

## CHARACTERIZATION OF THE REGULAR POLYGONAL WAVEGUIDE FOR THE RF EM SHIELDING APPLICATION

B. Ravelo and B. Mazari

IRSEEM/ESIGELEC

Technopole du Madrillet

Avenue Galilée-BP10024

Saint Etienne du Rouvray Cedex 76801, France

**Abstract**—This article presents a theoretical characterization of the regular polygonal waveguide (RPW) having  $n$ -sides. Based on the symmetrical circular symmetry of the RPW and the circular waveguide (CW), the analogy between the electromagnetic (EM) behaviors of these to waveguide (WG) is established. After a brief recall about the state of the art concerning the WG engineering and its application, we introduce a basic theory of the WG presenting a regular polygonal cross-section with  $n$ -sides. By considering, the fundamental mode  $TE_{11}$ , we develop the main mathematical formulas summarizing the different characteristics (cut-off frequency,  $f_c$ , propagating constant,  $k_{11}$  and  $S$ -parameters) appropriated to any RPW in function of its physical parameters (number of sides,  $n$ , diameter,  $D$  and height,  $h$ ). In order to verify the validation of the developed analytical expressions, comparisons between the HFSS simulated and theoretical dispersion diagrams of regular pentagonal ( $n = 5$ ), hexagonal ( $n = 6$ ), heptagonal ( $n = 7$ ) metallic (copper) WG with for example, 50 mm outer diameter are presented. So, it was demonstrated that very good correlation between the theoretical predictions ( $f_c(n)$ ,  $k_{11}(n)$ ) is found with a relative error less than 1%. As application of the present study in terms of EM wave shielding, simulation of metallic wall with hexagonal aperture is also performed. Finally, discussion about the future work is drawn in conclusion.

## 1. INTRODUCTION

Since the middle of the last century, the knowledge of the fundamental concepts about the electromagnetic (EM) waveguide theory have aided several engineers for the design of the devices or equipments used in the telecommunication areas [1, 2]. It is well-known that in this field of application, waveguide (WG) structure constitutes one of the key elements [3, 4]. In fact, the WG can be utilized at all levels of telecommunication architectures. As illustration, in microwave and optical domains, WG constitutes a flexible element which offers numerous possibilities to design performing passive (transition element, isolator, circulator, resonator, attenuators, filter, divider/combiner...) and active (amplifier, mixers...) devices [5–9]. Moreover, by using  $S$ -parameters and EM wave propagation theory, other applications of waveguide structure have been also proposed for example, for setting a measurement technique allowing the determination of the EM material characteristics such as permittivity and permeability [10, 11].

Since few decades earlier, because of the high pass behavior of the closed WG, many applications have been also proposed in terms of the high frequency EM shielding. Currently, due to the progress of the wireless communication system, mankind knows and lives in different types of unavoidable EM pollutions such as disturbing radiating artificial (satellite communication, radar...) and natural (lightning) fields which become more and more considerable for the modern electronic systems. In order to protect the electronic equipments for example, in the embedded systems (vehicle, avionic or aircraft systems) faced to the effects of this EM pollution, light and low cost shielding industrial metallic panels holed having different geometries of apertures have been used [12–17]. It is interesting to note that the design of the panel with well-appropriated dimensions depends deeply on the theory of WG. In this context of shielding process, the most used type of WG is that one presenting regular hexagonal cross-section such as the honeycomb composite structure [12, 13]. As pointed out in [13], these structures are particularly advantageous to shield openings for heating and ventilation against undesirable electro-magnetic waves. In other words, the hexagonal cross-section waveguides are susceptible to limit the certain frequency range of RF wave propagation and simultaneously, capable to evacuate the air as the ventilation system. Nonetheless, till now, few analytical studies [18, 19] have been made and published about the characterization of the regular hexagonal waveguides compared to the rectangular and circular ones.

