{6}$$

where d_0 is cavity length and R_0 the curvature radius of spherical mirror. From (6), we can get the beam waist at planar mirror. Also we can see (6) is similar to (5).

2.1. Resonance Frequency

A Gaussian beam propagate from w_i to w_o, the phase shift should be $(p + 1)\pi$, p is an integer. That is

$$\theta_{mn}(d_{in}) + \theta_{mn}(d_{out}) = (p + 1)\pi, \quad p = 0, 1, 2, \ldots \tag{7}$$

Resonance frequency is obtained from (7) and Gaussian beam theory [1,2]

$$f_{reso} = \frac{c}{2(d_{in} + d_{out})} \left\{ p + 1 + \frac{m+n+1}{\pi} \left[\arctan(d_{in}/z_0) + \arctan(d_{out}/z_0) \right] \right\} \quad (8)$$

where c is light velocity. In circular coordinate,

$$f_{reso} = \frac{c}{2(d_{in} + d_{out})} \left\{ (p+1) + \frac{(2n+i+1)}{\pi} \left[\arctan(d_{in}/z_0) + \arctan(d_{out}/z_0) \right] \right\} \quad (9)$$

where z_0 is defined as (6), and m, n, and i are integer number.

3. SIMULATION OF OPEN CAVITY

To verify whether the leak of power can be avoided, two types of open cavities are simulated with two dimensional FDTD methods. One is composed of spherical mirror and planar mirror and another is described above.

A reference plane, as shown in Fig. 2, is set in the open cavity to record the input wave and the wave limited by the open cavity in time domain and then they are transformed into frequency domain to get the input power P_i and limited power P_L at different frequencies. So the loss due to the power leakage is

$$L_{leakage} = \frac{P_L}{P_i} \quad \text{or} \quad L_{leakage}(\text{dB}) = 10 \log \left(\frac{P_L}{P_i} \right) \quad (10)$$

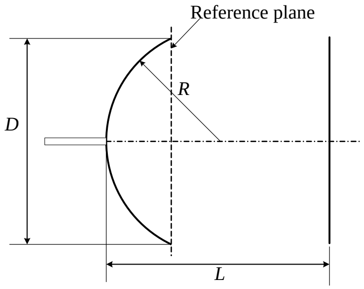

Figure 2. Configuration of the open cavity with spherical mirror and planar mirror.

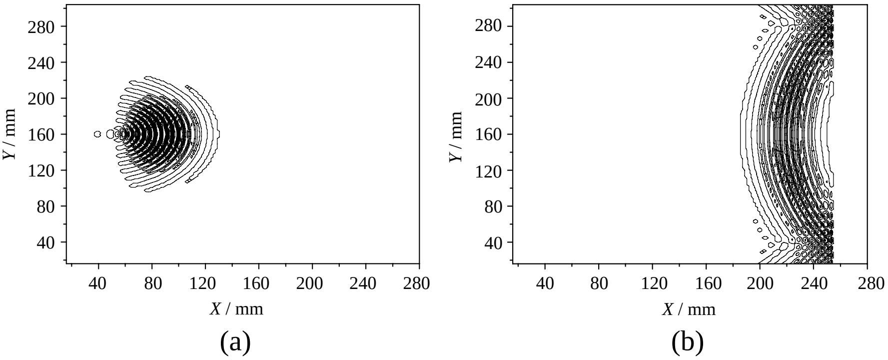

Figure 3. Pulse wave in conventional open cavity.

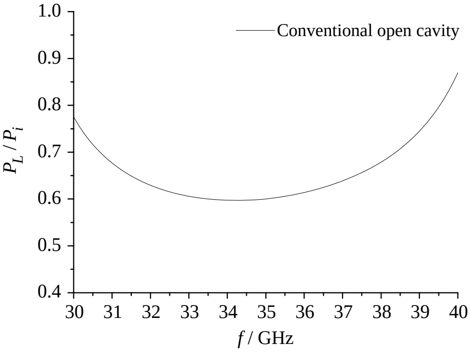

Figure 4. $L_{leakage}$ for conventional open cavity.

