Progress In Electromagnetics Research Letters, Vol. 44, 71–79, 2014

Quantum Analysis of Modified Caldirola-Kanai Oscillator Model for Electromagnetic Fields in Time-Varying Plasma

Jeong R. Choi[1, *], Samira Lakehal[2], Mustapha Maamache[2], and Salah Menouar[2]

Abstract—Quantum properties of a modified Caldirola-Kanai oscillator model for propagating electromagnetic fields in a plasma medium are investigated using invariant operator method. As a modification, ordinary exponential function in the Hamiltonian is replaced with a modified exponential function, so-called the q-exponential function. The system described in terms of q-exponential function exhibits nonextensivity. Characteristics of the quantized fields, such as quantum electromagnetic energy, quadrature fluctuations, and uncertainty relations are analyzed in detail in the Fock state, regarding the q-exponential function. We confirmed, from their illustrations, that these quantities oscillate with time in some cases. It is shown from the expectation value of energy operator that quantum energy of radiation fields dissipates with time, like a classical energy, on account of the existence of non-negligible conductivity in media.

1. INTRODUCTION

One of the main factors responsible for the complexity of media is the time dependence of electromagnetic parameters, such as permittivity, permeability, and conductivity. In case that at least one of these three parameters is not a positive scalar constant, the medium is classified as a complex one. Thanks to the development of materials science, it is now not difficult to obtain a complex plasma, which has required characteristics for a specific purpose, from the critical synthesis of plasma materials.

There are many scientific reports relevant to the characteristics of electromagnetic fields in complex media that are intrinsically space-varying, anisotropic, and dispersive. On the other hand, Maxwell equations for an electromagnetic field described by time-dependent Hamiltonian in plasma are in general very difficult to manage. It is recently known that Lewis-Riesenfeld invariant [1] is useful for deriving analytical solutions associated with quantum electromagnetic fields in time-varying media (see Ref. [2] and references there in). Indeed, the knowledge for characteristics of radiation fields propagating through an electromagnetic medium which has time-dependent parameters is crucial for analyzing and diagnosing plasma [3, 4].

One of simple systems that are described by a time-dependent Hamiltonian is Caldirola-Kanai (CK) oscillator [5, 6] that exhibit exponential decay characteristic of energy. If the conductivity in media is non-negligible, the electromagnetic fields can be modeled by the CK oscillator. In this paper, we study the quantum characteristics of a modified CK oscillator model of electromagnetic fields propagating through plasma. As a modification, we replace the normal exponential function in the CK Hamiltonian with a modified exponential function, so-called the q-exponential function. Usually, q-exponential function is used to study nonextensive features of dynamical systems [7–9]. The statistics of relativistic plasma and gas under an external electromagnetic field follows q-distribution and exhibit nonextensive properties [9].

Received 16 June 2013, Accepted 5 December 2013, Scheduled 19 December 2013

* Corresponding author: Jeong Ryeol Choi (choiardor@hanmail.net).

1 Department of Radiologic Technology, Daegu Health College, Yeongsong-ro 15, Buk-gu, Daegu 702-722, Republic of Korea.

2 Laboratoire de Physique Quantique et Systèmes Dynamiques, Département de Physique, Faculté des Sciences, Université Ferhat Abbas Sétif 1, Sétif 19000, Algeria.

In this paper, quantum characteristics of electromagnetic fields in plasma medium, such as quantum energy, quadrature fluctuations, and uncertainty relations, will be analyzed with consideration of nonextensivity of their modified CK oscillator model. To this end, we make an assumption that only the electromagnetic waves are quantized, and the medium is treated classically. Coulomb gauge will be taken when developing quantum theory of the system for convenience.

2. HAMILTONIAN DYNAMICS OF FIELDS

For the case that the parameters of a plasma medium vary with time, the relations between fields and current density inside it are given by $\mathbf{D} = \epsilon(t)\mathbf{E}$, $\mathbf{H} = \mathbf{B}/\mu(t)$, and $\mathbf{J} = \sigma(t)\mathbf{E}$, where ϵ is electric permittivity, μ magnetic permeability, and σ conductivity. The velocity of light is also dependent on time and is given by $c(t) = 1/\sqrt{\epsilon(t)\mu(t)}$.

Let us assume that there is no net free charge distribution in media. Then, the scalar potential vanishes in the Coulomb gauge and the vector potential can be written as

$$\mathbf{A}(\mathbf{r}, t) = \sum_l \mathbf{u}_l(\mathbf{r}) q_l(t), \tag{1}$$

where $\mathbf{u}_l(\mathbf{r})$ is a mode function determined from the geometric of the medium and $q_l(t)$ is a time function. Notice that $q_l(t)$ follows an equation of motion of the form [2, 10]

$$\ddot{q}_l + \frac{\sigma(t) + \dot{\epsilon}(t)}{\epsilon(t)} \dot{q}_l + \omega_l^2(t) q_l = 0, \tag{2}$$

where $\omega_l(t) = c(t)k_l$ and k_l is the lth mode wave number which is constant. In general, the wave number varies only when the electromagnetic wave passes through a boundary where spatial discontinuity exists [3]. On account of the second term associated with $\dot{q}_l$ in Eq. (2), the system dissipates with time. Hence, not only the existence of conductivity in media but also the steady increase of permittivity causes the dissipation of radiation fields.