For this reason this paper is proposed. Indeed, it is aimed

to analyze theoretically the properties of the RPW having  $n$ -sides. More precisely, it presents the determination of the dispersion diagram, cut off frequency and  $S$ -parameters of RPW transverse fundamental mode  $TE_{11}$ , according to its physical dimensions. For the better understanding, this report was structured as follows. Section 2 describes the introduced analytical study of RPW which was mainly deduced by analogy with cylindrical waveguide (CW). Then, Section 3 is consecrated to the validation of the established theoretical expressions with 3D EM simulations carried out with HFSS simulator. The last section draws the concluding summary and the prospects of this work.

## 2. ANALYTICAL DESCRIPTION

In geometrical point of view, we can find that a CW is also a RPW when the number of sides,  $n$  is equal to infinite. Meanwhile, these two types of WG present a circular symmetry. For this reason of symmetry, we propose in this section, to lead a study permitting to characterize and to establish the fundamental properties of the RPW by comparing with the CW. To do this, we will demonstrate the expressions of the propagation constant, cut-off frequency, and even the  $S_{11}$  or reflection- and  $S_{21}$  or transmission-parameters of the RPW.

### 2.1. Recall on the Circular Waveguide Theory

For starting, let us consider a metallic CW with diameter,  $D$  and filled by a dielectric medium with permittivity and permeability, respectively denoted  $\varepsilon$  and  $\mu$ . It is known that according to the classical WG theory, the longitudinal propagation constant,  $k_{mn}^{TE}$  and the cut-off frequency,  $f_{mn}^{TE}$  of the electrical transverse modes,  $TE_{mn}$  are expressed as:

$$k_{mn}^{TE}(\omega) = \sqrt{\frac{\omega^2}{v^2} - \left(\frac{2\alpha_{mn}}{D}\right)^2}. \quad (1)$$

$$f_{mn}^{TE} = \frac{\alpha_{mn}v}{\pi D}. \quad (2)$$

where  $\alpha_{mn}$  is the  $n$ -th root of the derivative of  $m$ -th order Bessel-function defined as  $J'_m(\alpha_{mn}) = 0$  and  $v$  is the wave velocity:

$$v = \frac{1}{\sqrt{\varepsilon\mu}}. \quad (3)$$

For example, by considering the lowest or fundamental mode,  $TE_{11}$ , knowing that the root of the equation,  $J'_1(\alpha_{11}) = 0$  is approximately

equal to  $\alpha_{11} \approx 1.84118$ . As a result, the cut-off frequency is written as:

$$f_{11}^{TE} \approx \frac{\alpha_{11}v}{\pi D}. \quad (4)$$

In this case, it is worth noting also that when  $f < f_{11}$ , we have the EM evanescent wave with attenuation,  $A_{z11}(f)$  at the height,  $z$  written as:

$$A_{z11}(f) = e^{-jk_{11} \cdot z} = e^{-z\sqrt{\left(\frac{2\alpha_{11}}{D}\right)^2 - \frac{4\pi^2 f^2}{v^2}}}. \quad (5)$$

In the continuation of this paper, we will serve to these expressions for investigating the EM characteristics of the PRW.

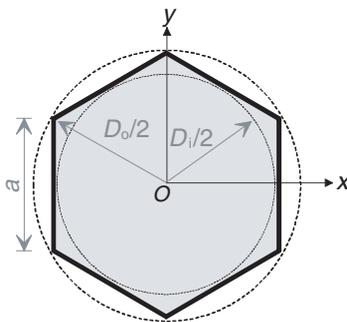
## 2.2. RPW Theory

For the illustration, let us consider the geometry of waveguide with regular polygonal cross-section with outer diameter,  $D_o$  with  $n$  sides as depicted in Fig. 1 ( $n = 6$ ) below.