3.1. Conventional Open Cavity

The configuration of the open cavity is shown in Fig. 2, and parameters of the open cavity are chosen as follows: curvature radius of the spherical mirror $R = 220\,\text{mm}$, cavity length $L = 200\,\text{mm}$, diameter of the cavity $D = 220\,\text{mm}$.

The input pulse wave is given in Fig. 3(a), and the pulse reaching the planar mirror is depicted in Fig. 3(b). It can be seen that some power has diffused out of the cavity. The simulated results of $L_{leakage}$ in Ka band are given in Fig. 4. It can be seen that more than 30% power has diffused out of the cavity over the band from 30.6 GHz to 38.4 GHz.

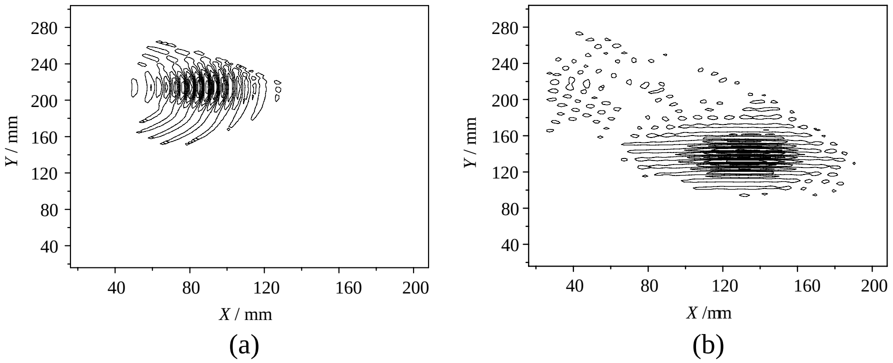

Figure 5. Pulse wave in proposed open cavity.

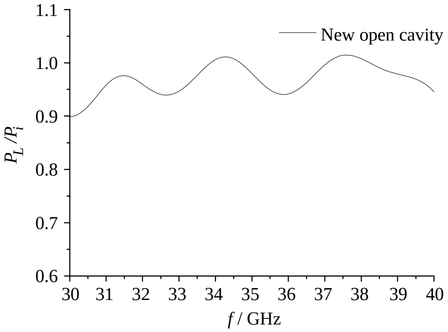

Figure 6. $L_{leakage}$ for proposed open cavity.

3.2. Open Cavity Composed of Ellipsoidal Mirror and Planar Mirrors

The configuration of the open cavity has been depicted in Fig. 1, and the parameters of the open cavity are: $a = 200\,\text{mm}$, $b = 141.42\,\text{mm}$. $R_1 = R_2 = R = 200\,\text{mm}$, $f = R/2 = 100\,\text{mm}$, $d_{in} = 110.0\,\text{mm}$, $d_{out} = 190.0\,\text{mm}$. And similar simulation is done for the proposed open cavity. The simulated results are depicted in Fig. 5. Fig. 5(a) shows the input pulse wave, and Fig. 5(b) is the limited pulse after many times reflection among the ellipsoidal mirror and the planar mirrors. The loss $L_{leakage}$ simulated is given in Fig. 6. It can be seen that the loss due to the power leakage is less than 10% over the band from 30 GHz to 40 GHz. Therefore, the proposed open cavity has lower power loss than the conventional open cavity.

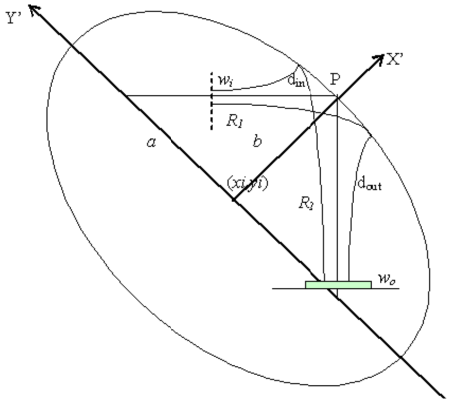

Figure 7. The new open cavity loaded with dielectric plate.