From now on, let us regard a particular mode and drop the subscript l from all subsequent equations for convenience. Then the corresponding Hamiltonian is expressed in the form

$$\hat{H}(\hat{q}, \hat{p}, t) = \frac{1}{2\epsilon_0} e^{-\Lambda(t)} \hat{p}^2 + \frac{1}{2} \epsilon_0 e^{\Lambda(t)} \omega^2(t) \hat{q}^2, \tag{3}$$

where $\hat{p} = -i\hbar(\partial/\partial q)$, ϵ_0 is a constant with dimension of permittivity and $\Lambda(t)$ is defined as

$$\Lambda(t) = \int_0^t \frac{\sigma(t') + \dot{\epsilon}(t')}{\epsilon(t')} dt' + \delta, \tag{4}$$

with a real constant δ. Using Hamilton's equation, you can easily check that Eq. (3) gives exact classical equation of motion represented in Eq. (2). Notice that this Hamiltonian is actually the same as that of the parametric harmonic oscillator treated by Maamache et al. [11]. It is proved that parametric harmonic oscillator is unitarily equivalent to a generalized time-dependent harmonic oscillator [11].

Considering the case that all parameters are constant, i.e., $\epsilon(t) = \epsilon_0$, $\mu(t) = \mu_0$, and $\sigma(t) = \sigma_0$, we obtain the CK Hamiltonian originally defined by Caldirola and Kanai [5, 6] such that

$$\hat{H}(\hat{q}, \hat{p}, t) = \frac{1}{2\epsilon_0} e^{-(\sigma_0 t/\epsilon_0 + \delta)} \hat{p}^2 + \frac{1}{2} \epsilon_0 e^{\sigma_0 t/\epsilon_0 + \delta} \omega_0^2 \hat{q}^2, \tag{5}$$

where $\omega_0 = k/(\mu_0 \epsilon_0)^{1/2}$.

An elegant generalization of the ordinary exponential function appeared in Eq. (5) is q-exponential function of the form [7]

$$\exp_{\mathrm{q}}(y) = [1 + (1 - \mathrm{q})y]^{1/(1-\mathrm{q})}, \tag{6}$$

where q is considered as a nonextensive parameter. Physically, q is the degree of nonextensivity, i.e., the degree of deviation of a system or physical quantities from extensivity. It is discovered by Tsallis [12] through his try to generalizing Boltzmann-Gibbs statistical mechanics considering the systems whose experimental data do not match with usual critical phenomena in statistical mechanics.

Clearly speaking, nonextensive thermostatistics is a generalization of the conventional Boltzmann-Gibbs statistics.

Comprehensive studies have been carried out for various physical properties of elementary excitations including light waves, such as thermodynamical characteristics, Tsallis nonextensivity, energy profiles, and so on [13]. In particular, for q $\to$ 1, Eq. (6) recovers to the ordinary exponential function. By replacing the exponential function with the q-exponential function from Eq. (5), we have a new Hamiltonian as

$$\hat{H}_{\mathrm{q}}(\hat{q},\hat{p},t) = \frac{\hat{p}^2}{2\epsilon_0 \exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)} + \frac{1}{2}\epsilon_0 \exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)\omega_0^2 \hat{q}^2. \tag{7}$$

Apparently, this modified Hamiltonian generalizes the original CK oscillator so that it can cover the nonextensive dynamical situations of electromagnetic phenomena. The system is sub-extensive when q $>$ 1 and, for q $<$ 1, it is super-extensive [14, 15]. Özeren [8] studied nonextensive quantum characteristics of the generalized CK oscillator in connection with SU(1,1) coherent states, on the basis of exactly the same Hamiltonian given in Eq. (7) (except for disregarding a trivial factor δ).

