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THE MULTIPLE ANTENNA INDUCED EMF METHOD FOR THE PRECISE CALCULATION OF THE COUPLING MATRIX IN A RECEIVING ANTENNA ARRAY

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Abstract—Practical antenna array designs generally require that the elements are separated by electrically short distances. The resultant mutual coupling often adversely affects the achievable performance. Various methods are available to quantify the effects of mutual coupling in arrays and improve performance through mutual coupling compensation. Mutual coupling is often described by a coupling matrix that relates the coupled and uncoupled quantities. Unfortunately, the accuracy with which the coupling matrix can be calculated is highly dependent on both the method selected and the frequency. This is a significant limitation for wideband analysis where the coupling matrix needs to be calculated accurately at all frequencies of interest. This paper introduces a novel method for the precise calculation of the coupling matrix at any frequency of interest. It is an extension of the induced EMF method to multiple array elements. The method has the important practical advantage of being independent of the numerical technique used in the analysis. Since the coupling matrix is calculated by exciting the elements in the transmission mode, the method resembles well-known network analysis. However, as outlined in the paper, there are subtle differences between the two approaches, which lead to more accurate results with the new proposed method. It is also demonstrated that antennas with arbitrary geometries and illuminations are handled accurately by the method.

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1. INTRODUCTION

The realization that mutual coupling can adversely affect the performance of practical antenna arrays has motivated extensive research into techniques for the analysis and compensation of mutual coupling. A recent review [1] has defined eight categories of such techniques. These are generally based on the use of a transfer matrix, commonly known as the coupling matrix.

There are several commonly used methods for calculating the coupling matrix. The widely known open-circuit voltage method [2] calculates the coupling matrix from the mutual impedances between the array elements. The self and mutual impedances can be obtained using the induced EMF method introduced by Carter in 1932 [3]. However, this approach depends on the assumption that the antenna excitation is present only at the discrete port locations. The full-wave method [4] avoids this limitation by using the method of moments (MoM) matrix to correctly take into account that the excitation in a receiving antenna is distributed over the entire antenna surface.

For receiving antenna arrays, most of the available mutual coupling compensation methods [2, 5–7] are only usable over a limited range of frequencies since the accuracy of the coupling matrix varies with the frequency. Currently, only the full-wave method can accurately calculate the coupling matrix at any frequency of interest and for any excitation. However, this method depends on the MoM electromagnetic (EM) numerical technique, and the MoM matrix cannot be easily obtained from most commercially available numerical EM tools.

The multiple antenna induced EMF method (MAIEM) proposed in this paper is an extension of the induced EMF method. By avoiding the need for calculating the MoM matrix, any EM numerical technique can be used for the calculation of the coupling matrix. Since the accuracy is equivalent to that of the full-wave method of [4], the MAIEM is suitable for the wideband analysis and compensation of mutual coupling in receiving antenna arrays. Since its inception, the application of the method has shown excellent agreement with both theoretical and measured performances of antenna arrays employed in various applications. It has become a very valuable tool in the accurate prediction of the performance of receiving antenna arrays of arbitrary geometries and in the complete elimination of mutual coupling effects.

The paper is organized as follows. In Section 2, the theory of the induced EMF method is reviewed. In Section 3, the MAIEM is explained, and its main differences with network analysis are highlighted to explain the performance improvement expected by its implementation. The suitability of the method for handling different elevation angles is evaluated in Section 4 and compared with that of the full-wave method. The MAIEM is validated in Section 5 using numerical techniques. In Section 6, the MAIEM is generalized to arrays of arbitrary elements and for arbitrary excitations. Finally, conclusions are presented in Section 7.