4. RESONANCE FREQUENCY OF LOADED OPEN CAVITY

The resonance frequency of vacuum open cavity of $\mathrm{TEM}_{00\mathrm{p}}$ modes of the new open cavity is given as follows,

$$f_{reso} = \frac{c}{2(d_{in} + d_{out})} \left\{ p + 1 + \frac{1}{\pi} \left[\arctan(d_{in}/z_0) + \arctan(d_{out}/z_0) \right] \right\} \tag{11}$$

The new open cavity loaded with dielectric plate is shown in Fig. 7. If both planar mirrors are perfectly electric wall, the resonance frequency can be solved from following transcendental equation,

$$\frac{1}{n} \tan\left(nk_0 t - \Phi_T\right) = -\tan\left[k_0\left(d + d_{in}\right) - \Phi_D\right] \tag{12}$$

where,

$$\Phi_T = \Phi_1(t) - \xi_1(t) = \tan^{-1}\left(\frac{t}{nz_0}\right) - \tan^{-1}\left(\frac{1}{nk_0 F_1(t)}\right),$$

$$\Phi_D = \Phi_2(t+d) - \Phi_2(t) + \Phi_i\left(d_{in}\right) - \xi_2(t+d) + \xi_2(t) - \xi_i\left(d_{in}\right),$$

$$\Phi_2(t+d) = \tan^{-1}\left(\frac{t+d-t'}{z_0}\right),$$

$$\xi_2(t+d) = \tan^{-1}\left(\frac{1}{k_0 F_2(t+d)}\right),$$

$$F_2(t+d) = t + d - t' + \frac{z_0^2}{(t+d-t')},$$

$$\begin{aligned}
\xi_2(t) &= \tan^{-1}\left(\frac{1}{k_0 F_2(t)}\right), F_2(t) = t - t' + \frac{z_0^2}{(t-t')}, \\
F_1(t) &= t + \frac{n^2 z_0^2}{t}, \\
\Phi_i(d_{in}) &= \tan^{-1}\left(\frac{d_{in}}{z_{0i}}\right), \\
F_i(d_{in}) &= d_{in} + \frac{z_{0i}^2}{d_{in}}, \\
\xi_i(d_{in}) &= \tan^{-1}\left(\frac{1}{k_0 F_i(d_{in})}\right), \quad t' = t(1 - 1/n), \\
z_{0i}^2 &= \frac{(1 - d_{in}/f)[d_{in} + (1 - d_{in}/f)d_{out}]^2}{(1 - d_{out}/f)[1 - (1 - d_{in}/f)(1 - d_{out}/f)]}, \\
d + t &= d_{out},
\end{aligned}$$

t is the thickness of the dielectric plate, z_0 is defined in (6), and k_0 is the wave number.

5. EXPERIMENT SIMULATION

In order to verify formulas (11) and (12), we simulate the vacuum and loaded cavity with the two dimensional FDTD method.

The parameters of the open cavity have been described in Section 3.2. A coupling hole is designed on one planar mirror, and a source is placed out of the cavity to excite the cavity through the coupling hole. After exciting, the field in time domain is recorded out

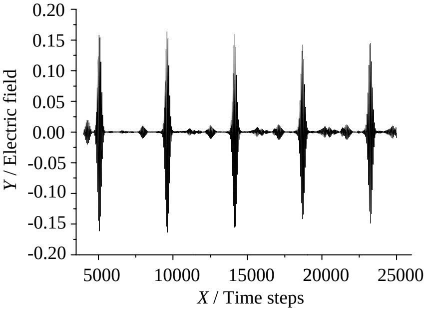

Figure 8. Time domain field recorded out of the coupling hole of the cavity.

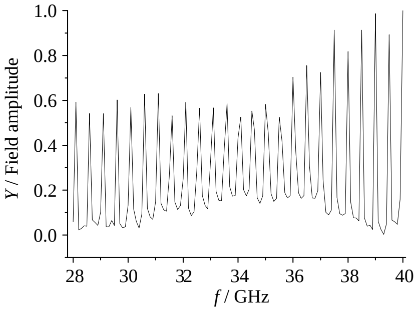

Figure 9. Frequency domain field transformed from the data in Fig. 8.

of the coupling hole, then the recorded field is transformed into the field in frequency domain, so the resonance frequency of the cavity can be obtained. Because simulation is done to cavity, enough time steps have to be necessary. The time domain field recorded out of the coupling hole of the cavity is given in Fig. 8, where time steps are 25000.