The equation of motion for q corresponding to the modified Hamiltonian is

$$\ddot{q}(t) + \frac{\sigma_0/\epsilon_0}{1+(1-\mathrm{q})(\sigma_0 t/\epsilon_0 + \delta)}\dot{q}(t) + \omega_0^2 q(t) = 0. \tag{8}$$

If we denote two linearly independent classical solutions of Eq. (8) as $s_1(t)$ and $s_2(t)$, they are easily derived to be

$$s_1(t) = s_{1,0}\sqrt{\frac{\pi\omega_0}{2\sigma_0(1-\mathrm{q})/\epsilon_0}}[\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)]^{-\mathrm{q}/2} \times J_\nu\left(\frac{\omega_0}{(1-\mathrm{q})\sigma_0/\epsilon_0} + \omega_0 t\right), \tag{9}$$

$$s_2(t) = s_{2,0}\sqrt{\frac{\pi\omega_0}{2\sigma_0(1-\mathrm{q})/\epsilon_0}}[\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)]^{-\mathrm{q}/2} \times N_\nu\left(\frac{\omega_0}{(1-\mathrm{q})\sigma_0/\epsilon_0} + \omega_0 t\right), \tag{10}$$

where $s_{1,0}$ and $s_{2,0}$ are integral constants, J_ν and N_ν are the first and the second kind Bessel functions, respectively, and $\nu = \mathrm{q}/[2(1-\mathrm{q})]$. Further, a general classical solution is represented as

$$q(t) = c_1 s_1(t) + c_2 s_2(t), \tag{11}$$

where c_1 and c_2 are arbitrary real constants.

3. ANALYSIS OF QUANTUM PROPERTIES

The Hamiltonian, Eq. (7), which we will manage in this section is time-dependent. Notice that the separation of time functions from others in that equation is impossible, leading to the failure of the conventional separation of variables method for solving the corresponding Schrödinger equation. Hence we need another method for quantum mechanical treatment of the system. One of the potential methods useful for this situation is the invariant operator method [1] which we will employ here.

From the Liouville-von Neumann equation,

$$\frac{d\hat{I}}{dt} = \frac{\partial \hat{I}}{\partial t} + \frac{1}{i\hbar}\left[\hat{I}, \hat{H}_{\mathrm{q}}\right] = 0, \tag{12}$$

we derive a quadratic invariant operator to be [2]

$$\hat{I} = \hbar\Omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right), \tag{13}$$

where Ω is a positive time-constant defined as

$$\Omega = (h_1 h_3 - h_2^2/4)^{1/2}\epsilon_0 \exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)[s_1\dot{s}_2 - \dot{s}_1 s_2], \tag{14}$$

with the real constants h_1, h_2, and h_3, and $\hat{a}$ and its Hermitian adjoint $\hat{a}^\dagger$ are the annihilation and the creation operators, respectively. If we regard Eqs. (9) and (10) with additional conditions that $s_{1,0} > 0$

and $s_{2,0} > 0$, $[s_1\dot{s}_2 - \dot{s}_1 s_2]$ is always positive. Hence, $h_1 - h_3$ should be taken to be $h_1 h_3 > h_2^2/4$ so that $\Omega > 0$. The definition of $\hat{a}$ (and, consequently, $\hat{a}^\dagger$) is more or less different from that of the simple harmonic oscillator and is given by

$$\hat{a} = \frac{1}{\sqrt{2\Omega}}\left[\left(\sqrt{\Omega\eta(t)} - i\frac{\dot{s}(t)\epsilon_0}{\sqrt{\hbar}}\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)\right)\hat{q} + i\frac{s(t)}{\sqrt{\hbar}}\hat{p}\right], \tag{15}$$

where $s(t)$ is a time function of the form

$$s(t) = \sqrt{h_1 s_1^2(t) + h_2 s_1(t) s_2(t) + h_3 s_2^2(t)}, \tag{16}$$

and $\eta(t) = \Omega/[s^2(t)\hbar]$.

The wave functions in the configuration space are represented as [1]

$$\langle q|\psi_n(t)\rangle = \langle q|\phi_n(t)\rangle \exp[i\theta_n(t)]. \tag{17}$$

Here, $\langle q|\phi_n(t)\rangle$ are the eigenstates of the invariant operator, that are given by

$$\langle q|\phi_n(t)\rangle = \sqrt[4]{\frac{\eta(t)}{\pi}}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\eta(t)}q\right) \times \exp\left[-\frac{1}{2}\left(\eta(t) - i\frac{\dot{s}(t)\epsilon_0}{s(t)\hbar}\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)\right)q^2\right], \tag{18}$$

and the phases $\theta_n(t)$ have the form

$$\theta_n(t) = -\left(n + \frac{1}{2}\right)\frac{\hbar}{\epsilon_0}\int_0^t \frac{\eta(t')dt'}{\exp_{\mathrm{q}}(\sigma_0 t'/\epsilon_0 + \delta)}. \tag{19}$$

For a system that is described by a time-dependent Hamiltonian, the energy operator is no longer the same as the Hamiltonian of the system. In this case, the energy operator is represented in terms of Hamiltonian such that [16]

$$\hat{E}_{\mathrm{q}} = \hat{H}_{\mathrm{q}}/\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta). \tag{20}$$

