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OPTICAL BANDPASS FILTER DESIGN USING SPLIT RING RESONATORS

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Abstract—Bandpass filter design using metallic split ring resonators (SRR) at optical frequencies is theoretically investigated in this paper. The transmission and reflection coefficients and the transmission phase of SRR array will be analyzed. We will show that there are many magnitude peaks in transmission coefficient that can be used as bandpass filter in 1550 nm-band. Increasing the number of SRR layers will cause increase in the number of magnitude peaks and narrowing of the bandwidth. The numerical analysis is performed using the transfer matrix method (TMM).

1. INTRODUCTION

Recently, optical properties of metamaterials have been considered theoretically and experimentally due to their potential applications as negative index materials. The theory of the propagation of electromagnetic waves in such media was developed more than three decades ago by Veselago [1]. The main ideas, which led to the fabrication of the first left-handed materials (LHM), were proposed by Pendry et al. [2–4]. This group claimed that a lattice of metallic split ring resonators (SRRs) with characteristic features in the millimeter range behaves as an effective medium with a negative magnetic permeability μ_{eff} [2]. After Pendry's work, Smith [5–7] made the array of SRRs and demonstrated experimentally that the SRR array works as the negative μ material in microwave region. As far as we know, this SRR array is the first negative μ material. Recently, experimental studies of magnetic properties of SRRs have been reported in the THz region [8], particularly at 30 THz (10 μm wavelength) [9], and 100 THz (3 μm wavelength) [10]. On the other hand, theoretical studies of SRR behavior have been done from THz to near-infrared region [11, 12]. Arrays of SRRs in thin metallic films show great promise in a number of different fields, due to their high transmission efficiencies at resonant wavelengths and strongly enhanced and localized optical fields. It has been shown both computationally and experimentally that such metallic structures display excellent band-pass characteristics. Also, considering potential technological applications, it would be equally important to fabricate SRRs at optical frequencies.

Recently, the interest in this field moves to the optical and visible light ranges [13, 14]. Realization of metamaterials possessing properties of an artificial magnetic material in the visible range would allow realization of artificial materials with negative index of refraction and open a way for many novel applications. One of the possible routes is the use of nano-sized resonant metal particles of complex shapes. Various shapes have been proposed which their operational principles are basically the same. In particular, several variants of split rings have been manufactured and measured in frequency range of hundreds of terahertz and in the visible range [15, 16]. When assembling the metamaterials at THz or higher frequencies by scaling down the dimensions of the resonators, the major inconvenience that we face is the difficulty of the small-scale manufacturing process. However, it has been noticed that the geometrical scaling of the particle resonant frequency breaks down when the working frequency gets higher, so that the resonant frequency saturates at the level of several hundreds of terahertz [17, 18]. This effect is explained by the plasmonic behavior of metals at visible frequencies, usually in terms of kinetic energy of electrons carrying current and an additional electron self-inductance [17]. Thus, numerical studies such as the present work can be a guide for future experimental efforts aimed for fabrication of SRRs that operate at optical frequencies. There are several reasons why this extension cannot be done simply by scaling down the characteristic dimensions of the building blocks of SRR. First, as the frequency approaches the optical spectrum, the frequency dispersion of the dielectric function of the metal becomes important. Secondly, as the frequency becomes comparable to the metallic plasma frequency, surface plasmons are excited, leading to significant changes of the electromagnetic properties of the metallic structure [19–21]. However the magnetic properties of a lattice of SRRs have previously been investigated and shown that reducing the dimensions and using silver for SRR are necessary to realize the negative μ and correct operation of SRR in the optical and visible light ranges [14].

2. ANALYSIS

In the present work, we focus on transmission and reflection coefficients of a double SRR in optical frequency range, that is, wavelengths between 1530 nm and 1570 nm ($f = 191$–196 THz). We will show that a lattice of double SRRs can be used as an optical filter in this frequency range. The geometrical feature and equivalent circuit of the symmetric transmission line (TL) of double SRR unit-cell are presented in Fig. 1. Here r is the inner radius of the ring, w is the width of the rings, d is the distance between two rings, a is the unit-cell dimension in the xy-plane, and l is the distance between adjacent planes of the SRRs along the z-axis. Here we used related formula for SRR [22] and pendry's recipe [23] to derive geometrical capacitance and inductance. Also in contrast to the microwave regime, in the optical range, metals do not behave as ideal conductors, and addition to parallel-plate capacitance and conducting inductance, additional capacitances and inductances of the rings should be considered (C_{add}, L_{add}) [24].

