Joint Beamforming and Power Splitting Design for Two Way AF Relay Networks with Energy Harvesting

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Abstract—In this paper, we focus on the beamforming design in a two way amplify-and-forward relay network with energy harvesting, in which a three-node system consisting of two transmitters and one relay is considered. Specifically, we investigate the joint beamforming and power splitting scheme to obtain the maximum weighted sum rate. The formulated problem is non-convex and challenging. We equivalently transform it into a more tractable problem via successive convex approximation and constrained concave-convex procedure. Then, an iterative algorithm is proposed. Numerical results demonstrate the superiority of the proposed method, as well as the effect of dynamic power splitting in improving the sum rate of relay network.

1. INTRODUCTION

Energy harvesting (EH) is considered as one of the effective approaches for improving the energy efficiency (EE) and prolonging the lifetime of wireless networks, since EH-enabled nodes can harvest energy from either radio-frequency (RF) signals or ambient energy sources such as solar panel and wind turbine, which enables them to operate continuously [1].

On the other hand, the recently development in metamaterial has promoted the implement of EH [2–5]. Specifically, in [2], the authors investigated an extremely-broad band metamaterial absorber for solar energy harvesting based on a star-shaped resonator. In [3], the authors investigated the implementation of a perfect metamaterial absorber into multi-functional sensor applications. Recently, in [4], the authors investigated mass-energy equivalence which appears in left-handed metamaterials, while in [5], the authors investigated the enhancement of image quality by using metamaterial inspired energy harvester.

Currently, there are two promising EH methods in wireless networks, termed as wireless-power communication network (WPCN) and simultaneous wireless information and power transfer (SWIPT), respectively [6]. The main difference between WPCN and SWIPT is that wireless power and information are transmitted in individual and common slot, while power splitting (PS) is a widely used method in SWIPT to coordinate information decoding (ID) and EH simultaneously [7].

Since relay is commonly an energy-constrained node in cooperative networks, it is critical to require ambient energy supply for supporting cooperative communication. An important application scenario for SWIPT lies in the energy-constrained relay networks, in which the relay node harvests energy from the RF signals transmitted by the source and then uses the harvested energy to forward the received signal [8–11]. Specifically, in [8], the authors proposed an EH relay scheme based on time switching (TS) relaying protocol. In [9], the authors proposed a secure beamforming in amplify-and-forward (AF) EH relay networks, with PS and TS schemes, respectively. In [10], the authors proposed an energy-efficient and secure beamforming for self-sustainable EH relay networks. Recently, in [11], the authors proposed a precoding design of multiple-input-multiple-output (MIMO) AF relay system with SWIPT.
Furthermore, for two way relay, in [12], the authors proposed a PS and relaying optimization method for two way relay SWIPT systems. In [13], the authors proposed a beamforming for information and energy cooperation in cognitive two way AF relay networks. In [14], the authors proposed an energy-efficient power allocation for two way EH relay systems, which was extended in [15] and [16] with considering beamforming in the source and relay node. Recently, in [17], the authors proposed an EE design for SWIPT in MIMO two way AF relay networks. In [18], the authors proposed a joint source and relay design for MIMO two way AF relay networks with SWIPT.

However, most of these works focus on the total power minimization design. On the other hand, the weighted sum rate design for two way EH relay has not been investigated. Motivated by these observations, in this paper, we focus on the optimization design in a two way EH AF relay network, in which a three node system consisting of two transmitters and one relay is considered. Specifically, we investigate the joint beamforming and PS design to maximize the weighted sum rate. The formulated problem is non-convex and challenging. To handle this problem, we equivalently transform it into a more tractable reformulation via successive convex approximation (SCA) and constrain concave-convex procedure (CCCP), then propose an iterative method to solve the approximated problem. Numerical results show the effectiveness of the proposed scheme.

The rest of this paper is organized as follows. A system model and problem statement is given in Section 2. Section 3 investigates the joint design problem, wherein an SCA-CCCP based iterative approach is proposed. Simulation results are illustrated in Section 4. Section 5 concludes this paper.