The quantized energy $E_{n,\mathrm{q}}$ in Fock state can be derived by evaluating the expectation value of this operator with the aid of Eq. (17), to be

$$E_{n,\mathrm{q}} = \langle\psi_n(t)|\hat{E}_{\mathrm{q}}|\psi_n(t)\rangle = \frac{\hbar}{2}\left\{\frac{\epsilon_0}{\Omega}\left[\dot{s}^2(t) + \omega_0^2 s^2(t)\right] + \frac{\Omega}{\epsilon_0 s^2(t)\left[\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)\right]^2}\right\}\left(n + \frac{1}{2}\right). \tag{21}$$

We have illustrated this quantum energy in Fig. 1, bearing in mind that the condition $h_1 h_3 > h_2^2/4$ which is previously mentioned should be preserved. When $h_1 = h_3$ and $h_2 = 0$, $E_{n,\mathrm{q}}$ dissipates with time in a monotonous manner. However, for the cases of $h_1 \neq h_2$ and/or $h_2 \neq 0$, $E_{n,\mathrm{q}}$ oscillates with time. By comparing Fig. 1(d) with Fig. 1(c), we see that the energy oscillation is amplified when $h_1 \times h_3$ approaches $h_2^2/4$. Fig. 1(f) shows that the phase of oscillation for $(h_1, h_2, h_3) = (x_a, x_0, x_b)$ is roughly out of phase with that for $(h_1, h_2, h_3) = (x_b, x_0, x_a)$ where x_a, x_b, and x_0 are arbitrary numbers, provided that x_a is sufficiently larger than x_0 and x_b. However, for the case $x_a \approx x_b$ with $x_0 \neq 0$, they keep nearly in phase with each other, provided that $x_0^2/4$ is sufficiently large (but not exceeds $x_a \times x_b$).

For a more simple case, i.e., the case of ordinary CK oscillator that is described by Eq. (5), the solutions, s_1 and s_2, are given by

$$s_1 = s_{1,0}e^{-(\sigma_0 t/\epsilon_0 + \delta)/2}\cos(\tilde{\omega}t + \varphi_1), \tag{22}$$

$$s_2 = s_{2,0}e^{-(\sigma_0 t/\epsilon_0 + \delta)/2}\sin(\tilde{\omega}t + \varphi_2), \tag{23}$$

where

$$\tilde{\omega} = \left[\omega_0^2 - \sigma_0^2/\left(4\epsilon_0^2\right)\right]^{1/2}. \tag{24}$$

Then, by choosing $s_{1,0} = s_{2,0}$, $\varphi_1 = \varphi_2$, $h_1 = h_3 = 1$, and $h_2 = 0$ and replacing the q-exponential function with the ordinary exponential function, the quantized energy, Eq. (21), reduces to

$$E_n = \hbar\frac{\omega_0^2}{\tilde{\omega}}e^{-(\sigma_0 t/\epsilon_0 + \delta)}\left(n + \frac{1}{2}\right), \tag{25}$$