2. THE INDUCED EMF METHOD

The equivalent circuit of a receiving antenna consists of a voltage source, v_{oc}, in series with the input impedance of the antenna, Z_{ANT}, and the load impedance, Z_L, connected at the terminals of the antenna [9], as illustrated in Figure 1. The induced EMF, v_{oc}, is the equivalent open-circuit voltage at the antenna terminals. It is equal to the voltage that would appear at the terminals if the load impedance were removed. For a vertical wire element centered at the origin and oriented in the z-direction, with currents assumed to be flowing only in the z-direction, the induced EMF method allows for the calculation of the value of v_{oc} using the following equation:

$$v_{oc} = -\frac{1}{I(0)} \int_{-L/2}^{L/2} I(z) E_z(z) dz \tag{1}$$

where L is the length of the wire; $E_z(z)$ is the z-component of the incident electric field; $I(z)$ is the current distribution along the wire; $I(0)$ is the input current at the antenna terminals. It must be noted that $I(z)$ and $I(0)$ are currents computed under transmission mode excitation of the wire. A voltage source is directly applied to the terminals to first compute the current distribution needed for the use of (1). When the voltage source is of unit amplitude, the value of the

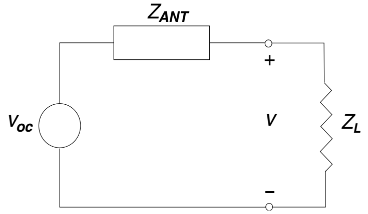


Figure 1. Equivalent circuit of a receiving antenna consisting of the equivalent open-circuit voltage, v_{oc}, the input impedance of the antenna, Z_{ANT}, the load impedance connected to the antenna, Z_L, and the terminal voltage v measured across the load impedance.

input impedance is given by:

$$Z_{ANT} = \frac{1}{I(0)} \tag{2}$$

Hence, (1) can be expressed as:

$$v_{oc} = -Z_{ANT} \int_{-L/2}^{L/2} I(z)E_z(z)dz \tag{3}$$

For uniform plane wave signals arriving in a direction orthogonal to the wire orientation, the incident electric field is independent of z, and (3) reduces to:

$$v_{oc} = -E_z Z_{ANT} \int_{-L/2}^{L/2} I(z)dz \tag{4}$$

where E_z is the constant electric field z-component. It is common to express (4) as [10]:

$$v_{oc} = hE_z \tag{5}$$

where h is generally known as the effective height of the antenna. The voltage across the load is finally given by:

$$v = \frac{Z_L}{Z_{ANT} + Z_L} v_{oc} \tag{6}$$

through a simple voltage divider applied to the circuit of Figure 1.

The current distribution, $I(z)$, is often assumed to be sinusoidal in amplitude and to have a constant phase [10]. Using this assumption, analytical equations can be derived to obtain (6). Unfortunately, the assumption is only valid for an asymptotically thin wire over a limited range of frequencies. This is due to the dependence of $I(z)$ upon the length and diameter of the wire and the frequency of operation [10]. It follows that $I(z)$ must be solved using numerical techniques for the precise calculation of (6) in practical wire antennas. Although the induced EMF method has been widely used in the calculation of self and mutual impedances [13–15], it has been very rarely used for the calculation of the induced EMF itself, v_{oc}, in the context of an external illumination by a plane wave for example. Most importantly, it has never been used in this context for multiple coupled antennas. In Section 3, this calculation is performed to ultimately yield a very accurate estimation of mutual coupling effects.

3. THE MULTIPLE ANTENNA INDUCED EMF METHOD

For the development of the MAIEM, the theory of Section 2 is extended to an antenna array. The equivalent circuit components of Figure 1 can be modified to represent the antenna array. For the array, Z_{ANT} and Z_L are square matrices, and $\mathbf{v_{oc}}$ and $\mathbf{v}$ are column vectors. The MAIEM is based upon the superposition of multiple induced EMF's. The total induced EMF of an element is equal to the sum of the induced EMF due to its own external excitation and the induced EMF due to the external excitation of the other elements. For clarity, an array of two vertical wire elements will be considered. Extending Equation (4), the open-circuit equivalent voltage of the first element can be expressed as:

$$v_{oc_1} = -E_{z_1} Z_{ANT_1} \int_{-L_1/2}^{L_1/2} I_{11}(z)dz - E_{z_2} Z_{ANT_1} \int_{-L_2/2}^{L_2/2} I_{21}(z)dz \quad (7)$$

where E_{z_1} and E_{z_2} are the incident electric fields on the first and second elements respectively, Z_{ANT_1} is the input impedance of the first element, L_1 and L_2 are the lengths of the two elements, I_{11} is the current distribution of the first element under unit-voltage excitation of itself in the presence of the other element, and I_{21} is the current distribution of the second element while the first element is excited by a unit-voltage source. Similarly, the open-circuit equivalent voltage of the second element is defined by:

$$v_{oc_2} = -E_{z_1} Z_{ANT_2} \int_{-L_1/2}^{L_1/2} I_{12}(z)dz - E_{z_2} Z_{ANT_2} \int_{-L_2/2}^{L_2/2} I_{22}(z)dz \quad (8)$$