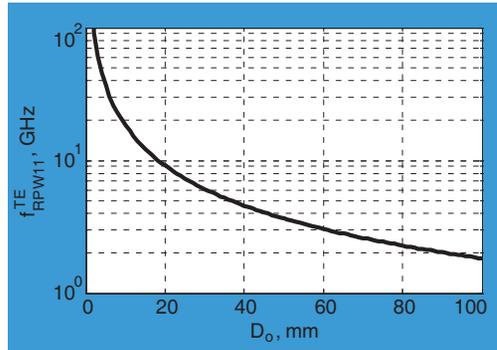
Through a simple geometrical analysis of the polygonal cross-section, one establishes that the maximal diameter denoted  $D_i$  of the inner circle and side value denoted  $a$  are related by:

$$D_i = a \cot\left(\frac{\pi}{n}\right) = D_o \cos\left(\frac{\pi}{n}\right). \quad (6)$$

Therefore, the  $TE_{11}$ -mode cut-off frequency of PRW,  $f_{c11}$  depends naturally, on the CWs with maximal inner- and minimal outer-diameters. In that case, one establishes that it is logically equal to



**Figure 1.** Cross-section of regular hexagonal ( $n = 6$ ) waveguide with outer diameter,  $D_o$ .



**Figure 2.** Hexagonal WG ( $n = 6$ ) cut-off frequency,  $f_{RPW11}^{TE}$  vs  $D_o$  with vacuum medium.

the cut-off frequency of the CW having equivalent diameter equal to  $D_i$  and  $D_o$  harmonic mean value:

$$\frac{2}{D_e} = \frac{1}{D_i} + \frac{1}{D_o} \Rightarrow D_e(n) = \frac{2}{1 + \sec\left(\frac{\pi}{n}\right)} D_o. \quad (7)$$

So, for the lowest mode,  $TE_{11}$ , the cut-off frequency should be merely written as:

$$f_{RPW11}^{TE} = \frac{\alpha_{11}v}{2\pi D_e}. \quad (8)$$

Substituting Equation (7) into (8), one gets:

$$f_c(n) = f_{RPW11}^{TE} = \frac{\alpha_{11}v}{4\pi D_o} \left[ 1 + \sec\left(\frac{\pi}{n}\right) \right]. \quad (9)$$

To get more insight about the interpretation of this formula, the semi-log graph of the regular hexagonal WG ( $n = 6$ ) cut-off frequency,  $f_{RPW11}^{TE}$  in function of the outer diameter,  $D_o$  varying from 1 to 100 mm when considering vacuum medium ( $\varepsilon = \varepsilon_0$ ) is plotted in Fig. 2 below.

Consequently, the longitudinal  $k_{11}^{TE}$ -wave vector modulus of the RPW with  $n$ -sides becomes:

$$k_{11}^{TE}(f) = \frac{2\pi}{v} \sqrt{f^2 - (f_{RPW11}^{TE})^2} = \sqrt{\frac{4\pi^2 f^2}{v^2} - \left(\frac{\alpha_{11}}{2D_o}\right)^2 \left[1 + \sec\left(\frac{\pi}{n}\right)\right]^2}. \quad (10)$$

It means that the corresponding guided wavelength versus frequency is defined as:

$$\lambda_g(f) = \frac{2\pi}{k_{11}^{TE}(f)} = \frac{v}{\sqrt{f^2 - (f_{RPW11}^{TE})^2}}. \quad (11)$$

So, when  $f < f_{RPW11}^{TE}$ , the EM-field attenuation,  $A_{z11}(f)$  is given by:

$$A_{z11}(f) = e^{-z \sqrt{\frac{\alpha_{11}^2}{4D_o^2} \left[1 + \sec\left(\frac{\pi}{n}\right)\right]^2 - \frac{4\pi^2 f^2}{v^2}}}. \quad (12)$$