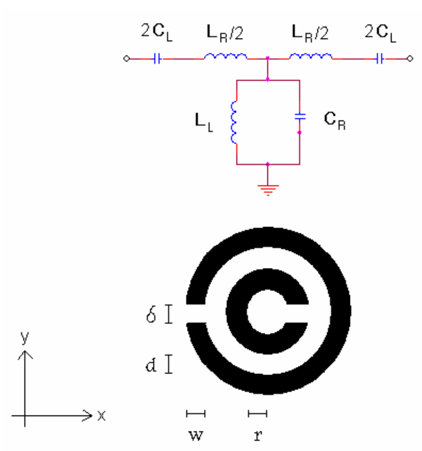


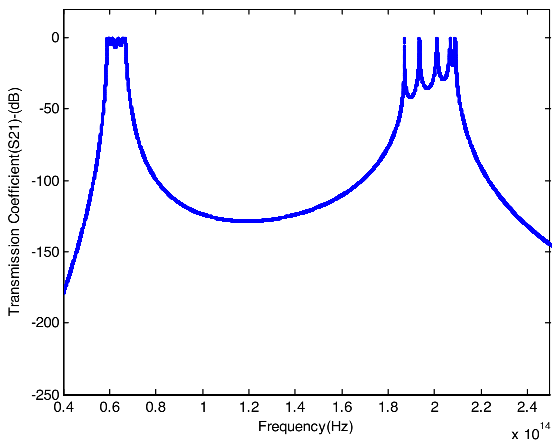
Figure 1. The geometrical feature and equivalent circuit model of the symmetric transmission line of double split ring resonator unit-cell.

These elements are due to magnetic and electric fields inside the metal rings at Terahertz frequencies. It is interesting to note that metal losses have negligible effect in this case so we use lossless model in our analysis.

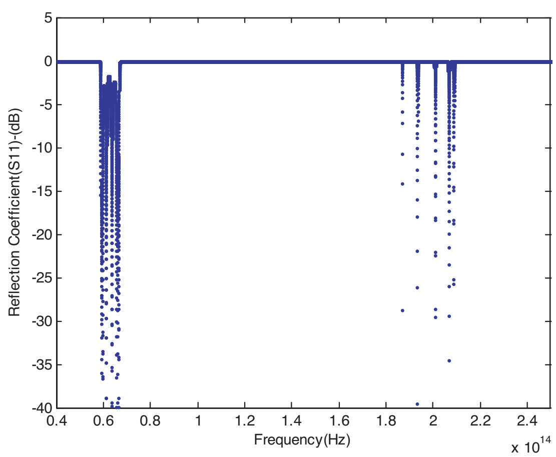
In analyzing the transmission properties of a SRR slab, we used transfer matrix method (TMM) [25]. A main advantage of using TMM approach is that it allows one to decompose large structures, for which the transfer matrix might be difficult to calculate, into smaller ones which can be calculated easier. Subsequently, from these matrices which characterize the subdivision of the large structure, one can calculate the transfer matrix for the entire structure. The reason why this approach works is that the transfer matrix obeys the multiplication rule ($T_{12} = T_2 \cdot T_1$), by which the transfer matrices of two adjacent subdivisions (T_1, T_2) multiply together and result the transfer matrix of the combined structure (T_{12}). However, repeated use of this procedure can lead to numerical instabilities. As one adds new layers, the numerical instabilities in the transfer matrix build up, leading to massive computational problems. To overcome this problem, and account for the combined effect of two layers of material, we used a similar procedure except that we constructed the scattering matrix of an ensemble of two layers from scattering matrices of each layer. The advantage of this approach is that the elements of the scattering matrix are of the order of unity; consequently, when applied to scattering matrices, this recursive process is numerically stable [26–28]. The drawback is that computing of the scattering matrix for two layers by using the scattering matrix of each layer require much computation. Therefore, in practice one can calculate first the transfer matrices for subsystems that are as large as possible; then determine the scattering matrices for these subsystems; and finally, from these scattering matrices, calculate the scattering matrix for the entire system.

In this case, the magnetic field of incident wave, is perpendicular to the SRRs plane and the propagation direction is along the plane and N layers of SRRs. By adjusting the geometrical parameters, including linewidth, radius, and the gap distance of the SRR structures, the resonance can be tuned to a specific frequency of interest. Also the SRR dimensions are, $a = 200\,\mathrm{nm}$, $r = 41.6\,\mathrm{nm}$, $w = \delta = 20.5\,\mathrm{nm}$, $d = 10\,\mathrm{nm}$ and $l = 40\,\mathrm{nm}$. There are some restrictions that should be considered. One is that, the unit-cell dimension should be smaller than incident wavelength ($a \ll \lambda$). If this condition were not complied, there would be the possibility that internal structure of the medium could diffract as well as refract radiation giving the game away immediately. Another restriction that is important, occurs during the decreasing SRR dimensions for reaching the higher resonant frequency. This resonance must be less than the plasma frequency which, is the resonant frequency of electrons oscillation inside metal ($\omega < \omega_p$). So we use silver ($\omega_p = 14 \times 10^{15}\,\mathrm{s}^{-1}$) for SRRs as a solution for this limitation.

