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Matrix Splitting Technique for Solving Electromagnetic Scattering Problems over a Wide Angle by Compressive Sensing

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Abstract—By combining the method of moments and the compressive sensing theory, a rapid scheme for analyzing the electromagnetic scattering problems over a wide incident angle has been developed, by which the calculation times of traditional method of moments can be decreased efficiently. To further reduce the calculation times, the matrix splitting technique is proposed to establish a new scheme in this paper. The basic principle is elaborated in detail, and the effectiveness of the new scheme is verified by numerical results.

1. INTRODUCTION

Method of moments (MoM) [1] is often used to analyze the electromagnetic scattering characteristics of objects having the advantage of high accuracy and naturally satisfying the radiation boundary condition. However, the calculation needs to be implemented repeatedly at every angle increment in analyzing the electromagnetic problems under a wide angle excitation, which leads to a huge amount of computation. Traditionally, MoM is combined with the approximate method (e.g., asymptotic waveform evaluation (AWE) [2]) to improve the computational efficiency in such problems.

Recently, a rapid scheme [3] is proposed by introducing the compressive sensing (CS) theory [4, 5] into MoM. In the scheme, a new kind of incident sources that includes much information from different incident angles is constructed, and the measurements of induced currents are obtained by solving the matrix equation of MoM excited by the new sources, then the original induced currents over the wide angle can be reconstructed with the help of the suitable sparse transform and recovery algorithm.

To further enhance the performance of the scheme, much effort has been devoted to the researches on basis function [6], hybrid compressive approach [7], target characterization [8], etc. In previous work, we found that the number of times to solve the matrix equation is limited by the rank of the current vector group. In this paper, a matrix splitting technique is proposed to break the limit from the rank and consequently to accelerate the measurement process.

2. FORMULATIONS AND EQUATIONS

The matrix equation of traditional MoM for electromagnetic problems over a wide angle can be written as

$$\mathbf{Z}\left[\mathbf{I}_1\ \mathbf{I}_2\ \cdots\ \mathbf{I}_n\right] = \left[\mathbf{V}_1\ \mathbf{V}_2\ \cdots\ \mathbf{V}_n\right] \tag{1}$$

in which $\mathbf{Z}$ is the impedance matrix, and $[\mathbf{I}_1\ \mathbf{I}_2\ \cdots\ \mathbf{I}_n]$ and $[\mathbf{V}_1\ \mathbf{V}_2\ \cdots\ \mathbf{V}_n]$ are the matrices constituted by n induced current vectors and the corresponding n excitation vectors over n different incident angles, respectively.

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First, in the conventional rapid scheme, M new incident sources based on CS theory are formed as

$$\mathbf{V}_i^{\mathrm{CS}} = c_{i1}\mathbf{V}_1 + c_{i2}\mathbf{V}_2 + \cdots + c_{in}\mathbf{V}_n \;\; (i = 1, 2, \cdots, M) \tag{2}$$

where c_{ij} is the random coefficient. Meanwhile, M corresponding current vectors based on CS theory can be solved from

$$\mathbf{Z}\left[\mathbf{I}_1^{\mathrm{CS}} \; \mathbf{I}_2^{\mathrm{CS}} \; \cdots \; \mathbf{I}_M^{\mathrm{CS}}\right] = \left[\mathbf{V}_1^{\mathrm{CS}} \; \mathbf{V}_2^{\mathrm{CS}} \; \cdots \; \mathbf{V}_M^{\mathrm{CS}}\right]. \tag{3}$$

Then, considering the linear identity of the problem, one can obtain

$$\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{Mn} \end{bmatrix} \left[\mathbf{I}_1 \; \mathbf{I}_2 \; \cdots \; \mathbf{I}_n\right]^T = \mathbf{\Phi}\mathbf{\Psi}\left[\boldsymbol{\alpha}_1 \; \boldsymbol{\alpha}_2 \; \cdots \; \boldsymbol{\alpha}_N\right] = \left[\mathbf{I}_1^{\mathrm{CS}} \; \mathbf{I}_2^{\mathrm{CS}} \; \cdots \; \mathbf{I}_M^{\mathrm{CS}}\right]^T \tag{4}$$

in which the matrix of random coefficients is denoted as $\mathbf{\Phi}$**;** $\mathbf{\Psi}$ represents the sparse transform matrix (e.g., fast Fourier transform (FFT), discrete wavelet transform (DWT), etc.); $[\boldsymbol{\alpha}_1 \; \boldsymbol{\alpha}_2 \; \cdots \; \boldsymbol{\alpha}_N]$ is the sparse matrix composed of the projection of each column of $[\mathbf{I}_1 \; \mathbf{I}_2 \; \cdots \; \mathbf{I}_n]^T$ in $\mathbf{\Psi}$; and N is the number of basis functions.