While in the opposite case  $f > f_{RPW11}^{TE}$  where the wave is propagating, the RPW behaves as a transmission line presenting a characteristic impedance equal to:

$$Z_c = \sqrt{\frac{\mu}{\varepsilon}}. \quad (13)$$

Thus, by definition, one demonstrates that the under study WG presents the reflection and transmission parameters respectively, written as:

$$S_{11}(f) = \frac{j(Z_c^2 - Z_0^2) \tan[-k_{11}^{TE}(f)z]}{2Z_c Z_0 + j(Z_c^2 + Z_0^2) \tan[-k_{11}^{TE}(f)z]}, \quad (14)$$

$$S_{21}(f) = \frac{2jZ_c Z_0 \cos[-k_{11}^{TE}(f)z]}{2Z_c Z_0 + j(Z_c^2 + Z_0^2) \tan[-k_{11}^{TE}(f)z]}, \quad (15)$$

where  $Z_0 = 50 \Omega$  is the reference impedance. Obviously, when assuming that the WG is unmatched ( $Z_c \neq Z_0$ ), the optimal frequency of the maximum transmitted guided wave are achieved when  $S_{11}(f) = 0$ . Mathematically, this condition is verified if:

$$\tan \left[ -\frac{2\pi z}{v} \sqrt{f_m^2 - (f_{RPW11}^{TE})^2} \right] = 0 \Leftrightarrow \frac{2\pi z}{v} \sqrt{f_m^2 - (f_{RPW11}^{TE})^2} = m\pi, \quad (16)$$

$$\Rightarrow f_m = \sqrt{\left(\frac{mv}{2z}\right)^2 + (f_{RPW11}^{TE})^2}, \quad (17)$$

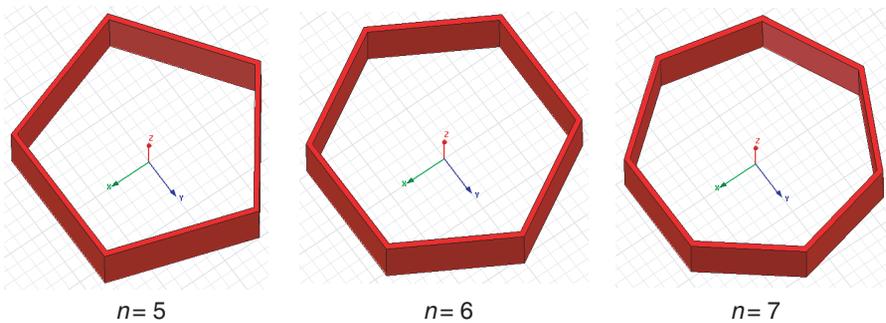
where  $m$  is a positive integer. In order to check the validation of this theoretical prediction, simulations performed with EM 3D tool are proposed in the next section.

### 3. SIMULATION RESULTS

Along this section, we point out that the simulation results discussed in this section was carried out with the HFSS<sup>TM</sup> FEM solver from Ansoft. The metallic parts of our WG are constituted by copper metal and filled with vacuum.

#### 3.1. Dispersion Diagrams of RPW with Different Number of Sides

In order to check the efficiency of the indicated above formulas, we simulated three metallic RPW with thickness,  $e = 0.5$  mm shown in Fig. 3 below for different values of side numbers  $n = 5, 6$  and  $7$ . We



**Figure 3.** HFSS<sup>TM</sup> design of the simulated regular polygonal WG with outer diameter,  $D_o = 50$  mm, height,  $h = 70$  mm and thickness,  $e = 0.5$  mm.

**Table 1.** Calculated outer diameter,  $D_e$  and cut-off frequency,  $f_c$  of the equivalent CW versus  $n$ .

| $n$      | 5        | 6        | 7        |
|----------|----------|----------|----------|
| $D_e(n)$ | 44.7 mm  | 46.4 mm  | 47.4 mm  |
| $f_c(n)$ | 3.93 GHz | 3.79 GHz | 3.71 GHz |

point out that the outer diameter and the height of these structures are respectively, set at  $D_o = 50$  mm and  $h = 70$  mm.

Therefore, by using Equations (7) and (9), one gets the calculated results shown in Table 1. It indicates the corresponding to CW with its equivalent diameter,  $D_e(n)$  and cut-off frequency,  $f_c(n)$  in function of the RPW side number,  $n = \{5, 6, 7\}$ .