The following matrix equation can then be formulated for the calculation of (7) and (8):

$$\mathbf{v_{oc}} = -Z_{ANT} I^T \mathbf{E_z} \quad (9)$$

where

$$\mathbf{v_{oc}} = \begin{bmatrix} v_{oc_1} \\ v_{oc_2} \end{bmatrix} \quad (10)$$

$$Z_{ANT} = \begin{bmatrix} Z_{ANT_1} & 0 \\ 0 & Z_{ANT_2} \end{bmatrix} \quad (11)$$

$$I = \begin{bmatrix} \int_{-L_1/2}^{L_1/2} I_{11}(z)dz & \int_{-L_1/2}^{L_1/2} I_{12}(z)dz \\ \int_{-L_2/2}^{L_2/2} I_{21}(z)dz & \int_{-L_2/2}^{L_2/2} I_{22}(z)dz \end{bmatrix} \quad (12)$$

$$\mathbf{E_z} = \begin{bmatrix} E_{z_1} \\ E_{z_2} \end{bmatrix} \quad (13)$$

I^T in (9) denotes the transpose of matrix I. It is important to note that the entries of Z_{ANT} in (11) are calculated in the presence of the other element. Therefore, the entries differ from the input impedances calculated when the elements are in isolation. Also, the entries of I in (12) are calculated when a single element is excited by a unit-voltage source while the other is terminated into the load impedance to be used in the receiving mode. The terminal voltage vector, $\mathbf{v}$, is given by:

$$\mathbf{v} = Z_L \left(Z_L + Z_{ANT}\right)^{-1} \mathbf{v_{oc}} \tag{14}$$

where Z_L is the load impedance matrix. Z_L is a diagonal matrix whose diagonal entries are the individual load impedances connected to the elements, and is expressed as:

$$Z_L = \begin{bmatrix} Z_{L_1} & 0 \\ 0 & Z_{L_2} \end{bmatrix} \tag{15}$$

Substituting (9) into (14) yields:

$$\mathbf{v} = -Z_L \left(Z_L + Z_{ANT}\right)^{-1} Z_{ANT} I^T \mathbf{E_z} \tag{16}$$

It is observed that (16) can be expressed as:

$$\mathbf{v} = C\mathbf{E_z} \tag{17}$$

where

$$C = -Z_L \left(Z_L + Z_{ANT}\right)^{-1} Z_{ANT} I^T \tag{18}$$

C is generally referred to as the coupling matrix. It can be verified that C can be calculated offline using current distributions computed under transmission mode excitations. Each element needs to be excited separately by a unit-voltage source in order to fill every row of the matrix C. Symmetry can be used advantageously to reduce the number of computations required to determine every row of the matrix. It is noted that the calculation of (18) does not require the knowledge of the MoM matrix as in [4], and therefore is not limited to the MoM. Once C is known, mutual coupling compensation is possible, and the incident fields can be retrieved using:

$$\mathbf{E_z} = C^{-1}\mathbf{v} \tag{19}$$

The MAIEM can be applied to more than two array elements in a straightforward manner by increasing the dimensions of the coupling matrix accordingly and carrying out the required calculations.

It is very important to note that the MAIEM departs from the idea that mutual coupling in receiving arrays is based on the concept of mutual impedances as defined in [3] and later described in [2] and [9, 10]. As a result, mutual impedances are not calculated as such in the MAIEM. There are three crucial differences in the formulation of the open-circuit voltage method introduced in [2], which is analogous to network analysis, and the MAIEM. They are listed below:

1. The mutual impedances originally defined by [3], and subsequently used in work done on the open-circuit voltage method, describe the ratio of the open-circuit voltage of an array element to the excitation current of another array element. Using (1) which gives the open-circuit voltage of element i, the self and mutual impedances are expressed as:

$$Z_{ij} = \frac{V_{oc_i}}{I_j(0)} = -\frac{1}{I_i(0)I_j(0)} \int_{-L_i/2}^{L_i/2} I_i(z) E_{zij}(z) dz \qquad (20)$$

   where $I_i(0)$ and $I_j(0)$ are the input currents of elements i and j when separately excited by a voltage source, and $E_{zij}(z)$ is the electric field at the surface of element i resulting from the excitation of element j by a voltage source at its terminals. It is important to note that the MAIEM uses the incident fields of the incoming signals in its formulation, as opposed to those originating from the other elements. Therefore, the column vector $\mathbf{E_z}$ in (16) is by no means comprised of the terms E_{zij} of (20).
2. The current distribution $I_i(z)$ in (20) along element i results from the excitation of this same element by a voltage source at its terminals. In the MAIEM, the matrix I in (16) is comprised of current distributions $I_{ij}(z)$ which are defined as the current distributions along element i due to the excitation of element j by a voltage source at its terminals.
3. The current distribution $I_i(z)$ in (20) along element i is determined when the terminals of the other elements are shorted [1]. The current distributions $I_{ij}(z)$ of the MAIEM are determined when the elements $i \neq j$ are terminated into their operating load impedances.

   These three major differences substantiate the important extension to Carter's theory introduced in this paper. As already pointed out in [1, 4] and [6–8], the concept of mutual impedances is inaccurate in estimating and compensating the effects of mutual coupling in receiving antenna arrays. The proposed MAIEM is therefore expected to yield superior performance to that of the open-circuit voltage method.

4. ELEVATION ANGLE CONSIDERATIONS

An important limitation of the full-wave method is that it accurately compensates for mutual coupling only if the elevation angle of incoming signals is known *a priori* [4]. Therefore, the performance of antenna arrays, such as smart antennas, operating in three dimensions can be problematic since the elevation angles are generally unknown. As illustrated in Figure 2, for an arbitrary value of elevation angle, a vertical phase shift has to be accounted for along each of the array elements. Therefore it is necessary to derive a matrix equation based on (3) where the incident fields are dependent on z. Defining the elevation angle, θ, as the angle between the positive z-axis and the direction of the transmitter, (12) can be modified to take into account the vertical phase shift in the MAIEM for an arbitrary value of θ to yield:

$$I = \begin{bmatrix} \int_{-L_1/2}^{L_1/2} I_{11}(z)e^{j\Phi_z}dz & \int_{-L_1/2}^{L_1/2} I_{12}(z)e^{j\Phi_z}dz \\ \int_{-L_2/2}^{L_2/2} I_{21}(z)e^{j\Phi_z}dz & \int_{-L_2/2}^{L_2/2} I_{22}(z)e^{j\Phi_z}dz \end{bmatrix} \tag{21}$$

where

$$\Phi_z = \frac{2\pi}{\lambda} z \cos\theta \tag{22}$$

and λ is the signal wavelength. Substituting (21) into (18) allows the calculation of the coupling matrix. Mutual coupling compensation is then possible using (19). It must be noted that the calculation of (21) is only possible if θ is known *a priori*. Therefore the MAIEM suffers from the same limitation as the full-wave method does in the sense that it is accurate only for a known elevation angle.

5. METHOD VALIDATION

In this section, the MAIEM is validated by predicting the terminal voltages under plane wave excitation at several frequencies for an array expected to be subject to strong mutual coupling due to the small electrical spacing of the elements. The terminal voltages are then compared with those computed by numerical techniques under plane wave excitation. For simplicity, the two-element linear array of Figure 2 is used for the validation where the elevation angle, θ, is set to 90°. The two elements are vertical dipoles of 2 m in length and 3 mm in diameter. They are centrally terminated into 50 Ω load impedances. The array is centered at the origin with the two elements located at a distance R from the array center.

