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An Unequal Divider with Different Terminated Impedances and Different Electrical Lengths of Four Uniform Transmission Lines

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Abstract—This paper proposes an unequal power divider with different terminated impedances and different electrical lengths for four uniform transmission lines. The proposed power divider consists of four transmission lines with different electrical lengths and an isolation resistor. Under different port impedances, the splitting ratio of the proposed divider can be adjusted to desired values by varying the electrical lengths of the divider transmission lines with uniform impedances. To verify the feasibility of the proposed divider, two circuits were designed with dividing ratios of 2 : 1 and 4 : 1 at an operating frequency of 2 GHz. The circuits used a uniform impedance of 40 Ω at terminated impedances of 50, 70, and 60 Ω. The performance showed excellent agreement between the simulated and experimental results.

1. INTRODUCTION

Power dividers are considered to be core components in wireless applications and are used for purposes such as sampling signals during measurements, monitoring, power amplifiers, antenna feeds, and beamforming [1, 2]. However, it is difficult to implement dividers with splitting ratios greater than unity using microstrip technology because of the narrow width of transmission lines. To overcome the implementation problem of high impedance lines, many researchers have focused on areas such as shorted coupled lines [3] and defected ground structures (DGSs) [4]. In addition, a design method for power dividers with high division ratios, which does not use high impedance lines, employs schemes such as adjusting the electrical length of a 50 Ω or 70.7 Ω uniform transmission line [5–8], and adjusting the lengths of specific lines [9]. These methods are implemented with 50 Ω or 70.7 Ω uniform transmission lines and a termination impedance of 50 Ω. Depending on the application, even if the termination impedance is not exactly 50 Ω, there is still need to vary the termination impedance to suit different applications. In this case, because an impedance transformer is not necessary, the method can be used more conveniently by adopting small sized circuits. This paper presents an unequal divider with different terminated impedances and uniform transmission lines with different electrical lengths. Under these conditions, the proposed power divider was analyzed using the scattering parameters. The electrical lengths between ports and isolation resistance of the proposed divider were calculated by numerical analysis after determining the values of the specific uniform impedance and the different termination impedances. To validate the proposed divider, we simulated and measured the unequal divider at a center frequency of 2 GHz.

2. THEORY AND DESIGN

The proposed divider consists of four uniform transmission lines with electrical lengths θ_1, θ_2, θ_3, and θ_4 and an isolation resistor R_{iso} with different termination impedances, R_{1T}, R_{2T}, and R_{3T}, as shown

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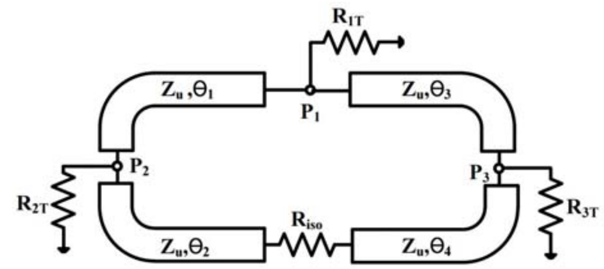
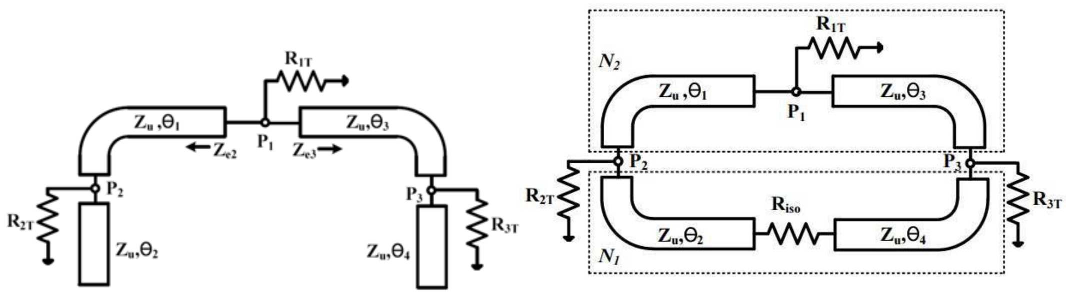


Figure 1. Schematic of the proposed power divider.