After simulations of the structures described in Fig. 3, we obtain the comparison results between the calculated (by using Equation (10)) and simulated dispersion diagrams plotted in Figs. 4(a) and 4(b) from 1 GHz to 8 GHz. Therefore, it is worth noting that due to the inaccuracy of HFSS-numerical calculation these dispersion diagrams presents relative errors widely lower than 1%.

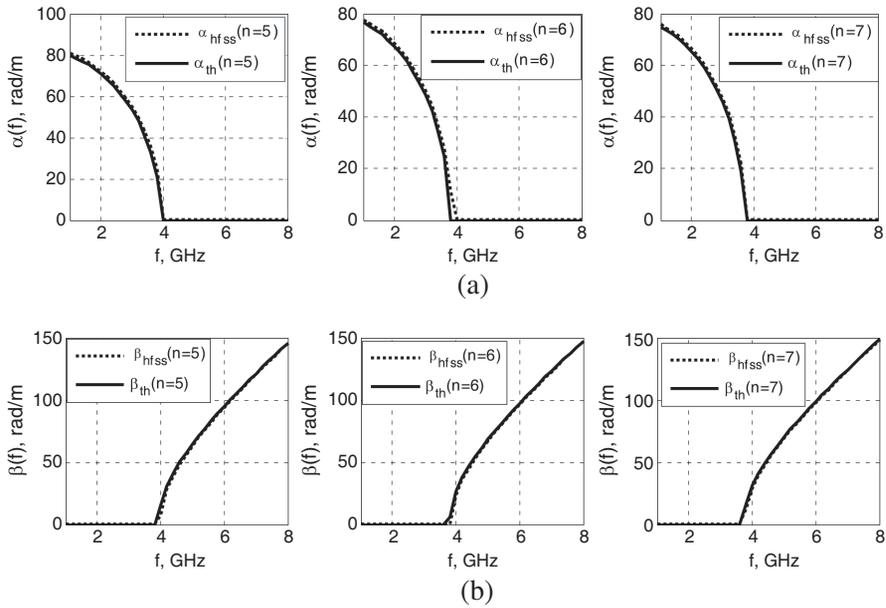
As depicted in Fig. 3, it is interesting to note that the cut-off frequency,  $f_c$  is inversely proportional to  $n$ . In addition, one can see clearly that the propagating mode exists only beyond the cut-off frequencies,  $f_c(n)$ .

### 3.2. Reflection and Transmission of Metallic Wall with Hexagonal Aperture

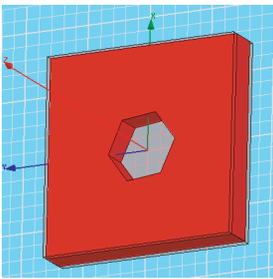
As aforementioned, one of the potential applications of the proposal study of RPW concerns the radiating field shielding techniques. In order to evidence the physical meaning of this industrial application, a  $150 \text{ mm} \times 150 \text{ mm}$  rectangular copper wall with thickness,  $z = 40$  mm and holed with hexagonal aperture having  $D_o = 50$  mm outer diameter was designed and simulated with HFSS as shown in Fig. 5 below.

According to the theoretical prediction of Equation (17), this structure should present the 6th first maximal transmission of the guided wavelength at the frequencies,  $f_m$  as explained in Fig. 6 in function of the metal (copper) thickness,  $z$ . Once again, as depicted in this figure, the transmission frequencies,  $f_m(t)$  are inversely proportional to the RPW height,  $z$ .