The transmission and reflection coefficient and the transmission phase for $N = 5$ layers of SRRs are presented in Fig. 2. As this figure shows, there is a gap in the transmission coefficient, which, being at 1.1×10^{14} Hz. Also there are many magnitude peaks in the measured transmission coefficient, which are essentially due to the impedance mismatching at the terminal ports. This mismatching is resulted from the wide variations in the frequency range of the incident wave and unbalanced condition of equivalent circuit of the SRR unit-cell. This condition occurs when the resonant frequencies of the shunt and serial capacitance and inductance in the equivalent circuit of transmission line are not equal, that is $\omega_{se} \neq \omega_{sh}$ or $L_L \cdot C_R \neq L_R \cdot C_L$. It



(a)



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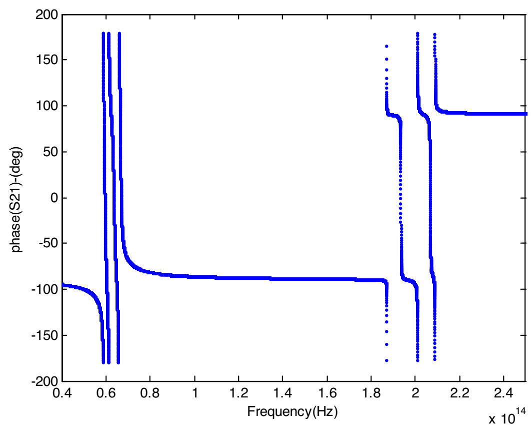


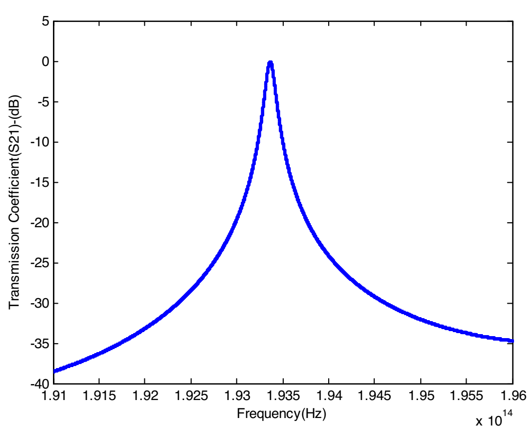
(c)

Figure 2. Transmission coefficient (a), reflection coefficient (b), and phase of S21 (c) for $N = 5$ layers of SRR.

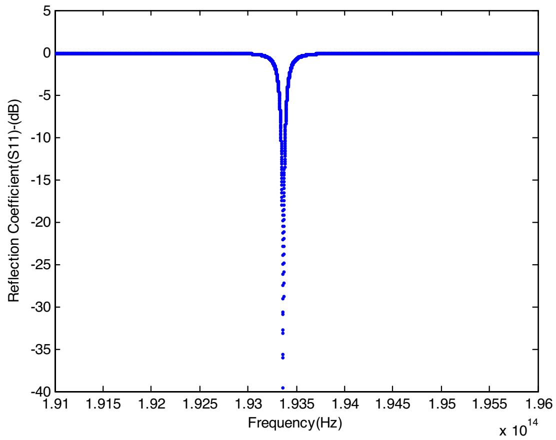
is noteworthy that the magnitude peaks exactly correspond to the resonance modes of the N-cell Resonators, which exist when the phase is $\phi = 0, \pi, -\pi, -2\pi, \ldots$.

We show that one can use each of these magnitude peaks as an optical bandpass filter. The magnitude of the transmission and reflection coefficients and the transmission phase for one of these peaks for $N = 5$ layers of SRR are presented in Fig. 3.