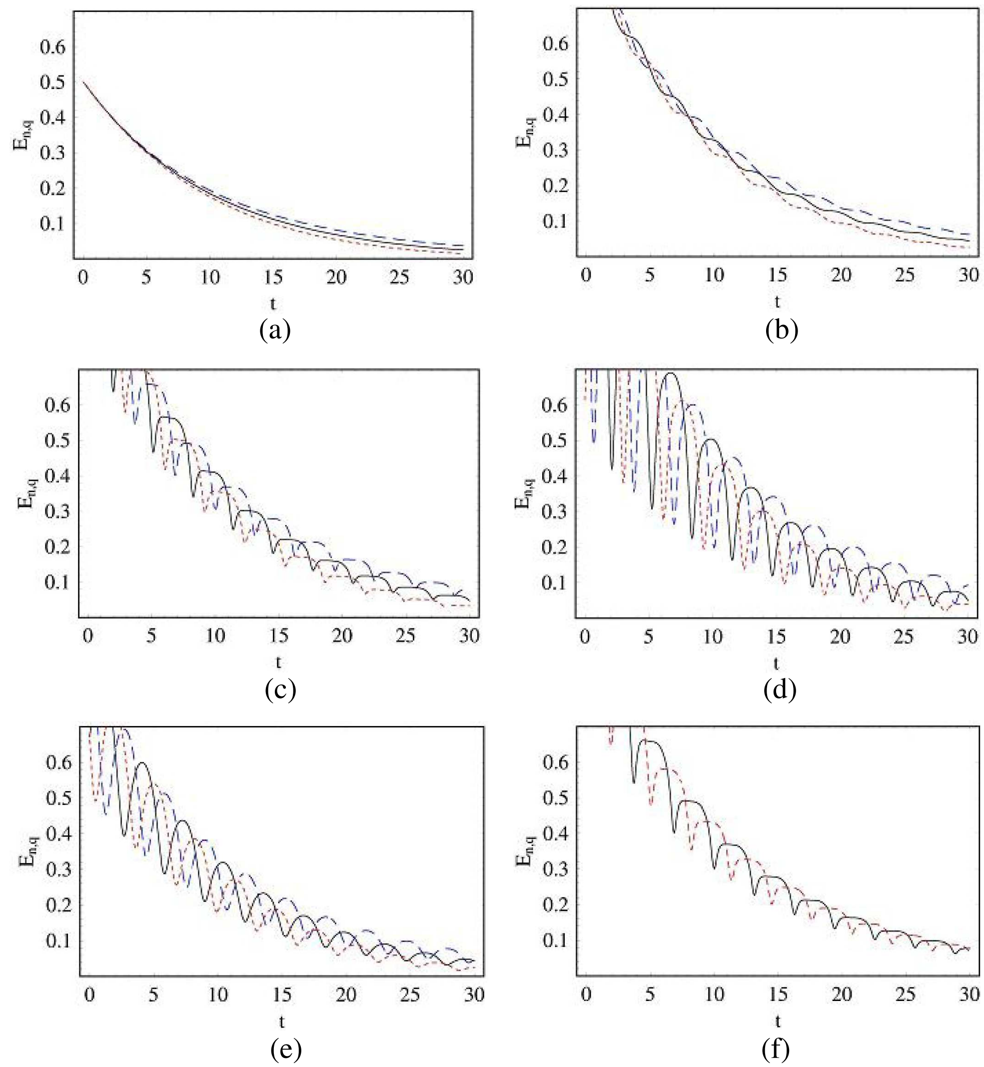

Figure 1. Expectation value of the energy operator, $E_{n,\mathrm{q}}$, given in Eq. (21). The values of (h_1, h_2, h_3) are (a) $(1, 0, 1)$, (b) (10,0,1), (c) $(10, 2, 1)$, (d) $(5, 2, 1)$, and (e) $(1, 0.9, 1)$. The values of q for (a)–(e) are 0.9 (long dashed line), 1.0 (solid line), and 1.1 (short dashed line). On the other hand, (f) is a comparison of energy oscillation between two cases of $(h_1, h_2, h_3) = (x_a, x_0, x_b)$ (solid line) and $(h_1, h_2, h_3) = (x_b, x_0, x_a)$ (dashed line) where $x_a = 10$, $x_b = 1$ and $x_0 = 2$ with the choice of q $= 0.9$. We used $n = 0$, $\sigma_0 = 0.1$, $\omega_0 = 1$, $\epsilon_0 = 1$, $\hbar = 1$, $s_{1,0} = s_{2,0} = 1$, and $\delta = 0$.

which agrees well with the previous result [17].

Now we turn our attention to the nonextensivity of fluctuations of the electromagnetic fields. The fluctuations of an arbitrary observable $\mathcal{O}$ are defined as

$$(\Delta\mathcal{O})_n = \left[\left\langle \psi_n(t)|\hat{\mathcal{O}}^2|\psi_n(t)\right\rangle - \left(\left\langle \psi_n(t)|\hat{\mathcal{O}}|\psi_n(t)\right\rangle\right)^2\right]^{1/2}. \tag{26}$$

For instance, we easily get the fluctuations of q and p, which are considered to have nonextensive characters, using Eq. (7):

$$(\Delta q)_n = \left[\eta^{-1}(t)\left(n + \frac{1}{2}\right)\right]^{1/2}, \tag{27}$$

$$(\Delta p)_n = \left[\frac{\hbar}{\Omega}\left(\dot{s}^2(t)\epsilon_0^2[\exp_{\mathrm{q}}(\sigma_0 t/\epsilon_0 + \delta)]^2 + \frac{\Omega^2}{s^2(t)}\right)\left(n + \frac{1}{2}\right)\right]^{1/2}. \tag{28}$$