Notations: Throughout the paper, we use upper case boldface letters for matrices and lower case boldface letters for vectors. Superscripts \((\cdot)^T, (\cdot)^\dagger, (\cdot)^H\) represent the transpose, conjugate, and conjugate transpose, respectively. The trace of matrix \(A\) is denoted as \(\text{Tr}(A)\). \(\mathbf{a} = \text{vec}(\mathbf{A})\) stands for stacking columns of matrix \(\mathbf{A}\) into a vector \(\mathbf{a}\). \(\text{vec}^{-1}(\cdot)\) is the inverse operation of vec(\(\cdot\)). \(\otimes\) denotes the Kronecker product. \(I\) denotes an identity matrix with appropriate size. \(\Re\{a\}\) denotes the real part of a complex variable \(a\). \(\mathcal{CN}(0, I)\) denotes a circularly symmetric complex Gaussian random vector with mean \(0\), and covariance matrix \(I\). \(\mathbb{E}[\cdot]\) stands for the statistical expectation.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

We consider a two way relay network for SWIPT as shown in Fig. 1, in which two transmitters A and B exchange information with the aid of an AF relay. We assume that the relay is equipped with \(N_r\) antennas, while other nodes are equipped with single antenna. Let \(\mathbf{f}_A \in \mathbb{C}^{N_r \times 1}\) and \(\mathbf{f}_B \in \mathbb{C}^{N_r \times 1}\) denote the channel from the transmitters to the relay, \(\mathbf{h}_A \in \mathbb{C}^{N_r \times 1}\) and \(\mathbf{h}_B \in \mathbb{C}^{N_r \times 1}\) denote the channel from the relay to the receivers, respectively. Since the relay operates in a half-duplex mode, one transmission round \(T\) is composed of two phases.

![Figure 1. System model for two-way AF relay with SWIPT.](image)

In the first phase, transmitters A and B broadcast their information \(s_A\) and \(s_B\) satisfying \(\mathbb{E}[|s_A|^2] = 1\) and \(\mathbb{E}[|s_B|^2] = 1\) to the relay. Thus, the received signal at the relay is given by

\[
y_r = \sqrt{P_A} \mathbf{f}_A s_A + \sqrt{P_B} \mathbf{f}_B s_B + n_r,
\]
where \( P_A \) and \( P_B \) are the transmit powers at A and B, respectively. \( n_r \in \mathbb{C}^{N_r \times 1} \) is the additive noise at the relay with variance \( \sigma_r^2 \). The reader can refer [12–15] for more details about this equation.

The received signal \( y_r \) at the relay is divided into two parts with PS ratios \( \rho (0 \leq \rho \leq 1) \) and \( 1 - \rho \) for ID and EH, respectively. Thus the signal for ID is

\[
y_r^{ID} = \sqrt{\rho} \left( \sqrt{P_A f_A s_A} + \sqrt{P_B f_B s_B} + n_r \right) + n_p,
\]

(2)

where \( n_p \in \mathbb{C}^{N_r \times 1} \) is the additional processing noise introduced by the PS circuit with variance \( \sigma_p^2 \).

On the other hand, the total energy harvested at the relay can be expressed as

\[
E_r = \frac{T}{2} \eta (1 - \rho) \left( P_A \| f_A \|^2 + P_B \| f_B \|^2 + \sigma_r^2 \right),
\]

(3)

where \( \eta \) denotes the EH efficiency. Without loss of generality, we assume \( \eta = 1 \).

Correspondingly, the maximum available power at the relay is given by

\[
P_r^{\text{max}} = \frac{E_r}{T/2} + P_0 = (1 - \rho) \left( P_A \| f_A \|^2 + P_B \| f_B \|^2 + \sigma_r^2 \right) + P_0,
\]

(4)

where \( P_0 \) is the relay initial power [9]. It should be noted that we set \( P_0 \) mainly for comparing our design with the traditional AF relay. In fact, \( P_0 \) tends to be small enough for EH relay.