Finally, in the view of CS theory, $\mathbf{\Phi}$ is the measurement matrix, and $[\mathbf{I}_1^{\mathrm{CS}} \; \mathbf{I}_2^{\mathrm{CS}} \; \cdots \; \mathbf{I}_M^{\mathrm{CS}}]^T$ can be regarded as the measurement results of the real induced current $[\mathbf{I}_1 \; \mathbf{I}_2 \; \cdots \; \mathbf{I}_n]^T$. By means of the recovery algorithm (e.g., orthogonal matching pursuit (OMP) [9]), the induced currents can be approximated as follows:

$$[\hat{\boldsymbol{\alpha}}_1 \; \hat{\boldsymbol{\alpha}}_2 \; \cdots \; \hat{\boldsymbol{\alpha}}_N] = \arg\min \left\|[\hat{\boldsymbol{\alpha}}_1 \; \hat{\boldsymbol{\alpha}}_2 \; \cdots \; \hat{\boldsymbol{\alpha}}_N]\right\|_L \;\; \text{s.t.} \;\; (\mathbf{\Phi}\mathbf{\Psi})\left[\boldsymbol{\alpha}_1 \; \boldsymbol{\alpha}_2 \; \cdots \; \boldsymbol{\alpha}_N\right] = \left[\mathbf{I}_1^{\mathrm{CS}} \; \mathbf{I}_2^{\mathrm{CS}} \; \cdots \; \mathbf{I}_M^{\mathrm{CS}}\right]^T \tag{5}$$

$$\left[\hat{\mathbf{I}}_1 \; \hat{\mathbf{I}}_2 \; \cdots \; \hat{\mathbf{I}}_n\right]^T = \mathbf{\Psi}\left[\hat{\boldsymbol{\alpha}}_1 \; \hat{\boldsymbol{\alpha}}_2 \; \cdots \hat{\boldsymbol{\alpha}}_N\right]. \tag{6}$$

Obviously, the M calculations of MoM to get $[\mathbf{I}_1^{\mathrm{CS}} \; \mathbf{I}_2^{\mathrm{CS}} \; \cdots \; \mathbf{I}_M^{\mathrm{CS}}]^T$ are dominating in the computational complexity of the conventional scheme. However, the number of measurements, i.e., the number of MoM computations, is limited by the rank of $[\mathbf{I}_1 \; \mathbf{I}_2 \; \cdots \; \mathbf{I}_n]^T$, since the rank means the number of vectors in maximum linearly independent group of matrix. In order to acquire a lower number of measurements, the rank needs to be reduced. For this purpose, a new scheme with the matrix splitting technique is proposed as follows:

Step 1: Eq. (1) is rewritten as

$$\mathbf{Z}\mathbf{I}_{\mathrm{total}} = \mathbf{V}_{\mathrm{total}} \tag{7}$$

in which $\mathbf{I}_{\mathrm{total}}$ and $\mathbf{V}_{\mathrm{total}}$ represent $[\mathbf{I}_1 \; \mathbf{I}_2 \; \cdots \; \mathbf{I}_n]$ and $[\mathbf{V}_1 \; \mathbf{V}_2 \; \cdots \; \mathbf{V}_n]$, respectively. Then, $\mathbf{Z}$ is split into two matrices: one is a block diagonal matrix which is constructed by the square submatrices on the diagonal of $\mathbf{Z}$; the other is composed of the rest submatrices of $\mathbf{Z}$. Taking two submatrices on the diagonal for an example, Z can be represented as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 & 0 \\ 0 & \mathbf{Z}_4 \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{Z}_2 \\ \mathbf{Z}_3 & 0 \end{bmatrix} \tag{8}$$

where $\mathbf{Z}_1$ to $\mathbf{Z}_4$ are the submatrices with the same dimension in $\mathbf{Z}$. For simplicity, the matrix including $\mathbf{Z}_1$ and $\mathbf{Z}_4$ in Eq. (8) is denoted as $\mathbf{Z}_{14}$, and the other is denoted as $\mathbf{Z}_{23}$.