The first step in the MAIEM is to excite one of the elements with a unit-voltage source at its terminals, while the other element is terminated into its load impedance, to calculate numerically the current distributions on both elements. Since EM numerical techniques require the finite discretization of the elements, (12) is approximated

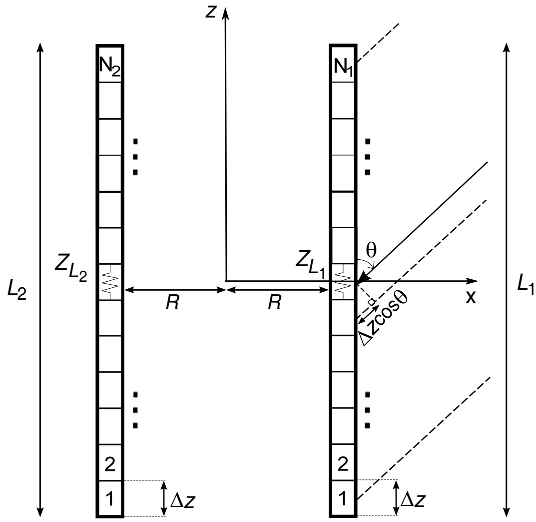


Figure 2. Array of two vertical wire elements of lengths L_1 and L_2, discretized into N_1 and N_2 segments and centrally terminated into load impedances Z_{L_1} and Z_{L_2} respectively. The two elements are separated by a distance of $2R$ and are illuminated by a signal having an elevation angle θ.

by:

$$I \approx \Delta z \begin{bmatrix} \sum_{i=1}^{N_1} I_{11}^i & \sum_{i=1}^{N_1} I_{12}^i \\ \sum_{i=1}^{N_2} I_{21}^i & \sum_{i=1}^{N_2} I_{22}^i \end{bmatrix} \tag{23}$$

where N_1 and N_2 are the numbers of discrete segments of the two elements, and Δz is the length of the segments assuming that the segments are all of the same size. It should be noted that the superscript i in (23) is only used to designate the segment number and is not used as an exponent. Exciting the first element with a unit-voltage source provides the entries of the first column of I in (23). By symmetry, since the dipoles have equal dimensions, the entries of the first column are reused to fill the second column of I. The value of current at the segment where the excitation is applied is the input current of the excited element. Therefore it is used to calculate the diagonal entries of the input impedance matrix, Z_{ANT}, as follows:

$$Z_{ANT} = \begin{bmatrix} \frac{1}{I_{11}(0)} & 0 \\ 0 & \frac{1}{I_{22}(0)} \end{bmatrix} \tag{24}$$

where $I_{11}(0)$ and $I_{22}(0)$ are the input currents of each element when they are separately excited by a unit-voltage source in the presence of the other element. Again, by symmetry, $I_{11}(0) = I_{22}(0)$, and the input current of the first element can be used as the input current of the second element. Since the two elements are terminated into $50\,\Omega$ load impedances, Z_L is simply given by:

$$Z_L = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \tag{25}$$

The end-fire excitation is selected to validate the method. Hence the incident fields are given by:

$$\mathbf{E_z} = \begin{bmatrix} e^{j\frac{2\pi}{\lambda}R} \\ e^{-j\frac{2\pi}{\lambda}R} \end{bmatrix} \tag{26}$$

To calculate the terminal voltages of the two array elements, (23)–(26) are substituted into (16) for all the frequencies of interest.

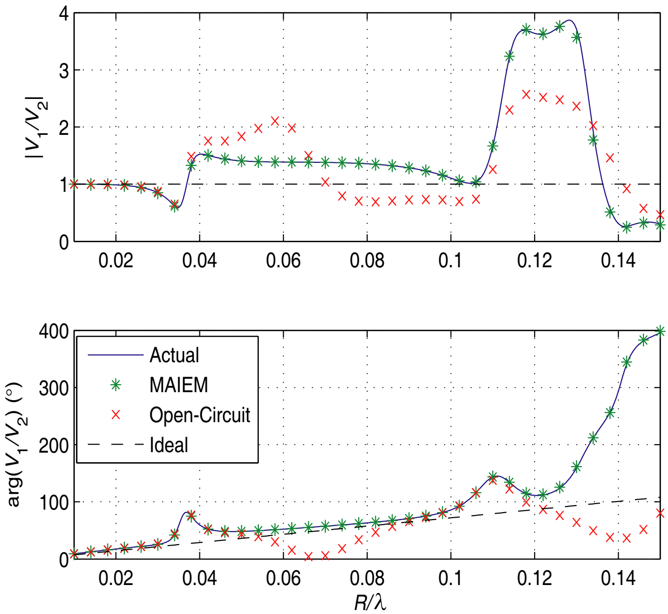


Figure 3. Wideband prediction of the amplitude ratio ($|V_1/V_2|$) and phase difference ($\arg(V_1/V_2)$) of the terminal voltages of two dipoles illuminated from the end-fire direction. Predictions are calculated using both the MAIEM and the open-circuit voltage method, the actual values are calculated using the MoM in the receiving mode, and the ideal values are those in the absence of mutual coupling.