In the second phase, the relay utilizes matrix \( \mathbf{W} \in \mathbb{C}^{N_r \times N_r} \) to forward the information to the respectively nodes. Thus, the received signals at nodes A and B are given, respectively, by

\[
y_A = \sqrt{\rho} \mathbf{h}_A^T \mathbf{W} \left( \sqrt{P_A f_A s_A} + \sqrt{P_B f_B s_B} + n_r \right) + \mathbf{h}_A^T \mathbf{W} n_p + n_A,
\]

(5a)

\[
y_B = \sqrt{\rho} \mathbf{h}_B^T \mathbf{W} \left( \sqrt{P_A f_A s_A} + \sqrt{P_B f_B s_B} + n_r \right) + \mathbf{h}_B^T \mathbf{W} n_p + n_B,
\]

(5b)

where \( n_A \) and \( n_B \) are additive noises at nodes A and B, with variances \( \sigma_A^2 \) and \( \sigma_B^2 \), respectively.

Since nodes A and B can remove its self-interference signal based on the priori knowledge [15], the remaining received signals at A and B, respectively, are given by

\[
\tilde{y}_A = \sqrt{\rho} \mathbf{h}_A^T \mathbf{W} \left( \sqrt{P_B f_B s_B} + n_r \right) + \mathbf{h}_A^T \mathbf{W} n_p + n_A,
\]

(6a)

\[
\tilde{y}_B = \sqrt{\rho} \mathbf{h}_B^T \mathbf{W} \left( \sqrt{P_A f_A s_A} + n_r \right) + \mathbf{h}_B^T \mathbf{W} n_p + n_B.
\]

(6b)

According to the received signal in q. (6), the instantaneous signal-to-interference-plus-noise-ratio (SINRs) at the two nodes can be expressed as

\[
\Gamma_A = \frac{\rho P_A \| \mathbf{h}_A^T \mathbf{W} f_B \|^2}{\rho \sigma_r^2 \| \mathbf{h}_A^T \mathbf{W} \|^2 + \sigma_p^2 \| \mathbf{h}_A^T \mathbf{W} \|^2 + \sigma_A^2} = \frac{\mathbf{w}^H \mathbf{Q}_A \mathbf{w}}{\mathbf{w}^H \mathbf{R}_A \mathbf{w} + \sigma_A^2},
\]

(7a)

\[
\Gamma_B = \frac{\rho P_B \| \mathbf{h}_B^T \mathbf{W} f_A \|^2}{\rho \sigma_r^2 \| \mathbf{h}_B^T \mathbf{W} \|^2 + \sigma_p^2 \| \mathbf{h}_B^T \mathbf{W} \|^2 + \sigma_B^2} = \frac{\mathbf{w}^H \mathbf{Q}_B \mathbf{w}}{\mathbf{w}^H \mathbf{R}_B \mathbf{w} + \sigma_B^2},
\]

(7b)

where \( \mathbf{w} = \text{vec} (\mathbf{W}) \), \( \mathbf{Q}_A = \rho P_A [(\mathbf{f}_B f_B^H) \otimes (\mathbf{h}_A h_A^H)]^T \), \( \mathbf{R}_A = [((\rho \sigma_r^2 \mathbf{I} + \sigma_p^2 \mathbf{I}) \otimes (\mathbf{h}_A h_A^H)]^T \), \( \mathbf{Q}_B = \rho P_B [(\mathbf{f}_A f_A^H) \otimes (\mathbf{h}_B h_B^H)]^T \), and \( \mathbf{R}_B = [((\rho \sigma_r^2 \mathbf{I} + \sigma_p^2 \mathbf{I}) \otimes (\mathbf{h}_B h_B^H)]^T \), respectively.