Step 2: Solve the following equation

$$\mathbf{Z}_{14}\mathbf{I}_{\mathrm{block}} = \mathbf{V}_{\mathrm{total}} \tag{9}$$

where $\mathbf{I}_{\mathrm{block}}$ is an unknown matrix.

Step 3: A new matrix equation is established as

$$\mathbf{Z}\left(\mathbf{I}_{\mathrm{block}} - \mathbf{I}_{\mathrm{total}}\right) = \left(\mathbf{V}_{\mathrm{block}} - \mathbf{V}_{\mathrm{total}}\right) \tag{10}$$

in which $\mathbf{V}_{\mathrm{block}} = \mathbf{Z}\mathbf{I}_{\mathrm{block}}$. Then, the current vectors group to be solved is converted from $\mathbf{I}_{\mathrm{total}}$ to $(\mathbf{I}_{\mathrm{block}} - \mathbf{I}_{\mathrm{total}})$. Combining Eq. (7) and Eq. (9), one will get

$$\mathbf{Z}_{14}\left(\mathbf{I}_{\mathrm{block}} - \mathbf{I}_{\mathrm{total}}\right) = \mathbf{Z}_{23}\mathbf{I}_{\mathrm{total}}. \tag{11}$$

Since $\mathbf{Z}_{14}$ is full rank, and $\mathbf{Z}_{23}$ is low rank, in general, the relationship between the ranks of these matrices in Eq. (11) can be described as

$$rank\left(\mathbf{I}_{\text{block}} - \mathbf{I}_{\text{total}}\right) = rank\left(\mathbf{Z}_{23}\mathbf{I}_{\text{total}}\right) < rank\left(\mathbf{I}_{\text{total}}\right). \tag{12}$$

Therefore, the rank of the unknown current vector group is reduced.

Step 4: Denoting $(\mathbf{I}_{\text{block}} - \mathbf{I}_{\text{total}})$ and $(\mathbf{V}_{\text{block}} - \mathbf{V}_{\text{total}})$ as $\mathbf{I}'$ and $\mathbf{V}'$, respectively, the conventional rapid scheme is applied to obtain $\mathbf{I}'$. M' excitation vectors based on CS theory are formed as

$$\mathbf{V'}_i^{\text{CS}} = c_{i1}\mathbf{V'}_1 + c_{i2}\mathbf{V'}_2 + \cdots + c_{in}\mathbf{V'}_n \;\; (i = 1, 2, \cdots, M') \tag{13}$$

in which $\mathbf{V'}_1$ to $\mathbf{V'}_n$ are the n columns of $\mathbf{V}'$. M' current vectors based on CS theory can be calculated by

$$\mathbf{Z}\left[\mathbf{I'}_1^{\text{CS}}\ \mathbf{I'}_2^{\text{CS}}\ \cdots\ \mathbf{I'}_{M'}^{\text{CS}}\right] = \left[\mathbf{V'}_1^{\text{CS}}\ \mathbf{V'}_2^{\text{CS}}\ \cdots\ \mathbf{V'}_{M'}^{\text{CS}}\right]. \tag{14}$$

where $\mathbf{I'}_1^{\text{CS}}$ to $\mathbf{I'}_{M'}^{\text{CS}}$ are actually M' measurements of each column in $\mathbf{I}'$. The measurement times could be reduced ($M' < M$), since the rank of $\mathbf{I}'$ is smaller than that of $\mathbf{I}_{\text{total}}$, as shown in Eq. (12). With the utilization of the solution similar to Eq. (5) and Eq. (6), the reconstruction of $\mathbf{I}'$ can be obtained, and the real induced currents can be approximated by

$$\hat{\mathbf{I}}_{\text{total}} = \mathbf{I}_{\text{block}} - \hat{\mathbf{I}}'. \tag{15}$$

The computational complexity of the conventional scheme mainly consists of two parts: one to obtain M measurements of the induced currents by using iteration method is $O(pMN^2)$, where p is the iteration counter, as shown in Eq. (3); the other to approximate the origin induced currents with OMP method is $O(nKMN)$, as shown in Eq. (5), where K is the sparsity of $\mathbf{I}_{\text{total}}$ after sparse transformation. Compared with the conventional scheme, the computational complexity of measurement and approximation in the proposed scheme has been decreased, since the measurement times is reduced to M'. Although an extra part of computational complexity for solving Eq. (9) is added, the proposed scheme is still more efficient because $\mathbf{Z}_{14}$ is a block diagonal matrix. Each submatrix in $\mathbf{Z}_{14}$ can be operated independently; furthermore, the number and dimension of the submatrices on the diagonal can be adjusted according to the object features.