(a) (b)

Figure 2. Equivalent circuit of proposed divider (a) when the port 1 excited and, (b) when the port 2 excited.

in Figure 1. For a power ratio of k^2 $(= P_2/P_3)$, the proposed divider should satisfy the S-parameters given in Equation (1).

$$S_{11} = S_{22} = S_{33} = S_{32} = 0\,, \quad S_{21} = k \cdot S_{31} \tag{1}$$

The conventional even-odd method cannot be used to analyze this divider because it has different termination impedances [10, 11]. Therefore, we can evaluate the S-parameters in each port under the given conditions and obtain the necessary conditions that they satisfy.

If the input power at port 1 is excited and the load power with the terminations, R_{2T}, R_{3T} satisfy the power ratio k^2, and the voltage between the left branch and ground is equal to that between the right branch and ground measured from port 1, then no current can flow between the isolation resistors. Figure 2(a) depicts the equivalent circuit of the proposed divider when port 1 is excited. The $ABCD$ parameters between port 1 and ports 2 and 3 are given below.

$$\begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & jZ_u\sin\theta_1 \\ j\frac{\sin\theta_1}{Z_u} & \cos\theta_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j\frac{\tan\theta_2}{Z_u} & 1 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} A_{31} & B_{31} \\ C_{31} & D_{31} \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & jZ_u\sin\theta_3 \\ j\frac{\sin\theta_3}{Z_u} & \cos\theta_3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j\frac{\tan\theta_4}{Z_u} & 1 \end{pmatrix} \tag{3}$$

Using Equations (2)–(3), we can derive S_{21} and S_{31} parameters at different termination impedances. In addition, using $S_{21} = k \cdot S_{31}$, we obtain the following equations:

$$\begin{aligned} & k^2 \cdot \sqrt{\frac{R_{3T}}{R_{2T}}} \cdot \left\{ (\cos\theta_1 - \sin\theta_1 \cdot \tan\theta_2) \cdot R_{2T} + \frac{1+k^2}{k^2} \cdot R_{1T} \cdot \cos\theta_1 \right\} \\ & = (\cos\theta_3 - \sin\theta_3 \cdot \tan\theta_4) \cdot R_{3T} + (1+k^2) \cdot R_{1T} \cdot \cos\theta_3 \end{aligned} \tag{4}$$

$$\begin{aligned} & k^2 \cdot \sqrt{\frac{R_{3T}}{R_{2T}}} \cdot \left\{ Z_u^2 \cdot \sin\theta_1 + \frac{1+k^2}{k^2} \cdot R_{1T} \cdot R_{2T} \cdot (\sin\theta_1 + \cos\theta_1 \cdot \tan\theta_2) \right\} \\ & = Z_u^2 \cdot \sin\theta_3 + (1+k^2) \cdot R_{1T} \cdot R_{2T} \cdot (\sin\theta_3 + \cos\theta_3 \cdot \tan\theta_4) \end{aligned} \tag{5}$$

Under the matched input condition ($S_{11} = 0$), we have $\frac{1}{R_{1T}} = \frac{1}{Z_{e2}} + \frac{1}{Z_{e3}}$, where Z_{e2} and Z_{e3} are the input impedances from port 1 to port 2 and from port 1 to port 3, respectively.

Based on the principle of conservation of energy and the ideal transmission lines in Figure 2(a), we obtain the following equations:

$$Re[Z_{e2}] = \frac{1+k^2}{k^2} \cdot R_{1T} = \frac{Z_u^2 \cdot R_{2T} \cdot (1+\tan^2\theta_1)}{R_{2T}^2 \cdot (\tan\theta_1 + \tan\theta_2)^2 + Z_u^2} \tag{6}$$

$$Re[Z_{e3}] = (1+k^2) \cdot R_{1T} = \frac{Z_u^2 \cdot R_{3T} \cdot (1+\tan^2\theta_3)}{R_{3T}^2 \cdot (\tan\theta_3 + \tan\theta_4)^2 + Z_u^2} \tag{7}$$

Figure 2(b) depicts the equivalent circuit when port 2 is excited. To find the conditions for the input and output matching and isolation at different terminated impedances, the networks N_1 and N_2 were connected in parallel. The N_1 network consists of an isolation resistor R_{iso} and transmission lines with electrical lengths θ_2, θ_4. The N_2 network consists of a termination resistor R_{1T} and transmission lines with electrical lengths θ_1, θ_3.