As we see, this peak is located at 1550 nm-band, and the -3 dB



(a)



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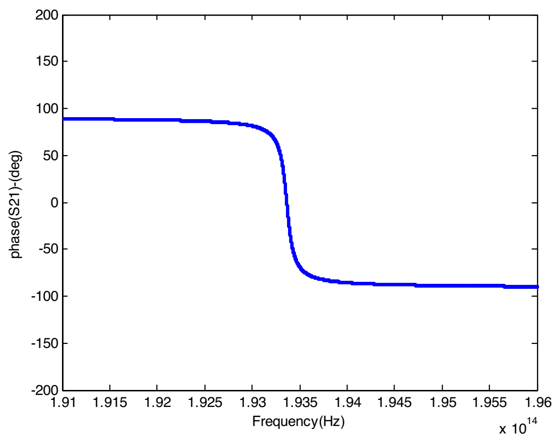


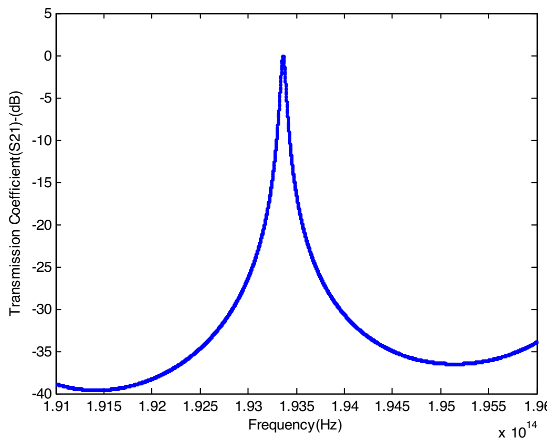
(c)

Figure 3. Transmission coefficient (a), reflection coefficient (b), and phase of S21 (c) for $N = 5$ layers of SRR, for one of the magnitude peaks at 1550 nm-band.

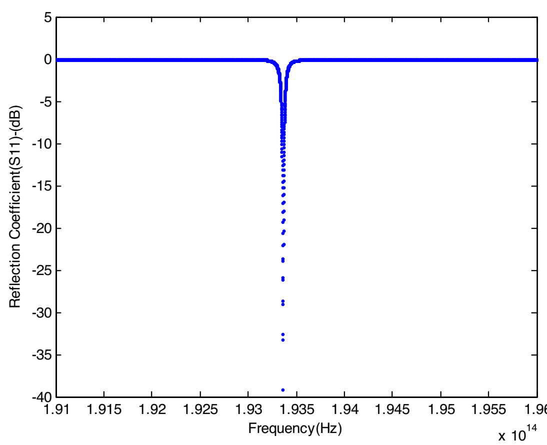
bandwidth of this bandpass filter is 100 GHz. If we want to have a filter with narrower bandwidth less than 100 GHz, we should use many more SRR layers. The related figures for a magnitude peak for $N = 10$ layers of SRR are presented in Fig. 4.

It is clear that when we increase the number of SRR layers, the number of magnitude peaks increases and the bandwidth of each peak decreases and becomes 50 GHz. Therefore by increasing the number of peaks, one can reduce the bandwidth of the filter.

However, there is a limitation to the number of peaks and it may that some adjacent peaks add to the 1550 nm-band. The other important parameter of this filter is central resonance frequency (f_0) which depends on the dimensions of SRRs and by changing the dimensions, f_0 is tuned to the selected frequency. As we see, this central resonance frequency is located at 193.3 THz.



(a)



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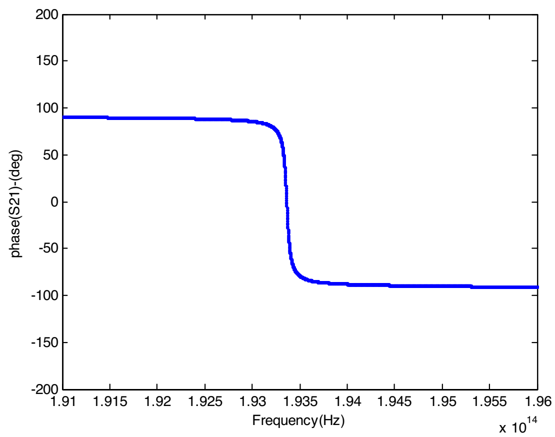

(c)

Figure 4. Transmission coefficient (a), reflection coefficient (b), and phase of S21 (c) for $N = 10$ layers of SRR, for one of the magnitude peaks at 1550 nm-band.

3. CONCLUSION

We demonstrated that the array of silver SRRs can be used as a bandpass filter at optical frequencies in the 1550 nm-band. The transmission and reflection coefficients and the transmission phase of SRR array were analyzed and it is shown that there are many magnitude peaks in transmission coefficient of these SRR arrays which enable us to use them as bandpass filters with acceptable bandwidths.

Increasing the number of resonant peaks by adding the number of SRR layers can be a useful way to narrow the filter bandwidth. Also this filter is tunable to the selected frequency by changing the dimensions of the SRR. Although, the dimension required for producing this effect can be challenging for most nanofabrication tools, they are still within the capability of state-of-the-art technology.

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