On the other hand, the transmit power of the relay is

\[
P_r = \rho \left( P_A \| \mathbf{W} f_A \|^2 + P_B \| \mathbf{W} f_B \|^2 + \sigma_r^2 \| \mathbf{W} \|^2 \right) + \sigma_p^2 \| \mathbf{W} \|^2,
\]

(8)

which is constrained as follows

\[
\text{Tr} (\mathbf{w}^H \mathbf{Cw}) \leq (1 - \rho) \left( P_A \| f_A \|^2 + P_B \| f_B \|^2 + \sigma_r^2 \right),
\]

(9)

where \( \mathbf{C} = (\rho (P_A f_A f_A^H + P_B f_B f_B^H + \sigma_r^2 \mathbf{I}) + \sigma_p^2) \mathbf{I} \otimes \mathbf{I} \).

For the convenient of the following deduction, we denote \( t = 1/\rho \) and equivalently rewritten (9) as

\[
\left(1 - \frac{1}{t}\right) \left( P_A \| f_A \|^2 + P_B \| f_B \|^2 + \sigma_r^2 \right) \geq \frac{\mathbf{w}^H \left( (P_A f_A f_A^H + P_B f_B f_B^H + \sigma_r^2 \mathbf{I}) \mathbf{I} \right) \mathbf{w}}{t} + \sigma_p^2 \mathbf{w} \mathbf{w}^H.
\]

(10)
Thus, the SINRs at the two nodes can be equivalently expressed as

\[
\Gamma_A = \frac{w^H \hat{Q}_A w}{w^H \hat{R}_A w + tw^H \hat{U}_A w + t\sigma_A^2},
\]

\[
\Gamma_B = \frac{w^H \hat{Q}_B w}{w^H \hat{R}_B w + tw^H \hat{U}_B w + t\sigma_B^2},
\]

where \( \hat{Q}_A = P_A[(f_B f_B^H) \otimes (h_A h_A^H)]^T, \hat{R}_A = [\sigma_A^2 I \otimes (h_A h_A^H)]^T, \hat{U}_A = [\sigma_A^2 I \otimes (h_A h_A^H)]^T, \hat{Q}_B = P_B[(f_A f_A^H) \otimes (h_B h_B^H)]^T, \hat{R}_B = [\sigma_B^2 I \otimes (h_B h_B^H)]^T, \) and \( \hat{U}_B = [\sigma_B^2 I \otimes (h_B h_B^H)]^T, \) respectively.

### 2.2. Problem Statement

In this paper, we investigate a weighted sum rate maximization design subject to the dynamic relay transmit power constraint. Mathematically, our problem can be expressed as

\[
\begin{align*}
\max_{w, t} & \quad \omega_1 \log_2 (1 + \Gamma_A) + \omega_2 \log_2 (1 + \Gamma_B) \\
\text{s.t.} & \quad \text{(10)}, \quad t \geq 1,
\end{align*}
\]

where \( \omega_1 \) and \( \omega_2 \) are the weighted factors satisfying \( 0 \leq \omega_1, \omega_2 \leq 1 \) and \( \omega_1 + \omega_2 = 1 \).

Eq. (12) is hard to handle directly due to the non-convex objective and constraint. In the next section, we propose an effective method to solve Eq. (12).

### 3. A SCA-CCCP BASED ITERATIVE ALGORITHM

In this section, we will derive an SCA-CCCP based optimization approach to Eq. (12).

#### 3.1. A Convex Transformation of the Relay Power Constraint

Firstly, we focus on the relay power constraint in Eq. (10). In fact, for Eq. (10), we have the following proposition.

**Proposition 1:** Eq. (10) is a convex constraint.

**Proof:** Firstly, the quadratic form \( z^H z \) is convex in \( z \). Besides, for any \( \rho > 0 \) and positive semi-definite matrix \( A \succeq 0 \), \( z^H Az / \rho \) is the perspective of \( z^H Az \) [19]. Since the perspective operation preserves convexity, \( z^H Az / \rho \) is jointly convex in \( (z, \rho) \) [19]. In addition, \( 1 - 1/t \) is concave in \( t \) (\( t \geq 1 \)). Thus, Eq. (10) is a convex constraint.