The $ABCD$ parameters between ports 2 and 3 for the N_1 and N_2 networks can be expressed as:

$$\begin{pmatrix} A_{N1} & B_{N1} \\ C_{N1} & D_{N1} \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & jZ_u\sin\theta_2 \\ j\frac{\sin\theta_2}{Z_u} & \cos\theta_2 \end{pmatrix} \begin{pmatrix} 1 & R_{iso} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_4 & jZ_u\sin\theta_4 \\ j\frac{\sin\theta_4}{Z_u} & \cos\theta_4 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} A_{N2} & B_{N2} \\ C_{N2} & D_{N2} \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & jZ_u\sin\theta_1 \\ j\frac{\sin\theta_1}{Z_u} & \cos\theta_1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R_{1T}} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_3 & jZ_u\sin\theta_3 \\ j\frac{\sin\theta_3}{Z_u} & \cos\theta_3 \end{pmatrix} \tag{9}$$

By evaluating Equations (8)–(9), we obtain the admittance parameters by converting $ABCD$ parameters to their equivalent Y-parameters $(Y)_{N1}$ and $(Y)_{N2}$ for the networks, N_1 and N_2. Finally, the total Y-parameters between port 2 and port 3 are obtained.

$$(Y)_T = (Y)_{N1} + (Y)_{N2} = \begin{pmatrix} y_{11T} & y_{12T} \\ y_{21T} & y_{22T} \end{pmatrix} \tag{10}$$

where $y_{11T} = \frac{\cos(\theta_2+\theta_4)+j\frac{R_{iso}}{Z_u}\sin\theta_2\cos\theta_4}{R_{iso}\cos\theta_2\cos\theta_4+jZ_u\sin(\theta_2+\theta_4)} + \frac{\cos(\theta_1+\theta_3)+j\frac{Z_u}{R_{1T}}\cos\theta_1\sin\theta_3}{-\frac{Z_u^2}{R_{1T}}\sin\theta_1\sin\theta_3+jZ_u\sin(\theta_1+\theta_3)}$, $y_{12T} = y_{21T} = \frac{-1}{R_{iso}\cos\theta_2\cos\theta_4+jZ_u\sin(\theta_2+\theta_4)} + \frac{-1}{-\frac{Z_u^2}{R_{1T}}\sin\theta_1\sin\theta_3+jZ_u\sin(\theta_1+\theta_3)}$, $y_{22T} = \frac{\cos(\theta_2+\theta_4)+j\frac{R_{iso}}{Z_u}\cos\theta_2\sin\theta_4}{R_{iso}\cos\theta_2\cos\theta_4+jZ_u\sin(\theta_2+\theta_4)} + \frac{\cos(\theta_1+\theta_3)+j\frac{Z_u}{R_{1T}}\sin\theta_1\cos\theta_3}{-\frac{Z_u^2}{R_{1T}}\sin\theta_1\sin\theta_3+jZ_u\sin(\theta_1+\theta_3)}$.