3. NUMERICAL RESULTS

To verify the effectiveness of the proposed scheme, three numerical examples are presented. For the convenience of comparison, the recovery error is defined as

$$\Delta = \frac{\left\|\hat{\mathbf{I}}_{\text{total}} - \mathbf{I}_{\text{total}}\right\|_2}{\left\|\mathbf{I}_{\text{total}}\right\|_2} \tag{16}$$

3.1. Numerical Example 1

First, a perfectly electrical conducting (PEC) sphere with a radius of 0.1 m is illuminated by the plane waves with the frequency of 3 GHz, and electric field integral equation (EFIE) is solved by MoM, in which the number of basis functions is 480. The incident waves are set in *xoy* plane, and the wide angle is divided into 1°, 2°, ..., 360°. Both the conventional scheme and the proposed scheme are applied to calculate the original induced currents, and Gaussian matrix, FFT basis, and OMP are taken as the measurement matrix, the sparse transform, and the recovery algorithm, respectively. In the proposed scheme, the impedance matrix is split into two matrices, as shown in Eq. (8). The effective rank of the unknown current vector group decreases from 28 to 24, and the relationships between the number of measurements and the recovery error is shown in Figure 1. It is clear that a lower number of measurements is required in the proposed scheme to achieve a similar precision to the conventional one. Setting the threshold of recovery error to 0.1%, the comparison of computing time is shown in Table 1.

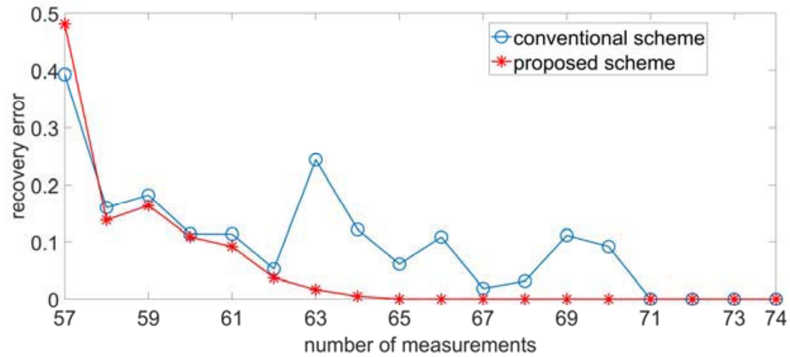

Figure 1. Variation of recovery errors with the number of measurements for the sphere.

Table 1. Comparison of computing time.

		Conventional	Proposed
Sphere (with FFT basis)		52.7 s	44.3 s
Cube (with excitation matrix)		27.1 s	19.9 s
Circle	FFT basis	3902.1 s	3000.6 s
	Excitation matrix	1951.6 s	1079.3 s

3.2. Numerical Example 2

Then, a PEC cube with a side length of 0.1 m under the same conditions is considered, and 450 basis functions are established on its surface for solving magnetic field integral equation (MFIE) by MoM. In this case, the excitation matrix is taken as the sparse transform by which the measurement times can be reduced to near the rank of the unknown current vector group [10]. Figure 2 depicts the change in the recovery error with measurement times, and it can be seen that the satisfactory accuracy is obtained by only 25 calculations of MoM in the proposed scheme while the conventional scheme needs 30 calculations (the effective rank decreases from 29 to 24). Comparing Figure 1 and Figure 2, we can see clearly that the relative degree of reduction of measurement times in the second example is higher when a suitable sparse transform is used. Furthermore, the measurement times are free from the limit of the rank of induced current vector group by the matrix splitting technique.

The bistatic radar cross sections (RCS) of the object illuminated by random incident angles (e.g., $0°$ and $45°$) are presented in Figure 3, which also agree very well with the results of traditional MoM. The computing time is also shown in Table 1 with the threshold of recovery error at 0.1%.