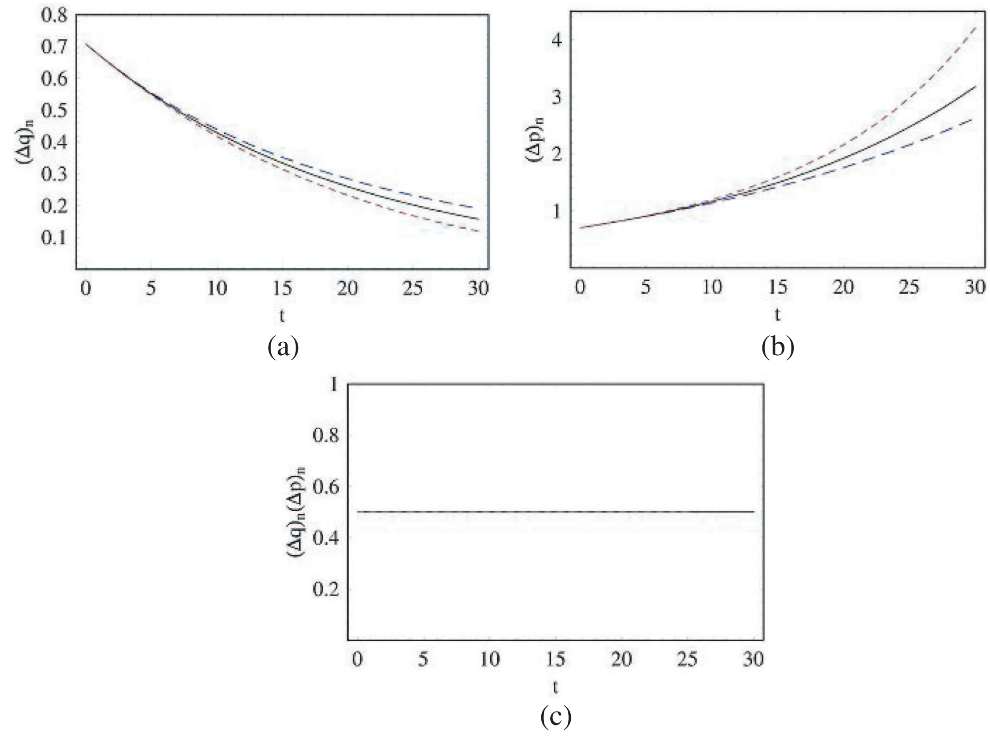

Figure 2. Fluctuations (a) $(\Delta q)_n$ and (b) $(\Delta p)_n$ and uncertainty product (c) $(\Delta q)_n(\Delta p)_n$. The values of (h_1, h_2, h_3) are $(1, 0, 1)$ and the values of q are 0.9 (long dashed line), 1.0 (solid line), and 1.1 (short dashed line). We used $n = 0$, $\sigma_0 = 0.1$, $\omega_0 = 1$, $\epsilon_0 = 1$, $\hbar = 1$, $s_{1,0} = s_{2,0} = 1$, and $\delta = 0$.

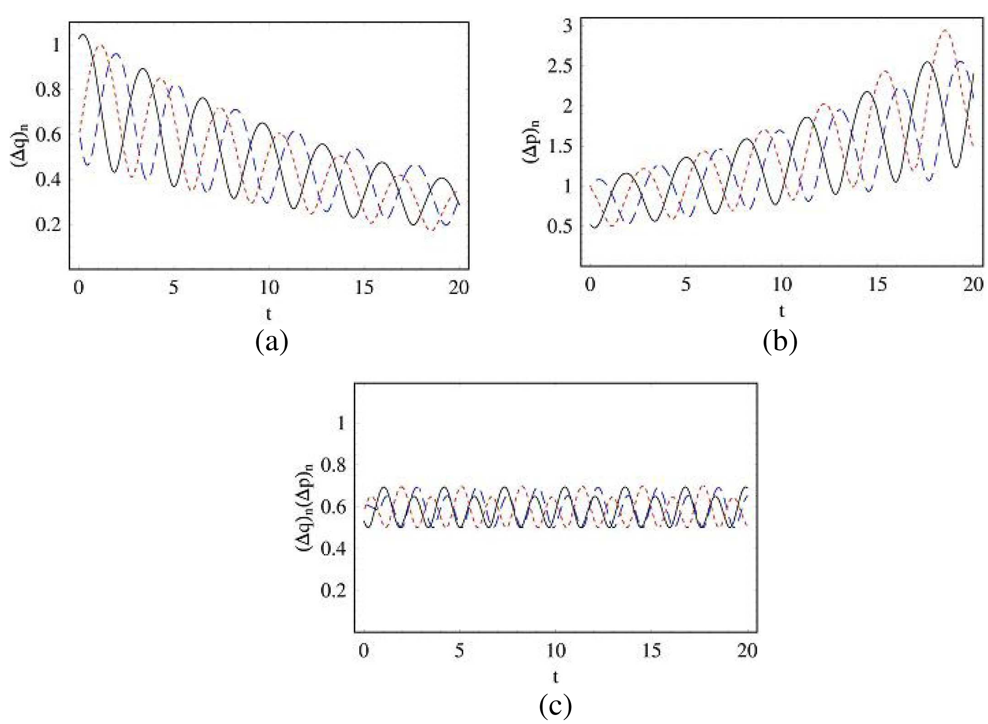

Figure 3. The same as Fig. 2 but the values of (h_1, h_2, h_3) are $(5, 0, 1)$.