Using Equation (10), the S_{22}, S_{33}, and S_{32} parameters between port 2 and port 3 can be expressed as

$$S_{22} = -\frac{(y_{11T} - \frac{1}{R_{2T}}) \cdot (y_{22T} + \frac{1}{R_{3T}}) - y_{12T} \cdot y_{21T}}{(y_{11T} + \frac{1}{R_{2T}}) \cdot (y_{22T} + \frac{1}{R_{3T}}) - y_{12T} \cdot y_{21T}} \tag{11}$$

$$S_{33} = -\frac{(y_{11T} + \frac{1}{R_{2T}}) \cdot (y_{22T} - \frac{1}{R_{3T}}) - y_{12T} \cdot y_{21T}}{(y_{11T} + \frac{1}{R_{2T}}) \cdot (y_{22T} + \frac{1}{R_{3T}}) - y_{12T} \cdot y_{21T}} \tag{12}$$

$$S_{32} = -\frac{2\sqrt{\frac{1}{R_{2T} \cdot R_{3T}}} \cdot y_{21T}}{(y_{11T} + \frac{1}{R_{2T}}) \cdot (y_{22T} + \frac{1}{R_{3T}}) - y_{12T} \cdot y_{21T}} \tag{13}$$

From Equations (4)–(7) and (11)–(13), we can finally solve for the isolation resistor R_{iso} and the electrical lengths θ_1, θ_2, θ_3, and θ_4 of the transmission lines to satisfy the conditions for input matching, insertion losses, output matching, and isolation.

3. SIMULATION AND EXPERIMENTAL RESULTS

To validate the performance of the proposed divider, we designed and simulated a 2 : 1 and a 4 : 1 unequal divider with a uniform impedance of 40 Ω and terminated impedances of 50, 70, and 60 Ω, at a center frequency of 2 GHz. The design parameters obtained by satisfying the conditions for S_{11}, S_{22}, and S_{33} were better than -20 dB, while the isolation of S_{32} was better than -25 dB using the derived parameters in Equations (4)–(7) and (11)–(13). Figure 3 depicts the variation in the design parameter values ($\theta_1 \sim \theta_4$, R_{iso}) with the splitting ratio ($k^2 = 2 \sim 9$) at $Z_u = 40\,\Omega$ and port impedances of 50, 70, and 60 Ω. In the graphs, the electrical lengths θ_1, θ_4 and the isolation resistance R_{iso} tend to increase as the splitting ratio increases, while the electrical lengths of θ_2, θ_3 tend to decrease. For the 2 : 1 unequal divider, the electrical lengths and isolation resistance were as follows: $\theta_1 = 154°$, $\theta_2 = 8.7°$, $\theta_3 = 143°$, $\theta_4 = 47°$, and $R_{iso} = 15\,\Omega$. For the 4 : 1 unequal divider, the electrical lengths and isolation resistance were as follows: $\theta_1 = 153°$, $\theta_2 = 5°$, $\theta_3 = 121°$, $\theta_4 = 53°$, and $R_{iso} = 22\,\Omega$. The proposed unequal divider was fabricated on a Teflon Taconic printed circuit board (PCB) with a dielectric constant, $\varepsilon_r = 2.5$, and thickness, $h = 0.787$ mm. These dividers were designed and optimized using the Microwave Office software developed by National Instruments.

Figure 4 shows photographs of the proposed 2 : 1 and 4 : 1 unequal power dividers. Figure 5 shows the simulated and measured S-parameters for 2 : 1 unequal divider. It was observed that the insertion losses were $|S_{21}| = 1.9$ dB and $|S_{31}| = 4.6$ dB; the input return loss for $|S_{11}|$ was better than -25 dB while the output losses for $|S_{22}|$ and $|S_{33}|$ were better than -16 dB, and the isolation of $|S_{32}|$