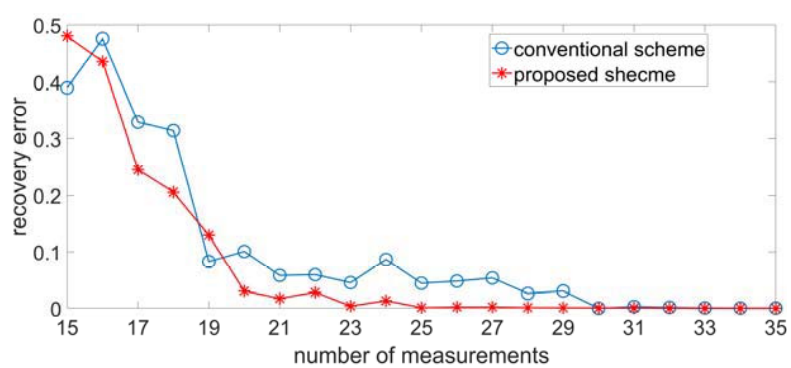

Figure 2. Variation of recovery errors with the number of measurements for the cube.

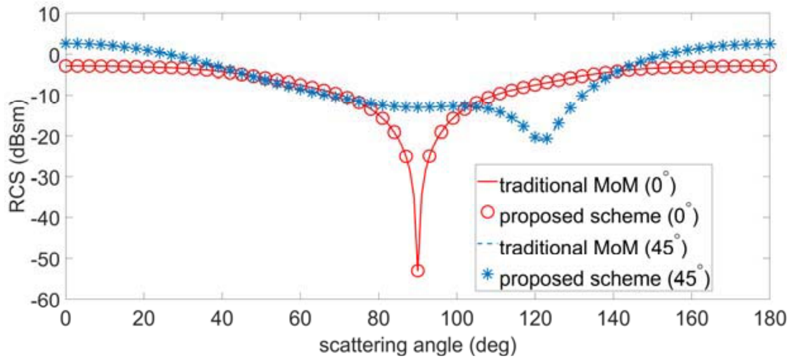

Figure 3. Comparisons of RCS for the cube under different incident angles.

3.3. Numerical Example 3

As the third example, an infinite PEC circle cylinder with a radius of 100 wavelengths (λ) is analyzed, which is illuminated by TM waves with the incident angles at 0.1°, 0.2°, ..., 360°. The perimeter of the cylinder is divided into 6280 equally spaced segments for solving combined field integral equation (CFIE) by MoM. To decrease the computational complexity of solving Eq. (9), the number of submatrices on the diagonal of the block diagonal matrix is set to 4. Selecting the FFT basis and excitation matrix as the sparse transforms, respectively, the comparison of the computing time is provided in Table 1 (the effective rank decreases from 1360 to 835).

To further prove the advantage of the proposed scheme, taking the excitation matrix as the sparse transform and assuming that the object changes from electrically small to electrically large, the relationship between the ratio of computation of the proposed scheme to the conventional scheme and the electric size of the circle cylinder is provided in Figure 4. It is clear that the computation of the proposed scheme is reduced with different electric sizes, and the relative reduction becomes more obvious with the increase of electric size of the object.

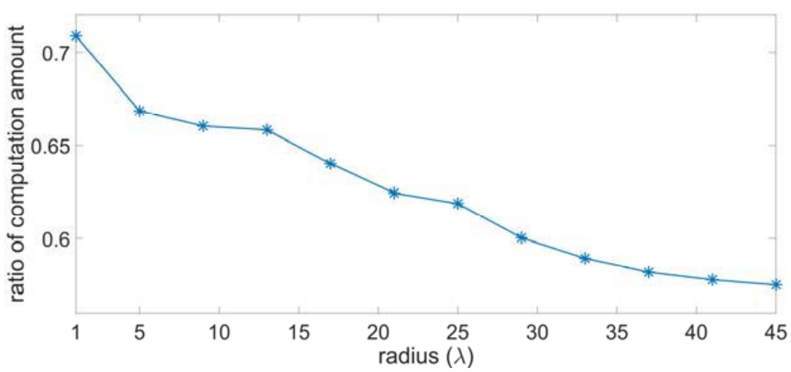

Figure 4. Variation of ratio of computation amount with the electric size of the object.

4. CONCLUSION

In order to improve the efficiency of the conventional rapid scheme for the fast analysis of the wide angle scattering problems, the matrix splitting technique is put forward. With the help of the technique, a novel scheme is established in which the limit from the rank of the induced current vector group is broken, and a lower number of measurements is required to achieve a satisfactory accuracy.

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