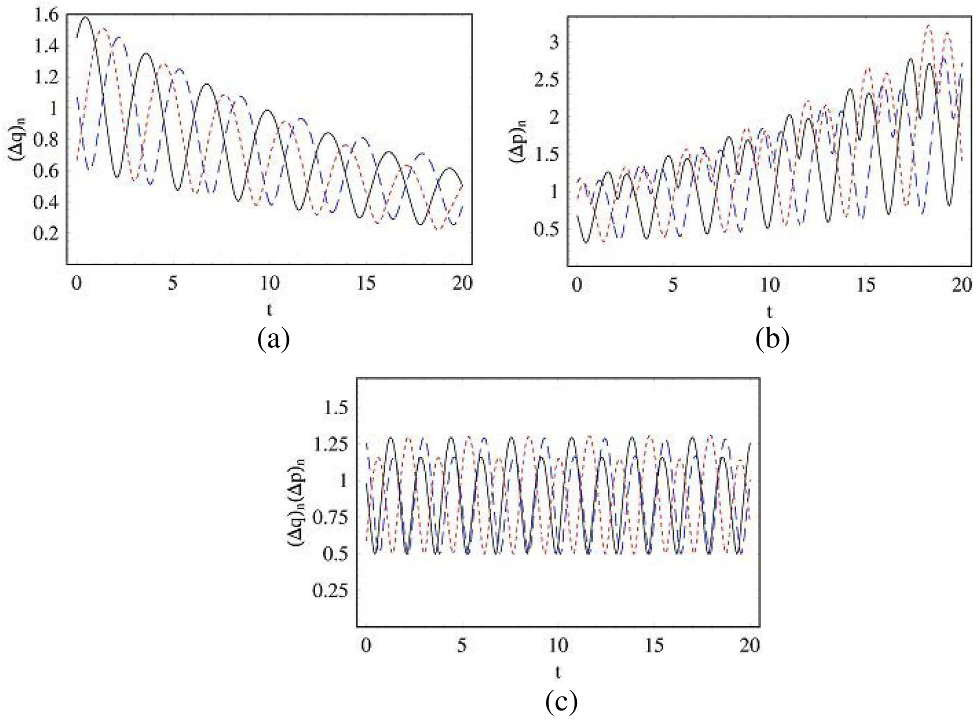

Figure 4. The same as Fig. 2 but the values of (h_1, h_2, h_3) are $(5, 2, 1)$.

The corresponding uncertainty product is also obtained by multiplying these two quantities. From Figs. 2–4, we see that the fluctuations and uncertainty products oscillate with time similarly to quantized electromagnetic energy. The envelop of $(\Delta q)_n$ decreases while that of $(\Delta p)_n$ increases with time, but the envelop of their product does not decrease or increase. The time behaviours of these quantities are slightly different depending on parameter q and this reflects delicate nonextensive characteristics of electromagnetic phenomena.

4. CONCLUSION

Quantum mechanical properties of the modified CK oscillator model for propagating electromagnetic fields in plasma medium, described in terms of q-exponential function, are investigated. The quantized energy, quadrature fluctuations, and the uncertainty product are analyzed by means of the invariant operator method.

In some cases, quantized electromagnetic energy and fluctuations of canonical variables oscillate with time. We confirmed that, when we are unable to neglect the conductivity in media, the quantum energy dissipates with time like the classical energy. The rate of energy dissipation is slightly different depending on the nonextensive parameter q. The quantum energy somewhat rapidly dissipates for large q as shown in Fig. 1. There are many reports in connection with quantum dissipation [5, 6, 8, 16–26]. Fujii and Suzuki studied Jaynes-Cummings model for quantum dissipative systems [18, 19]. The possibility of generation of squeezing by damping for amplitudes of quantum superposition of coherent states is found by Bužek et al. [20]. Choi used dissipation of the scalar field in inflation model of cosmology in order to fix the well known cosmological constant problem in cosmology [25, 26]. Özeren investigated the effects of nonextensive parameter q on time evolution of the complex SU(1,1) coherent state parameter for the modified CK oscillator [8]. However, the effect of q on the quantum energy dissipation, which is illustrated here, do not reported until now as far as we know.

The variation of both quadrature fluctuations and quantum energy increases as q become large due to nonextensive characteristics of the system. The envelop of the fluctuation $(\Delta q)_n$ decreases with time whereas that of $(\Delta p)_n$ increases. On the other hand, the corresponding uncertainty product is always larger than $\hbar/2$ which is the minimal value allowed in quantum harmonic oscillator and its envelop neither decays nor increases with time.

ACKNOWLEDGMENT

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grant No.: NRF-2013R1A1A2062907). The work of S. Lakehal, M. Maamache, and S. Menouar was supported by the Agence Thématique de Recherche en Sciences et Technologie (ATRST) contract of May 2, 2011 (code pnr: 8/u19/882).