The software package FEKO [11] is used to compute the current distributions required in (23) and (24). The predicted amplitude ratio and phase difference of the two terminal voltages calculated using the MAIEM are shown in Figure 3. To appreciate the improvement in accuracy provided by the MAIEM, the predictions using the open-circuit voltage method of [2] are also shown. Both sets of results are compared against the actual amplitude ratio and phase difference calculated numerically in the receiving mode under end-fire plane wave excitation. The ideal amplitude ratio and phase difference are shown as an indication of the assumed values when mutual coupling is not accounted for. The frequency dependent discrepancy between the actual and the ideal values is consistent with the performance degradation often observed in antenna arrays as a result of mutual coupling. It is seen that the MAIEM predicted values are in good agreement with the actual values. Only minor differences are observed mainly in the amplitude ratio at higher frequencies. This is explained by the finite nature of the discretization used in the calculation of the current distributions. As frequency increases, the discrete segments become electrically larger with the result that the approximation of continuous current distributions becomes less accurate. Consequently, the number of segments should be selected to provide the desired accuracy at the highest frequency of interest. The results confirm that the MAIEM is appropriate for calculating the coupling matrix in a very precise manner at any frequency of interest. It is verified that the accuracy of the method is superior to that of the open-circuit voltage method, especially as the electrical dimensions of the array become large. The method was also validated for more complex array configurations, involving multiple sub-arrays covering different frequency bands, primarily used for wideband direction finding in [12].

6. GENERALIZED MULTIPLE ANTENNA INDUCED EMF METHOD

In [4], the full-wave method is applied to an array of vertical wire elements for plane wave excitations of known elevation angles. In [8], the full-wave method is extended to an array of arbitrary elements under arbitrary excitation. Similarly, the MAIEM can be generalized to arrays of arbitrary elements and to arbitrary excitations. To this end, any summation found in (23) is removed and all the current distribution components are accounted for as follows for an array of two arbitrary elements:

$$I_{gen} \approx \Delta \begin{bmatrix} \mathbf{I_{11}}^T & \mathbf{I_{12}}^T \\ \mathbf{I_{21}}^T & \mathbf{I_{22}}^T \end{bmatrix} \tag{27}$$

where

$$\mathbf{I_{11}} = \left[I_{11}^{1x}\ I_{11}^{1y}\ I_{11}^{1z}\ I_{11}^{2x}\ I_{11}^{2y}\ I_{11}^{2z}\ \cdots\ I_{11}^{N_1x}\ I_{11}^{N_1y}\ I_{11}^{N_1z} \right] \quad (28)$$

$$\mathbf{I_{12}} = \left[I_{12}^{1x}\ I_{12}^{1y}\ I_{12}^{1z}\ I_{12}^{2x}\ I_{12}^{2y}\ I_{12}^{2z}\ \cdots\ I_{12}^{N_1x}\ I_{12}^{N_1y}\ I_{12}^{N_1z} \right] \quad (29)$$

$$\mathbf{I_{21}} = \left[I_{21}^{1x}\ I_{21}^{1y}\ I_{21}^{1z}\ I_{21}^{2x}\ I_{21}^{2y}\ I_{21}^{2z}\ \cdots\ I_{21}^{N_2x}\ I_{21}^{N_2y}\ I_{21}^{N_2z} \right] \quad (30)$$

$$\mathbf{I_{22}} = \left[I_{22}^{1x}\ I_{22}^{1y}\ I_{22}^{1z}\ I_{22}^{2x}\ I_{22}^{2y}\ I_{22}^{2z}\ \cdots\ I_{22}^{N_2x}\ I_{22}^{N_2y}\ I_{22}^{N_2z} \right] \quad (31)$$