The field in frequency domain after transform is depicted in Fig. 9. The resonance peak is not so sharp due to less time steps.

When the time steps are increased to 100000, and the time domain field recorded is shown in Fig. 10. The field in frequency domain is obtained from Fourier transform and given in Fig. 11. It is similar to that shown by VNA (vector network analyzer) in measurement. The peaks in Fig. 11 denote the resonance frequencies of the open cavity. We can see that the FDTD analysis can simulate the experiment well. Using this method, the resonance frequencies of the vacuum cavity and the loaded cavity can be obtained like in experiments.

Then, the simulation results are compared to Formulas (11) and (12). The resonance frequency of the vacuum cavity and loaded cavity, which are obtained from Formulas (11), (12) and FDTD simulation, are given in Table 1, where the dielectric constant of the loaded plate is 2.08. It is clear that the resonance frequencies of the cavity from FDTD are very similar with those from Formulas (11) and (12), and the interval of the resonance frequency of the vacuum cavity and loaded cavity computed by FDTD and formula respectively are the same. However, as the FDTD is operated in two dimensional space and the ellipsoid mirror surface is approximated by the staircase, the resonance frequencies of the cavity from FDTD have little difference compared to that from formula, i.e., it can not be exactly the same to that from Formulas (11) and (12).

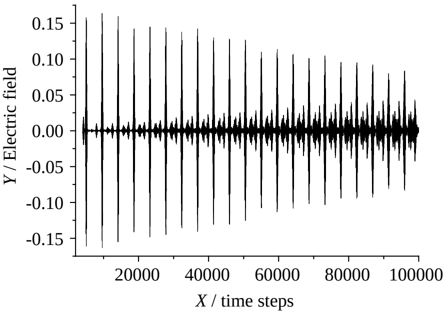

Figure 10. Time domain field recorded out of the coupling hole of the cavity.

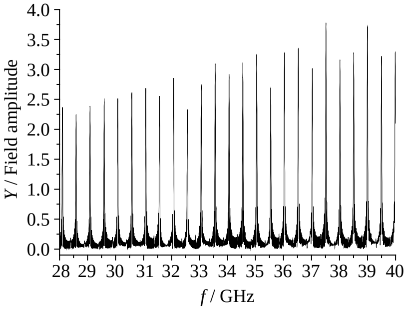

Figure 11. Frequency domain field transformed from the data in Fig. 10.

Table 1. Resonance frequency of the cavity.

Formula		FDTD	
f (GHz) vacuum	f (GHz) dielectric $t = 2.4$	f (GHz) vacuum	f (GHz) dielectric $t = 2.4$
30.77719	30.63965	30.583	30.433
31.27684	31.13745	31.080	30.936
31.77649	31.63537	31.569	31.420
32.27615	32.13343	32.072	31.921
32.77580	32.63164	32.565	32.417
33.27546	33.12998	33.060	32.909
33.77511	33.62845	33.557	33.405
34.27477	34.12704	34.054	33.902
34.77442	34.62575	34.546	34.395
35.27407	35.12456	35.042	34.888
35.77373	35.62348	35.542	35.388
36.27338	36.12248	36.042	35.881
36.77304	36.62158	36.527	36.372
37.27269	37.12075	37.028	36.874
37.77234	37.61999	37.513	37.362
38.27200	38.11931	38.013	37.865
38.77165	38.61868	38.506	38.356
39.27131	39.1181	38.996	38.843
39.77096	39.61758	39.496	39.342

6. CONCLUSION

An open cavity composed of ellipsoidal mirror and planar mirrors is proposed and simulated. It has lower loss of the power leakage than the conventional open cavity, which is demonstrated by FDTD method. The resonance frequencies of the vacuum and loaded cavities are computed by formulas and experiment simulation with FDTD and compared. The correctness of the formulas and simulation with FDTD verifies each other.

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