#### 3.2. A Convex Transformation of the Weighted Sum Rate

The remaining task is to handle the non-convex objective. Via introducing slack variables \( \alpha_1, \alpha_2, \beta_1, \) and \( \beta_2 \), Eq. (12) can be rewritten as

\[
\begin{align*}
\max_{w, t, \alpha_1, \alpha_2, \beta_1, \beta_2} & \quad \omega_1 \alpha_1 + \omega_2 \alpha_2 \\
\text{s.t.} & \quad \beta_1 \geq 2^{\alpha_1} - 1, \quad \beta_2 \geq 2^{\alpha_2} - 1, \\
& \quad \frac{w^H \hat{Q}_A w}{\beta_1} \geq \frac{w^H \hat{R}_A w + tw^H \hat{U}_A w + t\sigma_A^2}{\beta_1}, \\
& \quad \frac{w^H \hat{Q}_B w}{\beta_2} \geq \frac{w^H \hat{R}_B w + tw^H \hat{U}_B w + t\sigma_B^2}{\beta_2}, \\
& \quad \text{(10)}, \quad t \geq 1.
\end{align*}
\]

However, there still exist non-convex constraint in Eqs. (13c) and (13d). Since Eqs. (13c) and (13d) have the same form, in the following we will focus on Eq. (13c).
Specifically, via following the main idea of SCA, we approximate the left hand side of Eq. (13c) by the first order Taylor expansion as
\[
\frac{2\Re \left\{ \tilde{w}^H \hat{Q}_A w \right\}}{\beta_1} - \frac{\tilde{w}^H \hat{Q}_A \tilde{w}}{\beta_1^2} \beta_1 \geq \tilde{w}^H \hat{R}_A w + tw^H \hat{U}_A w + t\sigma_A^2, \tag{14}
\]
where \( \tilde{w} \) and \( \tilde{\beta}_1 \) are given point achieved by the previous iteration.

The most difficult part in the right hand side of Eq. (14) is the production term \( tw^H \hat{U}_A w \), which is non-convex. In the following, we will utilize CCCP to overcome this obstacle. Firstly, we introduce the following lemma.

**Lemma 1** [10]: Function in the form \( \xi(x, y) = xy \) is quasi-concave. For arbitrary \( \mu > 0 \), define a function
\[
\xi_u(x, y) = \frac{u}{2} x^2 + \frac{1}{2u} y^2,
\]
then \( \xi_u(x, y) \) is always an upper estimate of \( \xi(x, y) \) for a fixed \( \mu > 0 \) and is convex. Furthermore, \( \xi_u(x, y) \) also satisfies
\[
\xi_u(x, y) = \xi(x, y), \quad \nabla \xi_u(x, y) = \nabla \xi(x, y),
\]
when \( u = y/x \).

Following the idea of CCCP and Lemma 1, we introduce auxiliary variable \( \theta_A \). Thus, \( tw^H \hat{U}_A w \) can be upper bounded as
\[
\begin{cases}
    t\theta_A \leq \frac{\bar{u}_A t^2}{2} + \frac{1}{2\bar{u}_A} \theta_A^2, \\
    w^H \hat{U}_A w \leq \theta_A,
\end{cases} \tag{15}
\]
where \( \bar{u}_A = \tilde{\theta}_A/\tilde{t} \). Besides, \( \tilde{\theta}_A \) and \( \tilde{t} \) are the obtained optimal point in the previous iteration.

Combining these steps, Eq. (14) can be approximated as the following convex constraint
\[
\frac{2\Re \left\{ \tilde{w}^H \hat{Q}_A w \right\}}{\beta_1} - \frac{\tilde{w}^H \hat{Q}_A \tilde{w}}{\beta_1^2} \beta_1 \geq \tilde{w}^H \hat{R}_A w + \frac{\bar{u}_A t^2}{2} + \frac{1}{2\bar{u}_A} \theta_A^2 + t\sigma_A^2. \tag{16}
\]

The same procedure can be utilized to handle Eq. (13d), and we omit the details for brevity.