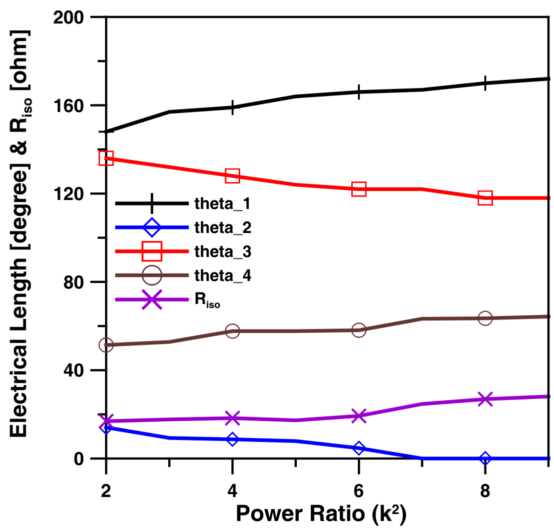
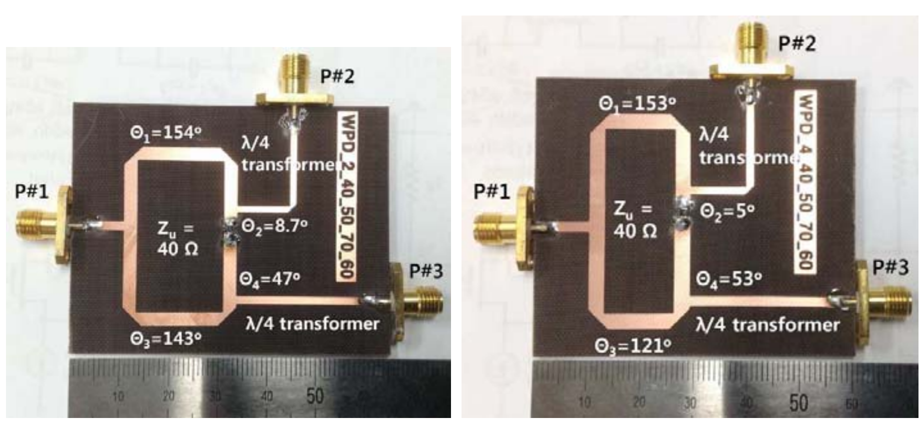


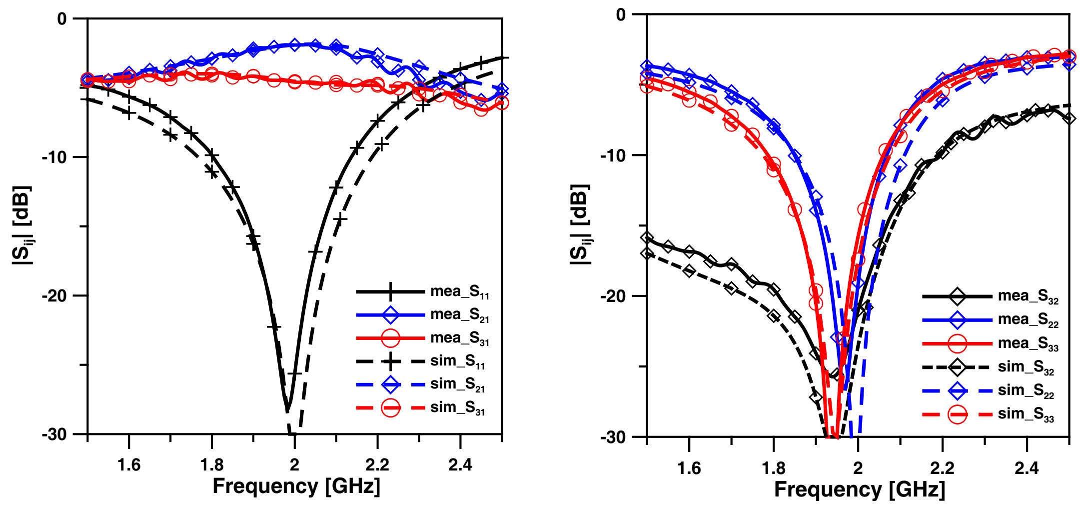
Figure 3. Graph showing variation in the design parameters for the splitting ratio with $Z_u = 40\,\Omega$ and port impedances of 50, 70, and 60 Ω.