In (27), Δ is the lateral dimension of the segments in the direction of the x, y and z axes, assuming it is equal for every segment in all three dimensions. In (28)–(31), the superscripts x, y and z denote the directional components of the currents. It is important to note that when the segments are considered thin enough that no current is assumed to flow in a certain direction, the entries in (28)–(31) corresponding to this direction are ignored, thereby reducing the size of the matrix I_{gen}. To account for every electric field component, the excitation vector takes the form:

$$\mathbf{E} = \begin{bmatrix} \mathbf{E_1}^T \\ \mathbf{E_2}^T \end{bmatrix} \quad (32)$$

where

$$\mathbf{E_1} = \left[E_1^{1x} E_1^{1y} E_1^{1z} E_1^{2x} E_1^{2y} E_1^{2z} \cdots\ E_1^{N_1x} E_1^{N_1y} E_1^{N_1z} \right] \quad (33)$$

$$\mathbf{E_2} = \left[E_2^{1x} E_2^{1y} E_2^{1z} E_2^{2x} E_2^{2y} E_2^{2z} \cdots\ E_2^{N_2x} E_2^{N_2y} E_2^{N_2z} \right] \quad (34)$$

Similar to I_{gen}, when no current is assumed to flow in a certain direction, the entries of (33), (34) associated with this direction can be deleted to reduce the size of the vector $\mathbf{E}$. Following (18), the generalized coupling matrix is given by:

$$C_{gen} = -Z_L \left(Z_L + Z_{ANT} \right)^{-1} Z_{ANT} I_{gen}^T \quad (35)$$

The terminal voltages can be calculated using:

$$\mathbf{v} = C_{gen}\mathbf{E} \quad (36)$$

In Section 6.1, the generalized MAIEM is validated for the two-dipole array studied in Section 5 under arbitrary excitation. In Section 6.2, the generalized MAIEM is validated for an array of arbitrary elements.

6.1. Arbitrary Excitation

Using (36), the terminal voltages can be calculated for excitations arriving from an arbitrary elevation angle. The amplitude ratio and

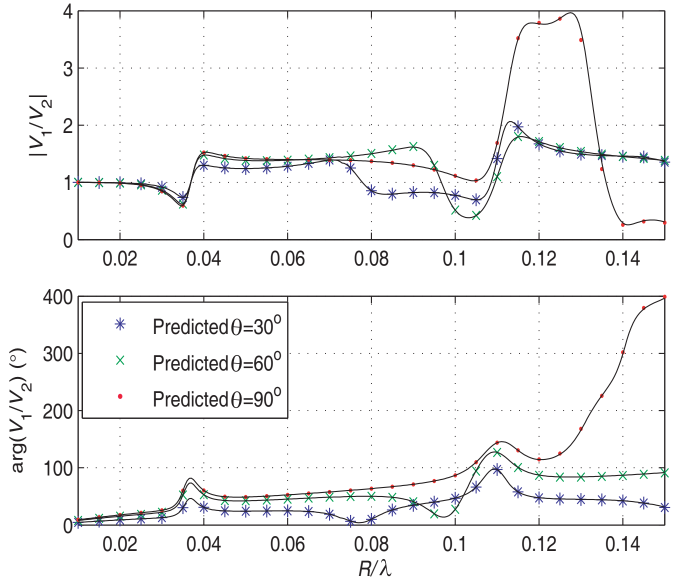
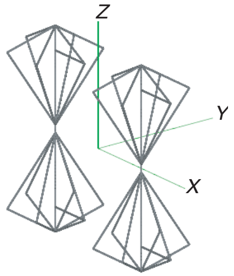


Figure 4. Wideband prediction of the amplitude ratio ($|V_1/V_2|$) and phase difference ($\arg(V_1/V_2)$) of the terminal voltages of two dipoles illuminated from three different elevation angles. The predicted values are calculated using the MAIEM and the actual values are calculated using the MoM in the receiving mode.

phase difference calculated for the two-dipole array when a plane wave signal arrives at elevation angles of 30°, 60°, and 90° are shown in Figure 4. It is verified that the use of the coupling matrix calculated using the generalized MAIEM accurately predicts the terminal voltages in a receiving array under arbitrary excitations. The coupling matrix obtained is therefore equivalent to the coupling matrix calculated based on [8].