REFERENCES

1. Lewis, Jr., H. R. and W. B. Riesenfeld, "An exact quantum theory of the time-dependent harmonic oscillator and of a charged particle in a time-dependent electromagnetic field," *J. Math. Phys.*, Vol. 10, No. 8, 1458–1473, 1969.
2. Choi, J. R., "Nonclassical properties of superpositions of coherent and squeezed states for electromagnetic fields in time-varying media," *Quantum Optics and Laser Experiments*, S. Lyagushyn, Ed., Chap. 2, 25–48, InTech, Rijeka, 2012.
3. Kalluri, D. K., *Electromagnetics of Time Varying Complex Media*, 2nd Edition, CRC Press, Boca Raton, 2010.
4. Heald, M. A. and C. B. Wharton, *Plasma Diagnostics with Microwaves*, Wiley, New York, 1965.
5. Caldirola, P., "Forze non conservative nella meccanica quantistica," *Il Nuovo Cimento*, Vol. 18, No. 9, 393–400, 1941.
6. Kanai, E., "On the quantization of the dissipative systems," *Prog. Theor. Phys.*, Vol. 3, No. 4, 440–442, 1950.
7. Tsallis, C., "What are the numbers that experiments provide?," *Quimica Nova*, Vol. 17, 468–471, 1994.
8. Özeren, S. F., "The effect of nonextensivity on the time evolution of the SU(1,1) coherent states driven by a damped harmonic oscillator," *Physica A*, Vol. 337, Nos. 1–2, 81–88, 2004.
9. Liu, Z. P., L. N. Guo, and J. L. Du, "Nonextensivity and the q-distribution of a relativistic gas under an external electromagnetic field," *Chin. Sci. Bull.*, Vol. 56, No. 34, 3689–3692, 2011.
10. Pedrosa, I. A. and A. Rosas, "Electromagnetic field quantization in time-dependent linear media," *Phys. Rev. Lett.*, Vol. 103, No. 1, 010402, 2009.
11. Maamache, M., J.-P. Provost, and G. Vallee, "Unitary equivalence and phase properties of the quantum parametric and generalized harmonic oscillators," *Phys. Rev. A*, Vol. 59, No. 3, 1777–1780, 1999.
12. Tsallis, C., "Possible generalization of Boltzmann-Gibbs statistics," *J. Stat. Phys.*, Vol. 52, Nos. 1–2, 479–487, 1988.
13. Albuquerque, E. L. and M. G. Cottam, "Theory of elementary excitations in quasiperiodic structures," *Phys. Rep.*, Vol. 376, Nos. 4–5, 225–337, 2003.
14. Anastasiadis, A. D. and G. D. Magoulas, "Particle swarms and nonextensive statistics for nonlinear optimisation," *The Open Cybernetics and Systemics Journal*, Vol. 2, 173–179, 2008.
15. McHarris, Wm. C., "Nonlinearities in quantum mechanics," *Braz. J. Phys.*, Vol. 35, No. 2B, 380–384, 2005.
16. Marchiolli, M. A. and S. S. Mizrahi, "Dissipative mass-accreting quantum oscillator," *J. Phys. A: Math. Gen.*, Vol. 30, No. 8, 2619–2635, 1997.
17. Choi, J. R., "The decay properties of a single-photon in linear media," *Chin. J. Phys.*, Vol. 41, No. 3, 257–266, 2003.
18. Fujii, K. and T. Suzuki, "An approximate solution of the Jaynes-Cummings model with dissipation," *Int. J. Geom. Methods Mod. Phys.*, Vol. 8, No. 8, 1799–1814, 2011.

19. Fujii, K. and T. Suzuki, "An approximate solution of the Jaynes-Cummings model with dissipation II: Another approach," *Int. J. Geom. Methods Mod. Phys.*, Vol. 9, No. 4, 1250036, 2012.
20. Bužek, V., A. Vidiella-Barranco, and P. L. Knight, "Superpositions of coherent states: Squeezing and dissipation," *Phys. Rev. A*, Vol. 45, No. 9, 6570–6585, 1992.
21. Ji, Y.-H. and M.-S. Lei, "Squeezing effects of a mesoscopic dissipative coupled circuit," *Int. J. Theor. Phys.*, Vol. 41, No. 7, 1339–1346, 2002.
22. Weiss, U., *Quantum Dissipative Systems*, 4th Edition, World Scientific, Singapore, 2008.
23. Garashchuk, S., V. Dixit, B. Gu, and J. Mazzuca, "The Schrödinger equation with friction from the quantum trajectory perspective," *J. Chem. Phys.*, Vol. 138, No. 5, 054107, 2013.
24. Stannigel, K., P. Rabl, and P. Zoller, "Driven-dissipative preparation of entangled states in cascaded quantum-optical networks," *New J. Phys.*, Vol. 14, No. 6, 063014, 2012.
25. Choi, J. R., "Quantum unitary transformation approach for the evolution of dark energy," *Dark Energy — Current Advances and Ideas*, J. R. Choi, Ed., Chap. 5, 117–134, Research SignPost, Kerala, 2009.
26. Choi, J. R., "SU(1,1) Lie algebraic approach for the evolution of the quantum inflationary universe," *Phys. Dark Univ.*, Vol. 2, No. 1, 41–49, 2013.