(a) (b)

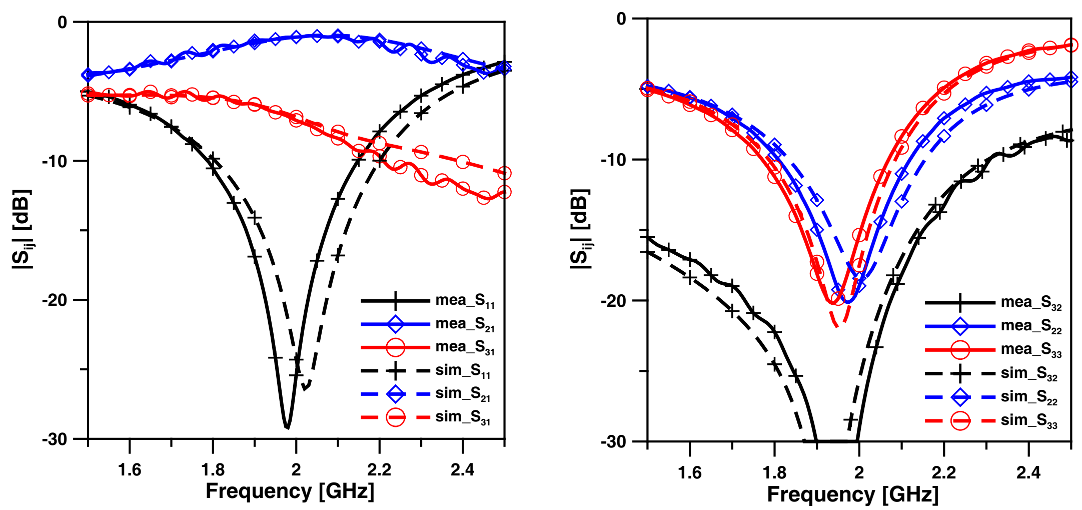
Figure 4. Photograph of the proposed (a) 2 : 1 and (b) 4 : 1, unequal power divider.

was better than 20 dB at a center frequency of 2 GHz. Figure 6 shows the simulated and measured S-parameters for the 4 : 1 unequal divider. It was observed that the insertion losses were $|S_{21}| = 1.1$ dB and $|S_{31}| = 7.1$ dB; the input return loss of $|S_{11}|$ was also better than -25 dB; the output losses of $|S_{22}|$ and $|S_{33}|$ were better than -15 dB while the isolation of $|S_{32}|$ was better than 25 dB at a center frequency of 2 GHz. In Figures 5 and 6, there was a difference between simulation and measurement due to errors in PCB fabrication. Table 1 shows a comparison of the reference and this work based on performance data.



(a) (b)

Figure 5. Measured and simulated S-parameters for the 2 : 1 unequal divider (a) $|S_{11}|$, $|S_{21}|$, $|S_{31}|$, and (b) $|S_{32}|$, $|S_{22}|$, $|S_{33}|$.



(a) (b)

Figure 6. Measured and simulated S-parameters for the 4 : 1 unequal divider (a) $|S_{11}|$, $|S_{21}|$, $|S_{31}|$, and (b) $|S_{32}|$, $|S_{22}|$, $|S_{33}|$.

Table 1. Comparison of the reference and this work based on performance data.

Item	Freq (GHz)	TL imp (Ω)	Term. (Ω)	$\vert S_{21}\vert/\vert S_{31}\vert$ (dB)	$\vert S_{11}\vert/\vert S_{22}\vert/\vert S_{33}\vert$ (dB)	$\vert S_{32}\vert$ (dB)	-10 dB Return loss BW (%)
[5]	1	70.7	50	2.0/5.0 1.0/7.0	25/25/25	30	75
[6]	60/90	50	50	3.3/3.3 3.7/3.7	23/23/23 18/18/18	18 22	39
[7]	3	50	50	2.0/5.0	more than 20	25	30
[8]	2	70.7	50	3.7/3.7	more than 15	25	96.5
[9]	1	50	50	0.47/10.36	27/25/25	41	20
This work	2	40	50/60/70	1.9/4.6 1.1/7.1	25/16/16 25/15/15	20 25	16 16

4. CONCLUSION

An unequal power divider with different terminated impedances and electrical lengths for four uniform transmission lines was proposed. At different termination impedances, the variations in the electrical lengths of the four transmission lines could be designed for the unequal divider using the desired splitting ratio. The performance of the proposed divider showed excellent agreement between the experimental and simulated results.

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