6.2. Arbitrary Elements

To verify that the generalized MAIEM is applicable to arrays of arbitrary elements, (27)–(36) are used to predict the terminal voltages of the two three-dimensional biconical wire elements depicted in Figure 5. Each element consists of two symmetrical arms comprised of six identical planar sections whose dimensions are shown in Figure 6. The two elements are centrally terminated into $50\,\Omega$ load impedances. The amplitude ratio and phase difference of the terminal voltages



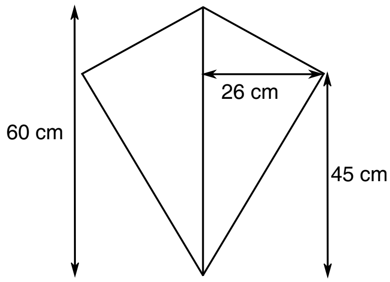


Figure 5. Array of two biconical elements having the dimensions shown in Figure 6.

Figure 6. Dimensions of a section of the biconical elements of Figure 5.

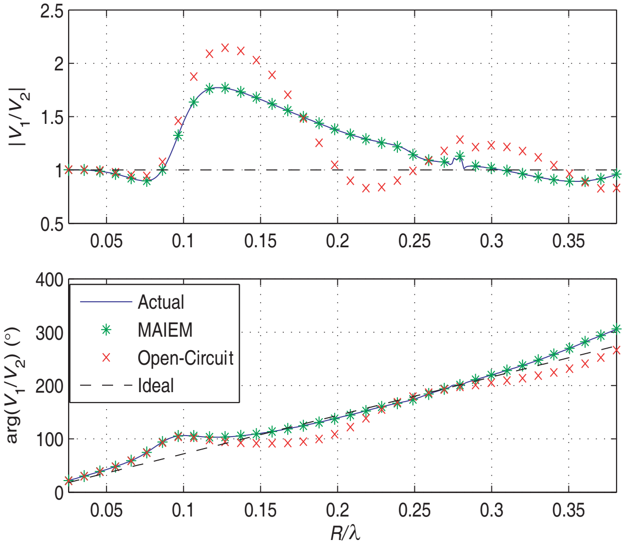


Figure 7. Wideband prediction of the amplitude ratio ($|V_1/V_2|$) and phase difference ($\arg(V_1/V_2)$) of the terminal voltages of two biconical elements illuminated from the end-fire direction. Predictions are calculated using both the MAIEM and the open-circuit voltage method, the actual values are calculated using the MoM in the receiving mode, and the ideal values are those in the absence of mutual coupling.

predicted under an end-fire plane wave excitation, having an elevation angle of 90°, are shown in Figure 7. Unlike the open-circuit voltage method, the generalized MAIEM is seen to provide accurate calculation of the terminal voltages for all the plotted frequencies. This confirms that the method can be used for arrays of arbitrary elements.

7. CONCLUSION

The multiple antenna induced EMF method (MAIEM) introduced in this paper, which presents a novel extension of the induced EMF method, can be used to precisely compute the coupling matrix of a receiving antenna array at any frequency. The main advantage of this new method is that the currents used to compute the coupling matrix can be obtained using any numerical technique. Therefore, it is the first method to offer the capability of accurately calculating the coupling matrix for receiving antenna arrays that lend themselves better to numerical analysis using techniques other than the MoM. Moreover, even if the MoM is used for the analysis, the MAIEM allows the accurate calculation of the coupling matrix without having to compute the MoM matrix. These are valuable practical features for antenna practitioners seeking the quick and accurate calculation of mutual coupling at any frequency. The MAIEM is applicable to mutual coupling compensation through the inversion of the coupling matrix. As the case for comparable mutual coupling compensation methods, the elevation angle of the incoming signal must be known. As a further development, the generalized MAIEM allows accurate mutual coupling analysis to be performed for antennas of arbitrary geometries and under arbitrary illuminations. Although it is not reported in this paper, this theory has been applied to other antenna geometries including planar surfaces. Further work is being done on its application to more complex antenna types. The results presented here should be useful in all applications involving receiving antenna arrays that are subject to mutual coupling.

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