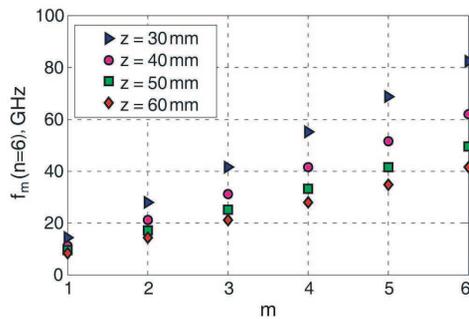
In order to confirm the certitude of the result indicated in Fig. 6, reflection and transmission parameters, respectively,  $S_{11}|_{\text{dB}}$  and  $S_{21}|_{\text{dB}}$  from 2 GHz to 12 GHz of the simulated structure with HFSS<sup>TM</sup>



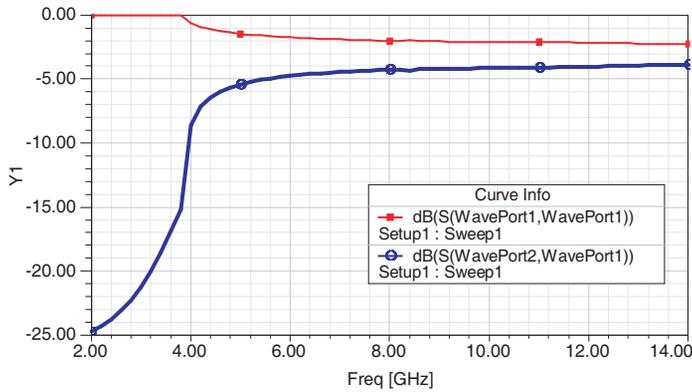
**Figure 4.** (a) Comparison of the theoretical ( $\alpha_{th}$ ) and HFSS ( $\alpha_{hfss}$ ) attenuation constants,  $\alpha(f)$  of the RPW shown in Fig. 3. (b) Comparison of the theoretical ( $\beta_{th}$ ) and HFSS ( $\beta_{hfss}$ ) simulated phase constants,  $\beta(f)$  of the RPW shown in Fig. 3.



**Figure 5.** HFSS design of rectangular metallic wall (150 mm  $\times$  150 mm  $\times$  40 mm) with polygonal aperture ( $D_o = 50$  mm).



**Figure 6.** Optimal frequencies,  $f_m$  of the guided wavelength of the RPW structure presented in Fig. 5 versus metal thickness,  $z$ .



**Figure 7.** HFSS<sup>TM</sup> simulated  $S$ -parameters,  $S_{11}|_{\text{dB}}$  and  $S_{21}|_{\text{dB}}$  of the structure presented in Fig. 5.

simulator were depicted in Fig. 7.

As described by this figure, similarly to the rectangular WG, the structure rectangular holed structure behaves generally as a high pass structure. Thus, we verify that the wavelengths below the cut-off frequency 3.79 GHz are attenuated below  $-10$  dB.

#### 4. CONCLUSION

The basic theory of the regular polygonal waveguide (RPW) are successfully presented and validated with the most popular 3D EM tool HFSS<sup>TM</sup>.

Based on the analogy with the classical CW theory, the main characteristics of RPW such as the cut-off frequency,  $f_c(n)$ , dispersion diagram  $k_{11}(n)$  and the  $S$ -parameters in function of RPW side number,  $n$  and its physical dimensions are established. In order to illustrate the evolution of these parameters according to the developed analytical expressions, graphical results were plotted and discussed. Furthermore, the validations of the theoretical prediction through simulations of pentagonal ( $n = 5$ ), hexagonal ( $n = 6$ ), heptagonal ( $n = 5$ ) metallic (copper) WG with 50 mm outer diameter were carried out by using the standard HFSS<sup>TM</sup> 3D EM simulators. Therefore, very good agreement between the theory and simulation was verified. It was shown that the RPW cut-off frequency is inversely proportional to  $n$ . Furthermore, to get more understandable idea about the application of the RPW for example, for the radiating EM shielding process, simulation of a rectangular metallic wall with hexagonal aperture is performed. So,

as forecasted in theory, this structure stops undoubtedly the EM wave with frequency below the specific RPW cut-off frequency.

As prospects to the present work, analytical and numerical studies of more realistic structures with hexagonal numerous apertures similar to [16,17,20] is in progress. Finally, we hope that this result is potentially necessary for the engineering designer of the electrical and electronic shielding with employing honeycomb panel [12,13].

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