Finally, around given point \( \{ \tilde{w}, \tilde{t}, \tilde{\beta}_1, \tilde{\beta}_2, \tilde{\theta}_A, \tilde{\theta}_B \} \), we formulate the following approximated problem
\[
\begin{align}
\max_{w, t, \alpha_1, \alpha_2, \beta_1, \beta_2, \theta_A, \theta_B} & \quad \omega_1 \alpha_1 + \omega_2 \alpha_2 \tag{17a} \\
\text{s.t.} & \quad (13b), (13c), (16), \tag{17b}
\end{align}
\]
which can be efficiently solved by the convex programming toolbox CVX [20]. Then, the optimal solution for Eq. (12) can be obtained in an iterative way.

### 4. SIMULATION RESULTS

In this section, we evaluate the performance of our proposed scheme through Monte Carlo simulations. The following parameters \( N_r = 5 \), \( P_A = P_B = P_S = 10 \) dBW, \( P_0 = 0 \) dBW, \( \omega_1 = \omega_2 = 0.5 \), \( \sigma_r^2 = \sigma_A^2 = \sigma_B^2 = -50 \) dBm, and \( \sigma_p^2 = -40 \) dBm are set for the following simulation unless specified. In addition, all the entries of \( f_A, f_B, h_A, \) and \( h_B \) are independent and identically distributed (i.i.d.) complex Gaussian random variables generated by \( \mathcal{CN}(0, 10^{-2}) \). It should be mentioned that our parameters mainly follow some related literatures such as [15–18]. In addition, we compare our algorithm with the following methods: 1) the traditional two way AF relay method, e.g., the relay only forward wireless information; 2) the two way WPCN AF relay with the optimal TS factor as in [9]; 3) the two way EH relay scheme with the fixed PS ratio \( \rho = 0.5 \); 4) the optimal PS radio with the singular-value-decomposition (SVD) based beamforming as in [13]. These designs are labeled as “the proposed
method”, “the traditional AF relay”, “the WPCN AF relay”, “the fixed PS scheme”, and “the SVD method”, respectively.

In Fig. 2, we show the weighted sum rate versus the source transmit power $P_s$. From this figure, we can see that our proposed method achieves better performance than other schemes. In addition, the two way WPCN relay with the optimal TS factor and the SVD method outperform the traditional two way AF relay and the fixed PS ratio method. These two methods suffer from performance loss compared with the other methods, which suggests the importance of the dynamic PS scheme. Besides, the proposed beamforming method is more effective than the SVD method, since the SVD only utilizes partial wireless channel to transmit information and energy.

In Fig. 3, we show the weighted sum rate versus the relay initial power $P_0$. Similar to the previous simulation, our proposed method achieves better performance than the other schemes. In addition, via comparing the curves in Fig. 3 with the curves in Fig. 2, we can see that the sum rate increases more quickly with the increase of $P_0$, rather than $P_s$, which suggest that the relay power is critical for the sum rate performance in the two way relay network. Since the amplify magnification of the signal is mainly determined by the relay power, the sum rate is mainly affected by the relay power threshold.

![Figure 2. The weighted sum rate versus the source transmit power.](image1)

![Figure 3. The weighted sum rate versus the relay initial power.](image2)

5. CONCLUSION

In this paper, a joint beamforming and PS design has been investigated for a two way AF relay network, where the relay is energy-constrained and powered by the RF signals. The formulated non-convex weighted sum rate maximization problem was handled by SCA and CCCP. Then, an iterative algorithm was proposed to solve the approximated problem. Numerical results show the superiority of the proposed method, as well as the effect of EH in improving the sum rate of relay network.

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